# Lifetimes of doubly charmed baryons: $\Xi_{cc}^{+}$ and $\Xi_{cc}^{++}$

V. V. Kiselev,* A. K. Likhoded, and A. I. Onishchenko
*State Research Center of Russia "Institute for High Energy Physics," Protvino, Moscow region, 142284 Russia*
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We perform a detailed investigation of total lifetimes for the $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$ baryons in the framework of operator product expansion over the inverse mass of the charmed quark, whereas, to estimate matrix elements of operators obtained in operator product expansion, some approximations of nonrelativistic QCD are used. This approach allows one to take into account the corrections to the spectator decays of $c$ quarks, which reflect the fact that these quarks are bound as well as the contributions connected to the effects of both the Pauli interference for the $\Xi_{cc}^{++}$ baryon and the weak scattering for the $\Xi_{cc}^{+}$ baryon. The realization of such a program leads to the following estimates for the total lifetimes of doubly charmed baryons: $\tau_{\Xi_{cc}^{++}}=0.43\pm0.11$ ps and $\tau_{\Xi_{cc}^{+}}=0.11\pm0.03$ ps. [S0556-2821(99)06309-2]

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## I. INTRODUCTION

A study on weak decays of doubly charmed baryons is of great interest for two reasons. The first one is connected to the investigation of the basic properties of weak interactions at the fundamental level, including the precise determination of Cabibbo-Kobayashi-Maskawa matrix parameters. The second reason is related to the possibility of exploring QCD as it is provided by the systems containing the heavy quarks. In the limit of a large scale given by the heavy quark mass, some aspects in the dynamics of strong interactions become simpler and one obtains the possibility of drawing definite model-independent predictions. Of course, both these topics appear in the analysis of weak decays for the doubly charmed baryons, whose dynamics is determined by an interplay between the strong and weak interactions. That is why these baryons are attractive and reasonable subjects for theoretical and experimental consideration.

The doubly charmed $\Xi_{cc}^{\diamond}$-baryon, where $\diamond$ denotes the electric charge depending on the valence light quark, represents an absolutely new type of object in comparison with ordinary baryons containing light quarks only. The basic state of such a baryon is analogous to a $(\bar{Q}q)$ meson, which contains a single heavy antiquark $\bar{Q}$ and a light quark $q$. In the doubly heavy baryon the role of the heavy antiquark is played by the $(cc)$ diquark, which is an antitriplet color state. It has a small size in comparison with the scale of the light quark confinement. Nevertheless the spectrum of $(ccq)$-system states has to differ essentially from the heavy meson spectra, because the composed $(cc)$ diquark has a set of the excited states (for example, $2S$ and $2P$) in contrast with the heavy quark. The energy of diquark excitation is twice less than the excitation energy of light quark bound with the diquark. So, the representation on the compact diquark can be straightforwardly connected to the level structure of the doubly heavy baryon.

Naive estimates for the lifetimes of doubly charmed baryons were done by the authors earlier [1]. A simple consideration of quark diagrams shows that in the decay of $\Xi_{cc}^{++}$ baryons, the Pauli interference for the decay products of the charmed quark and the valent quark in the initial state takes place in an analogous way to the $D^{+}$-meson decay. In the decay of $\Xi_{cc}^{+}$, the exchange by the $W$ boson between the valence quarks plays an important role as in the decay of $D^0$. These speculations and the presence of two charmed quarks in the initial state result in the following estimates for the lifetimes:

$$\tau(\Xi_{cc}^{++})\approx\frac{1}{2}\tau(D^{+})\simeq0.53 \text{ ps},$$

$$\tau(\Xi_{cc}^{+})\approx\frac{1}{2}\tau(D^{0})\simeq0.21 \text{ ps}.$$

In this work we discuss the systematic approach to the evaluation of total lifetimes for the doubly charmed baryons on the basis of both the optical theorem for the inclusive decay width and the operator product expansion (OPE) for the transition currents in accordance with the consequent nonrelativistic expansion of hadronic matrix elements derived in the OPE. Using the OPE at the first step, we exploit the fact that due to the presence of heavy quarks in the initial state, the energy release in the decay of both quarks is large enough in comparison with the binding energy in the state. Thus, we can use the expansion over the ratio of these scales. Technically, this step repeats an analogous procedure for the inclusive decays of heavy-light mesons as it was reviewed in [2]. Exploring the nonrelativistic expansion of hadronic matrix elements at the second step, we use the approximation of nonrelativistic QCD [3,4], which allows one to reduce the evaluation of matrix elements for the full QCD operators, corresponding to the interaction of heavy quarks inside the diquark, to the expansion in powers of $p_c/m_c$, where $p_c=m_cv_c\sim1$ GeV is a typical momentum of the heavy quark inside the baryon. The same procedure for the matrix elements, determined by the strong interaction of heavy quarks with the light quark, leads to the expansion in powers of $\Lambda_{\rm QCD}/m_c$.

*Email address: kiselev@mx.ihep.su

 

This way, taking into account the radiation of hard gluons in these decays, leads to the expansion in powers of $\alpha_s$, $v_c = p/m_c$ and $\Lambda_{\rm QCD}/m_c$. It is worth noting, that this expansion would be well defined, provided the expansion parameters are small. In the $c\to sud$ transition, the ratio of typical momentum for the heavy quark inside the hadron to the value of energy released in the decay, is not so small. We would like also to stress the important roles, played by both the Pauli interference and the weak scattering, suppressed as $1/m_c^3$ with respect to the leading spectator contribution, but the former are enhanced by a numerical factor, caused by the ratio of two-particle and three-particle phase spaces [5]. Numerical estimates show that the value of these contributions is considerably large, and it is of the order of 40–140 %. These effects take place in the different baryons, $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$, and, thus, they enhance the difference of lifetimes for these baryons. The final result for the total lifetimes of doubly charmed baryons is the following:

$$\tau_{\Xi_{cc}^{++}} = 0.43 \pm 0.11 \ \ \text{ps},$$

$$\tau_{\Xi_{cc}^{+}} = 0.11 \pm 0.03 \ \ \text{ps}.$$

The paper is organized in the following way. Section II contains a general application of the OPE in the framework of heavy quark expansion, basically repeating a common formulation given by [2–9]. We include also the corrections due to the nonzero strange quark mass as well as the logarithmic renormalization for effective nonrelativistic heavy-quark fields in the weak Lagrangian (''hybrid logs''). In Sec. III we evaluate different contributions to the inclusive decay rates of doubly charmed baryons. So, we apply the ideology of [9], wherein the differences between heavy-heavy and heavy-light mesons were stressed and investigated. The method allows a generalization to the heavy-heavy diquark system, interacting with a light quark, as it happens in the doubly heavy baryons. Numerical estimates are given in Sec. IV. Conclusions are drawn in Sec. V. Some formulas used in the calculations are collected in the Appendix for a reference.

## II. OPERATOR PRODUCT EXPANSION

Now let us start the description of our approach for the calculation of total lifetimes for the doubly charmed baryons. The optical theorem, taking into account the integral quark-hadron duality, allows us to relate the total decay width of the heavy quark with the imaginary part of its forward scattering amplitude. This relationship, applied to the $\Xi_{cc}^{\diamond}$-baryon total decay width $\Gamma_{\Xi_{cc}^{\diamond}}$, can be written down as

$$\Gamma_{\Xi_{cc}^{\diamond}} = \frac{1}{2M_{\Xi_{cc}^{\diamond}}} \langle \Xi_{cc}^{\diamond} | \mathcal{T} | \Xi_{cc}^{\diamond} \rangle, \tag{1}$$

where the $\Xi_{cc}^{\diamond}$ state in Eq. (1) has the ordinary relativistic normalization, $\langle \Xi_{cc}^{\diamond} | \Xi_{cc}^{\diamond} \rangle = 2EV$, and the transition operator $\mathcal{T}$ is determined by the expression

$$\mathcal{T} = \operatorname{Im} m \int d^4x \{ \hat{T} H_{\rm eff}(x) H_{\rm eff}(0) \}, \tag{2}$$

where $H_{\rm eff}$ is the standard effective Hamiltonian, describing the low-energy interactions of initial quarks with the decays products, so that

$$H_{\rm eff} = \frac{G_F}{2\sqrt{2}} V_{uq_1} V_{cq_1}^{*} [C_+(\mu) O_+ + C_-(\mu) O_-] + \text{H.c.}, \tag{3}$$

where

$$O_\pm = [\bar{q}_{1\alpha} \gamma_\nu (1-\gamma_5) c_\beta][\bar{u}_\gamma \gamma^\nu (1-\gamma_5) q_{2\delta}] \times (\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}),$$

and

$$C_+ = \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{6/(33-2f)}, \quad C_- = \left[ \frac{\alpha_s(M_W)}{\alpha_s(\mu)} \right]^{-12/(33-2f)},$$

where $f$ is the number of flavors.

Assuming that the energy release in the heavy quark decay is large, we can perform the operator product expansion for the transition operator $\mathcal{T}$ in Eq. (1). In this way we find a series of local operators with increasing dimension over the energy scale, wherein the contributions to $\Gamma_{\Xi_{cc}^{\diamond}}$ are suppressed by the increasing inverse powers of the heavy quark masses. This formalism has already been applied to calculate the total decay rates for the hadrons, containing a single heavy quark [2] (for most early work, having used similar methods, see also [6,8]). Here we would like to stress that the expansion applied in this paper is simultaneously in the powers of the inverse heavy quark mass and the relative velocity of heavy quarks inside the hadron. Thus, the latter points to the difference from the description of both the heavy-light mesons (the expansion in powers of $\Lambda_{\rm QCD}/m_c$) and the heavy-heavy mesons [9] (the expansion in powers of relative velocity of heavy quarks inside the hadron, where one can apply the scaling rules of nonrelativistic QCD [4]).

In this work we will extend this approach to the treatment of baryons, containing two heavy quarks. The operator product expansion applied has the form

$$\mathcal{T} = C_1(\mu) \bar{c}c + \frac{1}{m_c^2} C_2(\mu) \bar{c} g \sigma_{\mu\nu} G^{\mu\nu} c + \frac{1}{m_c^3} O(1). \tag{4}$$

The leading contribution in OPE is determined by the operator $\bar{c}c$, corresponding to the spectator decays of $c$ quarks. The use of the equation of motion for the heavy quark fields allows one to eliminate some redundant operators, so that no operators of dimension-four contribute. There is a single operator of dimension five, $Q_{GQ} = \bar{Q} g \sigma_{\mu\nu} G^{\mu\nu} Q$. As we will show below, significant contributions come from the operators of dimension-six $Q_{2Q2q} = \bar{Q}\Gamma q \bar{q} \Gamma' Q$, representing the effects of Pauli interference and weak scattering for $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$, correspondingly. Furthermore, there are also other operators of dimension-six $Q_{61Q} = \bar{Q}\sigma_{\mu\nu}\gamma_l D^\mu G^{\nu l} Q$ and $Q_{62Q} = \bar{Q} D_\mu G^{\mu\nu}\Gamma_\nu Q$. In what follows, we do not calculate the corresponding coefficient functions for the latter two operators, so that the expansion is certainly complete up to the second order of $1/m$ only. The reason for the restriction of dimension-six operators is twofold. First, the operators $Q_{61Q}$ and $Q_{62Q}$ do not contribute to the lifetime difference between the doubly charmed baryons under consideration, since they are independent of the quark contents of hadrons. Second, the four-quark operators are enhanced in comparison with the quark-gluon terms with the same dimension because the two-particle phase space integrated in the calculation of coefficients in front of the Pauli interference and weak rescattering has an additional factor of $16\pi^2$ in contrast to the three-particle phase space expressed in units of the heavy quark mass as it occurs for the coefficients of operators $Q_{61Q}$ and $Q_{62Q}$, so that they are suppressed (see a detailed discussion in [6,7]).

Further, the different contributions to OPE are given by the following:

$$\mathcal{T}_{\Xi_{cc}^{++}} = \mathcal{T}_{35c} + \mathcal{T}_{6,\mathrm{PI}},$$

$$\mathcal{T}_{\Xi_{cc}^{+}} = \mathcal{T}_{35c} + \mathcal{T}_{6,\mathrm{WS}},$$

where the first terms account for the operators of dimension three $O_{3Q}$ and five $O_{GQ}$, the second terms correspond to the effects of Pauli interference and weak scattering. The explicit formulas for these contributions have the following form:

$$\mathcal{T}_{35c} = 2\left(\Gamma_{c,\mathrm{spec}}\bar{c}c - \frac{\Gamma_{0c}}{m_c^2}[(2+K_{0c})P_1 + K_{2c}P_2]O_{Gc}\right), \tag{5}$$

where $\Gamma_{0c} = G_F^2 m_c^2/192\pi^3$ and $K_{0c} = C_-^2 + 2C_+^2$, $K_{2c} = 2(C_+^2 - C_-^2)$. This expression has been derived in [10] (see also [11]), and it is also discussed in [2]. The phase-space factors $P_i$ are [2,12]

$$P_1 = (1-y)^4, \quad P_2 = (1-y)^3,$$

where $y = m_s^2/m_c^2$.

$\Gamma_{c,\mathrm{spec}}$ denotes the contribution to the total decay width of the free decay for one of the two $c$ quarks, which is explicitly expressed below.

For the effects of Pauli interference and weak scattering, we find the following formulas:

$$\begin{aligned}\mathcal{T}_{\mathrm{PI}} = & -\frac{2G_F^2}{4\pi}m_c^2\left(1-\frac{m_u}{m_c}\right)^2\Bigg\{\Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right)[\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i][\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j] + \left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right) \\ & \times(\bar{c}_i\gamma_\alpha\gamma_5 c_i)[\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j]\Bigg]\left[(C_+ + C_-)^2 + \frac{1}{3}(1-k^{1/2})(5C_+^2 + C_-^2 - 6C_-C_+)\right] + \Bigg[\left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{4}\right) \\ & \times[\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j][\bar{q}_j\gamma^\alpha(1-\gamma_5)c_i] + \left(\frac{(1-z_-)^2}{2} - \frac{(1-z_-)^3}{3}\right)(\bar{c}_i\gamma_\alpha\gamma_5 c_j)[\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i]\Bigg] \\ & \times k^{1/2}(5C_+^2 + C_-^2 - 6C_-C_+)\Bigg\},\end{aligned} \tag{6}$$

$$\begin{aligned}\mathcal{T}_{\mathrm{WS}} = & \frac{2G_F^2}{4\pi}p_+^2(1-z_+)^2\Bigg[\left[C_+^2 + C_-^2 + \frac{1}{3}(1-k^{1/2})(C_+^2 - C_-^2)\right][\bar{c}_i\gamma_\alpha(1-\gamma_5)c_i][\bar{q}_j\gamma^\alpha(1-\gamma_5)q_j] + k^{1/2}(C_+^2 - C_-^2) \\ & \times[\bar{c}_i\gamma_\alpha(1-\gamma_5)c_j][\bar{q}_j\gamma^\alpha(1-\gamma_5)q_i]\Bigg],\end{aligned} \tag{7}$$

where $p_+ = p_c + p_q$, $p_- = p_c - p_q$ and $z_\pm = m_q^2/p_\pm^2$, $m_q = m_{u,d}$, $k = \alpha_s(\mu)/\alpha_s(m_c)$.

In the numerical estimates for the evolution of coefficients $C_+$ and $C_-$, we have taken into account the threshold effects, connected to the $b$ quark, as well as the threshold effects, related to the $c$-quark mass in the Pauli interference and weak scattering.

In expression (5), the scale $\mu$ is approximately equal to $m_c$. For the Pauli interference and weak scattering, this scale was chosen in such a way as to obtain an agreement between the experimental differences in the lifetimes of $\Lambda_c$, $\Xi_c^+$ and $\Xi_c^0$-baryons and the theoretical predictions, based on the effects mentioned above. This problem is discussed below. Anyway, the choice of these scales allows some variations, and a complete answer to this question requires calculations in the next order of perturbative theory.

The contribution of the leading operator $\bar{c}c$ corresponds to the imaginary part of the diagram in Fig. 1, as it stands in expression (4). The coefficient of $\bar{c}c$ can be obtained in the usual way by matching the Fig. 1 diagram, corresponding to the leading term in expression (4), with the operator $\bar{c}c$. This coefficient is equivalent to the free quark decay rate, and it is known in the next-to-leading logarithmic approximation of QCD [13–17], including the strange quark mass effects in

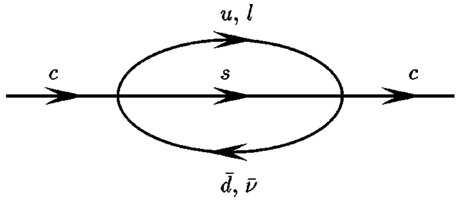


FIG. 1. The spectator contribution to the total decay width of doubly charmed baryons.

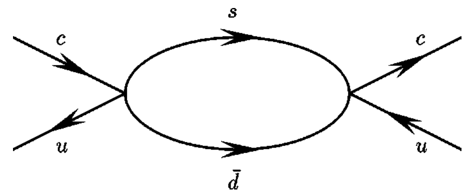


FIG. 2. The Pauli interference of $c$-quark decay products with the valence quark in the initial state for the $\Xi_{cc}^{++}$ baryon.

the final state [17]. To calculate the next-to-leading logarithmic effects, the Wilson coefficients in the effective weak Lagrangian are required at the next-to-leading accuracy, and the single gluon exchange corrections to the diagram in Fig. 1 must be considered. In our numerical estimates we use the expression for $\Gamma_{\rm spec}$, including the next-to-leading order corrections and $s$-quark mass effects in the final state, but we neglect the Cabibbo-suppressed decay channels for the $c$ quark. The bulky explicit expression for the spectator $c$-quark decay is placed in the Appendix.

Similarly, the contribution by $O_{GQ}$ is obtained when an external gluon line is attached to the inner quark lines in Fig. 1 in all possible ways. The corresponding coefficients are known in the leading logarithmic approximation. Finally, the dimension-six operators and their coefficients arise due to those contributions, wherein one of the internal $u$ or $\bar d$ quark line is ''cut'' in the diagram of Fig. 1. The resulting graphs are depicted in Figs. 2 and 3. These contributions correspond to the effects of Pauli interference and weak scattering. We have calculated the expressions for these effects which account for both the $s$-quark mass in the final state and the logarithmic renormalization of effective electroweak Lagrangian at low energies.

Since the simultaneous account for the mass effects and low-energy logarithmic renormalization of such contributions has been performed in this work, we would like to discuss this question in some detail.

The straightforward calculation of the diagrams in Figs. 2 and 3 with the account for the $s$-quark mass yields the following expressions:

$$\mathcal{T}_{\rm PI}=-\frac{2G_F^2}{4\pi}p_-^2\left\{\left[\frac{(1-z_-)^3}{12}g^{\alpha\beta}+\left(\frac{(1-z_-)^3}{2}-\frac{(1-z_-)^3}{3}\right)\frac{p_-^\alpha p_-^\beta}{p_-^2}\right]\right\}\{(C_++C_-)^2[\bar c_i\gamma_\alpha(1-\gamma_5)c_i][q_j\gamma_\beta(1-\gamma_5)q_j]$$
$$+(5C_+^2-6C_+C_-+C_-^2)[\bar c_i\gamma_\alpha(1-\gamma_5)c_j][q_j\gamma_\beta(1-\gamma_5)q_i]\}, \tag{8}$$

$$\mathcal{T}_{\rm WS}=\frac{2G_F^2}{4\pi}p_+^2(1-z_+)^2\{(C_+^2+C_-^2)[\bar c_i\gamma_\alpha(1-\gamma_5)c_i][q_j\gamma_\beta(1-\gamma_5)q_j]+(C_+^2-C_-^2)[\bar c_i\gamma_\alpha(1-\gamma_5)c_j][q_j\gamma_\beta(1-\gamma_5)q_i]\}. \tag{9}$$

For $p_+$ and $p_-$ we use their threshold values:

$$p_+=p_c\left(1+\frac{m_q}{m_c}\right),\quad p_-=p_c\left(1-\frac{m_q}{m_c}\right),$$

taking into account that the logarithmic renormalization of effective low-energy Lagrangian has the following form [5,6]:

$$L_{\rm eff,log}=\frac{G_f^2m_c^2}{2\pi}\left\{\frac{1}{2}\left[C_+^2+C_-^2+\frac{1}{3}(1-k^{1/2})(C_+^2-C_-^2)\right](\bar c\Gamma_\mu)(\bar d\Gamma^\mu d)+\frac{1}{2}(C_+^2-C_-^2)k^{1/2}(\bar c\Gamma_\mu d)(\bar d\Gamma^\mu c)+\frac{1}{3}(C_+^2-C_-^2)\right.$$
$$\times k^{1/2}(k^{-2/9}-1)(\bar c\Gamma_\mu t^a c)j_\mu^a-\frac{1}{8}\left[(C_++C_-)^2+\frac{1}{3}(1-k^{1/2})(5C_+^2+C_-^2-6C_+C_-)\right]\left(\bar c\Gamma_\mu c+\frac{2}{3}\bar c\gamma_\mu\gamma_5 c\right)(\bar u\Gamma^\mu u)$$
$$-\frac{1}{8}k^{1/2}(5C_+^2+C_-^2-6C_+C_-)\left(\bar c_i\Gamma_\mu c_k+\frac{2}{3}\bar c_i\gamma_\mu\gamma_5 c_k\right)(\bar u_k\Gamma^\mu u_i)-\frac{1}{8}\left[(C_+-C_-)^2+\frac{1}{3}(1-k^{1/2})\right.$$
$$\left.\times(5C_+^2+C_-^2+6C_+C_-)\right]\left(\bar c\Gamma_\mu c+\frac{2}{3}\bar c\gamma_\mu\gamma_5 c\right)(\bar s\Gamma^\mu s)-\frac{1}{8}k^{1/2}(5C_+^2+C_-^2+6C_+C_-)\left(\bar c_i\Gamma_\mu c_k+\frac{2}{3}\bar c_i\gamma_\mu\gamma_5 c_k\right)(\bar s_k\Gamma^\mu s_i)$$
$$\left.-\frac{1}{6}k^{1/2}(k^{-2/9}-1)(5C_+^2+C_-^2)\left(\bar c\Gamma_{mu}t^a c+\frac{2}{3}\bar c\gamma_\mu\gamma_5 t^a c\right)j^{a\mu}\right\}, \tag{10}$$

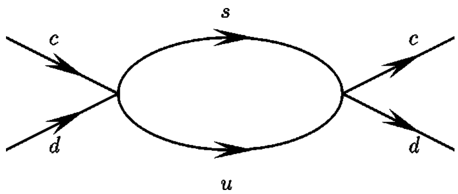


FIG. 3. The weak scattering of the valence quarks in the initial state for the $\Xi_{cc}^{+}$ baryon.

where $\Gamma_\mu=\gamma_\mu(1-\gamma_5)$, $k=[\alpha_s(\mu)/\alpha_s(m_c)]$ and $j_\mu^a=\bar{u}\gamma_\mu t^a u+\bar{d}\gamma_\mu t^a d+\bar{s}\gamma_\mu t^a s$ is the color current of light quarks ($t^a=\lambda^a/2$ being the color generators). Having performed the manipulations, we have obtained formulas (6), (7).

Here we would like to make a note, concerning the terms of the effective Lagrangian, containing the color current of light quarks. In the analysis below, we have omitted these terms, because they contribute in the Lagrangian with the strength factor $k^{-2/9}-1$, whose numerical value is equal to 0.054 (see below).

To calculate the contribution of semileptonic modes to the total decay width of $\Xi_{cc}^{\diamond}$ baryons (we have taken into account the electron and muon decay modes only), we use the following expressions [11] (see, also, [17]):

$$\begin{aligned}\Gamma_{sl}=4\Gamma_c(&\{1-8\rho+8\rho^3-\rho^4-12\rho^2\ln\rho\}\\&+E_c\{5-24\rho+24\rho^2-8\rho^3+3\rho^4-12\rho^2\ln\rho\}\\&+K_c\{-6+32\rho-24\rho^2-2\rho^4+24\rho^2\ln\rho\}\\&+G_c\{-2+16\rho-16\rho^3+2\rho^4+24\rho^2\ln\rho\}),\end{aligned} \quad (11)$$

where $\Gamma_c=|V_{cs}|^2G_F^2(m_c^5/192\pi^3)$, $\rho=m_s^2/m_c^2$. The quantities $E_c=K_c+G_c$, $K_c$ and $G_c$ are given by the expressions:

$$K_c=-\left\langle \Xi_{cc}^{\diamond}(v)\left|\bar{c}_v\frac{(iD)^2}{2m_c^2}c_v\right|\Xi_{cc}^{\diamond}(v)\right\rangle,$$

$$G_c=\left\langle \Xi_{cc}^{\diamond}(v)\left|\bar{c}_v\frac{gG_{\alpha\beta}\sigma^{\alpha\beta}}{4m_c^2}c_v\right|\Xi_{cc}^{\diamond}(v)\right\rangle, \quad (12)$$

where the spinor field $c_v$ in the effective heavy quark theory is defined by the form

$$c(x)=e^{-im_cv\cdot x}\frac{1+iD^\mu\gamma_\mu/m_c}{2}c_v(x). \quad (13)$$

Thus, we can see that the evaluation of total lifetimes for the doubly charmed baryons is reduced to the problem of estimation for the matrix elements of operators appearing in the above expressions, which is the topic of the next section.

### III. EVALUATION OF MATRIX ELEMENTS

Let us calculate the matrix elements for the operators, obtained as the result of OPE for the transitions under consideration. In general, it is a complicated nonperturbative problem but, as we will see below, in our particular calculation we can get some reliable estimates for the matrix elements of the required operators.

Using the equation of motion for the heavy quarks, the local operator $\bar{c}c$ can be expanded in the following series over the powers of $1/m_c$:

$$\langle\Xi_{cc}^{\diamond}|\bar{c}c|\Xi_{cc}^{\diamond}\rangle_{\rm norm}=1-\frac{\langle\Xi_{cc}^{\diamond}|\bar{c}[(i\boldsymbol{D})^2-((i/2)\sigma G)]c|\Xi_{cc}^{\diamond}\rangle_{\rm norm}}{2m_c^2}+O\left(\frac{1}{m_c^3}\right). \quad (14)$$

Thus, this evaluation can be reduced to the calculation of matrix elements for the following operators:

$$\bar{c}(i\boldsymbol{D})^2c,\quad \left(\frac{i}{2}\right)\bar{c}\sigma Gc,\quad \bar{c}\gamma_\alpha(1-\gamma_5)c\bar{q}\gamma^\alpha(1-\gamma_5)q,$$

$$\bar{c}\gamma_\alpha\gamma_5c\bar{q}\gamma^\alpha(1-\gamma_5)q.$$

The first operator corresponds to the time dilation connected to the motion of heavy quarks inside the hadron, the second is related to the spin interaction of heavy quarks with the chromomagnetic field of the light quark and the other heavy quark. Further, the third and fourth operators are the four-quark operators, representing the effects of Pauli interference and weak scattering.

In the system containing the nonrelativistic heavy quark, the quark-antiquark pairs with the same flavor can be produced with a negligible rate, since energy greater than $m_Q$ is required. In this situation, it is useful to integrate out the small components of the heavy-quark spinor field and to present the result in terms of the two component spinor $\Psi_Q$. Following this approach, we find that all contributions from virtualities greater than $\mu$, where $m_c>\mu>m_cv_c$, can be explicitly taken into account in the perturbative theory. This method is general and analogous to the effective heavy quark theory. So,

$$\begin{aligned}\bar{c}c=\Psi_c^\dagger\Psi_c-\frac{1}{2m_c^2}\Psi_c^\dagger(i\boldsymbol{D})^2\Psi_c\\+\frac{3}{8m_c^4}\Psi_c^\dagger(i\boldsymbol{D})^4\Psi_c-\frac{1}{2m_c^2}\Psi_c^\dagger g\boldsymbol{\sigma}\boldsymbol{B}\Psi_c\\-\frac{1}{4m_c^3}\Psi_c^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_c+\cdots,\end{aligned} \quad (15)$$

$$\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c=-2\Psi_c^\dagger g\boldsymbol{\sigma}\boldsymbol{B}\Psi_c-\frac{1}{m_c}\Psi_c^\dagger(\boldsymbol{D}g\boldsymbol{E})\Psi_c+\cdots. \quad (16)$$

In these expressions we have omitted the term $\Psi_c^\dagger\boldsymbol{\sigma}(g\boldsymbol{E}\times\boldsymbol{D})\Psi_c$, corresponding to the spin-orbital interaction, because it vanishes in the ground states of doubly charmed baryons. By definition, the two-component spinor $\Psi_c$ has the same normalization as $Q$,

$$\int d^3x\Psi_c^\dagger\Psi_c=\int d^3xQ^\dagger Q. \tag{17}$$

Then, with the required accuracy, $\Psi_c$ can be expressed through the big components of spinor $Q$

$$Q\equiv e^{-imt}\begin{pmatrix}\phi\\ \chi\end{pmatrix}, \tag{18}$$

due to the following formula:

$$\Psi_c=\left(1+\frac{(i\boldsymbol{D})^2}{8m_c^2}\right)\phi, \tag{19}$$

(this can be checked with the use of the equation of motion). Let us note that the covariant derivative should be taken in the adjoint representation, when it acts on the chromoelectric field,

$$(\boldsymbol{D}E)=(\boldsymbol{\partial}T^a-gf^{abc}T^b\boldsymbol{A}^c)\boldsymbol{E}^a. \tag{20}$$

Radiative corrections modify the coefficients of the chromomagnetic term $(\boldsymbol{\sigma}B)$ and ''Darwin'' term in Eq. (15). However, in the situation at hand, these effects can be consistently neglected.

Now let us consider the significance of different contributions to the expansions in Eqs. (15) and (16). Evaluating the contributions of chromomagnetic and ''Darwin'' terms, we have to take into account the interaction of the heavy quark with the light quark as well as the interaction with the other heavy quark. In the first case, the procedure of the calculation is analogous to that for the heavy-light mesons. So, the Darwin term is suppressed by a factor of $\Lambda_{\rm QCD}/m_c$ in comparison with the chromomagnetic term, and, thus, we neglect its contribution. In the second case, the analysis is analogous to that for the heavy-heavy mesons, so that we can use the scaling rules of the nonrelativistic QCD [4]. In this approach, the contributions of different operators can be estimated, using the following relations in the Coulomb gauge:

$$\Psi_c\sim(m_cv_c)^{3/2},\quad D\sim m_cv_c,\quad gE\sim m_c^2v_c^3,$$

$$gB\sim m_c^2v_c^4,\quad g\sim v_c^{1/2}.$$

From these scaling rules for the heavy-heavy interaction, we can deduce that the contribution of the ''Darwin'' term has the same order as that of chromomagnetic term.

Let us now start the calculation of matrix elements with the use of potential models for the bound states of hadrons. While estimating the matrix element value of the kinetic energy, we note that the heavy quark kinetic energy consists of two parts: the kinetic energy of the heavy quark motion inside the diquark and the kinetic energy, related to the diquark motion inside the hadron. According to the phenomenology of meson potential models, in the range of average distances between the quarks: 0.1 fm$<r<$1 fm, the average kinetic energy of quarks is constant and independent of both the quark flavors, constituting meson, and the quantum numbers, describing the excitations of the ground state. Therefore, we determine $T=m_dv_d^2/2+m_lv_l^2/2$ as the average kinetic energy of diquark and light quark with $m_l=m_q$, and $T/2=m_{c1}v_{c1}^2/2+m_{c2}v_{c2}^2/2$ as the average kinetic energy of heavy quarks inside the diquark (the coefficient 1/2 takes into account the antisymmetry of the color wave function for the diquark). Finally, we have the following expression: for the matrix elements of the heavy quark kinetic energy:

$$\frac{\langle\Xi_{cc}^\diamond|\Psi_c^\dagger(i\boldsymbol{D})^2\Psi_c|\Xi_{cc}^\diamond\rangle}{2M_{\Xi_{cc}^\diamond}m_c^2}\simeq v_c^2\simeq\frac{m_lT}{2m_c^2+m_cm_l}+\frac{T}{2m_c}. \tag{21}$$

We use the value $T\simeq0.4$ GeV, which results in $v_c^2=0.146$, where the dominant contribution comes from the motion of heavy quarks inside the diquark.

Now we would like to estimate the matrix element of the chromomagnetic operator, corresponding to the interaction of heavy quarks with the chromomagnetic field of the light quark. For this purpose, we will use the following definitions: $O_{\rm mag}=\Sigma_{i=1}^2(g_s/4m_c)\overline{c^i}\sigma_{\mu\nu}G^{\mu\nu}c^i$ and $O_{\rm mag}\sim\lambda[j(j+1)-s_d(s_d+1)-s_l(s_l+1)]$, where $s_d$ is the diquark spin (as was noticed by the authors earlier [1], there is only the vector state of the $cc$ diquark in the ground state of such baryons), $s_l$ is the light quark spin, and $j$ is the total spin of the baryon. Since both $c$ quarks additively contribute to the total decay width of baryons, we can use the diquark picture and substitute for the sum of $c$-quark spins, the diquark spin. This leads to the parametrization for $O_{\rm mag}$, as it is given above, and, moreover, it allows us to relate the value of the matrix element for this operator to the mass difference between the excited and ground states of the baryons:

$$O_{\rm mag}=-\frac{2}{3}(M^*_{\Xi_{cc}^\diamond}-M_{\Xi_{cc}^\diamond}). \tag{22}$$

Taking into account the interaction of heavy quarks inside the diquark leads to the following expressions for the chromomagnetic and Darwin terms:

$$\frac{\langle\Xi_{cc}^\diamond|\Psi_c^\dagger g\boldsymbol{\sigma}\cdot\boldsymbol{B}\Psi_c|\Xi_{cc}^\diamond\rangle}{2M_{\Xi_{cc}^\diamond}}=\frac{2}{9}g^2\frac{|\Psi(0)|^2}{m_c}, \tag{23}$$

$$\frac{\langle\Xi_{cc}^\diamond|\Psi_c^\dagger(\boldsymbol{D}\cdot g\boldsymbol{E})\Psi_c|\Xi_{cc}^\diamond\rangle}{2M_{\Xi_{cc}^\diamond}}=\frac{2}{3}g^2|\Psi(0)|^2, \tag{24}$$

where $\Psi(0)$ is the diquark wave function at the origin.

Collecting the results given above, we find the matrix elements of operators (15) and (16):

$$\frac{\langle \Xi_{cc}^{\diamond}|\bar{c}c|\Xi_{cc}^{\diamond}\rangle}{2M_{\Xi_{cc}^{\diamond}}}=1-\frac{1}{2}v_c^2-\frac{1}{3}\frac{M_{\Xi_{cc}^{\diamond}}-M_{\Xi_{cc}^{\diamond}}}{m_c}-\frac{g^2}{9m_c^3}|\Psi(0)|^2 -\frac{1}{6m_c^3}g^2|\Psi(0)|^2+\cdots \approx 1-0.074-0.004-0.003-0.005+\cdots. \tag{25}$$

We can see that the largest contribution to the decrease of the decay width comes from the time dilation, connected to the motion of heavy quarks inside the baryon. For the matrix element of the operator $\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c$, we get

$$\frac{\langle \Xi_{cc}^{\diamond}|\bar{c}g\sigma_{\mu\nu}G^{\mu\nu}c|\Xi_{cc}^{\diamond}\rangle}{2M_{\Xi_{cc}^{\diamond}}}=-\frac{4}{3}\frac{(M^*_{\Xi_{cc}^{\diamond}}-M_{\Xi_{cc}^{\diamond}})}{m_c} -\frac{4g^2}{9m_c^3}|\Psi(0)|^2-\frac{g^2}{3m_c^3}|\Psi(0)|^2. \tag{26}$$

Now let us continue with the calculation of the matrix elements for the four-quark operators, corresponding to the effects of Pauli interference and weak scattering. The straightforward calculation in the framework of nonrelativistic QCD gives

$$\langle \bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q\rangle=2m_cV^{-1}(1-4S_cS_q), \tag{27}$$

$$\langle \bar{c}\gamma_\mu\gamma_5c)(\bar{q}\gamma^\mu(1-\gamma_5)q\rangle=-4S_cS_q2m_cV^{-1}, \tag{28}$$

where $V^{-1}=|\Psi_1(0)|^2$, and $\Psi_1(0)$ is the light quark wave function at the origin of the two $c$ quarks. We suppose that $|\Psi_1(0)|$ has the same value as that in the $D$ meson. So, we find

$$|\Psi_1(0)|^2\approx\frac{f_D^2m_D^2}{12m_c}. \tag{29}$$

Then, again remembering that both $c$ quarks additively contribute to the total decay width and using the diquark picture, we can substitute for $S_{c_1}+S_{c_2}$ the $S_d$, where $S_d$ is the diquark spin. Thus, we have

$$\frac{\langle \Xi_{cc}^{\diamond}|(\bar{c}\gamma_\mu(1-\gamma_5)c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_{cc}^{\diamond}\rangle}{M_{\Xi_{cc}^{\diamond}}} =12m_c|\Psi_1(0)|^2, \tag{30}$$

$$\frac{\langle \Xi_{cc}^{\diamond}|(\bar{c}\gamma_\mu\gamma_5c)(\bar{q}\gamma^\mu(1-\gamma_5)q)|\Xi_cc^{\diamond}\rangle}{M_{\Xi_{cc}^{\diamond}}}=8m_c|\Psi_1(0)|^2. \tag{31}$$

The color antisymmetry of the baryon wave function results in relations between the matrix elements of operators with different sums over the color indexes:

$$\langle \Xi_{cc}^{\diamond}|(\bar{c}_iT_\mu c_k)(\bar{q}_k\gamma^\mu(1-\gamma_5)q_i|\Xi_{cc}^{\diamond}\rangle =-\langle \Xi_{cc}^{\diamond}|(\bar{c}T_\mu c)(\bar{q}\gamma^\mu(1-\gamma_5)q|\Xi_{cc}^{\diamond}\rangle,$$

where $T_\mu$ is any spinor structure. Thus, we completely derive the expressions for the evaluation of the required matrix elements.

## IV. NUMERICAL ESTIMATES

Now we are ready to collect the contributions, described above, and to estimate the total lifetimes of baryons $\Xi_{cc}^{++}$ and $\Xi_{cc}^{+}$. For the beginning, we list the values of parameters, which we have used in our calculations, and give some comments on their choice:

$$m_c=1.6\ \text{GeV},\quad m_s=0.45\ \text{GeV},\quad |V_{cs}|=0.9745,$$

$$M_{\Xi_{cc}^{++}}=3.56\ \text{GeV},\quad M_{\Xi_{cc}^{+}}=3.56\ \text{GeV},$$

$$M^*_{\Xi_{cc}^{\diamond}}-M_{\Xi_{cc}^{\diamond}}=0.1\ \text{GeV},$$

$$T=0.4\ \text{GeV},\quad |\Psi(0)|=0.17\ \text{GeV}^{3/2},$$

$$m_l=0.30\ \text{GeV}.$$

For the parameters $M_{\Xi_{cc}^{++}}$, $M_{\Xi_{cc}^{+}}$ and $M^*_{\Xi_{cc}^{\diamond}}-M_{\Xi_{cc}^{\diamond}}$ we use the mean values given in the literature. Their evaluation has been also performed by the authors in the potential model for the doubly charmed baryons with the Buchmüller-Tye potential, and also in Refs. [18–21]. For $f_D$ we use the value given in [6,22] and for $T$ we take it from [23]. The mass $m_c$ corresponds to the pole mass of the $c$ quark. For its determination we have used a fit of theoretical predictions for the lifetimes and the semileptonic width of the $D^0$ meson from the experimental data. This choice of $c$-quark mass seems effectively to include unknown contributions of higher orders in perturbative QCD to the total decay width of baryons under consideration.

In the operator product expansion, combined with the effective theory of heavy quarks, the heavy quark mass depends on the factorization scale for the Wilson coefficients standing in front of corresponding operators. Usually, this mass is associated with the pole mass, and not with the current mass in the perturbative QCD. While the current mass estimates ($m_c\approx1.25$ GeV) are less than those of the pole one ($m_c\approx1.4-1.8$ GeV), both contain the nonperturbative uncertainty connected to the region of low virtualities, which are beyond the perturbative theory ($\delta m_c\sim\Lambda_{\text{QCD}}\sim200$–300 MeV). Anyway, the charmed quark mass value determines the dominant term in the variation of theoretical predictions. To obtain reliable results, we apply the heavy quark effective theory to estimate the semileptonic widths of charmed mesons $D^+,D^0$, whose absolute values are approximately independent of the spectator contents. This procedure

TABLE I. The contributions of different modes to the total decay width of doubly charmed baryons.

| Mode or decay mechanism | Width ($ps^{-1}$) | Contribution in % ($\Xi_{cc}^{++}$) | Contribution in % ($\Xi_{cc}^{+}$) |
|---|---|---|---|
| $c_{\rm spec} \to s\bar{d}u$ | 2.894 | 124 | 32.4 |
| $c \to s e^+ \nu$ | 0.380 | 16 | 4.3 |
| $c \to s \mu^+ \nu$ | 0.380 | 16 | 4.3 |
| PI | −1.317 | −56 | |
| WS | 5.254 | | 59 |
| $\Gamma_{\Xi_{cc}^{++}}$ | 2.337 | 100 | |
| $\Gamma_{\Xi_{cc}^{+}}$ | 8.909 | | 100 |

leads to the value given in the above set of parameters. Moreover, the recent observation of the $B_c$ meson by the CDF collaboration [24] yielded the lifetime, which is in a good agreement with the predictions performed in the same OPE method [9], as it is applied in the present work, if the charmed quark mass is close to the value we have used. So, the region of low charmed quark mass values is reliably disfavored.

Next, the masses of light quarks are essential only in the estimation of kinematical characteristics such as the phase spaces in the intermediate states, when we have calculated the corresponding coefficients of operators, as well as in the evaluations of parameters of bound states such as the average square of heavy quark momentum or the wave functions in the nonrelativistic approach. In both cases, the use of constituent values of light quark masses is justified, since, first, the charmed quark mass is not so large to neglect the masses of hadrons in the final states, which can be effectively taken into account by the constituent values, and second, the potential picture is qualitatively and quantatively explored with the same phenomenological values of light quark masses.

The renormalization scale $\mu$ is chosen in the following way: $\mu_1 = m_c$ in the estimate of Wilson coefficients $C$ for the effective four-fermion weak Lagrangian with the $c$ quarks at low energies and $\mu_2 = 1.2$ GeV for the Pauli interference and weak scattering ($k$ factor). The latter value of the renormalization scale has been obtained from the fit of theoretical predictions for the lifetimes differences of baryons $\Lambda_c, \Xi_c^+, \Xi_c^0$ over the experimental data. Here we would like to note that the theoretical approximations used in [5] include the effect of logarithmic renormalization and do not take into account the mass effects, related to the $s$ quark in the final state. For the corresponding contributions to the decay widths of baryons with the different quark content we have

$$\triangle\Gamma_{nl}(\Lambda_c) = c_d \langle O_d \rangle_{\Lambda_c} + c_u \langle O_u \rangle_{\Lambda_c},$$
$$\triangle\Gamma_{nl}(\Xi_c^+) = c_s \langle O_s \rangle_{\Xi_c^+} + c_u \langle O_u \rangle_{\Xi_c^+}, \tag{32}$$
$$\triangle\Gamma_{nl}(\Xi_c^0) = c_d \langle O_d \rangle_{\Xi_c^0} + c_s \langle O_s \rangle_{\Xi_c^0},$$

where $\langle O_q \rangle_{X_c} = \langle X_c | O_q | X_c \rangle$, $O_q = (\bar{c}\gamma_\mu c)(\bar{q}\gamma^\mu q)$, and $q = u,d,s$. The coefficients $c_q(\mu)$ are equal to

$$c_d = \frac{G_f^2 m_c^2}{4\pi}\left[C_+^2 + C_-^2 + \frac{1}{3}(4k^{1/2}-1)(C_-^2 - C_+^2)\right],$$
$$c_u = -\frac{G_f^2 m_c^2}{16\pi}\left[(C_+ + C_-)^2 + \frac{1}{3}(1-4k^{1/2}) \times (5C_+^2 + C_-^2 - 6C_+C_-)\right], \tag{33}$$
$$c_s = -\frac{G_f^2 m_c^2}{16\pi}\left[(C_+ - C_-)^2 + \frac{1}{3}(1-4k^{1/2}) \times (5C_+^2 + C_-^2 + 6C_+C_-)\right].$$

We use the spin-averaged value of the $D$-meson mass for the estimation of the effective light quark mass $m_l$ as it stands below:

$$m_D = m_c + m_l + \frac{T m_l}{m_c + m_l} \approx 1.98 \ \text{GeV}. \tag{34}$$

The $s$-quark mass can be written down as

$$m_s = m_l + 0.15 \ \text{GeV}. \tag{35}$$

As we have already mentioned, the spectator decay width of the $c$ quark $\Gamma_{c,\rm spec}$ is known in the next-to-leading order of the perturbative QCD [13–17]. The most complete calculation, including the mass effects, connected to the $s$ quark in the final state is given in [17]. In the present work we have used the latter result for the calculation of the spectator contribution to the total decay width of doubly charmed baryons. In the calculation of the semileptonic decay width, we neglect the electron and muon masses in the final state. Moreover, we neglect the $\tau$-lepton mode.

Now, let us proceed with the numerical analysis of contributions by the different decay modes into the total decay width. In Table I we have listed the results for the fixed values of parameters described above. From this table one can see the significance of the effects caused by both the Pauli interference and the weak scattering in the decays of doubly charmed baryons. The Pauli interference gives the negative correction about 36% for the $\Xi_{cc}^{++}$ baryons, and the weak scattering increases the total width by 144% for $\Xi_{cc}^{+}$.

As it has been already noted in the Introduction, these effects take place differently in the baryons, and, thus, they enhance the difference of lifetimes for these hadrons.

It is worth here recalling that the lifetime difference of $D^+$ and $D^0$ mesons is generally explained by the Pauli interference of $c$-quark decay products with the antiquark in the initial state, while in the current consideration, we see the dominant contribution of weak scattering. This should not be surprising because under a more detailed consideration, we will find that the formula for the Pauli interference operator for the $D$ meson coincides with that for the weak scattering in the case of baryons, containing, at least, a single $c$ quark.

Finally, collecting the different contributions for the total lifetimes of doubly charmed baryons, we obtain the following values:

$$\tau_{\Xi_{cc}^{++}}=0.43\ \text{ps},\qquad \tau_{\Xi_{cc}^{+}}=0.11\ \text{ps}.$$

There are several sources of uncertainties in the above predictions. First, in this point, the most important value is the charmed quark mass, since the spectator-independent widths behave as $m_c^5$ and the spectator-dependent terms do as $m_c^3$. At low $m_c$ the destructive interference will be more essential. However, as we have already mentioned, the high values of $m_c$ are preferable for the reasonable agreement with the experimental data on the semileptonic width of $D$ mesons and $B_c$ lifetime. So, rather broad variations of both the $c$-quark mass in the range of 1.6–1.65 GeV and the mass difference for the strange and ordinary light quarks in Eq. (35) in the range of 0.15–0.2 GeV lead to the uncertainties in the lifetimes about 15%. Next, the dependence of lifetimes on the values of $f_D$ and the normalization point results in an uncertainty of less than 10%. Finally, as we have seen, the next-to-leading order terms of expansion in the heavy quark velocity $v$ can reach the relative value of 10% in the total widths, which can be used as the estimation of possible contributions from the higher-order terms in $v$. Summing up the various sources of uncertainties we find $\delta\tau_{\Xi_{cc}^{++}}=\pm 0.11$ ps, $\delta\tau_{\Xi_{cc}^{+}}=\pm 0.03$ ps.

## V. CONCLUSION

In this work we have performed a detailed investigation on the lifetimes of doubly charmed baryons $\Xi_{cc}^{++}$, $\Xi_{cc}^{+}$ on the basis of the operator product expansion for the transition currents. We have presented formulas taking into account both the mass effects and the low-energy logarithmic renormalization for the contributions to the total decay width of baryons containing heavy quarks, as is caused by the effects of Pauli interference and weak scattering. The usage of the diquark picture has allowed us to evaluate the matrix elements of operators derived. Further, we have discussed the procedure of choosing the values of parameters for the total lifetimes of these baryons. The obtained results show the significant role of both the Pauli interference and the weak scattering.


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## APPENDIX

In this Appendix we present the explicit formulas [17] for the spectator decay of the $c$ quark in next-to-leading order of the perturbation theory, taking into account the mass effects, related to the $s$ quark in the final state.

The coefficients $C_+$ and $C_-$ in the effective Lagrangian, taking into account the next-to-leading order in perturbative QCD, acquire the additional multiplicative factors:

$$F_{\pm}(\mu)=1+\frac{\alpha_s(m_W)-\alpha_s(\mu)}{4\pi}\,\frac{\gamma_{\pm}^{(0)}}{2\beta_0}\left(\frac{\gamma_{\pm}^{(1)}}{\gamma_{\pm}^{(0)}}-\frac{\beta_1}{\beta_0}\right)+\frac{\alpha_s(m_W)}{4\pi}B_{\pm},$$

where $\gamma_{\pm}^{(i)}$ are the coefficients of the anomalous dimensions for the operators $O_{\pm}$:

$$\gamma_{\pm}=\gamma_{\pm}^{(0)}\frac{\alpha_s}{4\pi}+\gamma_{\pm}^{(1)}\left(\frac{\alpha_s}{4\pi}\right)^2+O(\alpha_s^3)$$

with

$$\gamma_{+}^{(0)}=4,\quad \gamma_{-}^{(0)}=-8,\quad \gamma_{+}^{(1)}=-7+\frac{4}{9}n_f,$$

$$\gamma_{-}^{(1)}=-14-\frac{8}{9}n_f,$$

in the naive dimensional regularization (NDR) with the anticommutating $\gamma_5$, and $n_f$ is the number of flavors taken into account. $\beta_i$ is the initial two coefficients of the QCD $\beta$ function:

$$\beta=-g_s\left\{\beta_0\frac{\alpha_s}{4\pi}+\beta_1\left(\frac{\alpha_s}{4\pi}\right)^2+O(\alpha_s^3)\right\},$$

$$\beta_0=11-\frac{2}{3}n_f,\quad \beta_1=102-\frac{38}{3}n_f.$$

The coefficients $B_{\pm}$ are written down in accordance with the requirement of the agreement between the effective Lagrangian, evaluated at the scale $\mu=m_W$, and the Standard Model requirement up to terms of the order of $\alpha_s^2(m_W)$:

$$B_{\pm}=\pm B\frac{N_c\mp 1}{2N_c},$$

where $N_c=3$ is the number of colors. In the NDR scheme for $B$, we find $B=11$.

Using the effective Lagrangian in the next-to-leading order of the perturbative QCD and calculating the one-gluon corrections, we get the following expression for the spectator $c$-quark decay:

$$\Gamma(c\to s u\overline{d})=\Gamma_0\Bigg[2C_+^2(\mu)+C_-^2(\mu)+\frac{\alpha_s(m_W)-\alpha_s(\mu)}{2\pi}\{2C_+^2(\mu)R_+ +C_-^2(\mu)R_-\}+\frac{\alpha_s(\mu)}{2\pi}\{2C_+^2(\mu)B_+ +C_-^2(\mu)B_-\}$$
$$+\frac{3}{4}\{C_+(\mu)+C_-(\mu)\}^2\frac{2}{3}\,\frac{\alpha_s(\mu)}{\pi}\{G_a+G_b\}+\frac{3}{4}\{C_+(\mu)-C_-(\mu)\}^2\frac{2}{3}\,\frac{\alpha_s(\mu)}{\pi}\{G_c+G_d\}$$
$$+\frac{1}{2}\{C_+^2(\mu)-C_+^2(\mu)\}\frac{2}{3}\,\frac{\alpha_s(\mu)}{\pi}\{G_a+G_b+Ge\}\Bigg], \qquad \text{(A1)}$$

where

$$\Gamma_0=\frac{G_f^2 m_c^5}{192\pi^3}|V_{cs}|^2 f_1(m_s^2/m_c^2),$$

$$f_1(a)=1-8a+8a^3-a^4-12a^2\ln a,$$

and

$$R_\pm=B_\pm+\frac{\gamma_\pm^{(0)}}{2\beta_0}\left(\frac{\gamma_\pm^{(1)}}{\gamma_\pm^{(0)}}-\frac{\beta_1}{\beta_0}\right).$$

For $G_a$, $G_b$, $G_c$, $G_d$, and $G_e$ we have found:

$$(G_a+G_b)f_1(a)=\frac{31}{4}-\pi^2-a[80-\ln a]+32a^{3/2}\pi^2-a^2[273+16\pi^2-18\ln a+36\ln^2 a]+32a^{5/2}\pi^2-\frac{8}{9}a^3[118-57\ln a]$$
$$+O(a^{7/2}), \qquad \text{(A2)}$$

$$(G_c+G_d)f_1(a)=\frac{31}{4}-\pi^2-8a[10-\pi^2+3\ln a]-a^2[117-24\pi^2+(30-8\pi)\ln a+36\ln^2 a]$$
$$-\frac{4}{3}a^3[79+2\pi^2-62\ln a+6\ln^2 a]+O(a^4), \qquad \text{(A3)}$$

$$(G_a+G_b+G_e+B)f_1(a)=\left(6\ln\frac{m_c^2}{\mu^2}+11\right)f_1(a)-\frac{51}{4}-\pi^2+8a[21-\pi^2-3\ln a]+32a^{3/2}\pi^2$$
$$-a^2[111+40\pi^2-258\ln a+36\ln^2 a]+32a^{5/2}\pi^2$$
$$-\frac{4}{9}a^3[305+18\pi^2+30\ln a-54\ln^2 a]+O(a^{7/2}). \qquad \text{(A4)}$$

These approximations can be used in the range of $a$ values: $a<0.15$, where $a=(m_s/m_c)^2$, which indeed, takes place in the calculations under consideration.


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