CP violating $b\to s\gamma$ decay in supersymmetric models

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Supersymmetric models with nonuniversal squark masses can enrich the chiral structure and CP violating phenomena in $b\to s\gamma$ decays. Direct CP violation in $b\to s\gamma$ decay, mixing induced CP violation in radiative $B_{d,s}$ decays (such as $B_s\to\phi\gamma$ and $B_d\to K^*_{1,2}\gamma$), and Λ polarization in $\Lambda_b\to\Lambda\gamma$ decay can be substantially different from the standard model. Future experiments at e^+e^- and hadronic B factories will give important information on the underlying couplings for radiative b decays. [S0556-2821(99)05509-5]

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I. INTRODUCTION

The processes $B\to K^*\gamma$ and $b\to s\gamma$ are the first penguin processes to be observed in B decays [1]. As quantum loop effects, they provide good tests for the standard model (SM). The measured branching ratios [2] are in agreement with the SM predictions [3,4], although new physics effects are still allowed [5]. To further test SM, one must study the detailed structure of the $bs\gamma$ couplings. In the SM, the quark level $bs\gamma$ coupling is usually parametrized as

$$H_{\rm SM}=-c_7^{\rm SM}\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}V_{tb}V_{ts}^*\overline{s}[m_b(1+\gamma_5)+m_s(1-\gamma_5)]\sigma_{\mu\nu}F^{\mu\nu}b, \tag{1}$$

where $c_7^{\rm SM}\cong -0.3$ at the typical B decay energy scale $\mu\approx 5$ GeV. One notable feature is that the $1+\gamma_5$ chiral structure dominates, which reflects the left-handed nature of weak interactions. Although the branching ratio measurements are consistent with the SM, they can not determine the chiral structure of the couplings. In models beyond the SM, it is in principle possible that both chiralities are comparable, and the $1-\gamma_5$ component may even be the dominant one. It is therefore important to experimentally confirm the chiral structure.

The chirality structure can be tested by studying CP violation. If the $1+\gamma_5$ chiral structure dominates completely, then only direct CP violating rate asymmetries are possible. Since such asymmetries are small in the SM [6], their observation would indicate the presence of new physics. If both chiralities are present, possible only with new physics, mixing induced CP violation can occur as well [7]. Independent of CP violation, the chirality structure can also manifest itself in radiative b-flavored baryon decays, leading to different polarizations of the final state baryon [8]. When these asymmetries and polarizations are measured, they will provide useful information on the underlying couplings for radiative b decays.

In this paper we show that in supersymmetric models with nonuniversal squark mass matrices, the chiral structure for $b\to s\gamma$ can be very different from the SM. We then illustrate how the chiral structures can be studied by direct and mixing induced CP violation as well as Λ polarization in inclusive $b\to s\gamma$, exclusive $B\to M\gamma$, and $\Lambda_b\to\Lambda\gamma$ decays, respectively.

II. RADIATIVE B DECAY IN SUPERSYMMETRIC MODELS

Supersymmetry (SUSY) is one of the leading candidates for physics beyond the SM [9]. It can help resolve many potential problems when one extends beyond the SM, for example, the gauge hierarchy problem, unification of $SU(3)\times SU(2)\times U(1)$ gauge couplings, and so on. SUSY models also lead to many interesting low-energy phenomena. We will concentrate on flavor changing $b\to s\gamma$ decay due to nonuniversal squark masses.

Potentially large new flavor and CP violating sources may come from interactions between quarks, gauginos, Higgsinos, and squarks [10]. These interactions are given by

$$L=-\sqrt{2}g_s(\overline{d_L}\Gamma^\dagger_{DL}-\overline{d_R}\Gamma^\dagger_{DR})T^a\tilde{D}\tilde{g}_a -g\tilde{U}_k^*\overline{\tilde{\chi}_j^c}[(G_{UL}^{jki}-H_{UR}^{jki})P_L-H_{UL}^{jki}P_R]d_i+\text{H.c.}, \tag{2}$$

where P_L is the left-handed projection, $\tilde{Q}$, $\tilde{g}$, and $\tilde{\chi}$ are the squark, gluino, and chargino fields, and j, k, i are summed from 1–2, 1–6, and 1–3, respectively. The $\Gamma_{QL,R}$ matrices are the mixing matrices that relate the weak eigenstates $\tilde{Q}^i_{L,R}$ to the mass eigenstates $\tilde{Q}^k$:

$$(\tilde{Q}_L,\tilde{Q}_R)=(\Gamma^\dagger_{QL},\Gamma^\dagger_{QR})\tilde{Q}. \tag{3}$$

The matrices G, H are related to $\Gamma_{QL,R}$ and the chargino mixing matrices U and V [9] by

$$G^{jki}_{UL}=V^*_{j1}(\Gamma_{UL}V_{\rm CKM})^{ki},$$

$$H^{jki}_{UL}=U_{j2}(\Gamma_{UL}V_{\rm CKM}\hat{Y}_D)^{ki},$$

$$H^{jki}_{UR}=V^*_{j2}(\Gamma_{UR}\hat{Y}_U V_{\rm CKM})^{ki}, \tag{4}$$

where $V_{\rm CKM}$ is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix, $\hat{Y}_D=\mathrm{diag}(m_d,m_s,m_b)/(\sqrt{2}M_W\cos\beta)$, and $\hat{Y}_U=\mathrm{diag}(m_u,m_c,m_t)/(\sqrt{2}M_W\sin\beta)$. Note that, in contrast to Ref. [10], we have kept the $V_{\rm CKM}$ factor explicitly in G_{UL} and H_{UL} rather than absorbing it into Γ_{UL}.

Inspired by minimal supergravity models, the usual approach to SUSY modeling is to assume universal soft SUSY breaking masses. This certainly reduces the number of parameters, but it also removes soft squark masses as a potent source of flavor (and CP) violation. As we are concerned with the possible impact of SUSY models on $b\to s\gamma$ decay, we consider general low-energy mass mixings without assuming specific forms for the squark mass matrix at high energies. There is then no theoretical constraint on the form of $\Gamma_{QL,R}$ at the SUSY breaking scale.

One might expect that the dominant contributions come from gluino exchange because the coupling is stronger. However, it has been shown that chargino contributions can be important if flavor and CP violation in Γ_{QL} are large [11]. We will therefore include these contributions as well. There are also contributions from neutralino exchange. We have analyzed neutralino contributions and find their contributions to be about one order of magnitude smaller in the parameter space we consider.

The effective Hamiltonian due to gluino-squark and chargino-squark exchange for $b\to s\gamma, sg$ transitions is given by

$$\begin{aligned}H_{\rm eff}=&-\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}V_{tb}V^*_{ts}m_b\bar{s}\\&\times[c_7(1+\gamma_5)+c'_7(1-\gamma_5)]\sigma_{\mu\nu}F^{\mu\nu}b\\&-\frac{G_F}{\sqrt{2}}\frac{g}{8\pi^2}V_{tb}V^*_{ts}m_b\bar{s}[c_8(1+\gamma_5)\\&+c'_8(1-\gamma_5)]\sigma_{\mu\nu}T^aG^{\mu\nu}_a b,\end{aligned} \tag{5}$$

where we have neglected m_s, and $c_{7,8}=c^{\rm SM}_{7,8}+c^{\rm new}_{7,8}$ are the sum of the SM and new physics contributions, while $c'_{7,8}$ come purely from new physics. They are given by

$$\begin{aligned}c^{\rm new}_7(M_W)=&\frac{\sqrt{2}\pi\alpha_s}{G_F V_{tb}V^*_{ts}}\frac{Q_dC_2(R)}{m^2_{\tilde{D}_k}}\bigg\{(\Gamma^\dagger_{DL})^{sk}f_2(a_{\tilde{g}k})\Gamma^{kb}_{DL}\\&-\frac{m_{\tilde{g}}}{m_b}(\Gamma^\dagger_{DL})^{sk}f_4(a_{\tilde{g}k})\Gamma^{kb}_{DR}\bigg\}+\frac{1}{V_{tb}V^*_{ts}}\frac{M^2_W}{m^2_{\tilde{U}_k}}\\&\times\bigg\{(G^{jkb}_{UL}-H^{jkb}_{UR})(G^{jks}_{UL}-H^{jks}_{UR})^*[f_1(b_{jk})\\&+Q_uf_2(b_{jk})]-H^{jkb}_{UL}(G^{jks}_{UL}\\&-H^{jks}_{UR})^*\frac{m_{\tilde{\chi}^-_j}}{m_b}[f_3(b_{jk})+Q_uf_4(b_{jk})]\bigg\},\end{aligned} \tag{6}$$

$$\begin{aligned}c^{\rm new}_8(M_W)=&\frac{\sqrt{2}\pi\alpha_s}{G_F V_{tb}V^*_{ts}}\bigg\{\frac{2C_2(R)-C_2(G)}{2m^2_{\tilde{D}_k}}\\&\times\bigg[(\Gamma^\dagger_{DL})^{sk}f_2(a_{\tilde{g}k})\Gamma^{kb}_{DL}\\&-\frac{m_{\tilde{g}}}{m_b}(\Gamma^\dagger_{DL})^{sk}f_4(a_{\tilde{g}k})\Gamma^{kb}_{DR}\bigg]\\&-\frac{C_2(G)}{2m^2_{\tilde{D}_k}}\bigg[(\Gamma^\dagger_{DL})^{sk}f_1(a_{\tilde{g}k})\Gamma^{kb}_{DL}\\&-\frac{m_{\tilde{g}}}{m_b}(\Gamma^\dagger_{DL})^{sk}f_3(a_{\tilde{g}k})\Gamma^{kb}_{DR}\bigg]\bigg\}+\frac{1}{V_{tb}V^*_{ts}}\frac{M^2_W}{m^2_{\tilde{U}_k}}\\&\times\bigg\{(G^{jkb}_{UL}-H^{jkb}_{UR})(G^{jks}_{UL}-H^{jks}_{UR})^*f_2(b_{jk})\\&-H^{jkb}_{UL}(G^{jks}_{UL}-H^{jks}_{UR})^*\frac{m_{\tilde{\chi}^-_j}}{m_b}f_4(b_{jk})\bigg\},\end{aligned} \tag{7}$$

where $Q_{d,u}$ are the electric charges of the down and up type quarks, $a_{\tilde{g}k}\equiv m^2_{\tilde{g}}/m^2_{\tilde{D}_k}$, $b_{jk}\equiv m^2_{\tilde{\chi}^-_j}/m^2_{\tilde{U}_k}$, $C_2(G)=N=3$, and $C_2(R)=(N^2-1)/(2N)=4/3$ are Casimirs, and the functions $f_i(x)$ are given by

$$f_1(x)=\frac{1}{12(x-1)^4}(x^3-6x^2+3x+2+6x\ln x),$$

$$f_2(x)=\frac{1}{12(x-1)^4}(2x^3+3x^2-6x+1-6x^2\ln x),$$

$$f_3(x)=\frac{1}{2(x-1)^3}(x^2-4x+3+2\ln x),$$

$$f_4(x)=\frac{1}{2(x-1)^3}(x^2-1-2x\ln x), \tag{8}$$

which agree with Ref. [10]. The first term of Eqs. (6) and (7) comes from gluino exchange while the second term comes from chargino exchange. The chirality partners $c'_{7,8}$ from gluino exchange are obtained by interchanging Γ_{QL} and Γ_{QR}. The chargino contributions to $c'_{7,8}$ are suppressed by m_s/m_b. The $\Gamma^\dagger_{DL}(\cdots)\Gamma_{DL}$ terms arise from mixing among $\tilde{D}_L$ alone, while $\Gamma^\dagger_{DL}(\cdots)\Gamma_{DR}$ terms come from mixing between $\tilde{D}_L$ and $\tilde{D}_R$. We denote these as LL and LR mixing, respectively. Note the $M_{\tilde{g}}/m_b$ enhancement factor for LR mixing. The $m_{\tilde{\chi}_j^-}/m_b$ enhancement factor in the chargino contribution is softened by a factor of m_b/M_W in H^{jkb}_{UL}.

When running down to the B decay scale $\mu \approx m_b$, the leading order Wilson coefficients $c_i^{(\prime)}$ and next to leading order coefficients $c_7^{(\prime)(1)}$ are given by [4]

$$
\begin{aligned}
c_7(\mu=m_b) &= -0.31+0.67c_7^{\rm new}(M_W)+0.09c_8^{\rm new}(M_W),\\
c_8(\mu=m_b) &= -0.15+0.70c_8^{\rm new}(M_W),\\
c_7^{(1)}(\mu=m_b) &= +0.48-2.29c_7^{\rm new}(M_W)-0.12c_8^{\rm new}(M_W),
\end{aligned}
\tag{9}
$$

while for opposite chirality, which receives no SM contribution, one simply replaces $c^{\rm new}$ by c' and set the constant terms to zero.

In obtaining the above expressions we have assumed that SUSY breaking occurs at the TeV scale and the squark and gluino masses are in the few hundred GeV region. Therefore the gluino, squarks, top quark, and W boson are integrated out at $\mu \approx m_t$ at the same time. The coefficients obtained can be very different from the SM predictions, but they are of course subject to the constraint from the observed $b \to s\gamma$ branching ratio, in the form of

$$
\begin{aligned}
&Br(B\to X_s\gamma)|_{E_\gamma>(1-\delta)E_\gamma^{\max}}\\
&\quad \approx 2.57\times 10^{-3}\\
&\qquad \times K_{NLO}(\delta)\times\frac{Br(B\to X_c e\bar{\nu})}{10.5\%},
\end{aligned}
\tag{10}
$$

which should be compared to the most recent experimental result of Ref. [2] $(3.15\pm0.54)\times10^{-4}$, and δ is a parameter that defines the photon energy cut (ideally $\delta=1$). We take the last factor in Eq. (10) to be 1 and

$$
\begin{aligned}
K_{NLO}(\delta) = &\sum_{\substack{i,j=2,7,8\\ i\le j}} k_{ij}(\delta)\mathrm{Re}[c_i c_j^* + c'_i c'^*_j]\\
&+k_{77}^{(1)}(\delta)\mathrm{Re}[c_7^{(1)}c_7^* + c_7'^{(1)}c_7'^*],
\end{aligned}
\tag{11}
$$

where $c'_2=0$ and $k_{ij}(\delta)$ are known functions of δ, their values for some δ can be obtained by using the expressions given in Ref. [4]. We use $\delta=90\%$ which gives $Br(B\to X_s\gamma)\approx 3.3\times10^{-4}$ in the SM, in good agreement with data.

Because of the large number of parameters in the mixing matrices, it is not practical to perform a general analysis in the full parameter space. Our purpose is to demonstrate that in SUSY models, the prediction for CP violation and the chiral structure can be considerably different from SM predictions. We will restrict ourselves to some simple cases, and consider *mixing only between second and third generation down type squarks*. This has the advantage that the usual stringent constraints from processes involving the first generation, such as bounds from K^0-$\bar{K}^0$ mixing, neutron electric dipole moment (EDM), and so on can be evaded easily, and hence allow for large CP violation in B decays. In general, B_s-$\bar{B}_s$ mixing would also be different from the SM. Present limits do not impose strong constraints in the parameter space we consider, but may become more restrictive as experimental bounds improve.

Having decoupled the first generation, the 4×4 mixing matrix $(\Gamma_{DL},\Gamma_{DR})$ diagonalizes the squark mass matrix

$$
\tilde{M}^2_{\rm diag} = (\Gamma_{DL},\Gamma_{DR})\begin{pmatrix}\tilde{m}^2_{LL} & \tilde{m}^2_{LR}\\ \tilde{m}^{2\dagger}_{LR} & \tilde{m}^2_{RR}\end{pmatrix}\begin{pmatrix}\Gamma^\dagger_{DL}\\ \Gamma^\dagger_{DR}\end{pmatrix},
\tag{12}
$$

and must satisfy the following equations:

$$
\begin{gathered}
(\Gamma_{DL}\Gamma^\dagger_{DL}+\Gamma_{DR}\Gamma^\dagger_{DR})^{kl}=\delta^{kl},\\
\Gamma^{\dagger ik}_{DL(R)}\Gamma^{kj}_{DL(R)}=\delta^{ij},\quad \Gamma^{\dagger ik}_{DR(L)}\Gamma^{kj}_{DL(R)}=0,
\end{gathered}
\tag{13}
$$

where $i,j=2,3$ and $k,l=2,3,5,6$. We consider some simple cases for illustration. (a) LL or RR mixing: Mixing only in $\tilde{D}_L$ sector (LL) and/or $\tilde{D}_R$ sector (RR). With $\tilde{m}^2_{LR}=0$ while $\tilde{m}^2_{LL},\tilde{m}^2_{RR}$ are general 2×2 Hermitian matrices, one has $\Gamma^\dagger_{DL}=(u^\dagger,0),\Gamma^\dagger_{DR}=(0,v^\dagger)$. The unitary matrices u,v satisfy $u\tilde{m}^2_{LL}u^\dagger=\mathrm{diag}(\tilde{m}^2_{L,2},\tilde{m}^2_{L,3})$, $v\tilde{m}^2_{RR}v^\dagger=\mathrm{diag}(\tilde{m}^2_{R,2},\tilde{m}^2_{R,3})$, and

$$
\begin{aligned}
(\Gamma^\dagger_{DL})^{sk}f(a_k)\Gamma^{kb}_{DL} &= u^{\dagger s2}u^{2b}[f(a_2)-f(a_3)]\\
&= \cos\theta\sin\theta e^{i\sigma}[f(a_2)-f(a_3)],
\end{aligned}
\tag{14}
$$

while $(\Gamma^\dagger_{DL})^{sk}f(a_k)\Gamma^{kb}_{DR}=0$, with similar relations for v. There is one mixing angle θ and one physical phase σ for both u and v. Note that the *phase of v is not constrained* by $B\to X_s\gamma$. To further reduce the parameter space we take θ to be the same for u and v but allow the masses to be different. There are also two extreme cases of interest: LL only, i.e., no RR mixing, or $LL=RR$, i.e., $\tilde{m}^2_{LL}=\tilde{m}^2_{RR}$.

For simplicity, we take advantage of the fact that the mass matrices $\tilde{m}^2_{U,RR}$ and $\tilde{m}^2_{U,LR}$ are independent from $\tilde{m}^2_{RR}$ and $\tilde{m}^2_{LR}$, and assume no LR and RR mixings in the $\tilde{U}$ sector. That is, we take $\tilde{m}^2_{U,RR}$ to be diagonal and $\tilde{m}^2_{U,LR}=0$. However, since we have LL mixing in the $\tilde{D}$ sector, LL mixing in the $\tilde{U}$ sector will follow accordingly because of the $\mathrm{SU}_L(2)$ symmetry of the theory. That is,

$$
\begin{aligned}
\tilde{m}^2_{LL} &= \mathrm{diag}(m_d^2,m_s^2,m_b^2)+M'^2_{\tilde{Q}}-M_Z^2\left(\frac{1}{2}+Q_d s_W^2\right)\cos2\beta,\\
\tilde{m}^2_{U,LL} &= \mathrm{diag}(m_u^2,m_c^2,m_t^2)\\
&\quad + V_{\rm CKM}\left[M'^2_{\tilde{Q}}+M_Z^2\left(\frac{1}{2}-Q_u s_W^2\right)\cos2\beta\right]V^\dagger_{\rm CKM},
\end{aligned}
\tag{15}
$$

where $M_{\tilde{Q}}^{\prime 2}$ is the soft squark mass matrix. In our numerical study, we shall illustrate with $m^2_{\chi^+_{1,2}}=200{,}400$ GeV and $\tan\beta=2$. Up type squark masses will depend on down type squark masses and mixing angle. We apply a 100 GeV lower bound on up type squark masses, which further constrains the down squark mixing angle.

(b) LR mixing only. We consider an interesting case with $\tilde{m}^2_{LL}=\tilde{m}^2_{RR}=\mathrm{diag}(\tilde{m}^2,\tilde{m}^2)$ and neglect down-type quark masses, while $\tilde{m}^2_{LR}$ is a general 2×2 matrix. In this case, because $M_{\tilde{Q}}^{\prime 2}$ is proportional to the unit matrix, $\tilde{m}^2_{U,LL}$ is diagonal, as can be seen from Eq. (15). One then sees from Eq. (4) that the chargino contributions in Eqs. (7) and (8) are proportional to $V_{kb}V^*_{ks}$, and hence are much smaller than the gluino contributions.

Diagonalization $\tilde{M}^2_{\mathrm{diag}}=\mathrm{diag}(\tilde{m}^2_2,\tilde{m}^2_3,\tilde{m}^2_5,\tilde{m}^2_6)=\mathrm{diag}(\tilde{m}^2+\Delta\tilde{m}^2_2,\tilde{m}^2+\Delta\tilde{m}^2_3,\tilde{m}^2-\Delta\tilde{m}^2_2,\tilde{m}^2-\Delta\tilde{m}^2_3)$ is achieved via $\Gamma^\dagger_L=(u^\dagger,u^\dagger)/\sqrt{2},\Gamma^\dagger_R=(v^\dagger,-v^\dagger)/\sqrt{2}$, where u and v are unitary matrices satisfying $u\tilde{m}^2_{LR}v^\dagger=\mathrm{diag}(\Delta\tilde{m}^2_1,\Delta\tilde{m}^2_2)$. One then finds

$$
\begin{aligned}
2(\Gamma^\dagger_{DL})^{sk}f(a_k)\Gamma^{kb}_{DL}&=u^{\dagger s2}u^{2b}[f(a_2)-f(a_3)\\
&\qquad+f(a_5)-f(a_6)],\\
2(\Gamma^\dagger_{DR})^{sk}f(a_k)\Gamma^{kb}_{DR}&=v^{\dagger s2}v^{2b}[f(a_2)-f(a_3)\\
&\qquad+f(a_5)-f(a_6)],\\
2(\Gamma^\dagger_{DL})^{sk}f(a_k)\Gamma^{kb}_{DR}&=[u^{\dagger si}f(a_i)v^{ib}-u^{\dagger si}f(a_{i+3})v^{ib}],\\
2(\Gamma^\dagger_{DR})^{sk}f(a_k)\Gamma^{kb}_{DL}&=[v^{\dagger si}f(a_i)u^{ib}-v^{\dagger si}f(a_{i+3})u^{ib}],
\end{aligned}
\tag{16}
$$

where i is summed over 2 and 3, and

$$
u=\begin{pmatrix} ce^{i\tau} & se^{i\sigma}\\ -se^{i\tau} & ce^{i\sigma}\end{pmatrix},\quad v=\begin{pmatrix} c'e^{i\tau'} & s'e^{i\sigma'}\\ -s'e^{i\tau'} & c'e^{i\sigma'}\end{pmatrix}. \tag{17}
$$

We further simplify by assuming $\tilde{m}^2_{LR}$ to be Hermitian, hence $u=v$. Since a 2×2 Hermitian matrix has four independent real parameters, two would lead to eigenvalues $\Delta\tilde{m}^2_2$ and $\Delta\tilde{m}^2_3$, and again we have just one mixing angle and one phase.

Although these cases are rather simplified, they can still lead to phenomenological consequences that are very different from the SM. In the following sections, we proceed to study (i) direct CP violating partial rate asymmetry A_{CP}, (ii) mixing induced asymmetry A_{mix}, and (iii) final state Λ polarization α_Λ in $\Lambda_b\to\Lambda\gamma$, that follow from our model.

III. DIRECT *CP* VIOLATION

The CP violating partial rate asymmetry A_{CP} in $b\to s\gamma$ decay is defined as

$$
A_{CP}=\frac{\Gamma(b\to s\gamma)-\overline{\Gamma}(\bar{b}\to\bar{s}\gamma)}{\Gamma(b\to s\gamma)+\overline{\Gamma}(\bar{b}\to\bar{s}\gamma)}=\frac{|c_7|^2+|c'_7|^2-|\bar{c}_7|^2-|\bar{c}'_7|^2}{|c_7|^2+|c'_7|^2+|\bar{c}_7|^2+|\bar{c}'_7|^2}, \tag{18}
$$

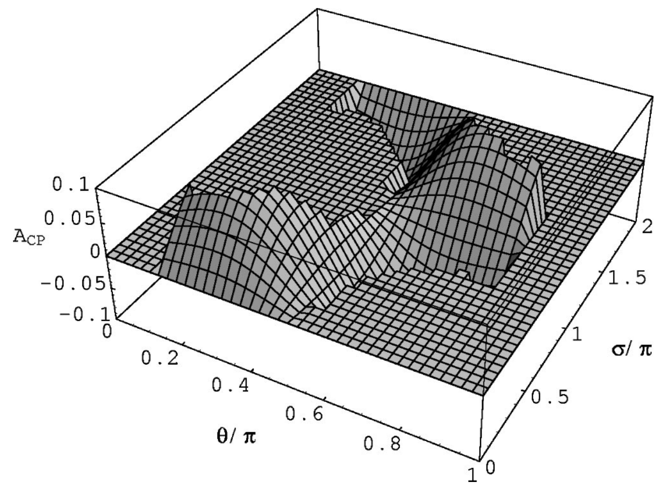

FIG. 1. A_{CP} vs mixing angle θ and phase σ for LL and RR but no LR mixings, with the mass values $m_{\tilde{g}}=200$ GeV and $(\tilde{m}_{L,2},\tilde{m}_{L,3},\tilde{m}_{R,2},\tilde{m}_{R,3})=(100,250,100,150)$ GeV. The flat surface corresponding to $A_{CP}=0$ is the parameter space forbidden by the $B\to X_s\gamma$ constraint, and the cut on θ is due to lower bound of the top mass squark.

where $\bar{c}^{(\prime)}_7$ are coefficients for $\bar{b}$ decay. In the SM, $A_{CP}\sim0.5\%$ [6] is very small, it is therefore a good place to look for deviations from the SM.

To have nonzero A_{CP}, apart from CP violating phases, one also needs absorptive parts. In the model under consideration, the absorptive parts come only from the the SM contribution with u and c quarks in the loop. Because of the left-handed nature of charged currents in the SM, the absorptive parts in $c'_{2,7,8}$ are suppressed by a factor of m_s/m_b which is small and therefore can be neglected. One finds [4]

$$
A_{CP}(\delta)=\frac{1}{|c_7|^2+|c'_7|^2}\{a_{27}(\delta)\mathrm{Im}[c_2c^*_7]+a_{87}(\delta)\mathrm{Im}[c_8c^*_7]+a_{28}(\delta)\mathrm{Im}[c_2c^*_8]\}, \tag{19}
$$

where the parameters $a_{ij}(\delta)$ depend on δ which defines the photon energy cut $E_\gamma>(1-\delta)E^{\max}_\gamma$, as mentioned earlier. From Ref. [4], we find that $a_{87}\sim-9.5\%$ is much larger than $a_{27}\sim1.06\%$ and $a_{28}\sim0.16\%$. Hence large A_{CP} is likely to occur when c_8 is sizable. We have carried out detailed studies and find that there is a large parameter space where A_{CP} can be substantially larger than the SM prediction. We give some special cases from (a) and (b) in Figs. 1 and 2.

Figure 1 shows A_{CP} vs θ and σ for the case with both LL and RR mixings but no LR mixing. We take the mass eigenvalues $m_{\tilde{g}}=200$ GeV, which we will use in all cases, and $(\tilde{m}_{L,2},\tilde{m}_{L,3},\tilde{m}_{R,2},\tilde{m}_{R,3})=(100,250,100,150)$ GeV. The asymmetry A_{CP} can reach 10%. If we only consider the gluino contribution, the allowed region is much reduced and A_{CP} can only reach a few percent. The chargino contribution is important in the sense that it can partially cancel against the gluino contribution, and the allowed region in the parameter space is enlarged. Naively one might think that the gluino contribution dominates over the chargino contribution

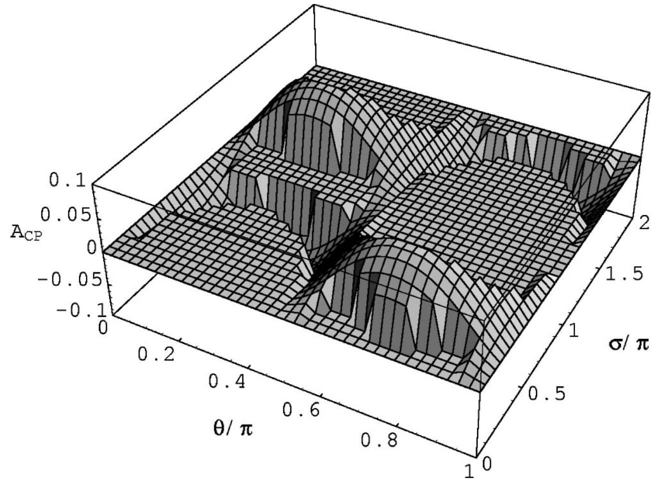

FIG. 2. A_{CP} for LR mixing with $u=v$ and $M_{\tilde{g}}=200$ GeV, $\tilde{m}=300$ GeV, $\Delta\tilde{m}_2^2=(20\text{ GeV})^2$, and $\Delta\tilde{m}_3^2=(30\text{ GeV})^2$.

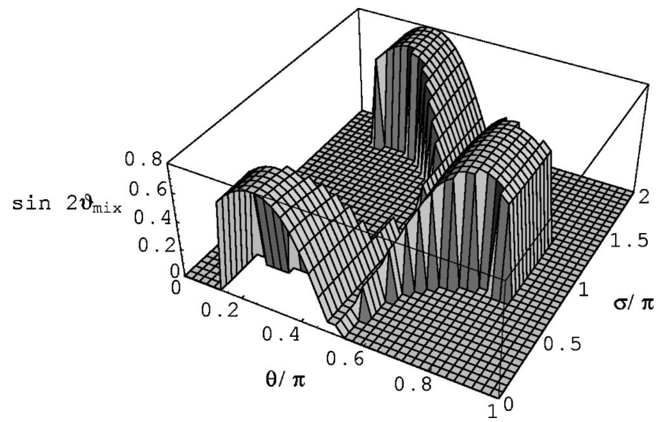

FIG. 3. $\sin(2\vartheta_{\text{mix}})$ in the LL and RR mixing cases with the same parameter space as in Fig. 1.

because α_s/α_w is large. However, this factor is only about 3 and is easily overcome by other enhancement factors in the chargino sector. In particular, the function f_3 in the chargino contribution is larger than f_1 in the gluino contribution. It turns out that both contributions are about the same order of magnitude and partially cancel against each other for the parameter space considered. Thus, $Br(B\to X_s\gamma)$ close to the SM result is easier to achieve, hence enlarging the allowed parameter space. For large $\tan\beta$, the parameter space is more restrictive because the chargino contribution tends to dominate over the gluino contribution. We have checked that the neutralino contribution is about one order of magnitude smaller compared to those from gluino or chargino interactions, thus does not make much impact.

Larger asymmetries exceeding 10% are attainable if one allows for only LL mixing, since the presence of RR mixing generates nonzero values for c_7', which contributes to the branching ratio but not to A_{CP}. For example, for $(\tilde{m}_{L,2},\tilde{m}_{L,3})=(100,300)$ GeV, A_{CP} can reach 15%.

It is of interest to note that, in the case with LR mixing only, the SUSY contribution has a large enhancement factor $m_{\tilde{g}}/m_b$. To satisfy the bound from observed branching ratio, the squark masses need to be nearly degenerate if the mixing angles are not small. Furthermore, because the chargino contribution is small as mentioned before, in this case it does not cancel against the gluino contribution. In Fig. 2, we show A_{CP} for LR mixing with $u=v$, and with $\tilde{m}=300$ GeV, $\Delta\tilde{m}_2^2=(20\text{ GeV})^2$, and $\Delta\tilde{m}_3^2=(30\text{ GeV})^2$. We see that A_{CP} can reach 10%. Thus, even if down squark masses are large and nearly degenerate (i.e., near universal squark masses), just some slight LR mixing could cause sizable A_{CP}. The B factories which would turn on soon will provide useful information about direct CP violation and can test the different models discussed here.

IV. MIXING INDUCED CP VIOLATION

For radiative $B_{d(s)}\to M_{d(s)}\gamma$ decays, where $M_{d(s)}$ is a $S=-1(0)$ CP eigenstate with eigenvalue $\xi=\pm$, it is possible to observe mixing induced CP violation [7]. Let $\Gamma(t)$ and $\overline{\Gamma}(t)$ be the time-dependent rate for $B^0\to M^0\gamma$ and $\overline{B}^0\to M^0\gamma$, respectively. One has

$$R_{CP}=\frac{\Gamma(t)-\overline{\Gamma}(t)}{\Gamma(t)+\overline{\Gamma}(t)}=-A_{\text{mix}}\sin(\Delta m t),$$

$$A_{\text{mix}}=\frac{2|c_7c_7'|}{|c_7|^2+|c_7'|^2}\,\xi\sin[\phi_B-\phi-\phi'], \tag{20}$$

where Δm and ϕ_B are the mass difference and phase in $B_{d(s)}$-$\overline{B}_{d(s)}$ mixing amplitude, and $\phi^{(\prime)}$ is the weak phase of $c_7^{(\prime)}$. In the B_d case ϕ_B is the same as in the SM since we do not consider squark mixings involving first generation.

In the SM, $c_7'/c_7=m_s/m_b$ hence $A_{\text{mix}}^{\text{SM}}$ is rather small. To obtain large A_{mix}, both c_7 and c_7' have to be simultaneously sizable. This can be easily achieved in SUSY models. In Figs. 3 and 4, we show some representative results using the same parameters as in Figs. 1 and 2. We find that in these cases, $\sin 2\vartheta_{\text{mix}}\equiv 2|c_7c_7'|/(|c_7|^2+|c_7'|^2)$ can reach 80, 90 %, respectively. It is interesting to note that large mixing induced CP violation (A_{mix}) does not necessarily imply large direct CP violation (A_{CP}), and vice versa. In Fig. 5 we show

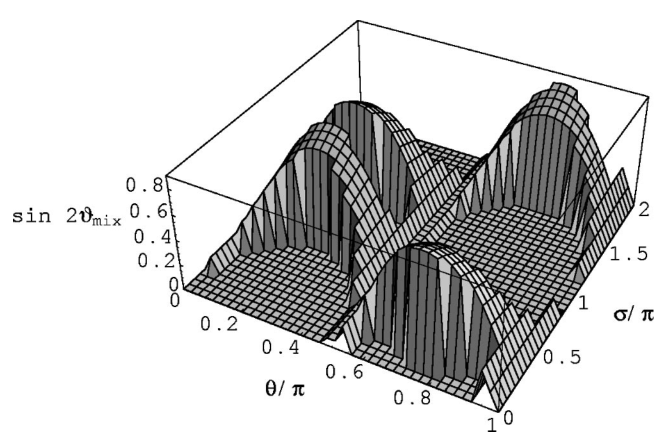

FIG. 4. $\sin(2\vartheta_{\text{mix}})$ in the LR mixing case with the same parameter space as in Fig. 2.

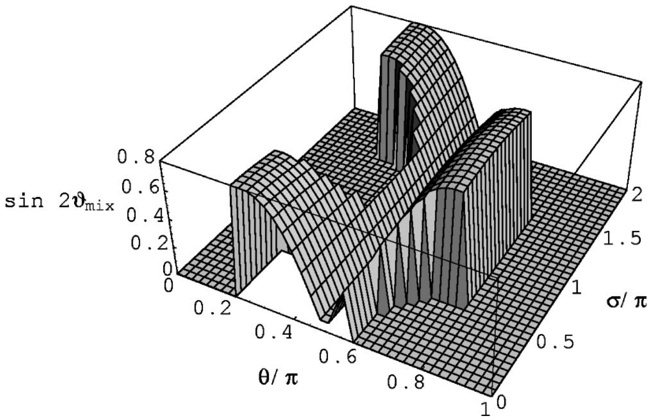

FIG. 5. $\sin(2\theta_{\rm mix})$ in the $LL=RR$ mixing case with $M_{\tilde{g}}=200$ GeV, $\tilde{m}_{L,2}=\tilde{m}_{R,2}=100$ GeV, and $\tilde{m}_{L,3}=\tilde{m}_{R,3}=150$ GeV.

the results in the $LL=RR$ mixing case with $\tilde{m}_{L,2}=\tilde{m}_{R,2}=100$ GeV and $\tilde{m}_{L,3}=\tilde{m}_{R,3}=150$ GeV, where $\sin 2\vartheta_{\rm mix}$ can reach 80%. We do not have large direct A_{CP} in this case, but one still can have large mixing induced CP violation. We note that the allowed region is rather large. However, the constraint from $Br(B\to X_s\gamma)$ does not favor large mass splitting in the $LL=RR$ case. For example, it does not allow the choice $\tilde{m}_{L,2}=\tilde{m}_{R,2}=100$ GeV and $\tilde{m}_{L,3}=\tilde{m}_{R,3}=250$ GeV.

To have large $A_{\rm mix}$, the phase combination $P=\sin(\phi_B-\phi-\phi')$ also needs to be large. This is easily achieved for the LL and RR mixing cases because the phase ϕ' is not constrained by the observed branching ratio. In the case with LR mixing only, because of the assumption of $u=v$, the phases are related and therefore are constrained from the interference with SM contribution in the branching ratio. One needs to make sure that the factor P is also large. We have checked in detail that this indeed happens in the cases considered. One can also relax the requirement to allow c_7' to have an independent phase. In this case the factor P can always be made large. Note that, even if ϕ and ϕ' vanish (no CP violation from soft squark masses), nonvanishing ϕ_B from SM contribution to B-$\overline{B}$ mixing can still lead to observable $A_{\rm mix}$, so long as c_7 and c_7' are comparable.

We have only assumed that the state M be a CP eigenstate, which can be $K_S\pi^0$ from K^{*0} or $K_{1,2}^{*0}$ for B_d decays, or ϕ for B_s decay. The expression for $A_{\rm mix}$ is process independent. However, because of the relatively long lifetime of K_S, and the fact that having γ and π^0 in the final state do not provide a good determination of the decay vertex position, $A_{\rm mix}$ for $B_d\to K^{*0}\gamma$ probably cannot be measured with sufficient accuracy. Perhaps the $B_d\to K_{1,2}^{*}\gamma$ situation would be better, but *these modes have to be measured first*. The situation for $B_s\to\phi\gamma$ is definitely better, but it can only be carried out at hadronic facilities such as the Tevatron or LHC, and only after B_s mixing is measured.

V. Λ POLARIZATION IN BEAUTY BARYON DECAY

The chiral structure can be easily studied in $\Lambda_b\to\Lambda\gamma$. The decay amplitude is given by

$$A(\Lambda_b\to\Lambda\gamma)=-\frac{G_F}{\sqrt{2}}\frac{e}{8\pi^2}V_{tb}V_{ts}^{*}C\overline{\Lambda}[c_7(1+\gamma_5)+c_7'(1-\gamma_5)]\sigma_{\mu\nu}F^{\mu\nu}\Lambda_b, \tag{21}$$

where C is a form factor which can in principle be calculated in heavy quark effective theory. The resulting branching ratio is of order 10^{-5} and should be measurable at future hadronic B factories. The chiral structure can be studied by measuring the polarization of Λ, via the angular distribution [8]

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}=\frac{1}{2}(1+\alpha_\Lambda\cos\theta),\quad \alpha_\Lambda=\frac{|c_7|^2-|c_7'|^2}{|c_7|^2+|c_7'|^2}, \tag{22}$$

where θ is the angle between the direction of the momentum of Λ in the rest frame of Λ_b and the direction of the Λ polarization in its rest frame. We emphasize that the parameter α_Λ does not depend on the hadronic parameter C, which makes it a good quantity for studying the chiral structure without any uncertainties from hadronic matrix elements. In the SM one has $\alpha_\Lambda=1$. Deviation from this value for α_Λ would be an indication of physics beyond the SM. Since Eq. (22) does not depend on the hadronic matrix element of the specific process, it can be applied to any other radiative beauty baryon decays.

It is clear that if c_7 and c_7' are of the same order, one would have substantial deviation from SM prediction. But unlike the case for $A_{\rm mix}$, large α_Λ does *not* require a large phase combination factor P. In fact, α_Λ is a measure of the chiral structure independent of CP violation. One can of course still study direct CP violation rate asymmetries. If the new physics contribution comes only from LL mixing, one would not have large deviation from the SM prediction. In Figs. 6, 7, and 8, we show the deviation from SM prediction $1-\alpha_\Lambda$ for the cases in Figs. 1, 2, and 5. We see that α_Λ can indeed be very different from the SM. Note that in these cases α_Λ has the same sign as in SM. For LL-RR mixing cases, this has to do with the compensating effect between chargino and gluino loops, while for LR mixing case it has to

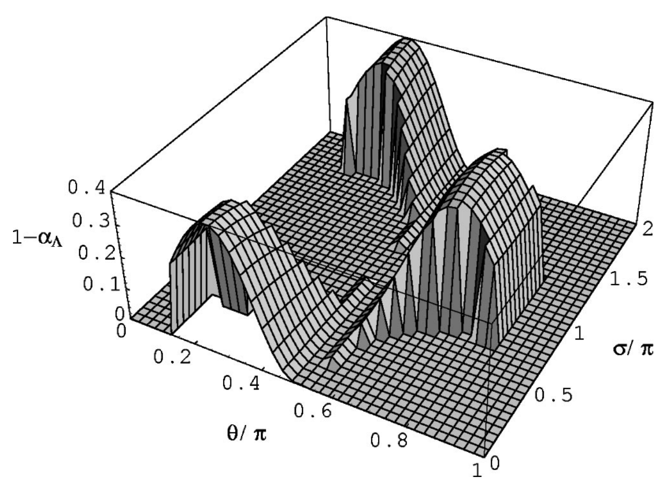

FIG. 6. $1-\alpha_\Lambda$ in the LL and RR mixing cases with the same parameter space as in Fig. 1.

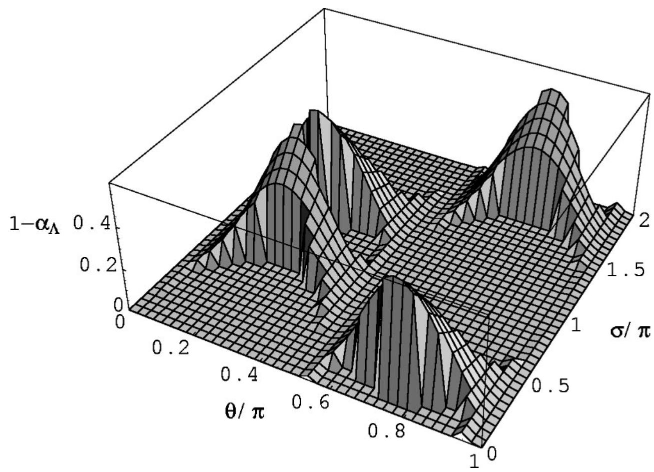

FIG. 7. $1-\alpha_\Lambda$ in the LR mixing case with the same parameter space as in Fig. 2.

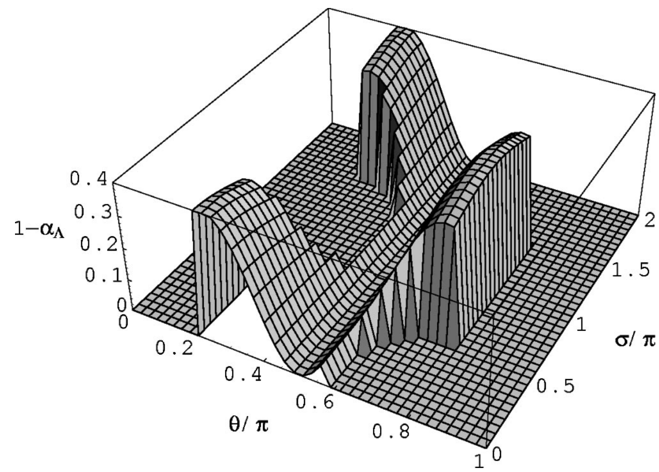

FIG. 8. $1-\alpha_\Lambda$ in the $LL=RR$ mixing case with the same parameter space as in Fig. 5.

do with the heaviness of squarks. We have only explored a small portion of the parameter space as a consequence of our many simplifications. Although we have not been able to identify parameter space where α_Λ flips sign, it does not mean that this is impossible.

VI. DISCUSSION AND CONCLUSIONS

In this paper we have shown that SUSY models with nonuniversal squark masses can give rise to rich phenomena in $b \to s\gamma$ decays. Indeed, such considerations received attention with the notion that $|c_8| \sim 2$ could help resolve [12,13] the long-standing low charm counting and semileptonic branching ratio problems, in the form of a rather enhanced $b \to sg \sim 5$–$10\,\%$. It has been shown that this is possible in SUSY models [13], but $b \to s\gamma$ provides a severe constraint. However, to have a large SUSY effect in $b \to s\gamma$ decay, $b \to sg$ need not be greatly enhanced. We have incorporated into the model the consideration of new CP phases, which naturally arise. Although we do not claim to have explored the full parameter space, we find that SUSY with nonuniversal squark masses could indeed lead to dramatic effects. The severe constraint from $Br(B \to X_s\gamma)$ does not exclude squark mixings. We find a cancellation effect between gluino and chargino contributions, which gives rise to rather large allowed regions for the mixing angle θ and phase σ, leading to interesting consequences for CP violation. The features exhibited in the present analysis is a common feature in SUSY models with low-energy flavor and CP violating squark mixings. A model with mixing only between second and third generation down type squarks can easily evade known low-energy constraints but give dramatic signals in $b \to s$ transitions.

Our purpose has been to illustrate such efficacy and hopefully motivate our experimental colleagues to perform detailed studies. We find that direct CP violating rate asymmetries can be as large as 10%, comparable to general multi-Higgs doublet models [14]. Purely LL mixing is favored in this case. Even more interesting would be the observation of mixing induced CP violation. Here, purely LL or RR mixing is insufficient, but LL and RR mixing or LR mixing models could lead to rather sizable effects. The observation of mixing induced CP violation immediately demonstrates that $b \to s\gamma$ decay has two chiralities. Independent of CP violation, however, the chirality structure can be tested by studying Λ polarization in $\Lambda_b \to \Lambda\gamma$ decay. The parameter α_Λ can deviate from the SM value of 1. The nonobservance of CP violation in B meson decays does not preclude surprises in the α_Λ measurement.

It is clear that $b \to s\gamma$ transitions provide good tests for new physics.

ACKNOWLEDGMENTS

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