

Heavy-to-light form factors in the final hadron large energy limit of QCD

J. Charles,* A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal

Laboratoire de Physique Théorique, Université de Paris-Sud, Bâtiment 210, F-91405 Orsay Cedex, France

(Received 15 December 1998; published 17 May 1999)

We argue that the large energy effective theory (LEET), originally proposed by Dugan and Grinstein, is applicable to exclusive semileptonic, radiative, and rare heavy-to-light transitions in the region where the energy release E is large compared to the strong interaction scale and to the mass of the final hadron, i.e., for q^2 not close to the zero-recoil point. We derive the effective Lagrangian from the QCD one, and show that in the limit of heavy mass M for the initial hadron and large energy E for the final one, the heavy and light quark fields behave as two-component spinors. Neglecting QCD short-distance corrections, this implies that there are only three form factors describing all the pseudoscalar to pseudoscalar or vector weak current matrix elements. We argue that the dependence of these form factors with respect to M and E should be factorizable, the M dependence (\sqrt{M}) being derived from the usual heavy quark expansion while the E dependence is controlled by the behavior of the light-cone distribution amplitude near the end point $u \sim 1$. The usual expectation of the $\sim(1-u)$ behavior leads to a $1/E^2$ scaling law, that is a dipole form in q^2 . We also show explicitly that in the appropriate limit the light-cone sum rule method satisfies our general relations as well as the scaling laws in M and E of the form factors, and obtain very compact and simple expressions for the latter. Finally we note that this formalism gives theoretical support to the quark model-inspired methods existing in the literature.

[S0556-2821(99)02309-7]

PACS number(s): 13.20.He, 12.39.Hg

I. INTRODUCTION

Nowadays, $|V_{cb}|$ is the third most accurately measured Cabibbo-Kobayashi-Maskawa (CKM) matrix element, and is quoted by the Particle Data Group [1] with less than 5% relative uncertainty. The Isgur-Wise symmetry [2], and the heavy quark effective theory (HQET) description of heavy-to-heavy semileptonic decays have permitted such a great success in heavy quark physics. Unfortunately, the HQET constraints on heavy-to-light decays are quite weak in their original form, and still do not allow a clean extraction of $|V_{ub}|$ from the present and future experimental data. The latter CKM coupling has currently a relative uncertainty of order 25%, depending on which model is used to evaluate the hadronic matrix elements [1]. It is thus very important to make theoretical progress in this field.

The peculiar feature of exclusive heavy-to-light transitions, the prototype of which is $B \rightarrow \pi l \nu_l$, is the large energy E given to the daughter by the parent hadron in almost the whole physical phase space except the vicinity of the zero-recoil point:

$$E = \frac{m_B}{2} \left[1 - \frac{q^2}{m_B^2} + \frac{m_\pi^2}{m_B^2} \right]. \quad (1)$$

As we shall see, one may assume that such transitions are dominated by soft gluon exchange, i.e., the Feynman mechanism, which is not power suppressed with respect to the hard contribution,¹ and does not suffer the α_s suppression charac-

teristic of the latter [3,4]; this has to be contrasted to the case of the pion electromagnetic form factor at very large $Q^2 = -q^2$. Then in the heavy-to-light case, the final active quark should carry most of the momentum of the light hadron, and the fast degrees of freedom become essentially classical. The large energy effective theory (LEET), originally introduced by Dugan and Grinstein [5], should be the correct tool to study such transitions: it could provide an operator product expansion the small parameter of which is $1/E$. As for the initial heavy quark, the assumption of the soft contribution dominance leads to an expansion in powers of the inverse heavy mass M , based on HQET. Our first result is that to leading order in $1/M$, $1/E$ and neglecting short-distance QCD corrections, all the weak current $P \rightarrow P(V)$ matrix elements can be expressed in terms of only three universal form factors. This implies relations between the usual semileptonic and penguin form factors which resemble the well-known Isgur-Wise relations in heavy-to-heavy transitions.

Then an interesting question is what the dependence of these form factors with respect to the large mass M and the large energy E is. From the usual heavy mass expansion of the initial hadron state, we obtain a factorization formula $\sim \sqrt{M} z(E)$. The asymptotic expansion of $z(E)$ is controlled by the behavior of the light-cone distribution amplitude of the final hadron near the end point $u \sim 1$. The usual assumption of the $\sim(1-u)$ behavior leads to a $z(E) \sim 1/E^2$ scaling law, which implies a $\sim M^{-3/2}/(1-q^2/M^2)^2$ dipole form for the three universal form factors.²

A strong support in favor of the HQET-LEET formalism for heavy-to-light form factors is given by the light-cone sum rules: indeed using the work of several groups [3,6–14] we

*Email address: Jerome.Charles@th.u-psud.fr

¹We neglect in the whole paper Sudakov effects, which are expected not to induce a large suppression at the physical m_B scale.

²The q^2 dependence of the form factors in the standard parametrization is discussed in Sec. V.

show explicitly that the latter method automatically satisfies the above relations and scaling laws. In addition we show in another paper [15] that the quark models based on the Bakamjian-Thomas formalism, which were shown to be covariant and to satisfy the Isgur-Wise relations in the heavy-to-heavy case [16], also become covariant in the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit. In agreement with our general results, these models do predict that there are only three independent form factors in heavy-to-light transitions, and that they scale as $\sqrt{M}z(E)$.

The paper is organized as follows. In Sec. II, we argue on the validity of LEET for the description of the Feynman mechanism and derive the correct form of the Lagrangian, as there is a subtlety concerning Dirac matrices which was missed in the literature. In Sec. III we show that the use of the HQET and LEET effective quark fields leads to express all the heavy-to-light ground state form factors in terms of only three universal functions. The asymptotic M and E dependence of these functions are discussed, and a $\sim \sqrt{M}/E^2$ form is shown to be the most plausible. Then in Sec. IV we derive explicitly the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit of the light-cone sum rule formulas for the weak current matrix elements and give the universal form factors in terms of \sqrt{M}/E^2 times integrals depending on the light-cone distribution amplitudes and the sum rule parameters, in a very simple and compact way. Finally in Sec. V we discuss the relation between the matrix elements parametrized in the standard way and the HQET-LEET universal form factors, and compare our results with previous approaches that were based on the constituent quark model.

II. THE LEET EFFECTIVE THEORY

LEET was introduced by Dugan and Grinstein [5] to study factorization of non-leptonic matrix elements in decays such as $B \rightarrow D^{(*)}\pi$, $D^{(*)}\rho$, \dots , where the light meson is emitted by the W boson. In this case, both quarks constituting the light energetic meson are fast. However, Aglietti *et al.* [17] have recently argued that such a situation could not be described by LEET, as the relative transverse momentum of the fast quarks may be hard. They proposed to use instead the LEET effective theory, a variant of LEET which takes into account hard transverse degrees of freedom. This seems to be similar to the description of the heavy quark systems: HQET is the appropriate theory for the heavy-light hadrons, while nonrelativistic QCD (NRQCD) should be used for the quarkonia. Conversely, Aglietti *et al.* found that LEET could be used in semi-inclusive nonleptonic decays such as $B \rightarrow DX_u$, where factorization should hold at the leading order [17].

Note also that the quark propagator in the LEET limit has gained further interest with the proposal of the Rome group [18] to use it to extract from the lattice the shape function for semileptonic inclusive B -decays, the structure functions in deep inelastic scattering, and the light-cone distribution amplitudes for exclusive hard processes.

However, no effort seems to have been devoted up to now to investigating how LEET could be used in processes where

only one quark is fast and the other partons are soft,³ similarly to the HQET description of heavy-to-heavy transitions. In this work, we will study amplitudes where an energetic hadron is connected to the decaying heavy hadron by a current operator, i.e., exclusive semileptonic, radiative, and rare heavy-to-light decays, such as $B \rightarrow \pi l \nu_l$, $B \rightarrow K^* \gamma$, and $B \rightarrow Kl^+l^-$. In the large recoiling region, i.e., for q^2 sufficiently far away from the zero-recoil point (of course the physical q^2 in radiative decays is exactly zero), the final active quark carries a large energy (in the rest frame of the parent hadron) and interacts mostly with soft degrees of freedom—the spectator quark and the gluons. Thus one may expect that the trajectory of the fast quark suffers only small fluctuations around the classical, almost lightlike, world line of the daughter hadron. Actually there are also hard gluon exchange contributions, through which the large momentum is shared by both the active and spectator quarks. However, the perturbative calculation of these diagrams typically leads to very small values compared to the dominant overlap diagram, due to the hard α_s suppression [3,4]. We will assume that these contributions are negligible, and we also neglect other radiative corrections for simplicity.

Let us now define the large energy effective theory in a more systematical way. From now on, we will refer to high-energy exclusive heavy-to-light decays, and consider only the ground state mesons. The appropriate kinematical variables for such decays are the following.

The four-momentum p , mass M , and four-velocity v of the heavy hadron

$$p \equiv Mv \quad (2)$$

The four-vector n and the scalar E defined by

$$p' \equiv En, \quad v \cdot n \equiv 1, \quad (3)$$

where p' is the four-momentum of the light hadron, $p'^2 = m'^2$. Thus

$$E = v \cdot p' \quad (4)$$

is just the energy of the light hadron in the rest frame of the heavy hadron.

In the following we will consider the limit of heavy mass for the initial hadron and large energy for the final one:

$$(\Lambda_{\text{QCD}}, m') \ll (M, E) \quad \text{with } v \text{ and } n \text{ fixed.} \quad (5)$$

³A potential problem of the LEET effective theory is the so-called “instability” phenomenon [17,19]: indeed Aglietti argues in Ref. [19] that the interaction of a LEET quark with a *massless* soft quark—that constitutes precisely the case that we are interested in—generates divergences in the forward direction. However, the mass of a quark in a bound state should rather be viewed as an *off shellness* of order Λ_{QCD} , and it can easily be checked that this instability problem in Aglietti’s argument does not occur for a non-vanishing mass of the soft quark [20].

Note that we do not assume anything for the ratios E/M and Λ_{QCD}/m' . As $n^2 = m'^2/E^2 \rightarrow 0$, n becomes lightlike in the above limit. In the rest frame of v , with the z direction along p' , one has simply

$$v = (1, 0, 0, 0), \quad n \simeq (1, 0, 0, 1). \quad (6)$$

In a general frame one has the normalization conditions

$$v^2 = 1, \quad v \cdot n = 1, \quad n^2 \simeq 0. \quad (7)$$

In a decay such as $B \rightarrow \pi$, not too close from $q^2 = q_{\text{max}}^2 = (m_B - m_\pi)^2$, the final active quark gets a very large energy and should form with the spectator a hadron of finite mass. Thus, neglecting as said above hard spectator effects, the momentum r of the active quark is close to the momentum of the hadron:

$$r = En + k, \quad \text{with } k \sim \Lambda_{\text{QCD}} \ll E \text{ as the residual momentum.} \quad (8)$$

Our goal is now to derive the LEET Lagrangian from the QCD one in the limit (8). We would like to separate the large components of the quark field from the small ones which, corresponding to the negative energy solutions, should be suppressed by $1/E$. To this aim we follow closely the simple demonstration given in Ref. [21] for the derivation of the HQET Lagrangian. We define the projectors

$$P_+ = \frac{\not{n}\not{\psi}}{2}, \quad P_- = \frac{\not{\psi}\not{n}}{2} \quad (9)$$

which indeed verify from Eq. (7)

$$P_\pm^2 = P_\pm, \quad P_\pm P_\mp = 0, \quad P_+ + P_- = \frac{\{\not{n}, \not{\psi}\}}{2} = 1. \quad (10)$$

From the full, four-component, quark field $q(x)$ one may define two two-component projected fields $q_\pm(x)$ by

$$q_\pm(x) \equiv e^{iEn \cdot x} P_\pm q(x). \quad (11)$$

Thus from the projector properties one has

$$q(x) = e^{-iEn \cdot x} [q_+(x) + q_-(x)] \quad (12)$$

with

$$P_\pm q_\pm = q_\pm, \quad P_\mp q_\pm = 0 \quad (13)$$

and

$$\bar{q}_\pm P_\mp = \bar{q}_\pm, \quad \bar{q}_\pm P_\pm = 0. \quad (14)$$

It follows immediately that the QCD Lagrangian for the quark q , $\mathcal{L}_{\text{QCD}} = \bar{q}(i\not{D} - m_q)q$ can be expressed in terms of the q_\pm fields:

$$\begin{aligned} \mathcal{L}_{\text{QCD}} = & \bar{q}_+ \not{\psi} in \cdot D q_+ + \bar{q}_+ (i\not{D} - m_q) q_- + \bar{q}_- (i\not{D} - m_q) q_+ \\ & + \bar{q}_- \not{\psi} (2E + 2iv \cdot D - in \cdot D) q_-. \end{aligned} \quad (15)$$

The equation of motion $(i\not{D} - m_q)q(x) = 0$, projected by P_\pm , reads

$$\not{\psi} in \cdot D q_+ + [i\not{D} - m_q - \not{\psi}(2iv \cdot D - in \cdot D)]q_- = 0, \quad (16)$$

$$(i\not{D} - m_q - \not{\psi} in \cdot D)q_+ + \not{\psi}(2E + 2iv \cdot D - in \cdot D)q_- = 0. \quad (17)$$

The latter equation can formally be solved to express q_- in terms of q_+ :

$$\begin{aligned} q_-(x) = & (2E + 2iv \cdot D - in \cdot D + i\epsilon)^{-1} \\ & \times \not{\psi}(i\not{D} - m_q - \not{\psi} in \cdot D)q_+(x). \end{aligned} \quad (18)$$

Thus the field $q_-(x)$, corresponding to the negative energy solutions⁴ is of order $1/E$ with respect to $q_+(x)$. Physically this means that the pair creation is suppressed in the effective theory.

To summarize we have obtained the result:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{LEET}} + \mathcal{O}(1/E) \quad (19)$$

with

$$\mathcal{L}_{\text{LEET}} = \bar{q}_n \not{\psi} in \cdot D q_n, \quad (20)$$

where we have defined $q_n(x) \equiv q_+(x)$ to recall the usual notation $h_v(x)$ for the effective field of HQET. In addition the two-component field $q_n(x)$ verifies the projection condition

$$q_n(x) = \frac{\not{n}\not{\psi}}{2} q_n(x) \quad (21)$$

which implies in particular $\not{n}q_n = 0$. The LEET equation of motion is just

$$n \cdot D q_n(x) = 0. \quad (22)$$

Note that in the literature [5, 17, 19], the LEET Lagrangian was quoted without the $\not{\psi}$ factor: indeed these authors have inferred the Lagrangian from the large energy limit of the QCD quark propagator. However, the limit of the propagator is not sufficient to define the effective field from the QCD field. We will see that the $\not{\psi}$ factor and the projection condition (21) have important consequences on the symmetries of the effective theory and on the constraints on the form factors.

Note also that the assumption of a massless quark, or even a light quark (compared to Λ_{QCD}), is not needed to write Eq. (19). The mass term $\bar{q}_n m_q q_n$ just vanishes because of the

⁴Since $P_\pm = \frac{1}{2}(1 \pm \alpha_z)$ in the rest frame of v . In this frame, the projectors P_\pm coincide with the $\pm \gamma_3 \gamma_0/2$ projectors that are useful for the light-cone formalism [22]. Another possibility is to define $P_+ = \gamma_* \gamma_\bullet/2$ and $P_- = \gamma_\bullet \gamma_*/2$ with $\gamma_* = \not{n} \cdot x$ and $\gamma_\bullet = \not{x}$: this makes apparent the resemblance with the light-cone formalism of Ref. [23] and leads to the Lagrangian $\mathcal{L}_{\text{LEET}} = \bar{q}_n [\not{x}/n \cdot x] in \cdot D q_n$. The latter exhibits the invariance under the collinear conformal group as defined in Ref. [23].

projector (21). As far as masses are concerned, we only need $m_q \ll E$ for the quark, and $m' \ll E$ for the hadron, in order for n to become a lightlike vector. Of course in phenomenological applications we will use LEET mainly for the light u , d , and s quarks. However, it is worth noting that if the b quark were much heavier, say ~ 20 GeV, the heavy c quark in the $B \rightarrow D$ transition around $q^2=0$ would have to be described by LEET rather than by HQET, as the latter theory fails for $w = E/m'$ too large [21].

Let us now discuss the global symmetries of LEET: the simplest one is the flavor symmetry, as there is no mass term in the Lagrangian, meaning that mass effects should be small for energetic particles. It is also immediately apparent that the LEET Lagrangian (20) as well as the projection condition (21) are invariant under the chiral transformation

$$q_n(x) \rightarrow e^{i\alpha\gamma^5/2} q_n(x). \quad (23)$$

As for massless quarks, it is straightforward to show that the helicity operator of the LEET quark is just $\gamma^5/2$. This feature, and the fact that there is no dynamical Dirac matrix in the LEET Lagrangian (coupled to the covariant derivative D^μ), should indicate that the U(1) chiral symmetry can be embedded in a larger symmetry group [24]. This is actually what happens. One defines⁵ in the rest frame of v

$$\begin{aligned} S^1 &\equiv \frac{1}{2} \gamma^0 \Sigma^1 = \frac{1}{2} \gamma^1 \gamma^5, & S^2 &\equiv \frac{1}{2} \gamma^0 \Sigma^2 = \frac{1}{2} \gamma^2 \gamma^5, \\ S^3 &\equiv \frac{1}{2} \Sigma^3 = \frac{1}{2} \gamma^5 \gamma^0 \gamma^3. \end{aligned} \quad (24)$$

In a general frame, one defines two four-vectors e^1 and e^2 transverse to both v and n and

$$S^1 = \frac{\gamma^5 \not{e}^1}{2}, \quad S^2 = \frac{\gamma^5 \not{e}^2}{2}, \quad S^3 = \frac{\gamma^5}{2} (1 - \not{v} \not{n}). \quad (25)$$

Thus $S^3 q_n = \gamma^5 q_n$ because of the projector, as said above. As the Σ^i generate the SU(2) group, and from $[\gamma^0, \Sigma^i] = 0$ and $(\gamma^0)^2 = 1$, the S^i operators also verify the SU(2) algebra

$$[S^i, S^j] = i \epsilon_{ijk} S^k. \quad (26)$$

Finally, it is simple to check that both the Lagrangian (20) and the projection condition (21) remain invariant under the transformation generated by \vec{S}

$$q_n(x) \rightarrow e^{i\vec{\alpha} \cdot \vec{S}} q_n(x). \quad (27)$$

Our conclusion is that to leading order, LEET has as much global symmetry as HQET, that is a flavor and SU(2) symmetry.⁶

Unfortunately, unlike the HQET case, the action of the generators (25) on the physical states is not straightforward

to find. Indeed, the HQET symmetry group holds whatever the internal kinematical configuration of the heavy hadron,⁷ and this is obviously not the case for LEET. In short, the physical states are not dominated by the ‘‘objects’’ that are expected to be described by LEET (for example, the LEET symmetry does not imply $m_\rho = m_\pi$). However, this problem, which we leave for further investigation, will not prevent us from finding very significant results by sticking to the quark current operators and simply replacing the QCD light quark field by the LEET one wherever it is justified, as we will see in Sec. III.

As for the space-time symmetry, it can be checked that while the HQET Lagrangian is invariant under the rotation group (more precisely the little group of v), the LEET Lagrangian (20) is invariant under the group of the collinear conformal transformations.⁸ Furthermore, it should be possible to ‘‘make covariant’’ the theory by summing on the four-vector n , similarly to Georgi’s procedure [25] concerning HQET. Finally the Feynman rules of the LEET effective theory look similar to the HQET ones [21],

$$\text{LEET quark propagator: } \frac{i \not{v} \not{n}}{n \cdot k + i\epsilon} \frac{\not{n}}{2}, \quad (28)$$

$$\text{LEET quark-gluon vertex: } -ig \not{v} T_a n^\mu. \quad (29)$$

III. THE UNIVERSAL FORM FACTORS $\zeta(M, E)$, $\zeta_{\parallel}(M, E)$, $\zeta_{\perp}(M, E)$ AND THEIR SCALING LAWS

Our purpose in this section is to find the constraints on the heavy-to-light form factors which may follow from LEET. For definiteness, we consider the decay of a B meson, although some of our results may apply for D decays. As for the final particle, $P(V)$ stands for a light pseudoscalar (vector) meson. We are interested in the following matrix elements:

$$\langle P | V^\mu | B \rangle, \quad \langle P | T^{\mu\nu} | B \rangle, \quad (30)$$

$$\langle V | V^\mu | B \rangle, \quad \langle V | A^\mu | B \rangle, \quad \langle V | T_5^{\mu\nu} | B \rangle, \quad (31)$$

where $V^\mu = \bar{q} \gamma^\mu b$, $A^\mu = \bar{q} \gamma^\mu \gamma_5 b$, $T^{\mu\nu} = \bar{q} \sigma^{\mu\nu} b$ and $T_5^{\mu\nu} = \bar{q} \sigma^{\mu\nu} \gamma_5 b$ are, respectively, the vector, axial, tensor, and pseudotensor weak currents, with q the appropriate active light flavor. The matrix elements

$$\langle P | T_5^{\mu\nu} | B \rangle, \quad \langle P | T^{\mu\nu} q_\nu | B \rangle, \quad (32)$$

$$\langle V | T^{\mu\nu} | B \rangle, \quad \langle V | T_{(5)}^{\mu\nu} q_\nu | B \rangle \quad (33)$$

($q_\mu = p_\mu - p'_\mu = M v_\mu - E n_\mu$ is the four-momentum transfer) can obviously be obtained from Eqs. (30),(31) by using

⁷Except for exchange of hard momenta ($k \gtrsim M$), which generates logarithmic corrections that are calculable in perturbation theory.

⁸These are defined in Ref. [23], within the context of the quantization on the light cone.

⁵We use $\vec{\Sigma} \equiv (\vec{\sigma} \ 0)$, where $\vec{\sigma}$ are the Pauli matrices.

⁶Note the important role played by the \not{v} factor in the Lagrangian.

$\gamma_5 \sigma^{\mu\nu} = (i/2) \epsilon^{\mu\nu\rho\sigma} \sigma_{\rho\sigma}$ and/or contracting with q_ν . Let us recall the relation between q^2 and $E = v \cdot p'$

$$q^2 = M^2 - 2ME + m'^2 \Leftrightarrow E = \frac{M}{2} \left(1 - \frac{q^2}{M^2} + \frac{m'^2}{M^2} \right). \quad (34)$$

As a starting point, we decompose in all generality the above matrix elements in terms of Lorentz invariant form factors. We adopt a parametrization that is convenient to

study the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit, i.e., we use the variables (v^μ, n^μ, E) rather than (p^μ, p'^μ, q^2) . Some caution is needed to treat the polarization vector ϵ^μ of the vector meson: indeed when $E \rightarrow \infty$, one has $\epsilon_\perp^\mu = \mathcal{O}(1)$ for a transverse meson while $\epsilon_\parallel^\mu = \mathcal{O}(E/m_V)$ for a longitudinal one. Thus we decompose the matrix elements on the Lorentz structures v^μ , n^μ , $\epsilon^{*\mu} - (\epsilon^* \cdot v)n^\mu$, $(m_V/E)(\epsilon^* \cdot v)n^\mu$, $(m_V/E)(\epsilon^* \cdot v)v^\mu$, and $\epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \epsilon_\sigma^*$ which are finite in the asymptotic limit $M \rightarrow \infty$ and $E \rightarrow \infty$:

$$\langle P | V^\mu | B \rangle = 2E [\zeta^{(v)}(M, E) n^\mu + \zeta_1^{(v)}(M, E) v^\mu], \quad (35)$$

$$\langle P | T^{\mu\nu} | B \rangle = i2E \zeta^{(t)}(M, E) (n^\mu v^\nu - n^\nu v^\mu), \quad (36)$$

$$\langle V | V^\mu | B \rangle = i2E \zeta_\perp^{(v)}(M, E) \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \epsilon_\sigma^*, \quad (37)$$

$$\langle V | A^\mu | B \rangle = 2E \zeta_\perp^{(a)}(M, E) [\epsilon^{*\mu} - (\epsilon^* \cdot v)n^\mu] + 2E \frac{m_V}{E} (\epsilon^* \cdot v) [\zeta_\parallel^{(a)}(M, E) n^\mu + \zeta_{1\parallel}^{(a)}(M, E) v^\mu], \quad (38)$$

$$\begin{aligned} \langle V | T_5^{\mu\nu} | B \rangle = & -i2E \zeta_{1\perp}^{(t_5)}(M, E) \{ [\epsilon^{*\mu} - (\epsilon^* \cdot v)n^\mu] v^\nu - [\epsilon^{*\nu} - (\epsilon^* \cdot v)n^\nu] v^\mu \} \\ & - i2E \zeta_\perp^{(t_5)}(M, E) (\epsilon^{*\mu} n^\nu - \epsilon^{*\nu} n^\mu) - i2E \zeta_\parallel^{(t_5)}(M, E) \frac{m_V}{E} (\epsilon^* \cdot v) (n^\mu v^\nu - n^\nu v^\mu). \end{aligned} \quad (39)$$

Some additional comments on Eqs. (35)–(39) are in order:⁹ the $2E$ overall normalization factor has been chosen to restore the dimensionality and for further convenience [cf. Eq. (99)]. The superscripts (v) , (a) , . . . , refer to the Dirac structure of the current operators. Furthermore it is clear that for a matrix element to a longitudinal (transverse) vector meson, only the form factors with a $\parallel(\perp)$ subscript contribute in the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit. Finally we have made explicit the dependence of the form factors with respect to M and E , although they also depend on $m' = m_p$ or m_V .

Let us now expose our argument, in the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit. On the one hand we use LEET to describe the final active quark. Thus the quark field $\bar{q}(0)$ in the current operators will be replaced by the effective LEET field $\bar{q}_n(0)$, with $\bar{q}_n \not{v} \not{h} / 2 = \bar{q}_n$. To exploit the latter equation, two useful Dirac identities are

$$\frac{\not{v} \not{h}}{2} \gamma^\mu = \frac{\not{v} \not{h}}{2} [n^\mu \not{v} + i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \gamma_\sigma \gamma_5], \quad (40)$$

$$\begin{aligned} \frac{\not{v} \not{h}}{2} \sigma^{\mu\nu} = & \frac{\not{v} \not{h}}{2} [i(n^\mu v^\nu - n^\nu v^\mu) - i(n^\mu \gamma^\nu - n^\nu \gamma^\mu) \not{v} \\ & + \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \gamma_5]. \end{aligned} \quad (41)$$

On the other hand, one may wonder about the initial heavy quark: should it be described by QCD or HQET? We have already noticed that to leading order LEET neglects hard spectator effects, and that the hard momenta are “integrated out,” leaving only soft, nonperturbative degrees of freedom. Thus for consistency one should use HQET to describe the initial heavy quark, and replace the quark field $b(0)$ by $b_v(0)$ with $\not{v} b_v = b_v$. This supports the proposed conjecture that HQET may be applied to the whole physical kinematical range in heavy-to-light semileptonic decays [4], and is not restricted to the small recoil region, that is the Isgur-Wise limit [2]. Note that this HQET-LEET formalism is not more than a *soft contribution dominance assumption*, that will have short-distance α_s corrections, and nonperturbative $1/M$ and $1/E$ corrections.

Now we are in position to reduce the number of independent form factors, by simply replacing the quark current operator $\bar{q} \Gamma b$ by the effective one $\bar{q}_n \Gamma b_v$, which is finite in the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit. From the constraints $\not{v} b_v = b_v$ and $\not{h} q_n = 0$ and from Eqs. (40),(41) we find the following relations between the currents, to leading order in $1/M$ and $1/E$:

$$\bar{q}_n b_v = v_\mu \bar{q}_n \gamma^\mu b_v, \quad (42)$$

⁹We use the usual relativistic normalization of states and $\epsilon^{0123} = +1$. Note that our phase convention for the vector mesons differs by a i factor from the one used in Refs. [7–9,13,26,27]: $|V\rangle_{\text{this work}} = +i|V\rangle_{\text{Ref. [7]}}$.

$$\bar{q}_n \gamma^\mu b_v = n^\mu \bar{q}_n b_v + i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q}_n \gamma_\sigma \gamma_5 b_v, \quad (43)$$

$$\bar{q}_n \gamma^\mu \gamma_5 b_v = -n^\mu \bar{q}_n \gamma_5 b_v + i \epsilon^{\mu\nu\rho\sigma} v_\nu n_\rho \bar{q}_n \gamma_\sigma b_v, \quad (44)$$

$$\begin{aligned} \bar{q}_n \sigma^{\mu\nu} b_v &= i [n^\mu v^\nu \bar{q}_n b_v - n^\nu \bar{q}_n \gamma^\nu b_v - (\mu \leftrightarrow \nu)] \\ &+ \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \bar{q}_n \gamma_5 b_v, \end{aligned} \quad (45)$$

$$\begin{aligned} \bar{q}_n \sigma^{\mu\nu} \gamma_5 b_v &= i [n^\mu v^\nu \bar{q}_n \gamma_5 b_v + n^\nu \bar{q}_n \gamma^\nu \gamma_5 b_v - (\mu \leftrightarrow \nu)] \\ &+ \epsilon^{\mu\nu\rho\sigma} v_\rho n_\sigma \bar{q}_n b_v. \end{aligned} \quad (46)$$

Reporting Eqs. (42)–(46) in Eqs. (35)–(39) we find

$$\zeta_1^{(v)} = \zeta_{1\parallel}^{(a)} = \zeta_{1\perp}^{(ts)} = 0, \quad (47)$$

$$\zeta^{(v)} = \zeta^{(t)} \equiv \zeta, \quad (48)$$

$$\zeta_{\perp}^{(v)} = \zeta_{\perp}^{(a)} = \zeta_{\perp}^{(ts)} \equiv \zeta_{\perp}, \quad (49)$$

$$\zeta_{\parallel}^{(a)} = \zeta_{\parallel}^{(ts)} \equiv \zeta_{\parallel}. \quad (50)$$

Thus to leading order in $1/M$, $1/E$, and α_s there are only three independent form factors in heavy-to-light $B \rightarrow P(V)$ transitions, which from now on we will denote by ζ (for $B \rightarrow P$), ζ_{\parallel} and ζ_{\perp} (for $B \rightarrow V$). This implies nontrivial relations between the usual form factors f_+ , A_1 , etc. (see Sec. V). Note that among these relations there are the well-known Isgur-Wise relations [2] between the penguin and semileptonic form factors which follow from $\not{v} h_v = h_v$ only, while the relations among the semileptonic form factors stemming from $\not{v} h_v = h_v$ and $\bar{q}_n = \bar{q}_n \not{v} \not{v} / 2$ are new (they resemble Stech's [29] and Soares's [30] quark model relations, as we discuss in Sec. V).

The $\zeta(M, E)$ functions have a simple dependence with respect to the large mass M . Indeed it is well known that the following relation between the QCD and HQET eigenstates holds in the heavy mass limit [21]:

$$|B, p_\mu\rangle_{\text{QCD}} = \sqrt{M} |B, v_\mu\rangle_{\text{HQET}}, \quad (51)$$

where $|B, v_\mu\rangle_{\text{HQET}}$ is independent of M . Thus the matrix elements (30),(31) become

$$\langle P(V) | \bar{q} \Gamma b | B, p_\mu \rangle_{\text{QCD}} = \sqrt{M} \langle P(V) | \bar{q}_n \Gamma b_v | B, v_\mu \rangle_{\text{HQET}}, \quad (52)$$

where the only Lorentz scalar arising in the covariant decomposition of the right-hand side matrix element is $E = v \cdot p'$ (and $\epsilon^* \cdot v$ in the $B \rightarrow V$ case). From Eqs. (52) and (35)–(39) one has

$$\zeta(M, E) = \sqrt{M} \left[z^{(0)}(E) + \sum_{k=1}^{\infty} \frac{z^{(k)}(E)}{M^k} \right], \quad (53)$$

where the $z^{(k)}(E)/M^k$, $k \geq 1$, stand for higher order terms stemming from the HQET Lagrangian and from the heavy-to-light current operator. Similar $1/M$ expansion apply for ζ_{\parallel} and ζ_{\perp} .

In the original Isgur-Wise derivation [2], M is sent to infinity while E is kept fixed, and the well-known $\sim \sqrt{M}$ scaling is obtained for $E \ll M$. Actually whatever the ratio E/M , we may also take $E \rightarrow \infty$ provided that the $z^{(k)}(E)$, $k \geq 1$, are not enhanced by powers of E with respect to $z^{(0)}(E)$, which is unlikely. Indeed the $z^{(k)}(E)$ are suppressed by some power of the large scale E , which is related to the suppression of the wave function of the light energetic meson when one quark carries most of the momentum of the hadron, i.e. for the Feynman variable $u \sim 1$. This suppression is *universal*, belongs to the properties of the final state, and thus should hold for all the operators contributing to the expansion (53). In the end one obtains a factorized scaling law for *any ratio* E/M :

$$\begin{aligned} \zeta(M, E) &= \sqrt{M} z(E), & \zeta_{\parallel}(M, E) &= \sqrt{M} z_{\parallel}(E), \\ \zeta_{\perp}(M, E) &= \sqrt{M} z_{\perp}(E). \end{aligned} \quad (54)$$

We expect that the potential $\sim E/M$ nonfactorizable corrections to Eq. (54) will be suppressed by an additional power of the large scales M or E , or by α_s .

The question of the definite asymptotic E dependence is more involved, because we have no relation for LEET comparable to Eq. (51). However, it is now well accepted that at $q^2=0$, the Feynman mechanism contribution to the form factors should behave as the hard one, that is $\sim M^{-3/2}$ [3,7,8,14], although there is no really rigorous proof of that. As $E \sim M$ at $q^2=0$, it implies that the $z(E)$ functions in Eq. (54) behave as $\sim 1/E^2$. As already said, this follows from the behavior of the final state light-cone wave function near the end point $u \sim 1$ [3]: it is argued in Ref. [28] that the light-cone twist-two distribution amplitude, renormalized at a low scale $\mu \geq 1$ GeV, vanishes linearly at $u \sim 1$ similar to each term of its expansion in the Gegenbauer polynomials. Integrating over a region shrunk to $\Delta u \sim 1/E$, which is the signature of the Feynman mechanism, one obtains the $1/E^2$ scaling law

$$z(E) \sim \int_{1-\Lambda/E}^1 du (1-u) \sim \frac{1}{E^2}. \quad (55)$$

Should the behavior of the distribution amplitudes near $u \sim 1$ be different from the usual $\sim (1-u)$ expectation, the power law in $1/E$ should be changed. We stress again that the $\sim 1/E^2$ suppression (or whatever $\sim 1/E^n$) should affect *all* the $z^{(k)}$ functions in Eq. (53). We will see in the next section that this is exactly what the light-cone sum rule method predicts.

Here we would like to make an important comment: it is sometimes said that because the asymptotic M dependence is different at the two particular points $q^2=0$ and $q^2=q_{\text{max}}^2 = (M-m')^2$, HQET could not be valid in the whole range of q^2 . However, the extended HQET scaling prediction (54) is

fully compatible with both the Isgur-Wise $\sim\sqrt{M}$ scaling for $E\ll M$ and the Chernyak-Zhitnitsky $\sim M^{-3/2}$ scaling at $q^2=0$ provided $z(E)\sim 1/E^2$ at large E . Note also that the dipole scaling of $z(E)$ at large E does not prevent it to be polelike at finite E , according to the idea of B^* meson dominance.

For clarity, we summarize here our results: in the $M\rightarrow\infty$ and $E\rightarrow\infty$ limit, the heavy-to-light weak current matrix elements depend on only three independent dimensionless form factors ζ , ζ_{\parallel} , and ζ_{\perp}

$$\langle P|V^{\mu}|B\rangle=2E\zeta(M,E)n^{\mu}, \quad (56)$$

$$\langle P|T^{\mu\nu}|B\rangle=i2E\zeta(M,E)(n^{\mu}v^{\nu}-n^{\nu}v^{\mu}), \quad (57)$$

$$\langle V|V^{\mu}|B\rangle=i2E\zeta_{\perp}(M,E)\epsilon^{\mu\nu\rho\sigma}v_{\nu}n_{\rho}\epsilon_{\sigma}^*, \quad (58)$$

$$\begin{aligned} \langle V|A^{\mu}|B\rangle=2E\left\{\zeta_{\perp}(M,E)[\epsilon^{*\mu}-(\epsilon^*\cdot v)n^{\mu}] \right. \\ \left. +\zeta_{\parallel}(M,E)\frac{m_V}{E}(\epsilon^*\cdot v)n^{\mu}\right\}, \quad (59) \end{aligned}$$

$$\begin{aligned} \langle V|T_5^{\mu\nu}|B\rangle=-i2E\zeta_{\perp}(M,E)(\epsilon^{*\mu}n^{\nu}-\epsilon^{*\nu}n^{\mu}) \\ -i2E\zeta_{\parallel}(M,E)\frac{m_V}{E}(\epsilon^*\cdot v)(n^{\mu}v^{\nu}-n^{\nu}v^{\mu}). \quad (60) \end{aligned}$$

These three universal form factors have a \sqrt{M} dependence

$$\begin{aligned} \zeta(M,E)=\sqrt{M}z(E), \quad \zeta_{\parallel}(M,E)=\sqrt{M}z_{\parallel}(E), \\ \zeta_{\perp}(M,E)=\sqrt{M}z_{\perp}(E), \quad (61) \end{aligned}$$

and we have argued that a $1/E^2$ dependence is very plausible, thus

$$\zeta(M,E)=C\frac{\sqrt{M}}{E^2}=4C\frac{M^{-3/2}}{(1-q^2/M^2)^2}, \quad (62)$$

$$\zeta_{\parallel}(M,E)=C_{\parallel}\frac{\sqrt{M}}{E^2}=4C_{\parallel}\frac{M^{-3/2}}{(1-q^2/M^2)^2}, \quad (63)$$

$$\zeta_{\perp}(M,E)=C_{\perp}\frac{\sqrt{M}}{E^2}=4C_{\perp}\frac{M^{-3/2}}{(1-q^2/M^2)^2}. \quad (64)$$

where C , C_{\parallel} , and C_{\perp} are unknown dimensionful constants, of order $\Lambda_{\text{QCD}}^{3/2}$.

Let us repeat, however, that Eqs. (62)–(64) are not on a solid ground as Eqs. (56)–(61). Accepting them nevertheless, everything but three normalization constants is known about the form factors, which may constitute an even more favorable situation compared to the heavy-to-heavy case, at least from the mathematical point of view. Ironically, a model-independent value of the form factors at some particu-

lar q^2 would have been more interesting than the q^2 dependence, from the point of view of the extraction of $|V_{ub}|$.

Before closing this section, we would like to make clear the expected region of validity of the final hadron large energy limit: from Eq. (34), it is not restricted, at least formally, to the small values of q^2 . Indeed, q^2 can even be $\mathcal{O}(M^2)$ (e.g., $q^2=\alpha M^2$), provided that $1-q^2/M^2=\mathcal{O}(1)$ (that is $\alpha\neq 1$) and that M is large enough to get from Eq. (34) the conditions $\Lambda_{\text{QCD}}\ll E$, $m'\ll E$. However, the region near the zero-recoil point, $q^2=M^2-M\chi$ with χ finite, is outside the LEET domain. Of course, the realistic world is much more complicated, and one may expect sizeable non-LEET effects for physical quark and hadron masses, as one is going from $q^2=0$ to $q^2=q_{\text{max}}^2$. For B decays, one may hope that LEET will be valid for $0\leq q^2\leq 10-15$ GeV² although a more precise answer cannot be given without a careful study of the subleading terms.

In addition, we are aware of the fact that Eqs. (56)–(64) should receive logarithmic radiative corrections [$\sim\ln(M)$ and $\sim\ln(E)$] [5,31], which might be computed by matching QCD onto HQET-LEET. These calculations are beyond the scope of this paper.

IV. LIGHT-CONE SUM RULES IN THE FINAL HADRON LARGE ENERGY LIMIT

In this section, we shall show that the light-cone sum rule (LCSR) method is fully compatible with the HQET-LEET formalism that we have discussed above; moreover we obtain below explicit expressions for the three universal form factors, that are strikingly simple. Chernyak and Zhitnitsky [3] were the first authors to use the LCSR method to calculate the heavy-to-light form factors in the region where the energy release is sufficiently large (actually at $q^2=0$). The basic idea is to describe the decaying heavy hadron by an interpolating local current and to use the quark-hadron duality and the Borel transformation to suppress the contribution of the excited B states and of the continuum. On the other hand, the light hadron enters the game through the light-cone distribution amplitudes, order by order in the twist expansion. Later, the method was developed for $q^2\neq 0$ by several groups who took into account higher-twist and radiative corrections effects [6–14]. It has to be considered as a QCD-based approximation, although, to our knowledge, there is not a well defined limit of the underlying theory in which the sum rules are exact.

Our purpose is thus to study the $M\rightarrow\infty$ and $E\rightarrow\infty$ limit of the LCSR expressions for the form factors, using mainly the explicit formulas of Refs. [6,9,14]. For simplicity we do not consider here the tensor form factors, only the vector and axial ones. Thanks to the considerable effort developed in the literature [3,6–14], our calculation does not pose any major problem and will not be reported here in detail; nevertheless some comments are in order.

The twist expansion does not match the $1/M$ and $1/E$ power expansion. We have checked that the leading order contributions to the $M\rightarrow\infty$ and $E\rightarrow\infty$ limit depend not only on the two-particle twist-two but also on the twist-three distribution amplitudes while higher-twist (≥ 4) and multipar-

ticle (≥ 3) distribution amplitudes are power suppressed. This was already found in Refs. [3,7,8,14] where the heavy quark expansion was considered at the particular point $q^2 = 0$. This is also compatible with the finding of Ref. [12] that twist-three terms are numerically as important as the twist-two ones, while the twist-four and multiparticle contributions are much smaller. Actually, for the twist expansion to make sense, the number of different twists contributing to a given order in $1/M$ and $1/E$ should be finite, which seems indeed to be the case.

According to the preceding point, the early calculations of the $B \rightarrow V$ form factors are not consistent from the point of view of the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit. Indeed the authors of Refs. [7–9] have not taken into account the contribution of the two-particle twist-three distribution amplitudes $h_{\parallel}^{(t)}(u)$ and $h_{\parallel}^{(s)}(u)$, as defined in Ref. [26], which, however, contribute to leading order on the same footing as the ones of twist-two. To our knowledge, only in the recent works [13] have these functions been considered. We calculate the corresponding terms below.

The ‘‘surface terms’’ should be kept systematically. These terms come from the integration by parts after the Borel transformation, as discussed in Appendix A of Ref. [8]. However, for our purpose we have found simpler and equivalent to perform the integration by parts *before* the Borel transformation; thus in the calculations below we will use

$$\int_0^1 \frac{du}{\Delta^2} f(u) = \frac{1}{2} \int_0^1 \frac{du}{\Delta} \frac{1}{q \cdot p + um'^2} \times \left[\frac{m'}{q \cdot p + um'^2} f(u) - f'(u) \right], \quad (65)$$

where $f(u)$ is a function of u which verifies $f(0) = f(1) = 0$ and

$$\Delta = m_b^2 - (q + up')^2. \quad (66)$$

Then the Borel transformation is done with respect to the variable $(q + p')^2$, according to

$$\frac{1}{\Delta} \rightarrow \frac{1}{u\mathcal{M}^2} \exp \left[- \frac{m_b^2 - (1-u)q^2 + u(1-u)m'^2}{u\mathcal{M}^2} \right], \quad (67)$$

where \mathcal{M}^2 is the Borel parameter.

In order to find the $m_b \rightarrow \infty$ limit of the LCSR, the sum rule parameters have to be rescaled. Following Refs. [3,7,8,14] we write the Borel parameter

$$\mathcal{M}^2 = m_b \mu_0 \quad (68)$$

and the continuum threshold

$$s_0^B = (m_b + \omega_0)^2, \quad (69)$$

where the rescaled parameters μ_0 and ω_0 are finite in the $m_b \rightarrow \infty$ limit.

Moreover we note [6,9,14] that the integration domain after the Borel transformation and the continuum subtraction is $u \in [u_{\min}, 1]$, where

$$u_{\min} = \frac{m_b^2 - q^2}{s_0^B - q^2} \quad (\text{for } m' \approx 0). \quad (70)$$

In the $m_b \rightarrow \infty$ and $E \rightarrow \infty$ limit, one has the expansion $u_{\min} \approx 1 - \omega_0/E$, which shows that this is indeed the size of E that selects the $u \sim 1$ region.

We have now all the elements to perform the calculation. We use the standard notation for the decay constants f_B, f_P, f_V , and f_V^\perp and for the distribution amplitudes,¹⁰ $\phi, \phi_p, \phi_\sigma, \phi_{\parallel}, h_{\parallel}^{(s)}, h_{\parallel}^{(t)}, \phi_{\perp}, g_{\perp}^{(v)}$ and $g_{\perp}^{(a)}$. The behavior of these functions near $u \rightarrow 1$ is assumed to be identical to each term of their conformal expansion in the Gegenbauer polynomials [28], that is, up to $\sim (1-u)\ln(1-u)$ terms proportional to the light quark masses [26]

$$\phi \sim \phi_\sigma \sim \phi_{\parallel} \sim \phi_{\perp} \sim h_{\parallel}^{(s)} \sim g_{\perp}^{(a)} \sim (1-u), \quad (71)$$

$$\phi_p \sim h_{\parallel}^{(t)} \sim g_{\perp}^{(v)} \sim C^{\text{st}}. \quad (72)$$

For the $B \rightarrow P$ transitions we simply use the correlators given by formulas (17) in Ref. [6] and Eq. (14) in Ref. [14] and perform the Borel transformation after using Eq. (65). The standard procedure to subtract the continuum is applied, and we find that in the limit $m_b \rightarrow \infty$ and $E \rightarrow \infty$ the $B \rightarrow P$ semileptonic matrix element can be written under the form (35) with

$$\zeta^{(v)}(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[-f_P \phi'(1) I_2(\omega_0, \mu_0) - \frac{f_P m_P^2}{6(m_{q_1} + m_{q_2})} \phi'_\sigma(1) I_1(\omega_0, \mu_0) \right], \quad (73)$$

$$\zeta_1^{(v)}(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \frac{f_P m_P^2}{m_{q_1} + m_{q_2}} \times \left[\phi_p(1) + \frac{1}{6} \phi'_\sigma(1) \right] I_1(\omega_0, \mu_0). \quad (74)$$

Here $m_{q_{1,2}}$ stand for the masses of the quarks making the light pseudoscalar $q_1 q_2$ meson, and the $I_j(\omega_0, \mu_0)$ are functions of the sum rule parameters (ω_0, μ_0) through

¹⁰All these functions are defined and discussed at length in Ref. [26].

$$I_j(\omega_0, \mu_0) = \int_0^{\omega_0} d\omega \omega^j \exp\left[\frac{2}{\mu_0}(\bar{\Lambda} - \omega)\right], \quad j=1,2, \quad (75)$$

where $\bar{\Lambda}$ is the binding energy of the heavy meson, $\bar{\Lambda} = m_B$

$-m_b$, which is finite in the $m_b \rightarrow \infty$ limit. For the particular point $q^2=0$, Eqs. (73),(74) agree with Ref. [14] in the $m_b \rightarrow \infty$ limit.

For the $B \rightarrow V$ transitions more work is needed. We have recalculated the correlator considered in Ref. [9]

$$\Pi_\mu(p', q) = i \int d^4x e^{iq \cdot x} \langle V, p' | T \bar{q}(x) \gamma_\mu (1 - \gamma_5) b(x) \bar{b}(0) i \gamma_5 q(0) | 0 \rangle \quad (76)$$

taking into account the contribution of the distribution amplitudes $h_{\parallel}^{(s)}(u)$ and $h_{\parallel}^{(t)}(u)$. Using the method and notations of Ref. [9], we find¹¹

$$\begin{aligned} \Pi_\mu(p', q) = & -m_b f_V m_V \int_0^1 \frac{du}{\Delta} \left[\epsilon_\mu^* g_\perp^{(v)} + 2(q \cdot \epsilon^*) p'_\mu \frac{1}{\Delta} (\Phi_{\parallel} - G_\perp^{(v)}) \right] - i \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p'^\rho q^\sigma \left[\frac{m_b}{2} f_V m_V \int_0^1 \frac{du}{\Delta^2} g_\perp^{(a)} + f_V^\perp \int_0^1 du \frac{\phi_\perp}{\Delta} \right] \\ & - f_V^\perp \int_0^1 du \frac{\phi_\perp}{\Delta} [\epsilon_\mu^*(p' \cdot q + p'^2 u) - p_\mu(q \cdot \epsilon^*)] - f_V^\perp m_V^2 \int_0^1 \frac{du}{\Delta} \epsilon_\mu^* \left[\left(1 + 2 \frac{m_b^2}{\Delta} \right) (H_{\parallel}^{(t)} - \Phi_\perp) + \frac{1}{2} h_{\parallel}^{(s)} \right] \\ & + 2 f_V^\perp m_V^2 \int_0^1 \frac{du}{\Delta^2} (q \cdot \epsilon^*) (q_\mu + u p'_\mu) \left(H_{\parallel}^{(t)} - \Phi_\perp - \frac{1}{2} h_{\parallel}^{(s)} \right), \quad (77) \end{aligned}$$

the uppercase notation meaning the primitive of a lower-case function

$$F(u) = - \int_0^u dv f(v). \quad (78)$$

Compared to Eq. (19) of Ref. [9], the last two terms of the above equation are new.¹²

In taking the Borel transformation of Eq. (77), a subtlety emerges: the standard procedure considers the Lorentz-invariant decomposition of Π_μ , that is the coefficients of ϵ_μ^* , $(q \cdot \epsilon^*) p'_\mu$ and $(q \cdot \epsilon^*) q_\mu$, and performs the Borel transformation on the variable $(p' + q)^2$ with $p'^2 = m_V^2$ and q^2 fixed to their physical value. This amounts in particular to considering $q \cdot \epsilon^*$ as a constant. While this is true for transverse mesons, for which $q \cdot \epsilon_\perp^* = 0$, it is clearly not possible for longitudinal mesons. Thus for longitudinal mesons we simplify the correlator Π_μ by expressing $q \cdot \epsilon_\parallel^*$ in terms of $p' \cdot q$ and q^2 before Borel transforming on $(p' + q)^2$:

$$q \cdot \epsilon_\parallel^* = \left(\frac{p' \cdot q}{m_V} + m_V \right) \sqrt{1 - \frac{m_V^2}{E^2}}. \quad (79)$$

In the limit $m_b \rightarrow \infty$ and $E \rightarrow \infty$ the $B \rightarrow V$ semileptonic matrix elements can finally be written under the form (37),(38) with

$$\begin{aligned} \zeta_{\parallel}^{(a)}(M, E) = & \frac{1}{f_B} \frac{1}{2E^2} [-f_V \phi_{\parallel}'(1) I_2(\omega_0, \mu_0) \\ & + f_V^\perp m_V h_{\parallel}^{(t)}(1) I_1(\omega_0, \mu_0)], \quad (80) \end{aligned}$$

$$\zeta_{\parallel}^{(a)}(M, E) L = \frac{1}{f_B} \frac{f_V^\perp m_V}{2E^2} \left[h_{\parallel}^{(t)}(1) + \frac{1}{2} h_{\parallel}^{(s)'}(1) \right] I_1(\omega_0, \mu_0), \quad (81)$$

$$\begin{aligned} \zeta_{\perp}^{(a)}(M, E) = & \frac{1}{f_B} \frac{1}{2E^2} [-f_V^\perp \phi_{\perp}'(1) I_2(\omega_0, \mu_0) \\ & + f_V m_V g_{\perp}^{(v)}(1) I_1(\omega_0, \mu_0)], \quad (82) \end{aligned}$$

$$\begin{aligned} \zeta_{\perp}^{(v)}(M, E) = & \frac{1}{f_B} \frac{1}{2E^2} \left[-f_V^\perp \phi_{\perp}'(1) I_2(\omega_0, \mu_0) \right. \\ & \left. - \frac{1}{4} f_V m_V g_{\perp}^{(a)'}(1) I_1(\omega_0, \mu_0) \right]. \quad (83) \end{aligned}$$

As for the transverse form factors at the particular point $q^2=0$, Eqs. (82),(83) agree with Ref. [8] in the $m_b \rightarrow \infty$ limit.

Let us now discuss our results, Eqs. (73),(74) and (80)–(83). First, as $f_B \sqrt{m_B} \sim C^{\text{st}}$ [2], the factorized scaling law $\sim \sqrt{M}/E^2$ is clearly seen, as anticipated in Sec. III. Note that

¹¹Recall our conventions, footnote No. 10.

¹²They are taken into account in Ref. [13], together with twist-four and radiative corrections.

the $1/E^2$ dependence holds despite the fact that ϕ_p , $g_\perp^{(v)}$, and $h_\parallel^{(t)}$ do not vanish at $u=0,1$, which is a hint that this scaling law may be independent of the LCSR calculation. Second it still seems that we have six independent form factors to describe the semileptonic $B \rightarrow P(V)$ transitions. However, using the results of Refs. [7,23,26] based on the conformal expansion of the distribution amplitudes and the equations of motion, the following relations hold exactly in QCD:

$$\phi_p(1) + \frac{1}{6} \phi'_\sigma(1) = 0, \quad (84)$$

$$h_\parallel^{(t)}(1) + \frac{1}{2} h_\parallel^{(s)'}(1) = 0, \quad (85)$$

$$g_\perp^{(v)}(1) + \frac{1}{4} g_\perp^{(a)'}(1) = 0. \quad (86)$$

Reporting the above relation in Eqs. (73),(74) and (80)–(83) we obtain $\zeta_1 = \zeta_{1\parallel} = 0$ and that the semileptonic $B \rightarrow P(V)$ matrix elements can finally be written under the form (56)–(60) with

$$\zeta(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[-f_P \phi'(1) I_2(\omega_0, \mu_0) + \frac{f_P m_P^2}{m_{q_1} + m_{q_2}} \phi_p(1) I_1(\omega_0, \mu_0) \right], \quad (87)$$

$$\zeta_\parallel(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[-f_V \phi'_\parallel(1) I_2(\omega_0, \mu_0) + f_V^\perp m_V h_\parallel^{(t)}(1) I_1(\omega_0, \mu_0) \right], \quad (88)$$

$$\zeta_\perp(M, E) = \frac{1}{f_B} \frac{1}{2E^2} \left[-f_V^\perp \phi'_\perp(1) I_2(\omega_0, \mu_0) + f_V m_V g_\perp^{(v)}(1) I_1(\omega_0, \mu_0) \right]. \quad (89)$$

It is interesting to note that Eqs. (84)–(86) can be derived simply using the LEET projection condition (21). Indeed the functions $\phi_p(u)$ and $\phi_\sigma(u)$ are defined by [with $\phi_\sigma(0) = \phi_\sigma(1) = 0$]

$$\frac{f_P m_P^2}{m_{q_1} + m_{q_2}} \phi_p(u) = \int \frac{En \cdot dx}{2\pi} e^{-iuEn \cdot x} \langle P | \bar{q}_1(x) i \gamma_5 \mathcal{A}(x|0) q_2(0) | 0 \rangle, \quad (90)$$

$$\frac{-f_P m_P^2}{6(m_{q_1} + m_{q_2})} \phi'_\sigma(u) n_\mu = \int \frac{En \cdot dx}{2\pi} e^{-iuEn \cdot x} \langle P | \bar{q}_1(x) \sigma_{\mu\nu} \gamma_5 n^\nu \mathcal{A}(x|0) q_2(0) | 0 \rangle \quad (91)$$

with $p'_\mu = En_\mu$ the hadron four-momentum, u the momentum fraction of the quark q_1 , and \mathcal{A} the path-ordered gluon operator ensuring the gauge invariance of the above matrix elements

$$\mathcal{A}(x|0) = \text{P exp} \left\{ i g \int_0^1 dw x_\mu A^\mu(wx) \right\}. \quad (92)$$

For $u=1$ one may replace in Eqs. (90),(91) $q_1(x)$ by the effective LEET field $q_{1n}(x)$ with $\not{n} q_{1n}(x) = 0$. Using $\sigma_{\mu\nu} = i(g_{\mu\nu} - \gamma_\nu \gamma_\mu)$ one immediately gets Eq. (84). Eqs. (85),(86) can be obtained similarly. Furthermore it can be checked that $1/E$ corrections to LEET generates $1-u$ corrections in the distribution amplitudes and thus vanish for $u=1$.

We would like to make here a last comment, of phenomenological interest. While we have shown that the LCSR approach satisfies the general relations and scaling laws among the form factors, the same approach also allows us to calculate some of the deviations to the asymptotic limit $M \rightarrow \infty$ and $E \rightarrow \infty$. As a first test, we have checked using the most recent calculations [13] that the relations between the form factors are quite robust, i.e., they are well satisfied even in the nonasymptotic regime. However, the $\sim \sqrt{M/E^2}$ scal-

ing law seems to be affected by large corrections (at $q^2=0$, this is discussed in Ref. [7]), which may appear surprising. We leave this interesting question for further investigation.

To conclude the light-cone sum rule method provides an explicit realization of the HQET-LEET formalism, that is it satisfies the predictions (56)–(64) exactly in the limit of heavy mass for the initial meson and large energy for the final one. This is a remarkable nontrivial result, and we would like to insist on the extreme simplicity of Eqs. (87)–(89).

In addition we show in Ref. [15] that the quark models based on the Bakamjian-Thomas formalism also verify the HQET-LEET relations (56)–(60) between the form factors, as well as the $\sqrt{M}z(E)$ scaling law of Eq. (61). This is another hint that the LEET formalism is well adapted to the description of heavy-to-light transitions.

V. PHENOMENOLOGICAL DISCUSSION

Here our purpose is to write down the standard form factors f_+ , A_1 , etc., in terms of the three universal functions ζ , ζ_\parallel , and ζ_\perp , a convenient way to compare our results with previous approaches, and to discuss some phenomenological applications. The former form factors are defined as follows:

$$\langle P|V^\mu|B\rangle = f_+(q^2) \left[p^\mu + p'^\mu - \frac{M^2 - m_P^2}{q^2} q^\mu \right] + f_0(q^2) \frac{M^2 - m_P^2}{q^2} q^\mu, \quad (93)$$

$$\langle P|T^{\mu\nu}q_\nu|B\rangle = i \frac{f_T(q^2)}{M + m_P} [q^2(p^\mu + p'^\mu) - (M^2 - m_P^2)q^\mu], \quad (94)$$

$$\langle V|V^\mu|B\rangle = i \frac{2V(q^2)}{M + m_V} \epsilon^{\mu\nu\rho\sigma} p^\nu p'^\rho \epsilon^{*\sigma}, \quad (95)$$

$$\langle V|A^\mu|B\rangle = 2m_V A_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu + (M + m_V) A_1(q^2) \left[\epsilon^{*\mu} - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right] - A_2(q^2) \frac{\epsilon^* \cdot q}{M + m_V} \left[p^\mu + p'^\mu - \frac{M^2 - m_V^2}{q^2} q^\mu \right], \quad (96)$$

$$\langle V|T^{\mu\nu}q_\nu|B\rangle = -2T_1(q^2) \epsilon^{\mu\nu\rho\sigma} p^\nu p'^\rho \epsilon^{*\sigma}, \quad (97)$$

$$\langle V|T_5^{\mu\nu}q_\nu|B\rangle = -iT_2(q^2) [(M^2 - m_V^2) \epsilon^{*\mu} - (\epsilon^* \cdot q)(p^\mu + p'^\mu)] - iT_3(q^2) (\epsilon^* \cdot q) \left[q^\mu - \frac{q^2}{M^2 - m_V^2} (p^\mu + p'^\mu) \right]. \quad (98)$$

Now we consider the matrix elements as given by their asymptotic expression (56)–(60) with (v^μ, n^μ, E) unambiguously defined in Eqs. (2)–(4); then we identify in these equations the coefficients of n^ν , v^μ , $\epsilon^{*\mu} - (\epsilon^* \cdot v)n^\mu$, etc., with the corresponding ones in the standard parametrization (93)–(98), keeping all the light mass terms although, strictly speaking, they are subdominant in the final hadron large energy limit. The point is that these kinematical mass corrections could be numerically very large, and thus should not be thrown away; for example, $M_K^*/M_D = 0.48$ and $M_K^*/M_B = 0.17$. Although it has a certain degree of arbitrariness and introduces some model-dependence, this procedure amounts to assume that the matrix elements are well approximated by their asymptotic value (56)–(60), while the form factors f_+ , A_1 , etc., are not, because of the light mass terms which appear in their definition. This was already postulated in Ref. [32], and this is in rough agreement with Ref. [27], where it is found that the main light meson mass corrections are purely kinematical.

We find, with the notation $E_P(E_V)$ for the value of E obtained by putting $m' = m_P(m_V)$ in Eq. (34),

$$f_+(q^2) = \zeta(M, E_P), \quad (99)$$

$$f_0(q^2) = \left(1 - \frac{q^2}{M^2 - m_P^2} \right) \zeta(M, E_P), \quad (100)$$

$$f_T(q^2) = \left(1 + \frac{m_P}{M} \right) \zeta(M, E_P), \quad (101)$$

$$A_0(q^2) = \left(1 - \frac{m_V^2}{ME_V} \right) \zeta_{\parallel}(M, E_V) + \frac{m_V}{M} \zeta_{\perp}(M, E_V), \quad (102)$$

$$A_1(q^2) = \frac{2E_V}{M + m_V} \zeta_{\perp}(M, E_V), \quad (103)$$

$$A_2(q^2) = \left(1 + \frac{m_V}{M} \right) \left[\zeta_{\perp}(M, E_V) - \frac{m_V}{E_V} \zeta_{\parallel}(M, E_V) \right], \quad (104)$$

$$V(q^2) = \left(1 + \frac{m_V}{M} \right) \zeta_{\perp}(M, E_V), \quad (105)$$

$$T_1(q^2) = \zeta_{\perp}(M, E_V), \quad (106)$$

$$T_2(q^2) = \left(1 - \frac{q^2}{M^2 - m_V^2} \right) \zeta_{\perp}(M, E_V), \quad (107)$$

$$T_3(q^2) = \zeta_{\perp}(M, E_V) - \frac{m_V}{E} \left(1 - \frac{m_V^2}{M^2} \right) \zeta_{\parallel}(M, E_V). \quad (108)$$

Equations (99)–(108) make apparent the fact that the form factors f_0 , A_1 , and T_2 have a “kinematical pole” $\sim (1 - q^2/M^2)$ with respect to the others, which is a finding that was described in Ref. [32] as being essentially a relativistic effect. In addition, the $\sim 1/E^2$ dependence of the ζ form factors, as discussed in Sec. III, imply a dipole behavior for

f_+ , f_T , A_0 , A_2 , V , T_1 , and T_3 and a pole one for f_0 , A_1 and T_2 with the heavy mass as the pole mass. This pole-dipole description of the form factors was a phenomenological ansatz made in Ref. [32]. Also Ref. [32] needed to introduce some unknown normalization constants, which we may now interpret as the constants C , C_{\parallel} and C_{\perp} of Eqs. (62)–(64).

Moreover, it becomes clear from Eqs. (99),(108) that the HQET-LEET predictions are close to the relations obtained by Stech [29] and Soares [30] who have used a constituent quark model approach. Except some ambiguities in the sub-leading terms $\sim m'/M$ or m'/E , our general relations (99),(108) coincide with Stech's and Soares's ones, if we impose $\zeta_{\perp} = \zeta_{\parallel}$. Note that we have found no general reason for $\zeta_{\perp} = \zeta_{\parallel}$, and it seems incompatible with the LCSR explicit expressions (88),(89), where the ratio $\zeta_{\perp}/\zeta_{\parallel}$, although constant, depends nontrivially on the sum rule parameters, the decay constants and the light-cone distribution amplitudes, i.e., on the dynamics. Similarly, in the explicit and covariant expressions for the form factors that we have obtained in Ref. [15] using the Bakamjian-Thomas quark model approach, it does not seem possible to have $\zeta_{\perp} = \zeta_{\parallel}$ without any assumption on the quark-quark potential. Nevertheless, the similarities between Stech's and Soares's predictions and ours is quite remarkable, and give strong support to these findings.

An interesting phenomenological discussion is done in Ref. [33], where some tests of the form factor relations, obtained by Stech and Soares, are performed or proposed. Moreover, these relations seem to agree quite well with lattice data, as the study of Ref. [34] shows. However, some of these applications are not possible in our case, for example the study of the longitudinal polarization of the light daughter meson, because the ratio $\zeta_{\perp}/\zeta_{\parallel}$ is not known from our formalism.¹³ Thus we consider here only the V/A_1 ratio, leaving other possible applications for further investigation. From Eqs. (103) and (105), this ratio is given by

$$\frac{V(q^2)}{A_1(q^2)} = \frac{(M + m_V)^2}{M^2 + m_V^2 - q^2}. \quad (109)$$

The knowledge of this ratio has important consequences: on the one hand, it is measured at $q^2=0$ in the decays $D \rightarrow K^* l \nu_l$ and $D_s \rightarrow \phi l \nu_l$ [1]; on the other hand it provides the ratio Γ_+/Γ_- of the width to the helicity eigenstates $\lambda = \pm 1$. The latter is also measured in $D \rightarrow K^* l \nu_l$ [1], as well as in $B \rightarrow K^* J/\psi$ [35] where it can be estimated if the factorization of the nonleptonic matrix elements is assumed [32,36].

¹³With $\zeta_{\perp} = \zeta_{\parallel}$, Soares finds a good agreement between his prediction and the data for $\Gamma_L/\Gamma_{\text{tot}}$ in $B \rightarrow K^* J/\psi$. We feel that in the LCSR expressions (88),(89), the numerical value of the ratio C_{\perp}/C_{\parallel} is accidentally close to 1, because the decay constants and the distribution amplitudes are not very different for the transverse and the vector meson.

For the semileptonic decay the ratio Γ_+/Γ_- reads, thanks to Eq. (109),

$$\frac{\Gamma_+}{\Gamma_-} = \left[\int_{m_V}^{E_{\text{max}}} \left| \sqrt{1 - m_V^2/E^2} - 1 \right|^2 dE \right] / \left[\int_{m_V}^{E_{\text{max}}} \left| \sqrt{1 - m_V^2/E^2} + 1 \right|^2 dE \right], \quad (110)$$

with

$$E_{\text{max}} = \frac{M}{2} \left(1 + \frac{m_V^2}{M^2} \right), \quad (111)$$

while for the nonleptonic decay $B \rightarrow V_1 V_2$ in the factorization assumption, after simplification by Eq. (109), it is given by [36]

$$\frac{\Gamma_+}{\Gamma_-} = \frac{|1 - \sqrt{1 - 1/x^2} + m_{V_1}/(x m_{V_2})|^2}{|1 + \sqrt{1 - 1/x^2} + m_{V_1}/(x m_{V_2})|^2} \quad (112)$$

with

$$x = \frac{M^2 - m_{V_1}^2 - m_{V_2}^2}{2m_{V_1} m_{V_2}}. \quad (113)$$

Note that in the strict $M \rightarrow \infty$ and $E \rightarrow \infty$ limit (that imply $x \rightarrow \infty$), one has $\Gamma_+ = 0$ in both cases, which is reminiscent of the fact that an ultrarelativistic quark produced by the $V-A$ current is purely left handed [33]; the HQET-LEET relation (109) implies that the naive picture at the quark level still holds at the hadron level.

The predictions (110) and (112) are compared with experimental data in Table I. The agreement is striking; as for the ratio Γ_+/Γ_- in $D \rightarrow K^*$, this might be accidental because the nonzero value of this ratio is obtained from the large, although formally subleading, kinematical terms in m_V , the reliability of which is not clear, as stressed above. Moreover the decay $D \rightarrow K^*$ is naively quite far from the $M \rightarrow \infty$ and $E \rightarrow \infty$ limit, and the relation (109) is assumed quite arbitrarily to hold in the whole range of q^2 , otherwise the integration in Eq. (110) could not be performed. Nevertheless these results are encouraging and appeal to investigate further the applications of the HQET-LEET formalism in heavy-to-light decays. Finally, let us stress that the general relations that we have found among the form factors could be very useful for extracting the CKM matrix elements. It has already been shown that the Isgur-Wise relations between the penguin and semileptonic form factors may allow the extraction of $|V_{ub}|$ [4], by looking at $B \rightarrow K^* \gamma$ and $B \rightarrow \rho l \nu_l$. Here we have much more constraints on the form

TABLE I. Predictions of HQET-LEET for the ratio V/A_1 and the ratio Γ_+/Γ_- of the width to the helicity eigenstates $\lambda = \pm 1$ in various decays. The second column quotes the strict $M \rightarrow \infty$ and $E \rightarrow \infty$ limit, obtained by putting $m_V = 0$ in Eqs. (109)–(113); the third is the result which incorporates the $m_V \neq 0$ kinematical mass corrections [Eqs. (110) or (112)], as explained in the text. As for the nonleptonic decays, naive factorization is assumed, along the line of Refs. [32,36].

Observable	$m_V=0$	Eq. (110) or (112)	Exp. data
		$D \rightarrow \rho l \nu_l$	
V/A_1 at $q^2=0$	1	1.70	
Γ_+/Γ_-	0	0.11	
		$D \rightarrow K^* l \nu_l$	
V/A_1 at $q^2=0$	1	1.78	1.85 ± 0.12 [1]
Γ_+/Γ_-	0	0.15	0.16 ± 0.04 [1]
		$D_s \rightarrow \phi l \nu_l$	
V/A_1 at $q^2=0$	1	1.82	1.5 ± 0.5 [1]
Γ_+/Γ_-	0	0.18	
		$B \rightarrow \rho l \nu_l$	
V/A_1 at $q^2=0$	1	1.29	-
Γ_+/Γ_-	0	0.02	-
		$B_s \rightarrow K^* l \nu_l$	
V/A_1 at $q^2=0$	1	1.32	
Γ_+/Γ_-	0	0.02	
		$B \rightarrow K^* J/\psi$ (factorization)	
Γ_+/Γ_-	0	0.005	0.03 ± 0.08 [35]
		$B_s \rightarrow \phi J/\psi$ (factorization)	
Γ_+/Γ_-	0	0.007	

factors, as only one function describe all the $B \rightarrow V_\perp$ transitions. Therefore, a reanalysis of the phenomenological methods already proposed in the literature to extract the CKM couplings could be very interesting in this respect.

VI. CONCLUSION

We have argued that the HQET-LEET formalism seems to be well adapted to the description of heavy-to-light transitions in the large recoil region. In the asymptotic limit of heavy mass M for the initial meson and large energy E for the final one, there are only three independent form factors describing all the ground state heavy-to-light weak current matrix elements. Moreover, a factorization formula $\sim \sqrt{M} z(E)$ is obtained, and a dipole scaling law $\sim 1/E^2$ should come from the usual expectation of the $\sim (1-u)$ behavior for the suppression of the Feynman mechanism. We have checked explicitly that the light-cone sum rule method verifies these constraints, and predicts very simple analytical expressions for the form factors. Finally, there is a first agreement with available experimental data, although more observables are needed to make definite conclusions.

It is clear that there is a lot of work to do in this field. From the theoretical point of view, one should establish the HQET-LEET formalism more firmly than we have done. In particular, the radiative corrections should be handled. Some interesting questions also concern the relations between the LEET Lagrangian and the light-cone quantization.

From the phenomenological point of view, one may look at some observables that are fully predictable in the final hadron large energy limit. The question of how to treat the main corrections is open. Finally it is tempting to search for new methods which would allow us to extract the CKM matrix elements, with the possibility of controlling the theoretical uncertainties thanks to the general constraints on the form factors.

ACKNOWLEDGMENTS

Laboratoire de Physique Théorique is Laboratoire associé au CNRS UMR 8627. We are grateful to L. Lellouch for pointing out Ref. [32b]. The authors acknowledge partial support from the EEC-TMR Program, Contract No. CT98-0169.

- [1] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[2] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); Phys. Rev. D **42**, 2388 (1990).
[3] V. L. Chernyak and I. R. Zhitnitsky, Nucl. Phys. **B345**, 137 (1990).
[4] G. Burdman and J. F. Donoghue, Phys. Lett. B **270**, 55 (1991).
[5] M. J. Dugan and B. Grinstein, Phys. Lett. B **255**, 583 (1991).
[6] V. M. Belyaev, A. Khodjamirian, and R. Rückl, Z. Phys. C **60**, 349 (1993).
[7] A. Ali, V. M. Braun, and H. Simma, Z. Phys. C **63**, 437 (1994).
[8] P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997).
[9] T. M. Aliev, A. Özpineci, and M. Savci, Phys. Rev. D **56**, 4260 (1997).
[10] T. M. Aliev, H. Koru, A. Özpineci, and M. Savci, Phys. Lett. B **400**, 194 (1997).
[11] A. Khodjamirian, R. Rückl, S. Weinzierl, and O. Yakovlev, Phys. Lett. B **410**, 275 (1997).
[12] E. Bagan, P. Ball, and V. M. Braun, Phys. Lett. B **417**, 154 (1998).
[13] P. Ball, J. High Energy Phys. **9809**, 005 (1998); P. Ball and V. M. Braun, Phys. Rev. D **58**, 094016 (1998).
[14] A. Khodjamirian, R. Rückl and C. W. Winhart, Phys. Rev. D **58**, 054013 (1998).
[15] J. Charles *et al.*, Phys. Lett. B **451**, 187 (1999).
[16] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Lett. B **365**, 319 (1996).
[17] U. Aglietti and G. Corbò, hep-ph/9712242; Phys. Lett. B **431**, 166 (1998).
[18] U. Aglietti *et al.*, Phys. Lett. B **432**, 411 (1998); **441**, 371 (1998).
[19] U. Aglietti, Phys. Lett. B **292**, 424 (1992).
[20] O. Pène thanks G. Martinelli for a similar argument.
[21] M. Neubert, Phys. Rep. **245**, 259 (1994).
[22] See Appendix A of G. P. Lepage and S. J. Brodsky, Phys. Rev. D **22**, 2157 (1980).

- [23] V. M. Braun and I. B. Filyanov, *Z. Phys. C* **48**, 239 (1990).
- [24] The SU(2) global symmetry of LEET was already noted in Ref. [5], although the generators were not given explicitly.
- [25] H. Georgi, *Phys. Lett. B* **240**, 447 (1990).
- [26] P. Ball, V. M. Braun, Y. Koike, and K. Tanaka, *Nucl. Phys.* **B529**, 323 (1998).
- [27] P. Ball and V. M. Braun, *Nucl. Phys.* **B543**, 201 (1999).
- [28] V. L. Chernyak and I. R. Zhitnitsky, *Phys. Rep.* **112**, 173 (1984).
- [29] B. Stech, *Phys. Lett. B* **354**, 447 (1995); see also the model of M. Neubert and B. Stech, in *Heavy Flavours II*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1997), p. 294, hep-ph/9705292.
- [30] J. M. Soares, *Phys. Rev. D* **54**, 6837 (1996); J. M. Soares, hep-ph/9810402.
- [31] G. P. Korchemsky and G. Sterman, *Phys. Lett. B* **340**, 96 (1994); A. G. Grozin and G. P. Korchemsky, *Phys. Rev. D* **53**, 1378 (1996).
- [32] R. Aleksan *et al.*, *Phys. Rev. D* **51**, 6235 (1995).
- [33] J. M. Soares, hep-ph/9810421.
- [34] UKQCD Collaboration, L. Del Debbio *et al.*, *Phys. Lett. B* **416**, 392 (1998).
- [35] CLEO Collaboration, C. P. Jessop *et al.*, *Phys. Rev. Lett.* **79**, 4533 (1997); following Ref. [33], the CLEO results that are expressed in the transversity basis are translated into the helicity basis.
- [36] G. Kramer and W. F. Palmer, *Phys. Rev. D* **45**, 193 (1992).