Lepton and quark mass matrices

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We propose a model that all quark and lepton mass matrices have the same zero texture. Namely, their (1,1), (1,3), and (3,1) components are zeros. The mass matrices are classified into two types. Type I is consistent with experimental data in the quark sector. For the lepton sector, if the seesaw mechanism is not used, type II allows a large ν_μ-ν_τ mixing angle. However, severe compatibility with all neutrino oscillation experiments forces us to use the seesaw mechanism. If we adopt the seesaw mechanism, it turns out that type I instead of type II can be consistent with experimental data in the lepton sector too. [S0556-2821(99)05613-1]

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One of the ultimate goals in particle physics is to construct a unified model of quarks and leptons. The phenomenological construction of quark and lepton mass matrices can be an important step toward this goal, which reproduces and predicts direct and indirect observed quantities such as quark and lepton masses, mixing angles, and CP violating phases. In this paper we propose a model that all quark and lepton mass matrices M_u, M_d, M_ν, and M_e [mass matrices of up quarks (u,c,t), down quarks (d,s,b), neutrinos (ν_e,ν_μ,ν_τ), and charged leptons (e,μ,τ), respectively] have the same zero texture [1]. Here $M_\nu=-M_D^T M_R^{-1} M_D$ is the mass matrix of light Majorana neutrinos, which is considered to be constructed via the seesaw mechanism [2] from the neutrino mass matrix

$$\begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix}, \tag{1}$$

where M_D is the Dirac neutrino mass matrix and M_R is the Majorana mass matrix of the right-handed components. M_D and M_R are furthermore assumed to have the same zero-texture matrix as M_ν. This assumption restricts the texture forms as follows:

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \tag{2}$$

$$\begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}, \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$\begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}, \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}.$$

Here *'s indicate suitable nonzero numbers. Among these forms we choose the first one because it is the closest to the nearest-neighbor interaction (NNI) form [3] in which the (2,2) component is also zero. Namely, our texture of the mass matrix is

$$\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \tag{3}$$

Indeed, this matrix leaves its form in the seesaw mechanism as

$$\underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}^T}_{M_D^T} \underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}^{-1}}_{M_R^{-1}} \underbrace{\begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}}_{M_D} = \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}. \tag{4}$$

The nonvanishing (2,2) component distinguishes our form from NNI's. This difference, as will be shown, makes it possible to treat quark and lepton mass matrices universally and consistently with experiments.

Now we assign quark and lepton mass matrices as follows:

$$M_u=\begin{pmatrix} 0 & A_u & 0 \\ A_u & B_u & C_u \\ 0 & C_u & D_u \end{pmatrix}, \quad M_\nu=\begin{pmatrix} 0 & A_\nu & 0 \\ A_\nu & B_\nu & C_\nu \\ 0 & C_\nu & D_\nu \end{pmatrix},$$

$$M_d=P_d\begin{pmatrix} 0 & A_d & 0 \\ A_d & B_d & C_d \\ 0 & C_d & D_d \end{pmatrix}P_d^\dagger$$

$$=\begin{pmatrix} 0 & A_d e^{i\alpha_{12}} & 0 \\ A_d e^{-i\alpha_{12}} & B_d & C_d e^{i\alpha_{23}} \\ 0 & C_d e^{-i\alpha_{23}} & D_d \end{pmatrix},$$

$$M_e=P_e\begin{pmatrix}0 & A_e & 0\\ A_e & B_e & C_e\\ 0 & C_e & D_e\end{pmatrix}P_e^\dagger$$

$$=\begin{pmatrix}0 & A_e e^{i\beta_{12}} & 0\\ A_e e^{-i\beta_{12}} & B_e & C_e e^{i\beta_{23}}\\ 0 & C_e e^{-i\beta_{23}} & D_e\end{pmatrix}, \qquad (5)$$

where $P_d \equiv \mathrm{diag}(e^{i\alpha_1},e^{i\alpha_2},e^{i\alpha_3})$, $\alpha_{ij}\equiv\alpha_i-\alpha_j$, and $P_e \equiv \mathrm{diag}(e^{i\beta_1},e^{i\beta_2},e^{i\beta_3})$, $\beta_{ij}\equiv\beta_i-\beta_j$.

Let us discuss the relations between the following texture's components of mass matrix M:

$$M=\begin{pmatrix}0 & A & 0\\ A & B & C\\ 0 & C & D\end{pmatrix} \qquad (6)$$

and its eigenmass m_i. They satisfy

$$B+D=m_1+m_2+m_3,$$

$$BD-C^2-A^2=m_1m_2+m_2m_3+m_3m_1,$$

$$DA^2=-m_1m_2m_3. \qquad (7)$$

Therefore, the mass matrix is classified into two types by choosing B and D as follows:

$$[\text{type I}](B\ \text{large})\qquad B=m_2,\ \ D=m_3+m_1,$$

$$[\text{type II}](B\ \text{small})\qquad B=m_1,\ \ D=m_3+m_2. \qquad (8)$$

Here we do not accept the case of $B=m_1+m_2$ and $D=m_3$ since in this case C becomes zero and this matrix is out of our texture. We adopt type I for quark mass matrices. For the lepton sector we adopt type I and type II mass matrices for the cases with and without the seesaw mechanism, respectively. We proceed to discuss this in detail.

Let us discuss the quark sector first. The mass matrices of type I ($B=m_2$, $D=m_3+m_1$) explain the quark sector consistently as will be shown. Assigning a definite value $B=m_2$ and $D=m_3+m_1$ in Eqs. (7) for type I, we obtain

$$A=\sqrt{\frac{(-m_1)m_2m_3}{m_3+m_1}},\qquad C=\sqrt{\frac{(-m_1)m_3(m_3-m_2+m_1)}{m_3+m_1}}. \qquad (9)$$

Then the mass matrix of type I becomes

$$M=\begin{pmatrix}0 & \sqrt{\frac{m_1m_2m_3}{m_3-m_1}} & 0\\ \sqrt{\frac{m_1m_2m_3}{m_3-m_1}} & m_2 & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{m_3-m_1}}\\ 0 & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{m_3-m_1}} & m_3-m_1\end{pmatrix}$$

$$\simeq\begin{pmatrix}0 & \sqrt{m_1m_2} & 0\\ \sqrt{m_1m_2} & m_2 & \sqrt{m_1m_3}\\ 0 & \sqrt{m_1m_3} & m_3-m_1\end{pmatrix}\quad (\text{for } m_3\gg m_2\gg m_1). \qquad (10)$$

Here we have transformed m_1 into $-m_1$ by rephasing. M is diagonalized by an orthogonal matrix O as

$$O^T\begin{pmatrix}0 & \sqrt{m_1m_2} & 0\\ \sqrt{m_1m_2} & m_2 & \sqrt{m_1m_3}\\ 0 & \sqrt{m_1m_3} & m_3-m_1\end{pmatrix}O=\begin{pmatrix}-m_1 & 0 & 0\\ 0 & m_2 & 0\\ 0 & 0 & m_3\end{pmatrix}, \qquad (11)$$

with

$$O=\begin{pmatrix} \sqrt{\frac{m_2m_3^2}{(m_2+m_1)(m_3^2-m_1^2)}} & \sqrt{\frac{m_1m_3(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)(m_3-m_1)}} & \sqrt{\frac{m_1^2m_2}{(m_3-m_2)(m_3^2-m_1^2)}} \\ -\sqrt{\frac{m_1m_3}{(m_2+m_1)(m_3+m_1)}} & \sqrt{\frac{m_2(m_3-m_2-m_1)}{(m_2+m_1)(m_3-m_2)}} & \sqrt{\frac{m_1m_3}{(m_3-m_2)(m_3+m_1)}} \\ \sqrt{\frac{{m_1}^2(m_3-m_2-m_1)}{(m_2+m_1)(m_3^2-m_1^2)}} & -\sqrt{\frac{m_1m_2m_3}{(m_3-m_2)(m_2+m_1)(m_3-m_1)}} & \sqrt{\frac{(m_3)^2(m_3-m_2-m_1)}{(m_3^2-m_1^2)(m_3-m_2)}} \end{pmatrix}$$

$$\simeq\begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1^2m_2}{m_3^3}} \\ -\sqrt{\frac{m_1}{m_2}} & 1 & \sqrt{\frac{m_1}{m_3}} \\ \sqrt{\frac{m_1^2}{m_2m_3}} & -\sqrt{\frac{m_1}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1). \tag{12}$$

The mass matrices for quarks, M_d and M_u, are assumed to be of type I as follows:

$$M_d\simeq P_d\begin{pmatrix} 0 & \sqrt{m_dm_s} & 0 \\ \sqrt{m_dm_s} & m_s & \sqrt{m_dm_b} \\ 0 & \sqrt{m_dm_b} & m_b-m_d \end{pmatrix}P_d^\dagger, \quad M_u\simeq\begin{pmatrix} 0 & \sqrt{m_um_c} & 0 \\ \sqrt{m_um_c} & m_c & \sqrt{m_um_t} \\ 0 & \sqrt{m_um_t} & m_t-m_u \end{pmatrix}, \tag{13}$$

where m_d, m_s, and m_b are down quark masses and m_u, m_c, and m_t are up quark masses. Those M_d and M_u are diagonalized by matrices P_dO_d and O_u, respectively. Here the orthogonal matrices O_d and O_u which diagonalize $P_d^\dagger M_d P_d$ and M_u are obtained from Eq. (12) by replacing m_1, m_2, m_3 by m_d, m_s, m_b and by m_u, m_c, m_t, respectively. In this case, the Cabibbo-Kobayashi-Maskawa (CKM) [4] quark mixing matrix V can be written as

$$V=P_q^{-1}P_d^{-1}O_u^TP_dO_dP_q\simeq\begin{pmatrix} |V_{11}| & |V_{12}| & |V_{13}|e^{-i\phi} \\ -|V_{12}| & |V_{22}| & |V_{23}| \\ |V_{12}V_{23}|-|V_{13}|e^{i\phi} & -|V_{23}| & |V_{33}| \end{pmatrix}, \tag{14}$$

where the P_d^{-1} factor is included to put V in the form with diagonal elements real to a good approximation. Furthermore, the P_q^{-1} and $P_q=\mathrm{diag}(e^{i\phi_1},e^{i\phi_2},e^{i\phi_3})$ with $\phi_1-\phi_2=\arg(P_d^{-1}O_u^TP_dO_d)_{12}$ and $\phi_1-\phi_3=\arg(P_d^{-1}O_u^TP_dO_d)_{23}$ are for the choice of phase convention as Eq. (14). The explicit forms and numerical center values of the components of V are

$$\begin{aligned} |V_{12}|=&\Bigg|\sqrt{\frac{m_d(m_b+m_d)(m_b-m_s-m_d)}{(m_s+m_d)(m_b^2-m_bm_s-m_d^2)}}-\sqrt{\frac{m_u(m_t+m_u)(m_t-m_c-m_u)}{(m_c+m_u)(m_t^2-m_tm_c-m_u^2)}}e^{-i\alpha_{12}} \\ &-\sqrt{\frac{m_u^2(m_t^2-m_u^2)(m_t-m_c-m_u)}{(m_c+m_u)(m_t^2-m_tm_c-m_u^2)^2}}\sqrt{\frac{m_d(m_b-m_d)}{m_b^2-m_bm_s-m_d^2}}e^{-i\alpha_{13}}\Bigg| \\ \simeq&\left|\sqrt{\frac{m_d}{m_s}}-\sqrt{\frac{m_u}{m_c}}e^{-i\alpha_{12}}\right|=0.17\text{–}0.28, \end{aligned}$$

$$\begin{aligned} |V_{23}|=&\left|\sqrt{\frac{m_u(m_t+m_u)(m_t-m_c-m_u)}{(m_c+m_u)(m_t^2-m_tm_c-m_u^2)}}\sqrt{\frac{m_d^2m_s}{(m_b-m_s)(m_b^2-m_d^2)}}e^{i\alpha_{12}}+\sqrt{\frac{m_d(m_b-m_d)}{m_b^2-m_bm_s-m_d^2}}-\sqrt{\frac{m_u(m_t-m_u)}{m_t^2-m_tm_c-m_u^2}}e^{-i\alpha_{23}}\right| \\ \simeq&\left|\sqrt{\frac{m_d}{m_b}}-\sqrt{\frac{m_u}{m_t}}e^{-i\alpha_{23}}\right|=0.036\text{–}0.043, \end{aligned}$$

$$|V_{13}|=\Bigg|\sqrt{\frac{m_d^2 m_s}{(m_b-m_s)(m_b^2-m_d^2)}}+\sqrt{\frac{m_u^2(m_t^2-m_u^2)(m_t-m_c-m_u)}{(m_c+m_u)(m_t^2-m_t m_c-m_u^2)^2}}e^{-\alpha_{13}}$$
$$-\sqrt{\frac{m_u(m_t+m_u)(m_t-m_c-m_u)}{(m_c+m_u)(m_t^2-m_t m_c-m_u^2)}}\sqrt{\frac{m_d(m_b-m_d)}{m_b^2-m_b m_s-m_d^2}}e^{-i\alpha_{12}}\Bigg|$$
$$\simeq\left|\sqrt{\frac{m_d^2 m_s}{m_b^3}}-\sqrt{\frac{m_u}{m_c}}\left(\sqrt{\frac{m_d}{m_b}}-\sqrt{\frac{m_u}{m_t}}e^{-i\alpha_{23}}\right)e^{-i\alpha_{12}}\right|=0.0021\text{–}0.0025,$$

$$\cos\phi\simeq\frac{|V_{13}|^2+|V_{12}|^2|V_{23}|^2-|V_{31}|^2}{2|V_{12}||V_{23}||V_{13}|}\simeq\frac{|V_{12}|^2+m_u/m_c-m_d/m_s}{2|V_{12}|\sqrt{m_u/m_c}}=-1\text{–}1. \tag{15}$$

Here we have used the running quark mass at $\mu=m_Z$ [5]:

$$m_u(m_Z)=2.33^{+0.42}_{-0.45}\ \text{MeV},\quad m_c(m_Z)=677^{+56}_{-61}\ \text{MeV},\quad m_t(m_Z)=181\pm13\ \text{GeV},$$
$$m_d(m_Z)=4.69^{+0.60}_{-0.66}\ \text{MeV},\quad m_s(m_Z)=93.4^{+11.8}_{-13.0}\ \text{MeV},\quad m_b(m_Z)=3.00\pm0.11\ \text{GeV}. \tag{16}$$

Let us compare Eqs. (15) with the experimental values [6]:

$$|V_{12}|_{\text{expt}}=0.217\text{–}0.224,\quad |V_{23}|_{\text{expt}}=0.036\text{–}0.042,$$
$$|V_{13}|_{\text{expt}}=0.0018\text{–}0.0045\quad (90\%\ \text{C.L.}). \tag{17}$$

It is remarkable that the very heavy top quark mass raises no inconsistency in our model. The reason is as follows. In $|V_{23}|$, the first term of right-hand side in Eq. (15) ($\sqrt{m_d/m_b}=0.034$) is nearly equal to the experimental value ($|V_{23}|_{\text{expt}}=0.036$–$0.042$); so a heavy top quark mass does not make any trouble. Whereas in the case of type II and also the Fritzsch model [7], the first term of V_{23} becomes $\sqrt{m_s/m_b}=0.18$. So in order to adjust to the experimental value, the second term must be of the same order as the first term to cancel a large part of the first term. Thus the top quark could not have a very heavy mass.

If we adopt only the central values of quark masses in Eq. (16), the compatibility of our prediction, Eq. (15), with the experimental values, Eq. (17), imposes some constraints on α_{ij}. They are depicted in Fig. 1 in the shaded strip in the α_{13}-α_{23} plane. In this figure we have superimposed the rephasing-invariant Jarlskog parameter J of the quark sector, $J=\text{Im}(V_{12}V_{22}^*V_{13}^*V_{23})$ [8]. However, these restrictions are very sensitive to the errors of mass values and are not affirmative at least at this stage. Contours represent the value of J from -2.3×10^{-5} to 2.3×10^{-5}. The above restriction on α_{ij}, therefore, gives the bound on J as

$$1.6\times10^{-5}\lesssim|J|\lesssim2.2\times10^{-5}. \tag{18}$$

Using the popular approximation due to Wolfenstein [9], the CKM quark mixing matrix can be written in terms of only four real parameters:

$$\begin{pmatrix} V_{11} & V_{12} & V_{13}\\ V_{21} & V_{22} & V_{23}\\ V_{31} & V_{32} & V_{33}\end{pmatrix}\simeq\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \mathcal{A}\lambda^3(\rho-i\eta)\\ -\lambda & 1-\frac{\lambda^2}{2} & \mathcal{A}\lambda^2\\ \mathcal{A}\lambda^3(1-\rho-i\eta) & -\mathcal{A}\lambda^2 & 1\end{pmatrix}. \tag{19}$$

The measurement of the ρ and η parameters is usually associated with the determination of the only unknown vertex of a triangle in the ρ-η plane whose other two vertices are in (0,0) and (1,0) [10]. This triangle is called the unitarity triangle. Changing freely α_{13} and α_{23} in Eq. (15), the predicted points sweep out the light and dark gray regions (Fig. 2).

Next let us discuss the lepton sector. We develop our arguments first without the seesaw mechanism. The mass matrix of leptons is assumed to be of type II. Assigning $B=m_1$ and $D=m_3+m_2$ (type II) in Eq. (8), we obtain, from Eq. (7),

$$A=\sqrt{\frac{m_1(-m_2)m_3}{m_3+m_2}},\quad C=\sqrt{\frac{(-m_2)m_3(m_3+m_2-m_1)}{m_3+m_2}}. \tag{20}$$

Then, we obtain the mass matrix M of type II and the orthogonal matrix O that diagonalize it, which are expressed in terms of the mass eigenvalue m_i as

$$M=\begin{pmatrix} 0 & \sqrt{\frac{m_1m_2m_3}{m_3-m_2}} & 0 \\ \sqrt{\frac{m_1m_2m_3}{m_3-m_2}} & m_1 & \sqrt{\frac{m_2m_3(m_3-m_2-m_1)}{m_3-m_2}} \\ 0 & \sqrt{\frac{m_2m_3(m_3-m_2-m_1)}{m_3-m_2}} & m_3-m_2 \end{pmatrix} \simeq \begin{pmatrix} 0 & \sqrt{m_1m_2} & 0 \\ \sqrt{m_1m_2} & m_1 & \sqrt{m_2m_3} \\ 0 & \sqrt{m_2m_3} & m_3-m_2 \end{pmatrix}, \tag{21}$$

$$O=\begin{pmatrix} \sqrt{\frac{m_2m_3(m_3-m_2-m_1)}{(m_1+m_2)(m_3-m_1)(m_3-m_2)}} & \sqrt{\frac{m_1m_3^2}{(m_1+m_2)(m_3^2-m_2^2)}} & \sqrt{\frac{m_2^2m_1}{(m_3-m_1)(m_3^2-m_2^2)}} \\ \sqrt{\frac{m_1(m_3-m_2-m_1)}{(m_1+m_2)(m_3-m_1)}} & -\sqrt{\frac{m_2m_3}{(m_1+m_2)(m_3+m_2)}} & \sqrt{\frac{m_2m_3}{(m_3-m_1)(m_3+m_2)}} \\ -\sqrt{\frac{m_2m_1m_3}{(m_3-m_1)(m_1+m_2)(m_3-m_2)}} & \sqrt{\frac{m_2^2(m_3-m_1-m_2)}{(m_1+m_2)(m_3^2-m_2^2)}} & \sqrt{\frac{(m_3)^2(m_3-m_1-m_2)}{(m_3-m_1)(m_3^2-m_2^2)}} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1 & \sqrt{\frac{m_1}{m_2}} & \sqrt{\frac{m_1m_2^2}{m_3^3}} \\ \sqrt{\frac{m_1}{m_2}} & -1 & \sqrt{\frac{m_2}{m_3}} \\ -\sqrt{\frac{m_1}{m_3}} & \sqrt{\frac{m_2}{m_3}} & 1 \end{pmatrix} \quad (\text{for } m_3 \gg m_2 \gg m_1), \tag{22}$$

with

$$O^TMO=\begin{pmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{23}$$

where we have transformed m_2 into $-m_2$. The components (2,3) and (3,2) of O are not small compared with $\sqrt{m_1/m_3}$ in type I. Therefore, as a result of this large mixing, type II can be consistent with the large ν_μ-ν_τ mixing angle solution in atmospheric neutrino experiments as shown later.

The mass matrices of charged leptons and neutrinos are assumed to be of type II as follows:

$$M_e \simeq P_e \begin{pmatrix} 0 & \sqrt{m_em_\mu} & 0 \\ \sqrt{m_em_\mu} & m_e & \sqrt{m_\mu m_\tau} \\ 0 & \sqrt{m_\mu m_\tau} & m_\tau-m_\mu \end{pmatrix} P_e^\dagger,$$

$$M_\nu \simeq \begin{pmatrix} 0 & \sqrt{m_1m_2} & 0 \\ \sqrt{m_1m_2} & m_1 & \sqrt{m_2m_3} \\ 0 & \sqrt{m_2m_3} & m_3-m_2 \end{pmatrix}, \tag{24}$$

where m_e, m_μ, and m_τ are charged lepton masses and m_1, m_2, and m_3 are neutrino masses. Those M_e and M_ν are diagonalized by matrices P_eO_e and O_ν, respectively. Here the orthogonal matrix O_ν is obtained from Eq. (22) by taking m_i as the neutrino mass and O_e by replacing m_1, m_2, m_3 by m_e, m_μ, m_τ. In this case, the lepton mixing matrix U [hereafter we call it the Maki-Nakagawa-Sakata (MNS) mixing matrix [11]] is given by

$$U=P_l^\dagger P_e^\dagger O_e^T P_e O_\nu P_l = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}, \tag{25}$$

where $P_l=\mathrm{diag}(1,i,1)$ is included to have positive neutrino mass. The $P_l^\dagger P_e^\dagger$ factor leads U to the form whose diagonal elements are real to a good approximation. We obtain the expressions of some elements of U as follows:

$$U_{12} \simeq i\left(\sqrt{\frac{m_1}{m_2}} - \sqrt{\frac{m_e}{m_\mu}}e^{i\beta_{12}}\right),$$

$$U_{23} \simeq -i\left(-\sqrt{\frac{m_2}{m_3}} + \sqrt{\frac{m_\mu}{m_\tau}}e^{i\beta_{23}}\right),$$

$$U_{13} \simeq \sqrt{\frac{m_e}{m_\mu}}e^{i\beta_{12}}\left(\sqrt{\frac{m_2}{m_3}} - \sqrt{\frac{m_\mu}{m_\tau}}e^{i\beta_{23}}\right). \tag{26}$$

For example, substituting the neutrino masses

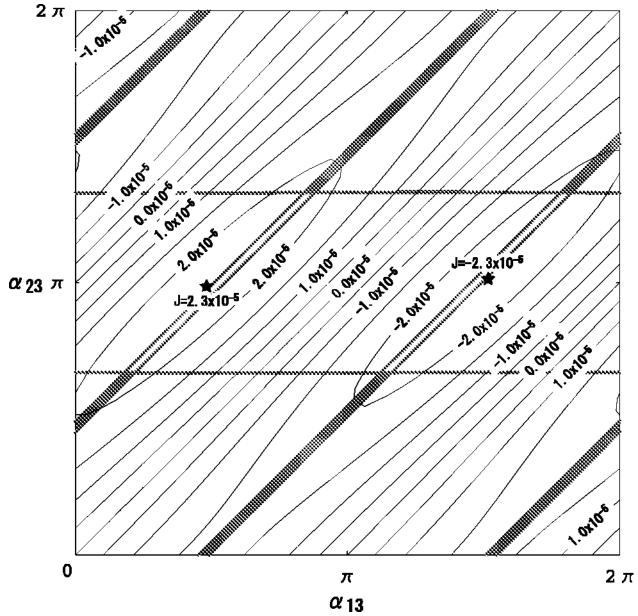

FIG. 1. The allowed region on the α_{13}-α_{23} plane is depicted by the shaded areas. In the allowed region, the contours indicate the rephasing invariant of the Jarlskog parameter J $[\equiv \mathrm{Im}(V_{12}V_{22}^{*}V_{13}^{*}V_{23})]$ of the quark sector.

$$m_1 = 1.4\times 10^{-4}\ \text{eV},\quad m_2 = 3.2\times 10^{-3}\ \text{eV},$$

$$m_3 = 7.1\times 10^{-2}\ \text{eV}, \tag{27}$$

and the charged lepton masses $m_e = 0.51$ MeV, $m_\mu = 106$ MeV, $m_\tau = 1.77$ GeV, into Eqs. (25) we obtain

$$|U_{12}| = 0.14\text{–}0.28,\quad |U_{23}| = 0.033\text{–}0.46,$$

$$|U_{13}| = 0.023\text{–}0.032. \tag{28}$$

Here we have used $\Delta m^2_{\text{atm}} = m_3^2 - m_2^2 = 5.0\times 10^{-3}$ eV^2 and $\Delta m^2_{\text{solar}} = m_2^2 - m_1^2 = 1.0\times 10^{-5}$ eV^2 with the assumption that $m_1 \ll m_2 \ll m_3$ and $m_1/m_2 = m_2/m_3$. Let us compare this prediction with the experimental values [12]:

$$|U_{12}|_{\text{expt}} = 0\text{–}0.71,\quad |U_{23}|_{\text{expt}} = 0.52\text{–}0.87,$$

$$|U_{13}|_{\text{expt}} = 0\text{–}0.22. \tag{29}$$

Here we have combined the constraints from a recent CHOOZ reactor experiment [13] and Super-Kamiokande atmospheric neutrino experiment [14].

Though the lepton mass matrices M_e and M_ν of type II lead to large ν_μ-ν_τ mixing, $|U_{23}|$ is still small compared with the experimental value. This trouble is resolved via the seesaw mechanism. In the seesaw mechanism, we have additional free parameters even in our model. So we set the following assumptions guided by the atmospheric neutrino oscillation experiments, which lead to a fairly large ν_μ-ν_τ mixing: (a) The mass matrices M_e, M_D, and M_R belong to type I, instead of type II, similar to quark mass matrices; (b) the mass eigenvalues of M_D and M_R satisfy

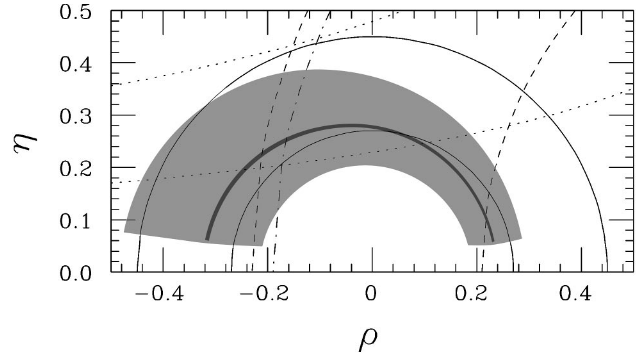

FIG. 2. The vertex position of unitarity triangle predicted by our model is superimposed on the diagram restricted by hadron experiments. Our predictions are obtained by changing α_{13} and α_{23} freely in Eq. (15) with no approximation. If each quark mass takes the center values in Eqs. (16), the dark gray region is allowed. On the other hand, taking the error of each quark mass into consideration, we obtain the light gray region.

$$m_{D3}:m_{D2}:m_{D1} = 1:x:x^2, \tag{30}$$

$$m_{R3}:m_{R2}:m_{R1} = 1:x^2:x^3. \tag{31}$$

Here m_{Di} and m_{Ri} are eigenvalues of M_D and M_R, respectively, and x is a small parameter.

It is noted from assumption (a) that M_ν itself is out of type I via the seesaw mechanism. If we use the assumption that M_e, M_D, and M_R belong to type II instead of type I, we cannot accommodate m_{R3}, m_{R2}, and m_{R1} to a large ν_μ-ν_τ mixing. Conversely, a large mixing enforces on us m_{R1} and m_{R2} of the same order, where we cannot distinguish type II from type I.

Using assumptions (a) and (b), we obtain

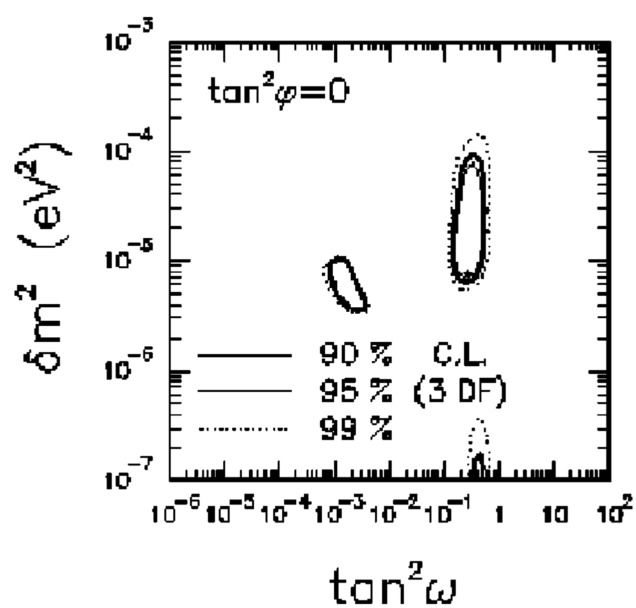

FIG. 3. The solid line and dotted line show 90% C.L. and 99% C.L., respectively, which were derived from a three-flavor analysis of solar neutrino deficit experiments [15]. The star indicates our prediction.

$$M_D \stackrel{\text{(a)}}{=} \begin{pmatrix} 0 & \sqrt{\frac{m_{D1}m_{D2}m_{D3}}{m_{D3}-m_{D1}}} & 0 \\ \sqrt{\frac{m_{D1}m_{D2}m_{D3}}{m_{D3}-m_{D1}}} & m_{D2} & \sqrt{\frac{m_{D1}m_{D3}(m_{D3}-m_{D2}-m_{D1})}{m_{D3}-m_{D1}}} \\ 0 & \sqrt{\frac{m_{D1}m_{D3}(m_{D3}-m_{D2}-m_{D1})}{m_{D3}-m_{D1}}} & m_{D3}-m_{D1} \end{pmatrix} \tag{32}$$

$$\stackrel{\text{(b)}}{\simeq} m_{D3} \begin{pmatrix} 0 & x\sqrt{x} & 0 \\ x\sqrt{x} & x & x \\ 0 & x & 1 \end{pmatrix} \tag{33}$$

and, similarly,

$$M_R \simeq m_{R3} \begin{pmatrix} 0 & x^2\sqrt{x} & 0 \\ x^2\sqrt{x} & x^2 & x\sqrt{x} \\ 0 & x\sqrt{x} & 1 \end{pmatrix}. \tag{34}$$

Then the neutrino mass matrix M_ν is given by

$$M_\nu = -M_D^T M_R^{-1} M_D = -\frac{(m_{D3})^2}{m_{R3}} \begin{pmatrix} 0 & \sqrt{x} & 0 \\ \sqrt{x} & 1+(\sqrt{x}-x)^2 & 1-(\sqrt{x}-x) \\ 0 & 1-(\sqrt{x}-x) & 1 \end{pmatrix}. \tag{35}$$

The orthogonal matrix which diagonalizes Eq. (35) is

$$O_\nu \simeq \begin{pmatrix} -\frac{1}{12}\frac{-72+48\sqrt{3}-9\sqrt{x}+5\sqrt{3}\sqrt{x}}{(\sqrt{3}-1)(3-\sqrt{3})^{3/2}} & \frac{1}{12}\frac{72+48\sqrt{3}+5\sqrt{3}\sqrt{x}+9\sqrt{x}}{(1+\sqrt{3})(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{4}x^{1/2}+\frac{\sqrt{2}}{8}x \\ \frac{1}{24}\frac{-72+48\sqrt{3}+21\sqrt{x}-7\sqrt{3}\sqrt{x}}{(3-\sqrt{3})^{3/2}} & \frac{1}{24}\frac{72+48\sqrt{3}-21\sqrt{x}-7\sqrt{3}\sqrt{x}}{(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{2}+\frac{5\sqrt{2}}{32}x \\ -\frac{1}{24}\frac{-72+48\sqrt{3}-15\sqrt{x}+5\sqrt{3}\sqrt{x}}{(3-\sqrt{3})^{3/2}} & -\frac{1}{24}\frac{72+48\sqrt{3}+15\sqrt{x}+5\sqrt{3}\sqrt{x}}{(3+\sqrt{3})^{3/2}} & \frac{\sqrt{2}}{2}-\frac{7\sqrt{2}}{32}x \end{pmatrix}, \tag{36}$$

and the eigenmass is

$$m_1 \simeq \frac{m_{D3}^2}{m_{R3}}\left\{\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\sqrt{x}-\left(\frac{3}{8}-\frac{\sqrt{3}}{24}\right)x\right\},$$

$$m_2 \simeq \frac{m_{D3}^2}{m_{R3}}\left\{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\sqrt{x}-\left(\frac{3}{8}+\frac{\sqrt{3}}{24}\right)x\right\}, \quad m_3 \simeq \frac{m_{D3}^2}{m_{R3}}\left\{2-\sqrt{x}+\frac{7}{4}x\right\}. \tag{37}$$

For numerical estimation we assume that the mass pattern, Eq. (30), is the same as that of an up quark;

$$m_t(m_Z):m_c(m_Z):m_u(m_Z) = 1:x:x^2 \quad (x \simeq 0.0036), \tag{38}$$

and, therefore, $m_{D3} = k \times m_t(m_Z)$, $m_{D2} = kx \times m_t(m_Z)$, and $m_{D1} = kx^2 \times m_t(m_Z)$. Using assumption (a), that M_e belongs to type I, the mass ratios of light Majorana neutrinos, the MNS matrix U, and the rephasing-invariant Jarlskog parameter J of the lepton sector become

$$m_3 : m_2 : -m_1 \simeq 1.0 : 0.04 : 0.01, \tag{39}$$

$$U=P_e^\dagger O_e^T P_e O_\nu \simeq \begin{pmatrix} -0.88-0.02e^{-i\beta_{12}} & 0.46-0.04e^{-i\beta_{12}} & 0.022-0.049e^{-i\beta_{12}} \\ 0.34-0.06e^{i\beta_{12}} & 0.62+0.03e^{i\beta_{12}}+0.01e^{-i\beta_{23}} & 0.71-0.01e^{-i\beta_{23}} \\ -0.31+0.01e^{i\beta_{23}} & -0.64+0.01e^{i\beta_{23}} & 0.71+0.01e^{i\beta_{23}} \end{pmatrix}, \tag{40}$$

and $|J|\lesssim 0.01$. Here we have assumed that the changes of lepton masses and the MNS mixing from $\mu=m_Z$ to $\mu=\mathrm{MeV}$ are very small. At this stage only one parameter, m_{R3}, still remains free. It will be determined from $\Delta m_{32}^2=5.0\times10^{-3}$ eV^2 as

$$m_{R3}=k^2\times(9.0\times10^{23})\ \ \mathrm{eV}. \tag{41}$$

Thus we have fixed parameters so as to adjust the atmospheric neutrino oscillation experiments. Assumptions (a) and (b) are not unique and their justification is checked by the compatibility with solar neutrino deficit experiments. From Eqs. (37), (40), and (41), we have the restrictive prediction

$$\Delta m_{21}^2\simeq 7.8\times10^{-6}\ \ \mathrm{eV}^2,\quad \tan^2\varphi\equiv\frac{|U_{13}|^2}{|U_{23}|^2+|U_{33}|^2}\simeq 0,$$

$$\tan^2\omega\equiv\frac{|U_{12}|^2}{|U_{11}|^2}\simeq 0.27, \tag{42}$$

which are superimposed on the analysis by Fogli *et al.* [15] (Fig. 3). The star in Fig. 3 indicates our prediction. The position of the star has been determined from atmospheric neutrino experiments and was free from solar neutrino deficit experiments. Nevertheless, its position is in the allowed region of solar neutrino experiments.

Concluding remarks are in order. We started with the same type of four texture zero-mass matrices both for quarks and leptons. They were classified into types I and II. Type I explains quark sector consistently. For the lepton sector type II, on the other hand, reproduces qualitatively large lepton mixing. However, the best fitting with experimental data requires the seesaw mechanism in the lepton sector with type I mass matrices similarly to quarks.

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