Bimaximal neutrino mixing and neutrino mass matrix

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We show that the bimaximal neutrino mixing pattern suggested by solar and atmospheric neutrino data can be derived from the maximal, symmetric, four-neutrino mixing in the limit that one of the neutrinos is made heavy. Imposing the constraints of no neutrinoless double beta decay and a 20% hot dark matter component of the universe leads to the three-neutrino mass matrix recently suggested by Georgi and Glashow. Our result can be useful in constructing theoretical models for the bimaximal pattern. We illustrate this by a simple example. [S0556-2821(99)00713-4]

PACS number(s): 14.60.Pq

The recent observation of neutrino oscillations at Super-Kamiokande [1] has been a great source of excitement in particle physics for two reasons: first it provides the best evidence to date for a nonzero neutrino mass which, in turn, is the first sign for new physics beyond the standard model; the second reason is that the Super-K analysis seems to require maximal mixing between ν_μ and ν_τ. The latter mixing pattern is very different from that observed in the quark sector, suggesting a more fundamental distinction between the physics of the neutrino and the charged fermion masses. Note that ultralightness of the neutrinos is another such distinction and seems to require the novel phenomena of the seesaw mechanism to understand it.

The apparent maximal mixing between the ν_μ and ν_τ has led to speculation that the entire mixing pattern in the three-neutrino sector may be essentially maximal. In this case, the solar neutrino puzzle will be solved either by the vacuum or large angle Mikheyev-Smirnov-Wolfenstein (MSW) mechanism [2]. The hot dark matter of the universe which may require a total neutrino mass in the range 4 – 5 eV will then imply that all three neutrino species are degenerate in mass [3]. A crucial test of this model will also be provided by further improvement of the solar neutrino data; e.g., day-night variation will test the large angle MSW mechanism which is one possibility in this case. The incorporation of the Liquid Scintillation Neutrino Detector (LSND) data [4] seems to require a fourth (sterile) neutrino, though there have been attempts to fit all data within a three-neutrino, roughly maximal, mixing framework, using ''indirect neutrino oscillations.'' For a recent version, see, e.g., [5].

Three possible neutrino mass matrices U_ν defined via

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_\nu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad (1)$$

have been widely discussed in the literature.

Case (A) [6]:

$$U_\nu = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 \\ \dfrac{1}{\sqrt{6}} & \dfrac{1}{\sqrt{6}} & -\dfrac{2}{\sqrt{6}} \\ \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}} & \dfrac{1}{\sqrt{3}} \end{pmatrix}. \qquad (2)$$

We will call this the democratic mixing.

Case (B) [7]:

$$U_\nu = \begin{pmatrix} \dfrac{1}{\sqrt{2}} & -\dfrac{1}{\sqrt{2}} & 0 \\ \dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{2} & \dfrac{1}{2} & -\dfrac{1}{\sqrt{2}} \end{pmatrix}. \qquad (3)$$

This has been called in the literature bimaximal mixing [7].

Case (C) [8]:

$$U_\nu = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \\ 1 & 1 & 1 \end{pmatrix}, \qquad (4)$$

where $\omega = e^{2\pi i/3}$; we will call this the maximal symmetric mixing. Such a mixing matrix $U_{(n)}$ emerges when we impose maximal ($1/n$) suppression of each neutrino flavor [9] for any prime n. If $n = p_1^{m_1} \cdots p_k^{m_k}$, where p_i are prime integers, then the maximal mixing symmetric (MMS) matrix will consist of a direct product of m_1 MMS matrices corresponding to p_1 with m_2 MMS matrices corresponding to p_2 etc.

$U_{(3)}$ [case (C)] may be marginal if we take the CHOOZ [10] and the atmospheric neutrino data into account. Cases

(A) and (B) are, however, fully consistent with the CHOOZ and all other data as long as LSND data are not included.

Should any of these mixing patterns be confirmed by further data, a key theoretical challenge would be to understand them from a fundamental gauge theory framework. Clearly, one must look for some underlying symmetry that would lead to the entries in the above mixing matrices. In a previous paper we showed that the democratic mixing matrix [11] can be understood if the lepton sector of the gauge model has a discrete interchange symmetry among three generations. No such simple symmetry among three generations of leptons is apparent for the bimaximal case (A). It is our goal in this paper to show that the bimaximal pattern emerges from a symmetric 4×4 maximal mixing matrix if a particular linear combination of the neutrinos is made heavy and decouples. Needless to say, the CERN e^+e^- collider LEP and SLAC Linear collider (SLC) data would imply that if there is a fourth generation, the neutrinos of the fourth generation must be heavy enough not to contribute to the Z width measured at LEP and SLC. Thus its decoupling from the light neutrino mass matrix is to be expected on phenomenological grounds. This result may provide us a clue regarding how one can proceed in constructing a gauge model for the bimaximal pattern. In particular, it highlights the need for the existence of a fourth generation for this purpose. We give an example of such a construction.

To introduce our line of reasoning, we note that for two neutrino flavors any mass matrix of the form

$$M_2 = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \tag{5}$$

leads to the maximal mixing matrix of the form

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \equiv U_2.$$

The unique maximal mixing 4×4 mixing matrix is then given by the direct product

$$U_4 = U_2 \otimes U_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \tag{6}$$

U_4 arises naturally in a diagonalization of a 4×4 mass matrix of the form $M_4 = M_2 \otimes M_2'$, and

$$M_2' = \begin{pmatrix} c & d \\ d & c \end{pmatrix}.$$

In the $(\nu_e^0, \nu_\mu^0, \nu_\tau^0, \nu_E^0)$ basis,

$$M_4 = \begin{pmatrix} A & B & C & D \\ B & A & D & C \\ C & D & A & B \\ D & C & B & A \end{pmatrix}, \tag{7}$$

with $A \equiv ac$, etc. Let us now add to the four-neutrino mass Lagrangian a heavy mass for the combination $\nu_e^0 - \nu_E^0$:

$$\mathcal{L} = M(\nu_e^0 - \nu_E^0)(\nu_e^0 - \nu_E^0). \tag{8}$$

Calling $\nu_\pm \equiv (\nu_e^0 \pm \nu_E^0)/\sqrt{2}$, we can rewrite the above mass matrix in terms of $\nu_+, \nu_\mu, \nu_\tau, \nu_-$ and decouple the field ν_-. Identifying $\nu_+ = \nu_e$, we get, for the ν_e, ν_μ, ν_τ mass matrix,

$$M_3 = \begin{pmatrix} A+D & F & F \\ F & A & D \\ F & D & A \end{pmatrix}, \tag{9}$$

where $F \equiv (C+B)/\sqrt{2}$. Diagonalizing this 3×3 mass matrix M_3, we find that the neutrino mixing matrix corresponds to the bimaximal case. This proves the main assertion of our model.

The eigenvalues are $m_1 = A + D + \sqrt{2}F$, $m_2 = A + D - \sqrt{2}F$, $m_3 = A - D$. We can accommodate a hot dark matter in this model if all three masses are almost degenerate. Combining this with the requirement that the model satisfy the neutrinoless double beta decay constraint [12] leads to the following ordering of the parameters:

$$A + D \simeq \delta_2 : F \simeq \sqrt{2}A + \delta_1, \tag{10}$$

where the S.K. atmospheric neutrino fit requires that $\delta_1 \simeq 10^{-3}$ eV (assuming that the overall common mass for the neutrinos is $\simeq 1-2$ eV). Similarly, the solution to the solar neutrino puzzle via the large angle MSW solution requires that $\delta \simeq 10^{-4} - 10^{-5}$ eV.

It is interesting to note that after enforcing the neutrinoless double beta decay and the hot dark matter constraints $\delta_1 \to 0$ $\delta_2 \to 0$, we obtain the following neutrino mass matrix:

$$M_\nu = m_0 \begin{pmatrix} 0 & \dfrac{1}{\sqrt{2}} & \dfrac{1}{\sqrt{2}} \\ \dfrac{1}{\sqrt{2}} & \dfrac{1}{2} & -\dfrac{1}{2} \\ \dfrac{1}{\sqrt{2}} & -\dfrac{1}{2} & \dfrac{1}{2} \end{pmatrix}. \tag{11}$$

This is precisely the mass matrix suggested by Georgi and Glashow [13] as a possible way to accommodate the neutrino data. We therefore have an alternative derivation of the same mass matrix.

Let us briefly mention the possible implications of our result. In addition to hinting at a deeper structure behind the bimaximal pattern, our result may have practical applications for building gauge models for this pattern. One avenue to explore would be to consider a four-generation version of the standard model [14] and decouple the heaviest Majorana combination as discussed above to obtain the three-generation model with bimaximal mixing. Below, we give an example of such a construction.

A four generation model for bimaximal mixing. Consider a four-generation extension of the standard model. Let us

TABLE I. Transformation properties of the fields under the interchange symmetries D_1 and D_2.

D_1	D_2
$L_1 \leftrightarrow L_2$	$L_1 \leftrightarrow L_3$
$L_3 \leftrightarrow L_4$	$L_2 \leftrightarrow L_4$
$\nu_1^c \leftrightarrow \nu_2^c$	$\nu_1^c \leftrightarrow -\nu_2^c$
$\phi_1 \leftrightarrow \phi_2$	$\phi_1 \leftrightarrow \phi_2$
$H_1 \leftrightarrow H_2$	$H_1 \leftrightarrow -H_2$
$H_3 \leftrightarrow H_4$	$H_3 \leftrightarrow H_4$
$e_{3,4}^c \leftrightarrow e_{3,4}^c$	$e_{3,4}^c \leftrightarrow -e_{3,4}^c$

denote the lepton doublets by L_i with $i=1,2,3,4$ and the four right handed charged leptons by e_i^c. Let us add two right handed $[\mathrm{SU}(2)_L$ singlet] neutral fermions denoted by (ν_1^c, ν_2^c). We omit the quark sector for simplicity. We also augment the Higgs sector of the model by adding an isotriplet $Y=2$ field (Δ), whose neutral component acquires a vacuum expectation value (VEV) which is seesaw suppressed to be of the order of eV's. We then add four new downlike Higgs doublets H_a $(a=1,2,3,4)$ and two new uplike Higgs pairs in addition to the two standard-model-like doublets ϕ_a $(a=1,2)$. The up and down like Higgs doublets can be distinguished from each other by a Peccei-Quinn-like symmetry (or supersymmetry) which we do not display since it can be softly broken. To extract the key features of our idea, we impose a discrete symmetry $D_1 \times D_2$ on our model. Under this symmetry, we assume the fields to transform as in Table I.

The rest of the fields are all assumed to be singlets under these symmetries. Let us now write down the invariant Yukawa couplings under these symmetries, $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3$:

$$\mathcal{L}_1 = f_1(L_1 L_1 + L_2 L_2 + L_3 L_3 + L_4 L_4)\Delta + f_2(L_1 L_4 + L_2 L_3)\Delta$$
$$+ f_3(L_1 L_3 + L_2 L_4)\Delta + f_4(L_1 L_2 + L_3 L_4)\Delta + \text{H.c.} \quad (12)$$

Note that after the Δ^0 field acquires a seesaw-suppressed VEV, it gives rise to the mass matrix M_4 in Eq. (7). Now we have to show that consistent with the discrete symmetry, we can decouple the combination of neutrinos in Eq. (8) and that the charged lepton mass matrices are such that they do not induce any contributions to mixings. To see the second point, let us write down $\mathcal{L}_\epsilon$:

$$\mathcal{L}_2 = h_1[(L_1 - L_4)H_3 + (L_2 - L_3)H_4]e_3^c + h_2[(L_1 + L_4)H_1$$
$$+ (L_2 + L_3)H_2]e_4^c] + h_3[(L_2 + L_3)H_3 + (L_1 + L_4)H_4]$$
$$\times (e_2^c + \alpha e_1^c) + h_4[(L_2 - L_3)H_1 + (L_1 - L_4)H_2]$$
$$\times (e_1^c + \beta e_2^c) + \text{H.c.} \quad (13)$$

We assume that $h_1 \gg h_3 \gg h_4 \gg h_2$. If we assume that only $H_{1,3}$ have, a VEV, then it keeps the $e_1 - e_4$ as a separate eigenstate, which we will assume to be heavy and make it decouple. The $e_1 + e_4$ will be the lightest eigenstate to be identified with the electron state.

Next let us show how one can decouple the correct heavy neutrino combination so as to implement our suggestion. For this purpose we note that there is the following Yukawa coupling allowed by the symmetries of our model:

$$\mathcal{L}_3 = f_5[(L_1 - L_4)\phi_1 \nu_1^c + (L_2 - L_3)\phi_2 \nu_2^c]$$
$$+ f_6[(L_1 - L_4)\phi_2 \nu_1^c + (L_2 - L_3)\phi_1 \nu_2^c]. \quad (14)$$

We add a soft breaking term to the theory of the form $M\nu_2^c \nu_2^c$ with $M \simeq 10^{11}$ GeV or so and assume that $f_6 \ll f_5$ and $\langle \phi_1 \rangle \neq 0$. This then leads to a partial seesaw that makes one of the light neutrinos, i.e., $\nu_2 - \nu_3$ very light, whereas it leaves the other combination, i.e., $\nu_1 - \nu_4$, with a mass in the 100 GeV range (not seesaw suppressed). As a result this combination decouples from our low energy neutrino spectrum as discussed. The three low energy neutrinos in our model have a bimaximal mixing pattern. We thus see that our decoupling scheme can be realized in explicit gauge models. We realize that the model just presented, though realistic, is given in the spirit of providing an existence proof of the kind of schemes we are advocating. However, they can perhaps be simplified and embedded into more clever schemes that will then provide a deeper origin for bimaximal models.

In conclusion, we have suggested a way to construct gauge models for bimaximal neutrino mixing starting with a four-generation model and illustrate this with the help of an extension of the standard model that incorporates four neutrino generations.

[1] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998); T. Kajita, presented at the ''Neutrino 98'' conference held in Takayama, Japan, 1998.

[2] See J. Bahcall, P. Krastev, and A. Smirnov, Phys. Rev. D **58**, 096016 (1998); for a recent review; see P. Langacker, hep-ph/9811460.

[3] D.O. Caldwell and R.N. Mohapatra, Phys. Rev. D **48**, 3259 (1993).

[4] C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995).

[5] R.P. Thun and S. McKee, Phys. Lett. B **439**, 123 (1998).

[6] H. Fritzsch and X. Xing, Phys. Lett. B **372**, 265 (1996); M. Fukugita, M. Tanimoto, and T. Yanagida, Phys. Rev. D **57**, 4429 (1998); Y. Koide, *ibid.* **39**, 1391 (1989).

[7] V. Barger, S. Pakvasa, T. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998); A. Baltz, A.S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998); M. Jezabek and A.

Sumino, Phys. Lett. B **440**, 327 (1998); G. Altarelli and F. Feruglio, *ibid.* **439**, 112 (1998).

[8] L. Wolfenstein, Phys. Rev. D **18**, 958 (1978); R.N. Mohapatra and S. Nussinov, Phys. Lett. B **346**, 75 (1995); P. Harrison, D. Perkins, and W. Scott, *ibid.* **349**, 137 (1995).

[9] S. Nussinov, Phys. Lett. **63B**, 201 (1976).

[10] M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).

[11] R.N. Mohapatra and S. Nussinov, Phys. Lett. B **441**, 299 (1998).

[12] L. Baudis *et al.*, Phys. Lett. B **407**, 219 (1997).

[13] H. Georgi and S.L. Glashow, hep-ph/9808293.

[14] P.Q. Hung, hep-ph/9803465 and refences therein.