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Another look at ''just-so'' solar neutrino oscillations

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We take another look at ''just-so'' solar neutrino oscillations, characterizing them by the energy $E_{\pi/2}$ at which the distance-varying angle is $\pi/2$, instead of by the usual Δm^2. The rising spectrum recently observed by the SuperKamiokande Collaboration is consistent with an $E_{\pi/2}\sim 6-9$ MeV and marginally with 48 MeV. The pp neutrinos must then be reduced to one-half the standard solar model prediction, and ^{7}Be neutrinos must make up a significant part of the SAGE and GALLEX gallium signal. For $E_{\pi/2}$ close to 9 and 48 MeV, the ^{7}Be neutrinos will also show a large seasonal variation, emphasizing the importance of direct measurements of the ^{7}Be neutrinos. [S0556-2821(99)50213-0]

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According to recent results from SuperKamiokande Collaboration [1], there is an excess of events at the high-energy end of the solar neutrino spectrum relative to the standard solar model (SSM) [2] and the small-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution of the solar neutrino problem [3]. In particular, the last three points above 12 MeV appear to rise more rapidly than a simple comparison with the SSM and MSW would indicate. This excess suggests that solar neutrinos may be undergoing ''just-so'' *in vacuo* oscillations with large mixing [4,5] instead of matter-enhanced MSW oscillations with small mixing, and that the relevant Δm^2 is of order 10^{-10} eV^2 rather than 10^{-5} eV^2.

The key point about ''just-so'' oscillations [5] is that the neutrino oscillation length is sufficiently close to the distance between the Sun and the Earth that the oscillation probability for the higher energy ^{8}B solar neutrinos is in the region of slow oscillations, rather than the rapid oscillations at the lower end of the solar neutrino spectrum. This means that instead of taking on the average value of one-half, the survival probability of ^{8}B neutrinos depends upon their energy and can run the gamut from zero to one. It, thus, becomes possible for the ''just-so'' electron-neutrino survival probability to exceed that of the small mixing-angle MSW case above some specific energy [6]; and this effect can then yield the qualitative behavior observed by the SuperKamiokande Collaboration.

To mark the division between slow and fast oscillations in terms of neutrino energy, it is convenient to characterize the oscillations by the energy at which the distance-dependent angle is $\pi/2$:

$$1.27\,\Delta m^2\,\frac{d_\odot}{E_{\pi/2}}=\frac{\pi}{2};\quad \Delta m^2=8.27\times10^{-12}\;\;E_{\pi/2}. \tag{1}$$

For a fixed solar distance $d_\odot$, 1.5×10^8 km chosen here, this is entirely equivalent to the usual Δm^2 characterization with energies in MeV and Δm^2 in eV^2. At $E_{\pi/2}$, the survival probability of solar electron-type neutrinos as they travel to Earth from the Sun,

$$P(\nu_e\rightarrow\nu_e;\;E_\nu)=1-\sin^2 2\theta\sin^2\left(\frac{\pi}{2}\,\frac{E_{\pi/2}}{E_\nu}\right), \tag{2}$$

reaches its minimum value of $(1-\sin^2 2\theta)$ and then increases monotonically to 1 as E_ν increases.

By choosing $E_{\pi/2}$ to be sufficiently below the endpoint energy of the ^{8}B neutrinos, we can ensure that the survival probability for ''just-so'' oscillations exceeds 0.4, the approximate value predicted by the small-angle MSW solution at the high-energy end of the solar neutrino spectrum [3]. This will give rise to a ratio of the ''just-so'' spectrum to the SSM spectrum which turns up more rapidly at the high-energy end, rather than the corresponding ratio for the MSW solution.

For neutrino energies smaller than $E_{\pi/2}$, oscillations set in and become more rapid as E_ν decreases. The survival probability reaches its maximal value at $E_{\pi/2}/2$ and its minimum value at $E_{\pi/2}/3$; the pattern then continues with $P(\nu_e\rightarrow\nu_e;\,E_\nu)$, maximal for E_ν, an even divisor of $E_{\pi/2}$, and becoming minimal for E_ν, an odd divisor. The smaller the energy, the more rapidly the oscillations take place. This behavior is illustrated in Fig. 1.

In Fig. 2 we plot the ratio of differential cross-sections for ^{8}B neutrinos for several choices of $E_{\pi/2}$ in the case of maximal mixing ($\sin^2 2\theta=1$) to the no-oscillation scenario. We use the standard cross-sections [7] and take into account the smearing effect of the energy resolution [7,8]. For the choice of 6 MeV, the curve begins to turn up at 4.5 MeV, and for the choice of 9 MeV, it begins to turn up at 8.5 MeV. For larger values of $E_{\pi/2}$, for example 12 and 15 MeV, the curves fail to increase at higher energy. However, for even higher $E_{\pi/2}$, for example 48 MeV, which corresponds to the best allowed parameter for ''just-so'' oscillations [9], the curve turns up at 8 MeV, but begins to turn over at 13.5 MeV. Of the $\sin^2 2\theta=1$ cases considered here that turn up at

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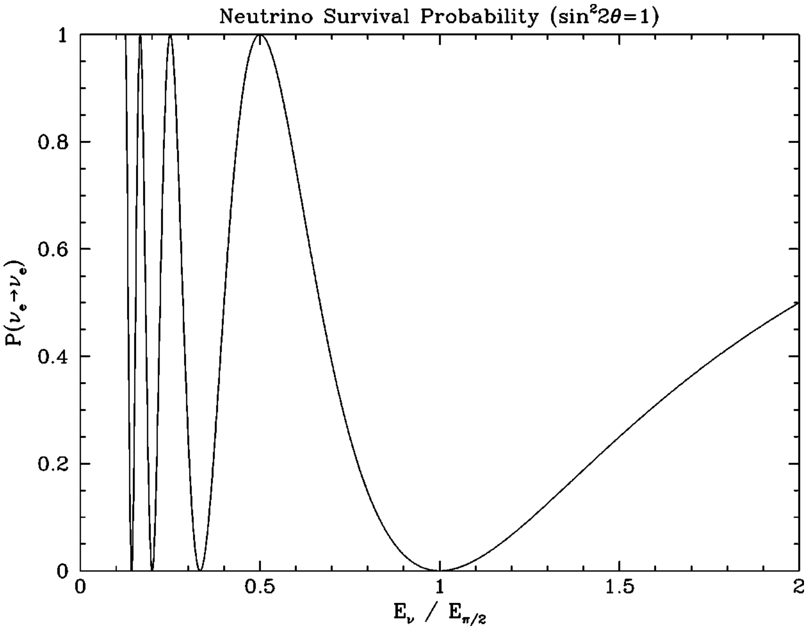

FIG. 1. Oscillation survival probability, ν_e to ν_e, as a function of $E_\nu/E_{\pi/2}$, where $E_{\pi/2}$ is the energy at which the distance-dependent angle equals $\pi/2$. It shows the slowly varying region for energies greater than $E_{\pi/2}$ and the rapidly varying region below $E_{\pi/2}/3$. (Values are not shown below $E_{\pi/2}/8$ for clarity.)

the high-energy end, the $E_{\pi/2}=$ 9 and 48 MeV cases are acceptable parameters for ''just-so'' oscillations, with the 6 MeV case marginally allowed [9].

It is not difficult to see that for the above $E_{\pi/2}$, both the *pp* and the ^{7}Be neutrinos, being of much lower energy, are in the region of rapid oscillations. For the *pp* neutrinos, this implies that their ν_e survival probability will reach the average value of

$$\langle P(\nu_e \to \nu_e;\ E_\nu)\rangle = 1 - \frac{1}{2}\sin^2 2\theta. \quad (3)$$

For ^{7}Be neutrinos, the value of the survival probability is a more delicate matter, since they are monoenergetic; it is given precisely by

$$P(\nu_e \to \nu_e; 0.86\,\mathrm{MeV}) = 1 - \sin^2 2\theta \sin^2\left(\frac{\pi}{2}\frac{E_{\pi/2}}{0.86}\right), \quad (4)$$

and will vary between its minimal and maximal values depending on the precise value of $E_{\pi/2}$. [It should be noted that variations in solar distances associated with the size of the ^{7}Be production zone ($\sim 10^5$ km) are too small compared to the mean Earth-Sun distance ($\sim 10^8$ km) to affect the survival probability by more than 1–2%.]

In order to estimate the survival probability for ^{7}Be neutrinos in this picture, we appeal to the results of the SAGE and GALLEX experiments [10] which indicate that the total signal is almost exactly equal to the signal predicted by the SSM for *pp* neutrinos alone [2]. Since the *pp* neutrinos in this ''just-so'' case are reduced to one-half their SSM value,

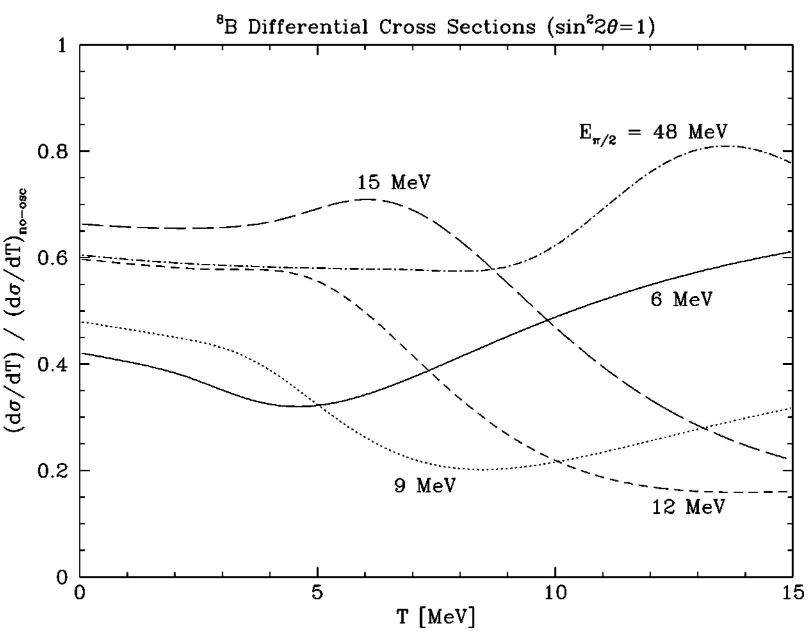

FIG. 2. The ratios of differential cross-sections for ^{8}B neutrino spectrum and SuperKamiokande resolution function for various values of $E_{\pi/2}$ to the SSM differential cross-sections are shown. The curves for $E_{\pi/2}$ in the range of 6 to 9 MeV (and possibly 48 MeV) appear to correspond to the energy spectrum observed by the SuperKamiokande Collaboration.

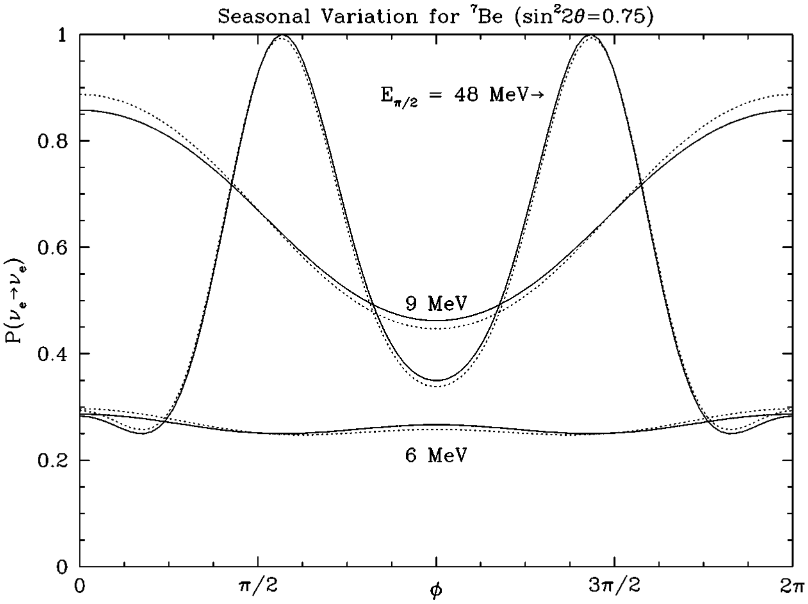

FIG. 3. Seasonal variation of the ^{7}Be survival probability for various values of $E_{\pi/2}$. In particular, the variation is large for $E_{\pi/2}$=9 and 48 MeV and small for 6 MeV. Solid lines are for $P(\nu_e \to \nu_e;\ d)$ alone [Eq. (5)] and dashed lines are for $P(\nu_e \to \nu_e;\ d)/d^2$, with d a function of orbital phase angle ϕ.

the ^{7}Be neutrinos must help to make up the difference. The SSM prediction for ^{7}Be neutrinos is approximately one-half the prediction for *pp* neutrinos [2] and so a large fraction of these, possibly all, will be needed to make up for the reduced signal from the *pp* neutrinos themselves. To meet this requirement, the value of $E_{\pi/2}$ must be ''fine-tuned'' so that its ratio to the ^{7}Be energy, 0.86 MeV, is close to an odd number—another ''just-so'' condition.

This prediction for the survival probability of ^{7}Be neutrinos should be contrasted with that of the small-angle MSW solution, which requires that they must be largely converted to non-electron neutrinos [11]. It should also be considered in light of phenomenological analyses [9] of the chlorine plus SuperKamiokande experiments, which indicates the possibility (but not the requirement) that the ^{7}Be contribution may not dominate the gallium signal. This would tend to disfavor the ''just-so'' explanation of the spectral effect, and to lend support to the small-angle MSW solution plus an unexpectedly large contribution from hep neutrinos at the high-energy end [12].

Another more subtle consequence of the ''just-so'' explanation is that, as has been pointed out by many authors [4,13], the survival probability for ^{7}Be neutrinos may have a significant seasonal variation due to the eccentricity of the orbit of the Earth. The variation in the orbit of the Earth due to its eccentricity is of order 10^6 km, and this can induce a large change in the distance-dependent angle, see Eq. (1), if the ratio $E_{\pi/2}/0.86$ happens to be large. Should this be the case, then gallium experiments such as SAGE and GALLEX [10] will also show a seasonal variation.

In terms of the eccentricity of the Earth's orbit around the Sun, $\epsilon = 0.017$, the oscillating factor in the probability is then given by:

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2\left[\frac{\pi}{2}\frac{E_{\pi/2}}{0.86}\frac{1-\epsilon^2}{1+\epsilon\cos\phi}\right]. \qquad (5)$$

Above we plot Eq. (5) and we also include curves with the $1/d^2$ variation in the neutrino flux which slightly enhances or diminishes the probability relative to oscillations alone [4]. If $E_{\pi/2}$ is in the neighborhood of 9 MeV, then the angular factor will be close to $5\pi + 2\pi/3$ and the probability will vary from 0.5 to 1, with a mean value of 0.75, during the year. On the other hand, if the factor is closer to 4.5π, the variation will only be about 5%.

In order to calculate the impact of this eccentricity dependence on gallium experiments, we need to choose the mixing angle. For the sake of illustration, we take the ''best value'' of Bahcall, Krastev, and Smirnov [9]:

$$\sin^2 2\theta = 0.75, \qquad (6)$$

with probabilities shown in Fig. 3 for three values of $E_{\pi/2}$. The gallium signal from *pp* neutrinos is then

$$S_{pp} = 72\times(1-0.75\times0.5) = 72\times(5/8)$$
$$= 45 \text{ solar neutrino units (SNU)}, \qquad (7)$$

where SNU is the solar neutrino unit and the yearly mean from ^{7}Be neutrinos is

$$S_{^7\mathrm{Be}} = 35\times(1-0.75\times0.75) = 16 \text{ SNU}. \qquad (8)$$

Taking an average signal of 73 SNU, we assume that the remaining 12 SNU come from a combination of about 5 SNU from ^{8}B and 7 SNU from other solar neutrinos [2], and that these neutrinos, which are generally higher in energy than ^{7}Be, do not vary appreciably with the seasons. The seasonal variation of ^{7}Be neutrinos runs from 22 SNU in winter to 9 SNU in summer, and so there could be a variation of $\pm 10\%$ in the total gallium signal [10]. (The effect is even larger for the 48 MeV case.)

Experiments designed to detect the ^{7}Be alone [14], should see the full extent of the seasonal variation. Alternatively, one could analyze the gallium results under the assumption that the observed signal consists of one-half the SSM prediction for *pp* neutrinos, about 36 SNU, plus about 10 from ^{8}B and other neutrinos, and the remainder from ^{7}Be. One could therefore consider the variation of the quantity

$$Q = \text{observed Ga signal} - 46 \text{ SNU} \tag{9}$$

which will amplify, on a percentage basis, the variation of the ^{7}Be signal in this picture.

Independently of possible seasonal variations, a key test of these ideas is provided by the average ^{7}Be neutrino signal. For ''just-so'' oscillations, ^{7}Be neutrinos must significantly remain as electron neutrinos and the signal will be large; whereas for the small-angle MSW solution, they must largely be converted to non-electron types and the signal will be small. Therefore, as long as the hints for ''just-so'' oscillations persist [1], experiments designed to detect ^{7}Be neutrinos will be highly important in confirming, or invalidating them.

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