

# PHYSICAL REVIEW D

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### Stellar Energy-Loss Rates in a Convergent Theory of Weak and Electromagnetic Interactions\*

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The stellar energy-loss rates due to the production of neutrino pairs are calculated in Weinberg's theory of electromagnetic and weak interactions. The ratio of the total rate calculated here to the rate calculated in the ordinary theory of weak interactions is  $10^{+0.93}_{-0.30}$ , where the uncertainty comes entirely from the lack of knowledge of the  $W$ -meson mass. The ratio of the experimental rate to the rate calculated in the ordinary theory is  $10^{0\pm 2}$ . Thus Weinberg's theory gives numbers well within the experimental limits for all values of the  $W$  mass.

#### I. INTRODUCTION

Weinberg has proposed a theory of leptons<sup>1</sup> which includes both the weak and electromagnetic interactions and is probably renormalizable.<sup>2,3</sup> Unfortunately, laboratory observations of differences between this theory and the usual four-point weak interaction<sup>4</sup> will be difficult since ordinary  $\beta$  decay is the same in both theories.

Processes which produce neutrino pairs are an extremely important energy-loss mechanism for stars in certain density and temperature ranges.<sup>5,6</sup> These processes have been calculated in detail in the point interaction by various authors.<sup>7-11</sup> In particular, Beaudet, Petrosian, and Salpeter<sup>12</sup> (BPS) have discussed the combined effect of several processes.

In this paper we calculate, in Weinberg's theory, the energy-loss rates for the three important ways of producing neutrino pairs,

$$\text{pair neutrino: } e^+e^- \rightarrow \nu + \bar{\nu}, \quad (1a)$$

$$\text{photoneutrino: } \gamma + e \rightarrow e + \nu + \bar{\nu}, \quad (1b)$$

$$\text{plasma neutrino: } \text{plasma} \rightarrow \nu + \bar{\nu}, \quad (1c)$$

and compare the results with the results obtained in the point interaction. In addition, energy-loss rates are given for two less-important processes that happen to be particularly easy to calculate:

$$\gamma + p \rightarrow p + \nu + \bar{\nu}, \quad (2a)$$

$$\gamma + \gamma \rightarrow \nu + \bar{\nu}. \quad (2b)$$

The reaction (2a) has been calculated in the ordinary weak-interaction theory by adding a neutral current,<sup>13</sup> and the reaction (2b) has been calculated in the ordinary  $W$ -meson theory.<sup>14</sup>

Each of the processes in (1) and (2) actually involves two reactions which must be added incoherently; one where the neutrino pair are electron neutrinos and another where the neutrinos are those associated with the muon. In the ordinary theory processes (1a), (1b), and (1c) cannot produce muon neutrinos (in lowest order). As a general procedure we will calculate the rate for electron neutrinos in detail. The rate for muon neutrinos can then be deduced by inspection.

## II. ENERGY-LOSS RATES

We will assume that the star consists of a completely ionized gas in thermal equilibrium at a temperature  $T$  with a density  $\rho$ . The number densities of the electrons and positrons are given by Fermi-Dirac distributions

$$n_{\mp} = \int dn_{\mp} = \frac{2}{(2\pi)^3} \int \frac{d^3p}{\exp(E/kT \mp \mu/kT) + 1}, \quad (3)$$

where  $\mu$  is the chemical potential of an electron (including the electron mass). The total gas is neutral; thus the number density of protons is  $n_- - n_+$ . If we neglect the weight of the electron-positron pairs then the mass density  $\rho$  of the plasma is

$$n_0 = n_- - n_+ = N \frac{\rho}{\mu_e}, \quad (4)$$

where  $N$  is Avogadro's number and  $\mu_e$  is related to the abundance  $X_i$ , the nuclear charge  $Z_i$ , and the atomic weight  $A_i$  of the  $i$ th atomic species in the plasma by

$$\frac{1}{\mu_e} = \sum_i X_i \frac{Z_i}{A_i}. \quad (5)$$

We shall only need the combination  $\rho/\mu_e$ .

Equations (3) and (4) provide a connection between the matter density  $\rho/\mu_e$ , the temperature, and the chemical potential. This relation is conveniently illustrated, for the temperatures and densities of interest to us, by Fig. 1 of Ref. 12.

For the processes of (1) and (2) we need only the following part of Weinberg's Hamiltonian<sup>1</sup>:

$$\begin{aligned} \mathcal{H} = & ieA^\alpha [W^\beta (2\partial_\beta W_\alpha^+ - \partial_\alpha W_\beta^+) + W_\alpha^+ \partial_\beta W^\beta - W^{\beta+} (2\partial_\beta W_\alpha - \partial_\alpha W_\beta) - W_\alpha \partial^\beta W_\beta^+] \\ & - e\bar{\psi}_e \gamma^\alpha \psi_e A_\alpha - \frac{g}{\sqrt{2}} \bar{\psi}_\nu \gamma^{\alpha\frac{1}{2}} (1 - \gamma_5) \psi_e W_\alpha^+ - \frac{g}{\sqrt{2}} \bar{\psi}_e \gamma^{\alpha\frac{1}{2}} (1 - \gamma_5) \psi_\nu W_\alpha \\ & - \frac{1}{4} \frac{1}{(g^2 + g'^2)^{1/2}} \bar{\psi}_e \gamma^\alpha (a + b\gamma_5) \psi_e Z_\alpha - \frac{1}{2} (g^2 + g'^2)^{1/2} \bar{\psi}_\nu \gamma^{\alpha\frac{1}{2}} (1 - \gamma_5) \psi_\nu Z_\alpha - \frac{1}{2} (g^2 + g'^2)^{1/2} \bar{\psi}_\mu \gamma^{\alpha\frac{1}{2}} (1 - \gamma_5) \psi_\mu Z_\alpha. \end{aligned} \quad (6)$$

Here  $g$  and  $g'$  are independent coupling constants and  $a$  and  $b$  are the combinations

$$a = 3g'^2 - g^2, \quad (7a)$$

$$b = g'^2 + g^2. \quad (7b)$$

The usual weak coupling constant  $G$ , the electric charge  $e$ , and the masses of the  $W$  and  $Z$  particles are related to  $g$  and  $g'$  by

$$e = \frac{gg'}{(g^2 + g'^2)^{1/2}}, \quad (8a)$$

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2}, \quad (8b)$$

$$m_Z^2 = \frac{g^2 + g'^2}{g^2} m_W^2. \quad (8c)$$

We shall often need the combinations

$$C_A \equiv 1 - \frac{b}{2m_Z^2} \frac{m_W^2}{g^2} \quad (9a)$$

and

$$C_V \equiv 1 + \frac{a}{2m_Z^2} \frac{m_W^2}{g^2}. \quad (9b)$$

From (8c) we see that  $C_A$  is always  $\frac{1}{2}$ , while  $C_V$  may be written as

$$\frac{1}{2} + \frac{e^2}{4m_W^2} \frac{\sqrt{2}}{G}. \quad (10)$$

$C_V$  is 2.5 if  $m_W$  is its minimum value, 37.3 GeV, and decreases to 0.5 if  $m_W$  becomes large. As is obvious from (9), in the usual point-interaction theory  $C_V$  and  $C_A$  are both equal to unity.

The rate of energy loss to neutrino pairs (in ergs/sec cm<sup>3</sup>) is simply the transition probability, multiplied by the energy of the neutrino pair and integrated over the density of states of all the particles, both initial and final. The density of states for an initial electron or positron is given by  $dn_{\mp}$  in (3). Photons in thermal equilibrium have a density of states

$$dn_\omega = \frac{1}{(2\pi)^3} \left( \exp \frac{\omega}{kT} - 1 \right)^{-1} d^3k, \quad (11)$$

where  $\omega$  is the energy of the photon and where

$$\omega^2 = |\vec{k}|^2 + \omega_p^2. \quad (12)$$

The plasma frequency  $\omega_p$  acts as a photon mass. For the neutrinos we have

$$dn_q = \frac{d^3q}{(2\pi)^3}, \quad dn_{q'} = \frac{d^3q'}{(2\pi)^3}, \quad (13)$$

while for an outgoing electron we have

$$dn_p = \frac{d^3p}{(2\pi)^3} \left\{ 1 - \left[ \exp \left( \frac{E}{kT} - \frac{\mu}{kT} \right) - 1 \right]^{-1} \right\}. \quad (14)$$

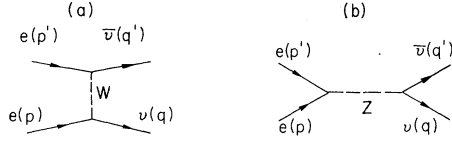


FIG. 1. The lowest-order Feynman diagrams for the process  $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$ . The symbols in parentheses are the momenta of the particles.

### A. Pair Annihilation

The lowest-order diagrams for the process  $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$  are given in Fig. 1. The  $W$ -exchange graph contributes a matrix element

$$-\frac{ig^2}{8m_W^2} \bar{u}_\nu(q) \gamma^\alpha (1 - \gamma_5) u_e(p) \bar{\nu}_e(p') \gamma_\alpha (1 - \gamma_5) v_\nu(q'), \quad (15a)$$

while the  $Z$ -exchange graph gives

$$\frac{i}{16m_Z^2} \bar{u}_\nu(q) \gamma^\alpha (1 - \gamma_5) v_\nu(q') \bar{\nu}_e(p') \gamma_\alpha (a + b\gamma_5) u_e(p). \quad (15b)$$

Making a Fierz transformation and using (9), we have the total matrix element

$$M = \frac{ig^2}{8m_W^2} \bar{u}_\nu(q) \gamma^\alpha (1 - \gamma_5) v_\nu(q') \times \bar{\nu}_e(p') \gamma_\alpha (C_V - C_A \gamma_5) u_e(p). \quad (16)$$

Squaring  $M$ , summing and averaging over the spins, and integrating over the final momenta by

$$Q = \frac{G^2 m^9}{18\pi^5} \{ (7C_V^2 - 2C_A^2) [G_0^- G_{-1/2}^+ + G_{-1/2}^- G_0^+] + 9C_V^2 [G_{1/2}^- G_0^+ + G_0^- G_{1/2}^+] + (C_V^2 + C_A^2) [4G_1^- G_{1/2}^+ + 4G_{1/2}^- G_1^+ - G_1^- G_{-1/2}^+ - G_{-1/2}^- G_1^+ - G_0^- G_{1/2}^+ - G_{-1/2}^- G_1^+] \}. \quad (22)$$

We should note that, from (3) and (4), the density can be written as

$$N \frac{\rho}{\mu_e} = \frac{m^3}{\pi^2} [G_0^- - G_0^+]. \quad (23)$$

When  $\nu \gg 1$ ,  $G_n^- \gg G_n^+$  and  $G_0^- \sim \rho/\mu_e$ . We will use this in the following calculations.

The integrals (21) cannot be done analytically for all  $\lambda$  and  $\nu$ . Therefore we cannot find an analytic expression for  $Q$  which holds for all temperatures and densities. However, since our primary purpose is to compare Weinberg's theory with the ordinary point-interaction theory, we shall content ourselves with evaluating (22) in various limiting regions of  $\nu$  and  $\lambda$ .

using Lenard's formula

$$\int \frac{d^3q}{2q^0} \frac{d^3q'}{2q'^0} \delta^4(P - q - q') q^\mu q'^\nu = \frac{1}{24} \pi (2P^\mu P^\nu + g^{\mu\nu} P^2), \quad (17)$$

we have<sup>15</sup>

$$\begin{aligned} \nu\sigma &\equiv \frac{1}{(2\pi)^2} \frac{1}{8EE'} \int \frac{d^3q}{2q^0} \frac{d^3q'}{2q'^0} \delta^4(p + p' - q - q') \sum_s |M|^2 \\ &= \frac{G^2}{12\pi} \frac{1}{EE'} \{ (C_V^2 + C_A^2) [m^4 + 3m^2 p \cdot p' + 2(p \cdot p')^2] \\ &\quad + 3(C_V^2 - C_A^2) [m^4 + m^2 p \cdot p'] \}, \quad (18) \end{aligned}$$

where  $E$  and  $E'$  are the energies of the electron and positron. This reproduces the point-interaction cross section when  $C_V = C_A = 1$ .

The rate of energy loss is given by

$$Q = \frac{4}{(2\pi)^6} \int \frac{d^3p}{e^{(E-\mu)/kT} + 1} \frac{d^3p'}{e^{(E'+\mu)/kT} + 1} (E + E') \nu\sigma. \quad (19)$$

Now, following Ref. 12, let us define  $\lambda$  and  $\nu$  as

$$\lambda = \frac{kT}{m}, \quad \nu = \frac{\mu}{kT}, \quad (20)$$

where  $m$  is the electron mass, and define the functions  $G_n^\pm(\lambda, \nu)$  as

$$G_n^\pm(\lambda, \nu) \equiv \lambda^{3+2n} \int_{\lambda^{-1}}^{\infty} dx \frac{x^{2n+1} (x^2 - \lambda^{-2})^{1/2}}{e^{x \pm \nu} + 1}. \quad (21)$$

The rate of energy loss (19) can now be written in terms of the integrals (21):

*Region I.*  $\lambda \ll 1$ ,  $\nu \ll 1/\lambda$ .

As long as  $\lambda \ll 1$ ,  $G_n^+$  is approximately equal to  $G_0^+$ , where

$$G_0^+ = \left(\frac{\pi}{2}\right)^{1/2} \lambda^{3/2} e^{-1/\lambda} e^{-\nu}. \quad (24)$$

In this region

$$G_n^- \approx G_0^- = \left(\frac{\pi}{2}\right)^{1/2} \lambda^{3/2} e^{-1/\lambda} e^{\nu}. \quad (25)$$

Then

$$Q_1 = \frac{G^2 C_V^2}{\pi^4} m^9 \left(\frac{kT}{m}\right)^3 e^{-2m/kT}. \quad (26)$$

This nonrelativistic, nondegenerate case holds

(roughly) for densities  $\rho/\mu_e \leq 10^5 \text{ g/cm}^3$  and temperatures between  $3 \times 10^8 \text{ }^\circ\text{K}$  and  $3 \times 10^9 \text{ }^\circ\text{K}$ , with higher densities requiring higher temperatures.

*Region II.*  $\lambda \ll 1$ ,  $1/\lambda \ll \nu \ll 2/\lambda$ .

$G_n^+$  is given by (24), while  $G_n^- \approx G_0^-$  with  $G_0^- \gg G_0^+$ . Using (23) we have

$$Q_{II} = \frac{\sqrt{2\pi}}{\pi^3} G^2 C_V^2 m^6 N \frac{\rho}{\mu_e} \left(\frac{kT}{m}\right)^{3/2} e^{-m/kT} e^{-\mu/kT}. \quad (27)$$

This nonrelativistic, mildly degenerate formula holds for temperatures  $T \leq 10^8 \text{ }^\circ\text{K}$  and densities between  $10^4 \text{ g/cm}^3$  and  $10^6 \text{ g/cm}^3$ .

*Region III.*  $\lambda \ll 1$ ,  $1 \ll \lambda\nu$ .

Again  $G_n^+$  is given by (24), and  $G_n^- \gg G_n^+$ . To highest power in  $\lambda\nu$

$$G_n^- \approx \frac{3}{2n+3} (\lambda\nu)^{2n} G_0^-. \quad (28)$$

Now

$$Q_{III} = \frac{G^2 \sqrt{2\pi}}{20\pi^3} (C_V^2 + C_A^2) \left(\frac{kT}{m}\right)^{3/2} \left(\frac{\mu}{m}\right)^2 m^6 e^{-m/kT} e^{-\mu/kT}. \quad (29)$$

Of course the chemical potential can be expressed as the Fermi energy. This relativistic and degenerate region holds for densities greater than  $10^7 \text{ g/cm}^3$  and temperatures greater than  $6 \times 10^7 \text{ }^\circ\text{K}$ , with the upper temperature limit given by  $\lambda \ll 1$ .

*Region IV.*  $\lambda \gg 1$ ,  $\nu \ll 1$ .

In this region we may ignore the chemical potential. To highest order in  $\lambda$

$$G_n^\pm \approx \lambda^{2n+3} (2n+2)! \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{S^{2n+3}}. \quad (30)$$

Then

$$Q_{IV} = 63.9 \frac{G^2}{\pi^5} (C_V^2 + C_A^2) m^9 \left(\frac{kT}{m}\right)^9. \quad (31)$$

This relativistic nondegenerate case only holds

for densities greater than  $10^7 \text{ g/cm}^3$ .

*Region V.*  $\lambda \gg 1$ ,  $\nu \gg 1$ .

Here  $G_n^- \gg G_n^+$ , so that we may use (23) with

$$G_n^+ \approx \lambda^{2n+3} (2n+2)! e^{-\nu}, \quad (32a)$$

$$G_n^- \approx \frac{3}{2n+3} (\lambda\nu)^n G_0^-. \quad (32b)$$

To highest power in  $\lambda\nu$  we have

$$Q_V = \frac{4}{5} \frac{G^2}{\pi^3} (C_V^2 + C_A^2) m^6 \left(\frac{\mu}{m}\right)^2 \left(\frac{kT}{m}\right)^4 N \frac{\rho}{\mu_e} e^{-\mu/kT}. \quad (33)$$

This degenerate relativistic region is restricted to densities greater than  $10^8 \text{ g/cm}^3$ , with a temperature of  $10^{10} \text{ }^\circ\text{K}$  at the lowest density extending to a range of  $10^{10} \text{ }^\circ\text{K}$  to  $10^{11} \text{ }^\circ\text{K}$  at a density of  $10^{10} \text{ g/cm}^3$ .

Thus we see that the ratio of the energy-loss rate as calculated here in Weinberg's theory to the energy-loss rate as calculated by BPS and others in the point-interaction theory is  $C_V^2$  for regions I and II and is  $\frac{1}{2}(C_V^2 + C_A^2)$  for regions III, IV, and V.

Because of the conservation of muon number the process  $e^+ + e^- \rightarrow \nu_\mu + \bar{\nu}_\mu$  can only go by the diagram in Fig. 1(b). Thus the matrix element for the production of muon neutrinos is given by (15b) alone. We can easily see that this has the effect of replacing  $C_V$  and  $C_A$  in the rates for  $e^+ + e^- \rightarrow \nu_e + \bar{\nu}_e$  by  $C_V - 1$  and  $-C_A$ , respectively. Since the rate of energy loss obviously depends on both processes, the ratio of the total energy-loss rate as calculated here to the old total energy-loss rate is

$$2C_V^2 - 2C_V + 1$$

for regions I and II and

$$\frac{1}{2}[2C_V^2 - 2C_V + 1 + 2C_A^2]$$

for regions III, IV, and V. We will discuss these ratios in Sec. III.

### B. Photoproduction

Photoproduction of neutrino pairs,  $\gamma + e \rightarrow e + \nu_e + \bar{\nu}_e$ , is thought to be important for low densities,  $\rho/\mu_e \leq 10^5 \text{ g/cm}^3$ , and relatively low temperatures,  $T \leq 4 \times 10^8 \text{ }^\circ\text{K}$ . If we neglect terms of order  $p/m_w$  in the matrix element, where  $p$  is the electron momentum, then we may neglect the diagrams where the photon is connected to the  $W$  meson. The other diagrams are shown in Fig. 2. The  $W$ -exchange diagrams give

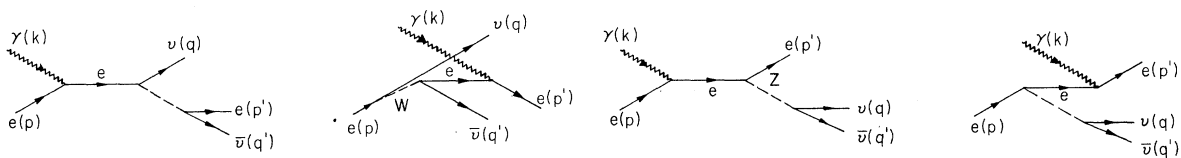


FIG. 2. The lowest-order diagrams for the photoproduction process  $\gamma + e \rightarrow e + \nu_e + \bar{\nu}_e$ .

$$\begin{aligned}
M_w = & -\frac{ie g^2}{8m_w^2} \bar{u}_\nu(q) \gamma^\alpha (1 - \gamma_5) (\not{p} + \not{k} - m_e)^{-1} \not{\epsilon} u_e(p) \bar{u}_e(p') \gamma_\alpha (1 - \gamma_5) v_\bar{\nu}(q') \\
& - \frac{ie g^2}{8m_w^2} \bar{u}_e(p') \not{\epsilon} (\not{p}' - \not{k} - m_e)^{-1} \gamma^\alpha (1 - \gamma_5) v_\bar{\nu}(q') \bar{u}_\nu(q) \gamma_\alpha (1 - \gamma_5) u_e(p),
\end{aligned} \tag{34}$$

where the momenta are defined in Fig. 2 and  $\epsilon^\mu$  is the photon polarization. The graphs with  $Z$  exchange contribute a matrix element equal to

$$\begin{aligned}
M_Z = & \frac{ie}{16m_Z^2} \bar{u}_e(p') \gamma^\alpha (a + b\gamma_5) (\not{p} + \not{k} - m_e)^{-1} \not{\epsilon} u_e(p) \bar{u}_\nu(q) \gamma_\alpha (1 - \gamma_5) v_\bar{\nu}(q') \\
& + \frac{ie}{16m_Z^2} \bar{u}_e(p') \not{\epsilon} (\not{p}' - \not{k} - m_e)^{-1} \gamma^\alpha (a + b\gamma_5) u_e(p) \bar{u}_\nu(q) \gamma_\alpha (1 - \gamma_5) v_\bar{\nu}(q').
\end{aligned} \tag{35}$$

These two contributions can be combined by a Fierz transformation to read

$$\begin{aligned}
M = & -\frac{ie g^2}{8m_w^2} \bar{u}_e(p') \gamma^\alpha (C_V - C_A \gamma_5) \frac{\not{p} + \not{k} + m_e}{2p \cdot k + \omega_0^2} \not{\epsilon} u_e(p) \bar{u}_\nu(q) \gamma_\alpha (1 - \gamma_5) v_\bar{\nu}(q') \\
& - \frac{ie g^2}{8m_w^2} \bar{u}_e(p') \not{\epsilon} \frac{\not{p}' - \not{k} + m_e}{-2p' \cdot k + \omega_0^2} \gamma^\alpha (C_V - C_A \gamma_5) u_e(p) \bar{u}_\nu(q) \gamma_\alpha (1 - \gamma_5) v_\bar{\nu}(q'),
\end{aligned} \tag{36}$$

where we have used (9) and, to include plasma effects, have set  $k^2 = \omega_0^2$ .

The next steps are obvious, although long and tedious, and we will not give the details. In order to compare easily with the point-interaction theory we shall follow BPS closely. We square (36), sum and average over the electron and neutrino spins, and integrate over the neutrino momenta, again using (17). Finally we must sum over the polarization of the photon. We can choose  $\epsilon^\mu = (0, \vec{\epsilon})$  so that in the rest frame of the initial electron

$$\begin{aligned}
(\epsilon \cdot p) &= 0, \\
\sum (\epsilon \cdot p')^2 &= |\vec{p}'|^2 - \frac{(\vec{p}' \cdot \vec{k})^2}{|\vec{k}|^2}.
\end{aligned} \tag{37}$$

Making a Lorentz transformation to the laboratory frame whose velocity is  $\vec{P}/E$  relative to the electron's rest frame, we have

$$\begin{aligned}
\sum (\epsilon \cdot p)^2 &= \sum (\epsilon \cdot p) (\epsilon \cdot p') = 0, \\
\sum (\epsilon \cdot p')^2 &= \frac{1}{(k \cdot p)^2 - m^2 \omega_0^2} \left[ \frac{1}{2} (2p \cdot k + \omega_0^2) (2p' \cdot k - \omega_0^2) (m^2 - p' \cdot p) \right. \\
&\quad \left. + m^2 (k \cdot (p + k - p'))^2 - \frac{1}{2} \omega_0^2 (m^2 + p \cdot p') (p + k - p')^2 \right].
\end{aligned} \tag{38}$$

The final result for the matrix element is

$$\begin{aligned}
I &\equiv \int \frac{d^3 q}{2q^0} \frac{d^3 q'}{2q'^0} (2\pi)^4 \delta^4(P - q - q') \sum_\epsilon \sum_S |M|^2 \\
&= \frac{2G^2 e^2}{3\pi} (C_V^2 + C_A^2) \left\{ 4P^2 + 2\beta\gamma(P^2 + m^2)(k \cdot P) + 2\beta\gamma\omega_0^2 P^2 [P^2 - 2m^2 - k \cdot P + \frac{1}{2}\omega_0^2] + \frac{1}{2}\omega_0^2 (\beta - \gamma)^2 P^2 (P^2 - m^2) \right. \\
&\quad \left. + [\gamma^2 P^2 (P^2 - m^2) + \beta\gamma\omega_0^2 (P^2 + m^2)] \frac{1}{(k \cdot p)^2 - m^2 \omega_0^2} [2(\beta\gamma)^{-1} (m^2 - p \cdot p') + m^2 (k \cdot P)^2 - \frac{1}{2}\omega_0^2 P^2 (m^2 + p' \cdot p)] \right\} \\
&+ \frac{2G^2 e^2}{3\pi} (C_V^2 - C_A^2) m^2 \left\{ 6\gamma^2 \omega_0^2 P^2 + 6\beta^2 \omega_0^2 P^2 + 4\beta\gamma P^2 \omega_0^2 - 8\beta\gamma (k \cdot P)^2 \right. \\
&\quad \left. + \frac{3\gamma^2 P^2 - \beta\gamma\omega_0^2}{(k \cdot P)^2 - m^2 \omega_0^2} [2(\beta\gamma)^{-1} (m^2 - p' \cdot p) + m^2 (k \cdot P)^2 - \frac{1}{2}\omega_0^2 P^2 (m^2 + p' \cdot p)] \right\},
\end{aligned} \tag{39}$$

$$P = p + k - p', \quad \beta^{-1} = k \cdot p + \frac{1}{2}\omega_0^2, \quad \gamma^{-1} = k \cdot p' - \frac{1}{2}\omega_0^2.$$

This reduces to the result of BPS when  $C_V = C_A = 1$ . The energy-loss rate for electrons is now given by

$$Q = \frac{1}{(2\pi)^7} \int_0^\infty \frac{p^2 dp}{E} (e^{(E-\mu)/kT} + 1)^{-1} \int_0^\infty \frac{k^2 dk}{\omega} (e^{\omega/kT} - 1)^{-1} \int_{-1}^1 dz \int \frac{d^3 p'}{E'} [1 - (e^{(E'-\mu)/kT} + 1)^{-1}] (E + \omega - E') I. \tag{40}$$

There is a corresponding rate for positrons.

In general these integrals must be done numerically. A method for doing the final five-dimensional integral is given in Ref. 11. We, however, will be content with evaluating (40) in two regions where it can be done analytically, namely, for nonrelativistic and extreme relativistic electrons. Our result for the nonrelativistic case, where photoproduction is thought to be important, must be used with caution since Ref. 11 demonstrated that relativistic effects become important at surprisingly low temperatures. (The total cross section for nondegenerate electrons can be completely evaluated analytically since it does not involve the integrals over the distribution factors. It is given in the Appendix.)

For extremely relativistic electrons the coefficient of  $C_V^2 - C_A^2$  is unimportant compared to the coefficient of  $C_V^2 + C_A^2$ . The result, therefore, is simply  $\frac{1}{2}(C_V^2 + C_A^2)$  times the usual result. When we add the rate for the emission of muon neutrinos the *total* rate is

$$\frac{1}{2}[2C_V^2 - 2C_V + 1 + 2C_A^2]$$

times the usual rate. The usual energy-loss rate in this region is given in BPS, where it is shown to be much less than the rate for the plasma neutrino process (1c).

For nonrelativistic electrons  $kT \ll m$  and, for densities below  $10^6$  g/cm<sup>3</sup>,  $\omega_0 \lesssim kT$ . Thus we may drop the photon mass terms in (39) and further evaluate (39) in the center-of-mass system. Since the densities are relatively low we may drop the Fermi distribution factor of the final electron relative to the 1 in the integrand of (40).

The integral of (39) over the momentum of the final electron is

$$\int \frac{d^3p'}{E'} I = \frac{32}{35} \frac{e^2 G^2}{m} \omega^5 [(C_V^2 + C_A^2) - \frac{2}{3}(C_V^2 - C_A^2)], \quad (41)$$

where  $\omega$  is the c.m. energy of the photon. Now, with the help of (4), we have the rate

$$Q = \frac{2}{105} \frac{\alpha G^2}{\pi^4} 7! \zeta(8) [C_V^2 + 5C_A^2] N \frac{\rho}{\mu_e} m^8 \left(\frac{kT}{m}\right)^8, \quad (42)$$

where  $\alpha = (137)^{-1}$  and  $\zeta$  is the Riemann  $\zeta$  function.

The ratio of (42) to the usual nonrelativistic nondegenerate rate is  $\frac{1}{6}[C_V^2 + 5C_A^2]$ . After we add the rate for muon neutrinos the ratio of the total rates is

$$\frac{1}{6}[2C_V^2 - 2C_V + 1 + 10C_A^2]. \quad (43)$$

We will discuss (43) in Sec. III.

### C. Plasma Neutrino

The plasma neutrino process,  $\gamma \rightarrow \nu + \bar{\nu}$ , is allowed if the electron gas has a dielectric constant  $e(\omega, \vec{k})$  which is less than unity. Then the relation

$$\omega^2 e(\omega, \vec{k}) = |\vec{k}|^2, \quad (44)$$

with

$$e(\omega, \vec{k}) \equiv 1 - \omega_p^2/\omega^2, \quad (45)$$

implies that the photon behaves as if it had a rest mass equal to the plasma frequency [as defined by (45)],  $\omega_p$ .

This process has been calculated in ordinary weak-interaction theory by calculating the graph of Fig. 3(a), where for the electron lines we use the electron Green's function in a sea of particles. The matrix element of the photon is given by

$$\langle 0 | A^\mu(0) | \gamma(k) \rangle = \epsilon^\mu \left/ \left[ \frac{\partial}{\partial \omega} (\omega^2 e^t) \right]^{1/2} \right. \quad (46)$$

for the transverse part of the photon, where  $\epsilon^\mu$  is the polarization vector and  $e^t$  is the transverse part of the dielectric constant. For longitudinal photons

$$\langle 0 | A^\mu(0) | \gamma(k) \rangle = \epsilon^\mu \left/ \left[ k^2 \frac{\partial e^l}{\partial \omega} \right]^{1/2} \right., \quad (47)$$

where  $k^2$  is the square of the four-vector.

In our theory we wish to calculate the diagrams in Figs. 3(b) and 3(c). However, if we neglect terms of order  $p^2/m_w^2$ , where  $p$  is the momentum of the electron in the closed loop of Fig. 3(b), then because of Fig. 3(c) we are just calculating Fig. 3(a) with a modified coupling constant. Since the transition rate due to the axial-vector current is much smaller than the rate due to the vector current, the modification is simply to multiply the usual rate by  $C_V^2 + (C_V - 1)^2$ . This is the sum of the rates for the two kinds of neutrinos.

For the usual rate we will use the expressions in BPS. The energy-loss rate for transverse photons is

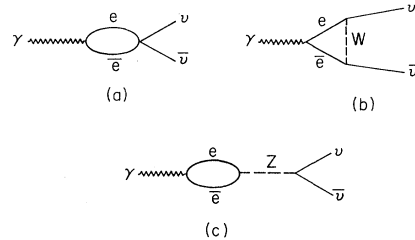


FIG. 3. (a) The lowest-order diagram for the decay of a plasmon in the ordinary theory of weak interactions. (b), (c) The lowest-order diagrams for plasmon decay in Weinberg's theory.

$$Q_T = \frac{G^2[C_V^2 + (C_V - 1)^2]}{48\pi^4\alpha} m^9 \gamma^6 \lambda^9 \int_\gamma^\infty \frac{x(x^2 - \gamma^2)^{1/2}}{e^x - 1} dx, \quad (48)$$

with

$$\gamma = \frac{\omega_0}{kT}, \quad \lambda = \frac{kT}{m}, \quad (49)$$

where  $\omega_0$  is a frequency given by

$$\omega_0^2 = \frac{4\alpha m^2}{3\pi} [2G_{-1/2}^+ + 2G_{-1/2}^- + G_{-3/2}^+ + G_{-3/2}^-]. \quad (50)$$

$$Q_L = \frac{G^2[C_V^2 + (C_V - 1)^2]}{48\pi^4\alpha} \left(\frac{\omega_0}{\omega_1}\right)^{7/2} m^9 \gamma^9 \lambda^9 \left(\frac{\omega_0}{\omega_1}\right)^7 \int_1^\infty \frac{y^{10}(y^2 - a^2)^2(y^2 - 1)^{1/2}}{e^{\gamma y} - 1} dy, \quad (52)$$

where  $\omega_1$  is given by

$$\omega_1^2 = \frac{4\alpha m^2}{3\pi} [2G_{-1/2}^+ + 2G_{-1/2}^- + G_{-3/2}^+ + G_{-3/2}^- - 3G_{-5/2}^+ - 3G_{-5/2}^-] \quad (53)$$

and  $a$  is the combination

$$a \equiv 1 + \frac{3}{5} \left(\frac{\omega_1}{\omega_0}\right)^2. \quad (54)$$

Again in the limit  $\gamma \gg 1$

$$Q_L = \frac{G^2[C_V^2 + (C_V - 1)^2]}{48\pi^4\alpha} m^9 \gamma^9 \lambda^9 \left(\frac{\pi}{8}\right)^{3/2} e^{-\gamma} \gamma^{-3/2}. \quad (55)$$

#### D. Photoproduction from Protons

Photoproduction of neutrino pairs from protons,  $\gamma + p \rightarrow p + \nu + \bar{\nu}$ , is not expected to be an important energy-loss mechanism. Nevertheless, we include it here because the result may be immediately read off from the result of photoproduction from electrons if we assume the proton couples to the  $Z$  with the same coupling constants as the electron. The relevant graphs are just those with a  $Z$  exchange, as in Fig. 2. Therefore the matrix element is the same as (36) with the electron mass replaced by the proton mass and  $C_V$  and  $C_A$  replaced by  $C_V - 1$  and  $-C_A$ . The protons are always nonrelativistic and we can read off the rate from (42).

The ratio of the total proton rate (including both types of neutrinos) to the total electron rate [as given by (43)] is

$$\frac{2[(C_V - 1)^2 + 5C_A^2]}{2C_V^2 - 2C_V + 1 + 10C_A^2} \left(\frac{m_e}{m_p}\right)^2, \quad (56)$$

which is always of the order of  $(m_e/m_p)^2 \approx 3 \times 10^{-7}$ .

#### E. Photon-Photon Scattering

The lowest-order diagrams for two free photons to produce neutrinos,  $\gamma + \gamma \rightarrow \nu + \bar{\nu}$ , are given in Fig. 4. The first three graphs are the same as in

$G_n^\pm$  are the integrals defined by (21).

In the region  $\gamma \gg 1$  (where the plasma process is expected to dominate)  $Q_T$  is given by

$$Q_T = \frac{G^2[C_V^2 + (C_V - 1)^2]}{48\pi^4\alpha} m^9 \gamma^6 \lambda^9 e^{-\gamma} \gamma^{3/2} \left(\frac{\pi}{2}\right)^{1/2}. \quad (51)$$

The energy-loss rate for longitudinal photons is

the ordinary  $W$ -meson theory and are gauge-invariant by themselves. The second three graphs, which involve a  $Z$  exchange, are zero. To see this we simply note that

$$(k_1 + k_2)^\alpha \bar{u}_\nu(q_1) \gamma_\alpha (1 - \gamma_5) u_\nu(q_2) = 0, \quad (57)$$

where  $k_1$  and  $k_2$  are the momenta of the two photons and  $k_1 + k_2 = q_1 + q_2$ . But it is impossible to construct a tensor for the closed-loop part of the diagrams which is simultaneously symmetric under interchange of the photons, is gauge-invariant, and is not proportional to  $(k_1 + k_2)^\alpha$ .

The cross section and the rate of energy loss are therefore the same as in the ordinary  $W$ -meson theory.

### III. RESULTS AND CONCLUSIONS

We have recalculated, in Weinberg's theory, the rate of energy loss in an electron gas due to the production of neutrino pairs. In particular we have considered the three processes which are thought to be dominant: pair annihilation (1a),

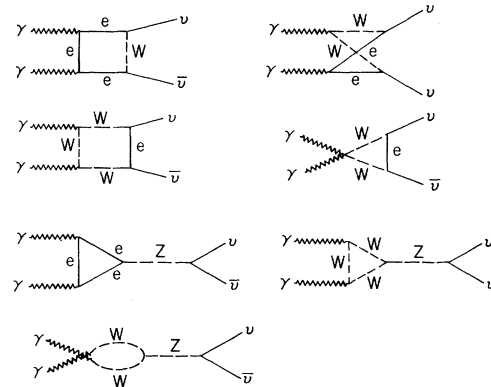


FIG. 4. The lowest-order diagrams for  $\gamma + \gamma \rightarrow \nu + \bar{\nu}$ .

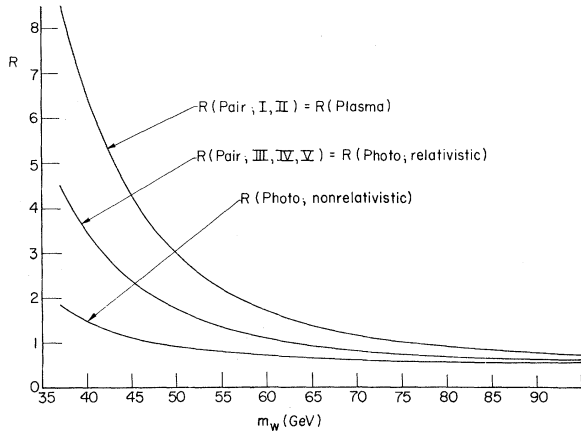


FIG. 5. The ratios of the energy-loss rates as calculated here to the energy-loss rates calculated in the ordinary theory vs the mass of the  $W$  meson.  $R$  (Pair, I, II) is the ratio for the pair annihilation in the regions I and II discussed in the text. All three lines approach  $R = \frac{1}{2}$  as  $m_w$  approaches  $\infty$ .

photoproduction (1b), and plasmon decay (1c). Although in each case the equations cannot be completely evaluated analytically for all temperatures and densities, we have evaluated them for special ranges of temperatures and densities. Within these special ranges the changes in the rates compared to energy-loss rates previously calculated in the point-interaction theory (or ordinary  $W$ -meson theory) are substantial. The ratios of the rates calculated here to the old rates are shown in Fig. 5. If the mass of the  $W$  meson is near its minimum value (37.3 GeV) some of the rates are as much as a factor of 8.50 larger. If the  $W$ -meson mass is very large all the rates are half their old values. Weinberg's theory decreases the importance of the photoproduction process relative to the other two processes for all  $W$ -

meson masses.

If we let  $R$  stand for the sum of the ratios in Fig. 5 then the experimental situation has been summarized by Stothers<sup>16</sup> as

$$R = 10^{0 \pm 2}. \quad (58)$$

In this theory we find

$$R = 10^{0 +0.93}_{-0.80}, \quad (59)$$

where the uncertainty is due to the unknown mass of the  $W$  meson. Since the limits in (59) are well within the present experimental limits we cannot put any bounds on the  $W$ -meson mass. Further, whether these changes in the rates are enough to be seen by observing the relative numbers of certain types of stars and thereby determining their lifetimes will depend on the precise value of that mass. If  $m_w$  is greater than 60 GeV the change in the rate is less than a factor of 2 and would require a very accurate stellar model. Current results from the laboratory experiments on the scattering  $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$  indicate that  $m_w$  is greater than 60 GeV.<sup>17,18</sup> On the other hand, if  $m_w$  is near its minimum value the large change in the plasma process should be observable in the number of white dwarfs.<sup>16</sup>

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#### APPENDIX: TOTAL CROSS SECTION FOR PHOTOPRODUCTION

If the electrons are nondegenerate the total cross section for photoproduction of electron neutrinos, (1b), can be calculated analytically for all photon energies. It is given by

$$\begin{aligned} \sigma = & \frac{e^2 G^2 m^2}{192 \pi^3} (C_V^2 + C_A^2) \frac{1}{x[x + (x^2 + 1)^{1/2}]} \left\{ \left[ \frac{16}{3} x^4 + \frac{20}{3} x^2 + \frac{4}{3} - \frac{5}{2} \frac{1}{x^2} + \left( \frac{16}{3} x^3 + 4x - \frac{25}{3} \frac{1}{x} \right) (x^2 + 1)^{1/2} \right] \ln(x + (x^2 + 1)^{1/2}) \right. \\ & \left. - \left[ \frac{70}{9} x^4 + \frac{11}{9} x^2 - \frac{25}{3} + \left( \frac{40}{9} x^3 + 3x - \frac{5}{2} \frac{1}{x} \right) (x^2 + 1)^{1/2} \right] \right\} \\ & + \frac{e^2 G^2 m^2}{96 \pi^3} (C_V^2 - C_A^2) \frac{1}{x[x + (x^2 + 1)^{1/2}]} \left\{ \left[ -4x^2 - 11 - 5 \frac{1}{x^2} - \left( 4x + \frac{5}{x} \right) (x^2 + 1)^{1/2} \right] \ln(x + (x^2 + 1)^{1/2}) \right. \\ & \left. + \left[ \frac{2}{3} x^4 + \frac{17}{3} x^2 + 5 + \left( -\frac{2}{3} x^3 + \frac{23}{3} x + 5 \frac{1}{x} \right) (x^2 + 1)^{1/2} \right] \right\}, \quad (A1) \end{aligned}$$

where  $x = \omega/m$  and  $\omega$  is the photon energy in the c.m. system.

Figure 6 shows the ratio of (A1) to the total

cross section in the usual theory, as given by (A1) with  $C_V = C_A = 1$ .

In the extreme relativistic (E. R.) region (A1)



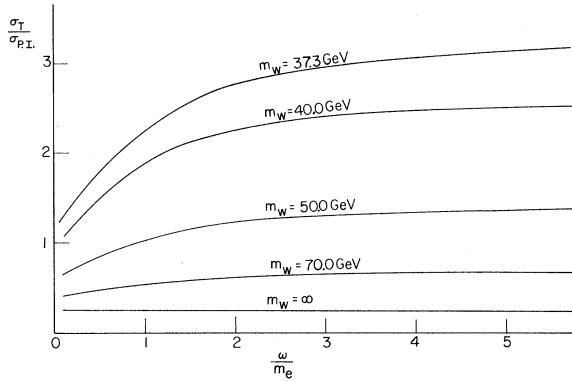


FIG. 6. The ratio of the total cross section for photoproduction of  $\nu_e \bar{\nu}_e$  as calculated here to the total photoproduction cross section calculated in the point-interaction theory vs the photon energy in the c.m. system (in units of electron mass).  $m_W$  is the  $W$ -meson mass.

gives

$$\sigma(\text{E. R.}) = \frac{e^2 G^2 m^2}{36\pi^3} (C_V^2 + C_A^2) x^2 \left[ \ln 2x - \frac{55}{48} \right]. \quad (\text{A2})$$

In the nonrelativistic (N. R.) region,  $x \ll 1$ , (A1) gives

$$\sigma(\text{N. R.}) = \frac{e^2 G^2 m^2}{630\pi^3} [C_V^2 + 5C_A^2] x^4. \quad (\text{A3})$$

The cross section for the photoproduction of muon neutrinos is again given by (A1), with  $C_V$  and  $C_A$  replaced by  $C_V - 1$  and  $-C_A$ , respectively.

We will not give the cross section for the pair-annihilation process (1a), since it has been calculated in detail by 't Hooft.<sup>15</sup>

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