PHYSICAL REVIEW D VOLUME 6, NUMBER 3 1 AUGUST 1972

t^{-2} Law for Electromagnetic Form Factors

P. Yodzis
School of Theoretical Physics, Dublin Institute for Advanced Studies, Dublin 4, Ireland
(Received 25 June 1971)

We show that, within the context of frame-dependent cutoff quantum field theory, the slowest possible asymptotic decrease, for $-t$ large, of the effective Sachs electromagnetic form factors is $O(t^{-2})$, where t is the invariant momentum transfer squared. The argument is model-independent and exact; it requires but a single analyticity assumption.

In a previous paper,[1] a new method was introduced for studying high-energy behavior within the context of frame-dependent cutoff quantum field theory, and it was shown that this method implies asymptotic constancy of hadron total cross sections, as an "upper bound," and the Pomeranchuk theorem. In the present work, we use this method to find the slowest possible asymptotic decrease of the effective Sachs electromagnetic form factors. We shall assume that the reader is acquainted with the first four sections of Ref. 1.

It should be pointed out that this work is done within the context of "stochastic" quantum field theory – a frame-dependent cutoff field theory which is free of ultraviolet divergences and, indeed, generally less "singular" than is local field theory. As explained in Ref. 1, we do not expect that our method is applicable to local field theory.

We use the graph shown in Fig. 1 to treat electron-proton scattering; note that it contains the complete electron vertex instead of the usual lowest-order electron vertex. We define the proton dynamical form factors by the standard canonical form[2]

$$(\Psi(P'),j^\mu(0)\Psi(P)) = \frac{i}{(2\pi)^3}\,\bar{u}(P')(\gamma^\mu F_1+\sigma^\mu{}_\nu q^\nu F_2)u(P),\qquad q\equiv P'-P,\tag{1}$$

and write the dynamical Sachs form factors as

$$G_M\equiv F_1+2MF_2\,,$$

$$G_E\equiv F_1+\frac{t}{2M}F_2\,,$$

where M is the proton mass, and $t\equiv -q^2$. From the graph in Fig. 1 we obtain, using the modified Feynman rules of stochastic field theory,[3] the Rosenbluth formula with one important change: Now the form factors occurring in that formula are not the dynamical ones $G_{E,M}$ but rather the effective form factors

$$(G_{E,M})_{\text{eff}}\equiv F_1^e|g(q)|^2G_{E,M}\,,\tag{2}$$

where F_1^e is the dynamical electron form factor, and g is the universal kinematical form factor of the stochastic theory.[4] Thus, it is the effective form factors $(G_{E,M})_{\text{eff}}$ which are obtained from the standard analyses of experimental data. In particular, it is well known that $(G_M)_{\text{eff}}$ falls off like t^{-2} for large $-t$.

We take the measuring frame $\mathcal{L}$ to be one in which $\vec{P}=\vec{0}$; then the matrix element in Eq. (1) is the one which is actually measured in laboratory electron-proton scattering experiments. We shall work in laboratory coordinates in which $n^\mu(\mathcal{L})=(\vec{0},1)$. Then the dynamical $G_{E,M}$ are functions of $-q^2\equiv t$ and $-n\cdot P'\equiv M-t/2M$.

Let

$$J_{\mu\nu}\equiv\sum_{\text{spins}}(\Psi(P'),j_\mu(0)\Psi(P))(\Psi(P),j_\nu(0)\Psi(P'))\,,$$

where $\Psi(P)$ is a one-proton state with momentum P, and the sum is over the initial and final proton spins. Then straightforward calculation yields

$$J_{\mu\nu}=\frac{-2M}{(2\pi)^6(t-4M^2)}\left[\left(-t\,\frac{4M^2-t}{4M^2}g_{\mu\nu}+\frac{4M^2-t}{2M^2}(P'_\mu P_\nu+P_\mu P'_\nu)-(P_\mu+P'_\mu)(P_\nu+P'_\nu)\right)G_M{}^2+(P_\mu+P'_\mu)(P_\nu+P'_\nu)G_E{}^2\right].\tag{3}$$

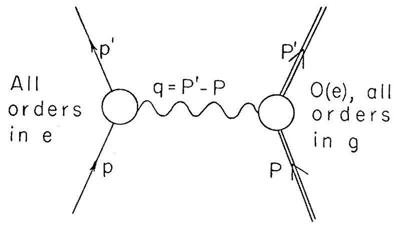

FIG. 1. *ep* scattering via single-photon exchange.

On the other hand, from the reduction formalism[5]

$$(\Psi(P'), j_\mu(0)\Psi(P)) = \frac{i}{(2\pi)^{3/2} g(P')} I_\mu ,$$

$$I_\mu \equiv \int d^4x\, e^{iP'\cdot x} F_\mu(x),$$

$$F_\mu(x) \equiv \theta(x)\bar{u}(P')(\Psi_0, [j_\mu(0), J(-x)]\Psi(P)),$$

where J is the proton source current and Ψ_0 is the vacuum. Let

$$f_{\mu+}(\xi) \equiv f_\mu(\xi), \quad \xi > 0, \quad f_{\mu-}(\xi) \equiv f_\mu(-\xi), \quad \xi > 0$$

$$f_\mu(\xi) \equiv \int d^3x\, F_\mu(\vec{x}, \xi + \hat{P}' \cdot \vec{x}),$$

where $\hat{P}'$ is the unit vector in the direction of $\vec{P}'$, and assume that $f_{\mu\pm}(\xi)$ admit power-series expansions which converge in the interval $\xi = [0, \infty)$. Then from Eq. (7) of Ref. 1, for large $-t$ we have

$$I_\mu \sim i \frac{2M}{-t} \sum_{m=0}^{\infty} \left(\frac{2Mi}{-t}\right)^m C_\mu^{(m)} ,$$

$$C_\mu^{(m)} \equiv \frac{d^m}{d\xi^m} f_\mu(0+) - \frac{d^m}{d\xi^m} f_\mu(0-).$$

To get an upper bound on the asymptotic behavior, we must assume that the leading terms $C_\mu^{(0)}$ are nonzero. This implies

$$J_{\mu\nu} \sim \frac{1}{(2\pi)^3 |g(P')|^2} \left(\frac{2M}{-t}\right)^2 \sum_{\text{spins}} C_\mu^{(0)} C_\nu^{(0)*} ,$$

where the asterisk (*) denotes complex conjugate. Carrying out the spin sum,

$$\sum_{\text{spins}} C_\mu^{(0)} C_\nu^{(0)*} \sim \frac{-t}{2M} C_{\mu\nu} ,$$

with $C_{\mu\nu}$ = constants. From Eq. (3) of Ref. 1,

$$|g(P')|^2 \sim 4M^2/2\pi\lambda^2 t^2 ,$$

where M is the proton mass and $\lambda \approx 10^{-14}$ cm is the universal "fundamental length." So

$$J_{\mu\nu} \sim \frac{2\pi\lambda^2}{(2\pi)^3} \left(\frac{-t}{2M}\right) C_{\mu\nu} . \tag{4}$$

From Eq. (3) above we find that

$$J_{44} = \frac{4M^2 - t}{2(2\pi)^6 M} G_E^2 ,$$
$$g^{ij} J_{ij} = \frac{-t}{2(2\pi)^6 M} (2G_M^2 + G_E^2) , \tag{5}$$

where i and j are summed only over the values 1, 2, 3. Equations (4) and (5) imply that the slowest asymptotic decrease allowed for the dynamical form factors $G_{E,M}$ is asymptotic constancy.

The dynamical electron form factors are treated the same way, except that the incoming particle is contracted out instead of the outgoing one, and the kinematics is different. One notes that, for $E \gg M$,

$$|\vec{P}'| \approx M/(1 - \cos\theta_{\text{lab}}), \quad -t \approx 2ME ,$$

where E is the lab energy of the incoming electron. Thus, if we stay within any fixed angular region $\theta_{\text{lab}} \geq \theta_{\min} > 0$, we find that $|\vec{P}| \approx E$, $|\vec{P}'| \approx 0$ [i.e., $\approx O(M)$] as E or $-t \to \infty$. It follows that the slowest asymptotic decrease allowed for F_1^e as $-t$ tends to infinity, with $\theta_{\text{lab}} \geq \theta_{\min}$, is asymptotic constancy.

Therefore, from Eq. (2), the slowest asymptotic decrease allowed for the effective form factors is

$$(G_{E,M})_{\text{eff}} \sim K |g(q)|^2 ,$$

where K is a constant. But from Eq. (3) of Ref. 1,

$$|g(q)|^2 \sim [2\pi\lambda^2 (\vec{q}^{\,2})_{\text{lab}}]^{-1} ,$$

and since $(\vec{q}^{\,2})_{\text{lab}} = t[(t/4M^2) - 1]$, we obtain the slowest possible asymptotic decrease of the effective form factors

$$(G_{E,M})_{\text{eff}} \sim O(t^{-2}) .$$

It is interesting that, both in the above result and in Ref. 1, the slowest asymptotic decrease allowed by this theory seems to be realized in nature. In the present case one can of course assert this only for G_M: The asymptotic behavior of G_E is not yet known. Our theory predicts that when G_E is measured at very large t, it too will be found to drop off like t^{-2}. Finally, it should be noted that the same analysis as the above applies also to electron-electron scattering. Thus, the present theory predicts that *at extremely high energies ee* elastic scattering should closely resemble *ep* elastic scattering.

[1]P. Yodzis and R. L. Ingraham, Phys. Rev. D 2, 1493 (1970).

[2]In the stochastic theory, there are some additional terms in Eq. (1). They are, however, $O(\lambda)$, where $\lambda \approx 10^{-14}$ cm is the universal "fundamental length." We therefore feel justified in neglecting these terms.

[3]R. L. Ingraham, Nuovo Cimento 24, 1117 (1962); 27, 303 (1963); R. L. Ingraham, *Renormalization Theory of Quantum Field Theory With a Cut-off* (Gordon and Breach, London, 1967).

[4]In this case the factor $|g(q)|^2$ comes from the photon propagator. For the barest sketch of the role of this kinematical form factor in the theory, see Ref. 1.

[5]For the necessary modifications to the usual Lehmann-Symanzik-Zimmermann rules, see Ref. 1. There is also a term involving an equal-time commutator, which can be shown to be asymptotically negligible compared with the displayed term.

PHYSICAL REVIEW D VOLUME 6, NUMBER 3 1 AUGUST 1972

Superconvergence Relations for Inclusive Reactions*

J. Finkelstein†

Institute of Theoretical Physics, Department of Physics, Stanford University, Stanford, California 94305

(Received 1 May 1972)

Sum rules for inclusive reactions, which are the analogs of superconvergence relations for two-body reactions, are discussed. A local-saturation approximation to one of these sum rules yields $(d\sigma/dt)_{\pi^+p\to\pi^0\Delta^{++}} = \frac{3}{2}(d\sigma/dt)_{\pi^-p\to\pi^0 n}$, which is in good agreement with experiment.

It has recently been shown[1,2] that one may write sum rules in the missing-mass variable of inclusive reactions, which are the analogs of finite-energy sum rules for two-body reactions. The zeroth-moment sum rule takes the form[2]

$$\int_0^N dM^2\left[\left(\frac{d^2\sigma}{dt\,dM^2}\right)_{ab\to cx}-\left(\frac{d^2\sigma}{dt\,dM^2}\right)_{cb\to ax}\right]$$

$$=\frac{1}{s}\sum_{i,j,k}(1-\tau_i\tau_j\tau_k)\left(\frac{s}{N}\right)^{\alpha_i(t)+\alpha_j(t)-1}N^{\alpha_k(0)}\xi_i(t)\xi_j^*(t)\frac{\beta^i_{a\bar c}(t)\beta^{j*}_{a\bar c}(t)\beta^k_{b\bar b}(0)g^k_{ij}(t)}{\alpha_k(0)+1-\alpha_i(t)-\alpha_j(t)}. \tag{1}$$

Here, as in Ref. 2, $s=(p_a+p_b)^2$, $t=(p_a-p_c)^2$, and M is the missing mass; the β's are the ordinary Regge, and g^k_{ij} the triple-Regge, couplings in the configuration shown in Fig. 1; $\xi_i(t)=(\tau_i+e^{-i\pi\alpha_i(t)})/\sin\pi\alpha_i(t)$, and $\tau_i=\pm1$ is the Regge signature. The inclusive cross sections in (1) are to be evaluated with s large and (approximately) fixed, and with t small and fixed.

It is the purpose of this note to suggest that it might be particularly interesting to look at those special cases of this type of sum rule in which the terms on the right-hand side either are entirely absent or are negligible. In these cases, we would have sum rules relating integrals over the (fairly low) missing-mass spectrum at each fixed t, which would not involve any triple (or even ordinary) Regge parameters. These sum rules could enable us, for example, to predict the size of certain inclusive cross sections, once certain others had been measured. Below, we illustrate with a few examples how such sum rules may be obtained, and also attempt to assess the utility of these sum rules by saturating them with experimental values for the low-M^2 spectrum, in cases for which these data are available.

One way to construct finite-mass sum rules in which no Regge parameters appear is to take linear combinations of sum rules of the type of Eq. (1) for different reactions, in such a way that all of the ugly Regge terms on the right-hand side would cancel. This is equivalent to constructing an amplitude with exotic quantum numbers in the $b\bar b$ channel, and then writing a superconvergence relation for that amplitude. The inclusive cross section for $ab\to cx$ can be written, in the Regge region, as

$$\frac{d\sigma}{dt\,dM^2}=\frac{1}{s^2}\sum_{i,j}(s)^{\alpha_i(t)+\alpha_j(t)}h_{ij}(M^2,t)\times\xi_i(t)\xi_j^*(t)\beta^i_{a\bar c}(t)\beta^{j*}_{a\bar c}(t), \tag{2}$$