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$$\bar{x} = \left(x^2 + \frac{4(p_\perp{}^2 + m^2)}{s} \right)^{1/2}$$

by $|x|$, which is valid for $s \rightarrow \infty$.

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PHYSICAL REVIEW D VOLUME 6, NUMBER 3 1 AUGUST 1972

Breakdown of Hadronic Scaling or Evidence for Clustering?

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(Received 9 March 1972)

We argue that the result of a recent comparison of single-particle spectra at accelerator and cosmic-ray energies is evidence for clustering effects in individual events rather than for a breakdown of the hadronic scaling, as originally suggested. It is pointed out that clustering on rapidity plots corresponding to individual events results, as a kinematic reflection, from the small inelasticity of the through-going particles. A simple model of inclusive spectra (suggested by multiperipheralism and gas-liquid analogy), with kinematic clustering built into it from the outset, is proposed.

In a recent issue of the *Physical Review Letters*, von Lindern *et al.*[1] compare 28-GeV bubble-chamber data on pp collisions with cosmic-ray interactions observed at $E_{\text{lab}} \gtrsim 10^4$ GeV in nuclear emulsions. The principal purpose of their work is to test whether single-particle spectra approach finite, energy-independent limits, as they should do according to the hadronic scaling hypothesis.[2,3] The conclusion of the paper is that at primary energies as high as 10^4 GeV the scaling limit is very far from being reached (e.g., the expected plateau in the central region of the one-particle rapidity distribution is apparently absent). If this were true the usefulness of the scaling hypothesis could legitimately be questioned. Since, otherwise, this hypothesis has much appeal, it is important to inquire whether a different interpretation can be given to the startling result of von Lindern *et al.* (and also whether some biases are not at the origin of their conclusion). In fact, it is easy to criticize an emulsion cosmic-ray experiment, but we shall not repeat the standard objections, certainly well known to the authors of the above-mentioned paper. We believe that the result of von Lindern *et al.* may have deeper roots than just the obvious biases associated with emulsion experiments at high energies. Indeed, we shall argue that this result can be regarded as an evidence for an interesting physical phenomenon (clustering) without necessarily contradicting the scaling hypothesis.

Let us remark that the present emphasis on the one- or two-particle inclusive spectra leaves aside the very important question: What do the individual events look like? To avoid misunderstanding let us stress that we are not interested here in the comparison of the inclusive spectra with their exclusive components. We would like rather to emphasize the importance of parameters describing *events as a whole* and of the *inclusive* spectra of these parameters. For definiteness, we focus our attention on the rapidity distributions. With regard to the question of the structure of individual events the following two alternative cases can be distinguished:

(i) The structure of individual rapidity plots is not very different from the average situation (as described by the one-particle spectrum).

(ii) Particles have a tendency to cluster[4] on rapidity plots representing individual events, the position and size of the cluster(s) varying from one event to another.

Notice that if the case (ii) is true, large fluctuations of the position and size of the cluster(s) from one event to another can easily result in a uniform average rapidity distribution, in agreement with Feynman's scaling hypothesis.[3]

One can give a simple argument to show that (ii) should be preferred to (i) if one wants to avoid a contradiction between scaling and cosmic-ray data. In fact, the distribution of the dispersion σ of individual rapidity plots, defined by

$$\sigma = \left[\frac{1}{N-1}\sum_{j=1}^{N}(\bar{y}-y_j)^2\right]^{1/2}, \qquad (1)$$

where y_j is the rapidity of the jth secondary and

$$\bar{y} = \frac{1}{N}\sum_{j=1}^{N} y_j, \qquad (2)$$

is obviously sensitive to clustering. Although cosmic-ray data yield angular rather than rapidity distributions, the approximate equivalence between the lab rapidity $y_{\rm lab}$ and the $\log_{10}\tan\theta_{\rm lab}$ variable[5]

$$\log_{10}\tan\theta_{\rm lab} \approx \log_{10}2 - y_{\rm lab}, \quad \theta_{\rm lab} \neq 0 \qquad (3)$$

permits one to use these data to get information on the structure of rapidity plots at ultrahigh energies. From the transformation properties of y one easily deduces that the average lab rapidity in one event is given by

$$\bar{y}_{\rm lab} \approx \log_{10}(2\gamma^*), \qquad (4)$$

where γ^* is the Lorentz factor for the transformation from the lab frame to the frame where the event is forward-backward-symmetric (in practice, this symmetry can only be approximate, of course). Using (3) one can rewrite (4) as (Castagnoli's equation[6])

$$\langle \log_{10}\tan\theta_{\rm lab}\rangle_{\rm av} \approx -\log_{10}\gamma^*. \qquad (5)$$

If (i) were true, nucleon-nucleon collision products would be distributed approximately symmetrically in the center-of-mass frame and one would have

$$\gamma^* \approx \sqrt{s}/2m. \qquad (6)$$

The value of $\langle\sigma\rangle$ at a given energy could thus be obtained, averaging over events characterized by the corresponding value of $\langle \log_{10}\tan\theta_{\rm lab}\rangle_{\rm av}$ (at highest energies the energy of the collision is usually unknown). The parameter $\langle\sigma\rangle$ calculated in this manner[7] increases slowly with $|\langle \log_{10}\tan\theta_{\rm lab}\rangle_{\rm av}|$, without bypassing 0.8, although $|\langle \log_{10}\tan\theta_{\rm lab}\rangle_{\rm av}|$ rises above 2:

$$\langle\sigma\rangle \lesssim 0.8. \qquad (7)$$

This figure for $\langle\sigma\rangle$ appears rather surprisingly small when (i) and scaling are postulated.[8] Indeed, assuming uniform rapidity distribution within an interval of length L one easily finds

$$\langle\sigma\rangle = L/2\sqrt{3}. \qquad (8)$$

Now, if L is approximately equal to the length of the available rapidity interval (or, using Feynman's gas-liquid analogy,[9] to the space between the "walls" within which the gas is contained), i.e., if $L \approx \log_{10}(s/m^2)$, one expects to have

$$\langle\sigma\rangle \approx 1 \text{ to } 1.5, \qquad (9)$$

for primary energies ranging from 10^3 to 10^5 GeV. The discrepancy between (7) and (9) is not astonishing, however, if (ii) is true. Clustering reduces the effective length of individual rapidity plots (if one forgets the leading particles which are close to the "walls"); also the collective motion of other-than-leading particles makes $\langle \log_{10} \tan\theta_{\text{lab}} \rangle_{\text{av}}$ a poor estimate of the primary energy (see later).

It seems that the apparent inconsistency with scaling, reported by von Lindern *et al.* is closely related to the otherwise observed smallness of $\langle \sigma \rangle$ [the dispersion of the compound $\log_{10} \tan\theta$ plot presented in Ref. 1 is, as far as one can see, compatible with (7)]. One is thus led to inquire whether again the clustering effects cannot be regarded as being responsible for the result.

Von Lindern *et al.* claim that individual events are symmetric in the center-of-mass frame and use Castagnoli's technique [Eq. (5) with γ^* given by Eq. (6)] to determine the primary energy. It seems to us that this claim does not rest on a very solid ground. The check of Castagnoli's[10] technique at about 30 GeV, quoted in Ref. 1, is not conclusive. This technique was found acceptable for events with the number of charged secondaries $\geqslant 4$ only, that is, equal or higher than the average. The fluctuations of $\langle \log_{10} \tan\theta_{\text{lab}} \rangle_{\text{av}}$ are suppressed for purely kinematic reasons at relatively low energies, where there is not enough energy left for production to allow a group of secondaries to have rather large values for both the effective mass (multiplicity) and the kinetic energy of collective motion in the center-of-mass system. At cosmic-ray energies the presence of strongly asymmetric events has repeatedly been reported.[11] In fact, we feel that a reliable estimation of the importance of asymmetries in individual cosmic-ray interactions can hardly be given at present, even using the so-called unbiased samples, because of measurement errors, poor statistics, uncertainties about the elementary nature of the interaction and other typical diseases of cosmic-ray experiments. On the other hand, assuming symmetric emission in c.m. system is particularly dangerous when one attempts to test scaling. In the presence of important clustering [case (ii)] scaling requires strong fluctuations of γ^* around its average value and $\langle \log_{10} \tan\theta_{\text{lab}} \rangle_{\text{av}}$ (or equivalently $\bar{y}_{\text{lab}}$) is expected to be a poor estimate of the primary energy. It is thus important to realize that the histogram obtained by adding plots with a given $\bar{y}_{\text{lab}}$ but unknown primary energy is different from the histogram one gets when plots corresponding to a fixed collision energy but fluctuating $\bar{y}_{\text{lab}}$ are added (especially if one recalls what the spectrum of energy of the cosmic radiation is).

In other words, *we suspect that the histogram presented by von Lindern et al. does not represent* $\int d^2q\,(E d\sigma/d^3q)$, *the quantity which is expected to scale*. The possible bias is visualized in Fig. 1: In order to get the inclusive one-particle spectrum $\int d^2q\,(E d\sigma/d^3q)$ one should add plots (a) and (b) at energy E_1 and plots (c) and (d) at energy E_2. The use of Castagnoli's technique implies adding (a) + (c) and (b) + (d). The nature of the deformation of the one-particle spectrum introduced in this manner is quite obvious.

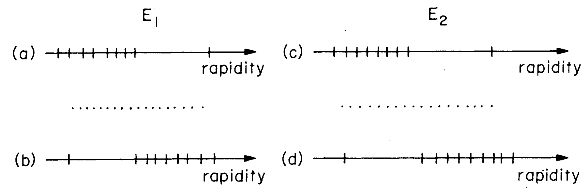

FIG. 1. The bias introduced by the use of Castagnoli's technique to estimate the primary energy, in the presence of important clustering effects, and its influence on the one-particle spectrum obtained: instead of adding (a) + (b) and (c) + (d) (corresponding to the same energy E_1 and E_2, respectively) one rather adds (a) + (c) and (b) + (d).

At this point of our discussion, the breakdown of scaling and clustering in individual events appear as two more or less alternative interpretations of the result of Ref. 1 and of the smallness of $\langle \sigma \rangle$ (of course, one can have clustering *and* no scaling; we are just discussing the simplest picture of what is happening). One can argue, however, that over-all scaling but with clustering in individual events is more likely to be a realistic picture.

First, if the genuine one-particle rapidity (or $\log_{10} \tan\theta$) distribution were nearly Gaussian (as is the spectrum presented in Fig. 2 of Ref. 1) a nontrivial dynamical mechanism would have to be postulated to account for the relatively abundant appearance at these energies of the so-called "two-center" (or "two-fireball") events.[7] On the other hand, it was pointed out several years ago by Czyzewski and one of us[12] that the frequent appearance of the two-fireball events can be considered as a rather straightforward consequence of scaling. The fact that the two-fireball events begin to show up above 10^3 GeV, that is, exactly at energies where a plateau in the $\log_{10} \tan\theta$ distributions is expected to set in, is perhaps not a pure coincidence.

Second, the kinematic reflection of the existence of the leading particles provides a simple mechanism for clustering. This has already been discussed by Berger and one of us[13] in a slightly different context. Denoting by $x_j = 2q_{\parallel}^j/\sqrt{s}$ the Feynman variables of the leading particles and assuming that the latter are nonwee, one easily obtains the following expressions for the effective mass squared s' and the average rapidity of other-than-leading particles,

$$s' \approx s(1-x_1)(1+x_2)\,,$$
$$\bar{y}_{\text{lab}} \approx \log_{10}(\sqrt{s}/m) + \tfrac{1}{2}\log_{10}\left(\frac{1-x_1}{1+x_2}\right)\,. \quad (10)$$

Notice that the deviation of $\bar{y}_{\text{lab}}$ from its over-all average value $\langle \bar{y}_{\text{lab}} \rangle$ [$\approx \log_{10}(\sqrt{s}/m)$ for identical incident particles] is a measure of the velocity of the collective motion of other-than-leading particles with respect to the c.m. system (this follows directly from the transformation properties of the rapidity variable). The observed weak dependence of the leading particle differential cross sections on x_j (provided x_j is not too small) indicates large fluctuations of both s' and $\bar{y}_{\text{lab}}$ if super-long-range correlation between leading particles at ultrahigh energies is absent:

$$\frac{d^2\sigma}{dx_1 dx_2}\, dx_1 dx_2 \approx \text{const}\, dx_1 dx_2$$
$$\propto \frac{1}{s}\, ds'\, d(\bar{y}_{\text{lab}} - \langle \bar{y}_{\text{lab}} \rangle)\,. \quad (11)$$

When s' is sufficiently smaller than s, the other-than-leading particles are confined, on the rapidity plot, to an interval smaller than the *a priori* available space. When the multiplicity is not too small[14] significant accumulations of the density of points in individual rapidity plots are expected. Using the gas-liquid analogy one would say that the fluctuations of the positions of particles close to the "walls" determine the collective motion of other particles within the container. The important point to realize is that there are (almost) always particles close to the "walls," namely, the leading particles.

The size on the rapidity plot of a cluster of other-than-leading particles is determined by the value of $\sqrt{s'}$/(multiplicity). If in the rest frame of the cluster the average momentum of a secondary particle is comparable to the average transverse momentum $\langle q \rangle \approx \langle q_T \rangle$ (the three-dimensional regime of Ref. 13), points on the rapidity plot are distributed normally with dispersion 0.4. If on the other hand $\langle q \rangle \gg \langle q_T \rangle$ (the one-dimensional regime of Ref. 13), the density within the cluster becomes roughly uniform and its size is proportional to $\log(s'/\langle q_T \rangle^2)$. Hence the kinematic clustering effect produces correlations in the rapidity space with characteristic length presumably increasing asymptotically like $\log s$ (cf. Ref. 13). It is interesting that recent analysis of the moments of multiplicity distributions suggests the existence of long-range correlations.[15] The kinematic clustering effect could easily be the needed correlation (the presence of long-range dynamic correlations is possible, of course, although it does not fit with the current models of multiparticle production). Incidentally, let us mention that assuming Regge-pole saturation of Mueller's optical formula[16] one gets an energy-independent two-particle correlation length.[13]

The long-range correlations associated with clustering effects, and especially with the kinematic clustering, presumably correspond in Mueller's theorem language to Regge cuts. We think that instead of eventually attempting fits with Regge cuts, etc., as people did for the two-body scattering, it is more interesting, as far as inclusive phenomenology is concerned, to try to understand the clustering effects. The inclusive spectra *relative to clusters* should be studied, and in particular those describing the cluster of other-than-leading particles.

To be more precise, let us define besides the conventional one-particle inclusive spectrum (p_j are the momenta of the incident particles)

$$f(qp_1p_2) = \frac{E\,d\sigma}{\sigma_{\text{in}} d^3q}\,, \quad (12)$$

the "conditional probability" $f(q\,|\,q'q''p_1p_2)$ (also normalized to the number of particles) that a non-leading particle has momentum q when the leading particles momenta are q' and q''. The challenging questions which, we believe, can be at least partially tested with data accumulated on the existing Data Summary Tapes, are: What does the distribution of $f(q\,|\,q'q'')$ look like? Does it depend on the total energy $\sqrt{s}$ or only on the effective mass $\sqrt{s'}$? How does $f(q\,|\,q'q'')$ compare to $f(q)$ measured at (total c.m. energy) $=\sqrt{s'}$?

As far as the last question is concerned, let us remark that the multiperipheral model and the gas-liquid analogy suggest that $f(q\,|\,q'q'')$ and $f(q)$ at (total c.m. energy) $=\sqrt{s'}$ look quite similar. This statement can be a starting point for a simple model of inclusive spectra, with the kinematic clustering effect built in it from the outset. In fact, assuming

$$f(q\,|\,q'q''p_1p_2) \approx f(q, p_1 - q', p_2 - q'') \quad (13)$$

and neglecting correlations between leading particles, one obtains the following integral equation[17] (we consider the case of identical spinless particle scattering, for simplicity):

$$f(qp_1p_2) = \sum_{j=1}^{2} h_j(qp_1p_2) + \int (dq')(dq'')h_1(q'p_1p_2) \times h_2(q''p_1p_2)\, f(q, p_1 - q', p_2 - q'')\,, \quad (14)$$

where $h_j(qp_1p_2)$ are defined analogously to (12) but for leading particles[18] (and are invariantly normalized to unity). The kinematic sum rule[19]

$$\int (dq)q_\mu f(qp_1p_2) = p_{1\mu} + p_{2\mu}$$

is identically satisfied by the solution of Eq. (14), as it should be. Furthermore if $h_j(q)$, considered as input functions, scale asymptotically so does $f(q)$. Equation (13) can be called the *recursion conjecture* since it permits one to express the inclusive spectrum at a given energy by the corresponding spectra at all lower energies. Equation (14) is expected to hold (if at all) at not-too-small energies. It can be used to study the evolution with energy of the rapidity plots, which are initially (at lower energies) approximately Gaussian. However, a more detailed discussion of this point is outside the scope of this paper.

We are indebted to Professor R. P. Feynman for reading the manuscript of this paper.

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†Laboratory associated with the CNRS. Work supported in part by the U. S. Atomic Energy Commission. Prepared under contract AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

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[3]R. P. Feynman, in *High Energy Collisions,* Third International Conference held at the State University of New York, Stony Brook, 1969, edited by C. N. Yang *et al.* (Gordon and Breach, New York, 1970).

[4]Essentially, we have in mind the clustering of other-than-leading secondaries. The leading particles are, on the rapidity plot and at very high energies, quite close to the kinematic limits.

[5]We define here the rapidity using the decimal logarithm. A derivation of (3) can be found in Ref. 1.

[6]C. Castagnoli *et al.*, Nuovo Cimento 10, 1539 (1953).

[7]Cf. M. Miesowicz, *Progress in Elementary Particle and Cosmic Ray Physics* (North Holland, Amsterdam, 1971), Vol. X, p. 103. Although this review paper is mainly devoted to a discussion of a particular model of multiparticle production, it has been written by an outstanding experimenter and contains a critical discussion of many experimental data and a rather complete list of references.

[8]The smallness of $\langle\sigma\rangle$ does not seem to result from the bias associated with the compositeness of the targets. This is a conclusion, perhaps hasty, one gets from a perusal of the distribution of σ corresponding to events with a large number of evaporation prongs, which obviously correspond to nonelementary interactions.

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[13]E. L. Berger and A. Krzywicki, Phys. Letters 36B, 380 (1971).

[14]Clustering can be avoided if multiplicity "conspires" with the effective mass $\sqrt{s'}$, so as to maintain the relative momenta between secondaries at very high values. This is not a realistic picture but this happens in the multiperipheral model with the dominant multiperipheral configuration.

[15]Z. Koba, H. B. Nielsen, and P. Olesen, Nucl. Phys. B40, 317 (1972); A. Białas and K. Zalewski, Cracow report, 1971 (unpublished).

[16]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[17]Essentially the same integral equation (up to a contribution to the inhomogeneous term, equal to the exclusive one-particle spectrum corresponding to 3- and 4-particle final states) holds in the multiperipheral model.

[18]Functions h_1 and h_2 are obtained, one from the other, by a reflection with respect to the plane $q_{\parallel}^{\text{c.m.}} = 0$.

[19]T. T. Chou and C. N. Yang, Phys. Rev. Letters 25, 1072 (1970).