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Some Aspects of (8, 8) Chiral Symmetry Breaking

H. Genz
Institut für Theoretische Kernphysik, Universität Karlsruhe, Germany

and

J. Katz
Institut für Theoretische Physik, Freie Universität Berlin, Germany
(Received 10 March 1972)

It is assumed that the chiral-symmetry-breaking Hamiltonian density transforms as part of the (8,8) representation of SU(3)⊗SU(3). We then derive a spectral-function sum rule for the model and make use of SU(3) assumptions to determine the symmetry-breaking parameter z. We find $z \approx -0.565$, in agreement with previous evaluations which made use of different techniques. The analogy to the $(\overline{3},3)\oplus(3,\overline{3})$ model of SU(3)⊗SU(3)-symmetry breaking is stressed. Other values of z for which the (8,8) model exhibits distinct qualitative features are also discussed. We conclude by presenting the soft-pion calculation of the K_{l3} form factors in the (8,8) model.

I. INTRODUCTION

There has recently been interest in the possibility that the chiral-symmetry-breaking Hamiltonian H' is part of the (8,8) representation of SU(3) ⊗ SU(3).[1-5] We shall not discuss here the (different) motivations of the authors of those references to abandon the $(3,\overline{3})\oplus(\overline{3},3)$ Gell-Mann–Oakes–Renner (GMOR, Refs. 6 and 7) scheme. We merely state that there are some apparent experimental difficulties of that model[4,8,9] and thus the study of more complicated schemes is certainly of interest.

If one abandons the $(3,\bar{3})\oplus(\bar{3},3)$ model the next simplest possibility is the assumption that H' transforms entirely as part of the $(8,8)$ representation of $SU(3)\otimes SU(3)$. Therefore, this alternative should be thoroughly investigated. Some steps in this direction have already been undertaken.[1-5]

We assume in the present paper that H' belongs to the $(8,8)$ representation of $SU(3)\otimes SU(3)$. It is then our first task in Sec. III to derive a spectral-function sum rule (SFSR) for such a model. In this way we make explicit that this particular SFSR (which was first derived by the authors in Ref. 2 under more restrictive assumptions) holds in any $(8,8)$-symmetry-breaking scheme. We then first make use of this SFSR to express the vacuum expectation value (VEV) of H' in terms of the VEV of the σ terms. This is of interest[10,11] for theories of scale symmetry breaking with a c-number δ.

In Sec. IV we employ this SFSR and in addition make use of SU(3) arguments in order to determine the value of the symmetry-breaking parameter z of the $(8,8)$ model. It is a remarkable fact that the value of z so obtained, namely

$$z=-\frac{1}{4}\,\frac{2(m_K/m_\pi)^2+1}{(m_K/m_\pi)^2-1}\approx -0.565\,, \tag{1.1}$$

is the same as that found in Refs. 1 and 5 making use of different assumptions.

For instructive purposes we always emphasize the analogy to the $(3,\bar{3})\oplus(\bar{3},3)$ model by first carrying out our calculations in that well-known scheme. Our discussion in that case partly overlaps that of Refs. 12 and 13. However, we feel that some of the points in Sec. IV that concern the GMOR model are presently of particular interest since the assumptions of GMOR are influenced by the possible contributions of ϵ poles to the matrix elements of the scalar densities u_i between pseudoscalar-meson states.[14,15] We, on the other hand, only make use of assumptions which are apparently not influenced by the ϵ contribution. As our main result in this case, SU(3) for $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$ yields (by use of a spectral-function sum rule for the vacuum expectation value of the σ terms[12,13,16]) *two* possible values of the symmetry-breaking parameter c of that model, namely $c\approx -1.25$ and $c\approx 0.05$. As is well known, the latter value implies a strong breaking of SU(3) for $\langle\Omega|v_\alpha|P_\beta\rangle$. Vice versa, SU(3) for $\langle\Omega|v_\alpha|P_\beta\rangle$ yields $c\approx -1.25$ and $c\approx 0.15$, the latter value implying a strong breaking of SU(3) for $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$. From SU(3) symmetry for *both* $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$ and $\langle\Omega|v_\alpha|P_\beta\rangle$ one of course immediately obtains $c=-1.25$, together with the Gell-Mann–Okubo mass formula for the η. However, let us stress once again that our main interest in this paper lies in the $(8,8)$ model.

In current-algebra models it is sometimes assumed that the pion σ terms are isoscalar, a hypothesis which might be tested in $\pi\pi$ scattering. However, nothing conclusive about them is yet known. If it would turn out that the pion terms have a considerable isospin-two part then, of course, the $(\bar{3},3)\oplus(3,\bar{3})$ model would be ruled out. It is also interesting to ask the converse question within our context: Would the absence of an isospin-two part of the pion σ term necessarily rule out the $(8,8)$ model? The first part of Sec. V is devoted to a study of this question. Namely, for $z=-1$ it is seen (Appendix B) that the pion σ terms are isoscalar. This value is not too far from the value $z=-0.565$, obtained in Sec. IV from SU(3) assumptions. It is, however, seen (Sec. V) that the SU(3) assumptions of Sec. IV are strongly violated for $z=-1$. Thus, isoscalar pion σ terms at an operator level together with the approximate validity of our SU(3) assumptions in Sec. IV would exclude the $(8,8)$ model.

Furthermore, we comment in Sec. V on the question (frequently asked and criticized in the literature) of whether the Hamiltonian density can be invariant under Kuo transformations. We shall see that if in the $(8,8)$ model one requires invariance under such transformations one is led to $z=\frac{1}{2}$ and, therefore, to a very large violation of SU(3). It is thus concluded that in the $(8,8)$ scheme the Hamiltonian density cannot be (neither exactly nor approximately) invariant under Kuo transformations.

We conclude by investigating in Sec. VI the K_{l3} form factors (see Ref. 17 for a review of the experimental situation) under the SU(3) assumptions in Eq. (4.17) and recover the well-known Callan-Treiman relation. We then present an argument as to why this relation should in general be expected to be valid if both the vacuum and the pseudoscalar meson states are approximately SU(3) symmetric.

In the K_{l3} calculations of Sec. VI a particular difference between the $(3,\bar{3})\oplus(\bar{3},3)$ and the $(8,8)$ model will become apparent. Namely, in the $(8,8)$ model the commutators between the axial charges Q_A^π and the vector current divergences are *not* proportional to linear combinations of current divergences. On the other hand, in the $(3,\bar{3})\oplus(\bar{3},3)$ model this proportionality holds for all values of the symmetry-breaking parameter.

II. DEFINING EQUATIONS FOR THE MODEL

For the convenience of the reader we present in this section the defining equations of the $(8,8)$ representation, essentially following Ref. 1. The (8,8) has 64 operators, which we call $S^{\alpha\beta}$ with $\alpha,\beta=1,\ldots,8$, transforming as

$$[Q^\alpha, S^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S^{\beta\delta}(x) + f^{\alpha\beta\delta}S^{\delta\gamma}(x)] \tag{2.1a}$$

under the SU(3) charges Q^α, and as

$$[Q_A^\alpha, S^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S^{\beta\delta}(x) - f^{\alpha\beta\delta}S^{\delta\gamma}(x)] \tag{2.1b}$$

under the axial SU(3)$\otimes$SU(3) charges Q_A^α. Defining symmetric and antisymmetric combinations of the $S^{\alpha\beta}$ by

$$S_s^{\alpha\beta} = S^{\alpha\beta} + S^{\beta\alpha} \tag{2.2a}$$

and

$$S_a^{\alpha\beta} = S^{\alpha\beta} - S^{\beta\alpha}, \tag{2.2b}$$

we note that these transform as

$$[Q^\alpha, S_s^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S_s^{\beta\delta}(x) + f^{\alpha\beta\delta}S_s^{\delta\gamma}(x)], \tag{2.3a}$$

$$[Q_A^\alpha, S_s^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S_a^{\beta\delta}(x) + f^{\alpha\beta\delta}S_a^{\gamma\delta}(x)], \tag{2.3b}$$

$$[Q^\alpha, S_a^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S_a^{\beta\delta}(x) - f^{\alpha\beta\delta}S_a^{\gamma\delta}(x)], \tag{2.3c}$$

and

$$[Q_A^\alpha, S_a^{\beta\gamma}(x)] = i\sum_{\delta=1}^{8}[f^{\alpha\gamma\delta}S_s^{\beta\delta} - f^{\alpha\beta\delta}S_s^{\gamma\delta}(x)]. \tag{2.3d}$$

Assuming that $S_s^{\alpha\beta}$ and $S_a^{\alpha\beta}$ have definite parities we see that the parity of $S_s^{\alpha\beta}$ is opposite to that of $S_a^{\alpha\beta}$. Analogously, we first see that the operators a_α and b_α defined by

$$a_\alpha = \sum_{\beta,\gamma=1}^{8} d_{\alpha\beta\gamma}S_s^{\beta\gamma} \tag{2.4a}$$

and

$$b_\alpha = \sum_{\beta,\gamma=1}^{8} f_{\alpha\beta\gamma}S_a^{\beta\gamma} \tag{2.4b}$$

transform as octets under SU(3). For such octets (as well as for singlets), the meaning of definite behavior under charge conjugation is well defined.[18] The commutation relations then imply that the C parities of a_α and b_α are the same. In order that the Hamiltonian has a part transforming as the eighth component of an octet, C must be positive. In order that the singlet can also be present, the parity of $S_s^{\alpha\beta}$ must be positive. Thus also the a's have positive parity. The most general SU(3) $\otimes$SU(3)-symmetry-breaking Hamiltonian constructed out of the (8,8) representation may then be written as

$$\begin{aligned} T_{00}(x) &\equiv H(x) \\ &= \hat{T}_{00}(x) + T'_{00}(x) - \text{VEV}, \end{aligned} \tag{2.5a}$$

with

$$T'_{00}(x) = \frac{A}{2\sqrt{8}}\sum_{\alpha=1}^{8} S_s^{\alpha\alpha}(x) + \frac{B\sqrt{3}}{2\sqrt{5}}\sum_{\alpha,\beta=1}^{8} d_{8\alpha\beta}S_s^{\alpha\beta}(x). \tag{2.5b}$$

In Eq. (2.5), $\hat{T}_{00}$ is invariant under SU(3)$\otimes$SU(3) and T'_{00} is [as seen from (2.5b)] a scalar (assuming, of course, that $S_s^{\alpha\beta}$ is). It will be convenient to define a quantity z by

$$z = \left(\frac{5}{8}\right)^{1/2}\frac{A}{B}. \tag{2.6}$$

The current divergences may then be obtained from the above by use of the relation

$$\partial^\mu J_\mu^a(x) = [iT'_{00}(x), Q^a(x_0)], \tag{2.7}$$

where $a=1,\ldots,16$ and J_μ^a denotes the vector and axial-vector currents. Since the explicit form of these divergences is sometimes useful, we have listed them in Appendix A.

III. SUM RULES FOR SPECTRAL FUNCTIONS

For the $(3,\bar{3})\oplus(\bar{3},3)$ model the vacuum expectation values of the σ terms $[Q^a, \partial^\mu J_\mu^b]$ have been thoroughly discussed in Refs. 12 and 13. As has first been pointed out in Ref. 12, a relation involving only the VEV of the σ terms follows in that model. That relation reads

$$3\sigma_A(8) + \sigma_A(3) = 4[\sigma_V(4) + \sigma_A(4)]. \tag{3.1}$$

[See Eq. (3.5) for the definition of the σ's.] In addition one has[12,13]

$$\sigma_V(4) = \frac{3c}{2}\left[\frac{1}{c+\sqrt{2}}\sigma_A(3) + \frac{2}{c-\sqrt{2}}\sigma_A(4)\right]. \tag{3.2}$$

It should be noticed that (3.1) does *not* depend on the value of the symmetry-breaking parameter c. As has already been pointed out in Ref. 2, in the (8,8) model there is just one relation between the VEV of the σ terms. It reads

$$\begin{aligned} \sigma_A(4) = &-\frac{3(1-2z)}{4(1+z)}\sigma_A(3) + (3-4z)\sigma_V(4) \\ &-\frac{1+6z-4z^2}{4(1-z-2z^2)}\sigma_A(8). \end{aligned} \tag{3.3}$$

The main difference between (3.1) and (3.3) is that (3.3) [in analogy to (3.2)] depends on the symmetry-breaking parameter z of the model. It is one of the purposes of this section to show that Eq. (3.3) holds in any (8,8)-symmetry-breaking scheme and does not depend on the further assumptions made in Ref. 2.

In order to derive Eq. (3.3) we first have to express the VEV of the $S_s^{\alpha\beta}$ in terms of the VEV of the σ terms. The resulting expressions also deserve some interest by themselves since, e.g., the VEV of T'_{00} may be expressed in terms of them [Eqs. (3.13) below]. We start by defining

$$\langle\Omega|S_s^{\alpha\beta}(x)|\Omega\rangle=\delta^{\alpha\beta}S(\alpha) \tag{3.4}$$

and

$$-i\langle\Omega|[Q_J^\alpha(x_0),\partial^\mu J_\mu^\beta(x)]|\Omega\rangle=\delta^{\alpha\beta}\sigma_J(\alpha)\,. \tag{3.5}$$

Equation (3.4) follows from $SU(2)_I\otimes Y$ invariance, which also yields

$$S(1)=S(2)=S(3) \tag{3.6a}$$

and

$$S(4)=S(5)=S(6)=S(7)\,. \tag{3.6b}$$

It is then very easy to obtain Eq. (3.5) together with the further properties

$$\sigma_A(1)=\sigma_A(2)=\sigma_A(3)\,, \tag{3.7a}$$

$$\sigma_A(4)=\sigma_A(5)=\sigma_A(6)=\sigma_A(7)\,, \tag{3.7b}$$

$$\sigma_V(1)=\sigma_V(2)=\sigma_V(3)=\sigma_V(8)=0\,, \tag{3.7c}$$

and

$$\sigma_V(4)=\sigma_V(5)=\sigma_V(6)=\sigma_V(7)\,. \tag{3.7d}$$

Since the commutators in Eq. (3.5) are linear forms in the $S_s^{\alpha\beta}$ we see that this equation provides us with an expression for the $\sigma_J(\alpha)$ in terms of the $S(\alpha)$. (The explicit forms may be read off in Appendix B.) Noticing that we have four independent $\sigma_J(\alpha)$ and only three independent $S(\alpha)$, we will have [solving for the $S(\alpha)$ in terms of the $\sigma_J(\alpha)$] three expressions for the $S(\alpha)$ and a relation between the $\sigma_J(\alpha)$.

These are

$$S(3)=\frac{1}{B}\frac{\sqrt5}{12(1+z)}[\sigma_A(8)-3\sigma_A(3)]\,, \tag{3.8a}$$

$$S(4)=\frac{1}{B}\frac{\sqrt5}{3(1-2z)}\sigma_A(8)\,, \tag{3.8b}$$

$$S(8)=\frac{\sqrt5}{B}\left[-\frac{3}{4(1+z)}\sigma_A(3)+\frac{8}{3}\sigma_V(4)-\frac{5+14z}{12(1+z)(1-2z)}\sigma_A(8)\right]\,, \tag{3.8c}$$

and

$$\sigma_A(4)=-\frac{3(1-2z)}{4(1+z)}\sigma_A(3)+(3-4z)\sigma_V(4)-\frac{1+6z-4z^2}{4(1+z)(1-2z)}\sigma_A(8)\,. \tag{3.9}$$

We want to note at this point that saturation of the σ terms by the appropriate particles leads to the formulas

$$\sigma_A(3)=\frac{f_\pi{}^2}{m_\pi{}^2}\,,\quad \sigma_A(4)=\frac{f_K{}^2}{m_K{}^2}\,,\qquad \sigma_A(8)=\frac{f_\eta{}^2}{m_\eta{}^2}\,,\quad \sigma_V(4)=\frac{f_\kappa{}^2}{m_\kappa{}^2}\,, \tag{3.10}$$

where we have neglected (following GMOR) $\eta\eta'$ mixing. In the above, the f_i are defined by (with P_β denoting the pseudoscalar mesons)

$$\langle\Omega|A_\mu^\alpha(0)|P_\beta\rangle=\delta_{\alpha\beta}\frac{f_\alpha}{m_\alpha{}^2}ip_\mu\,. \tag{3.11}$$

One might think that the positivity of the $\sigma_J(\alpha)$ alone via Eq. (3.9) restricts the possible values of z. This is, however, not the case.

We conclude this section by expressing the VEV of the $SU(3)\otimes SU(3)$-symmetry-breaking Hamiltonian in terms of the $\sigma_J(\alpha)$. This is of interest in the context of broken scale symmetry due to the following reason: If δ (for further reference and details the reader should consult Refs. 11 and 14) is a c number, then the commutator between the dilatation charge Q_D and T_μ^μ is given by ($H'\equiv T'_{00}$)

$$[Q_D(0),T_\mu^\mu(0)]=+il(l-4)H'(0)\,. \tag{3.12}$$

As in the $(3,\bar3)\oplus(\bar3,3)$ model, one may now take the VEV of this relation and express $\langle H'\rangle_0$ in terms of the $\sigma_J(\alpha)$. Using Eqs. (3.10) and saturating $\langle[Q_D,T_\mu^\mu(0)]\rangle_0$ by the ϵ contribution a relation between G_ϵ (defined by $\langle0|T_\mu^\mu|\epsilon\rangle=G_\epsilon$) and the f^α is obtained (involving also l and z). Similar analysis applies for the well-known $(3,\bar3)\oplus(\bar3,3)$ scheme. In this way the result of Ref. 10 (obtained there by using an equivalent Ward identity) for the $(3,\bar3)\oplus(\bar3,3)$ model is generalized to the (8,8). Precisely our technique was employed in Ref. 19 in order to analyze the commutator $[Q_D(0),\sigma(0)]=il\sigma(0)$ which is the same as Eq. (3.12) for $SU(2)\otimes SU(2)$ with σ defined by

$$\sigma(0)=\sum_{\alpha=1}^{3}[Q_A^\alpha,\partial^\mu A_\mu^\alpha(0)]$$

and H' belonging to the $(\frac12,\frac12)$ representation. In the present case of (8,8) symmetry breaking the VEV of H' expressed in terms of the $\sigma_J(\alpha)$ may be written as

$$\langle H'\rangle_0=\frac{-3z}{4(1+z)}\sigma_A(3)-\frac{4(1-z)}{3}\sigma_V(4)+\frac{z(7-2z)}{12(1+z)(1-2z)}\sigma_A(8) \tag{3.13a}$$

or equivalently [using (3.9)] as

$$\langle H'\rangle_0 = \frac{4z^2+3z-4}{4(1+z)(3-4z)}\sigma_A(3) - \frac{4}{3}\frac{1-z}{3-4z}\sigma_A(4) + \frac{-8z^3+6z^2+z-4}{12(1+z)(1-2z)(3-4z)}\sigma_A(8). \qquad (3.13b)$$

For the purposes of broken scale symmetry [as described after Eq. (3.12)] the form in (3.13b) seems more convenient. The above analysis for the (8,8) model is given elsewhere by Genz, Handschig, and Katz.[20]

IV. DETERMINATION OF z FROM SU(3) ARGUMENTS

We wish to make use in this section of SU(3) arguments to determine the symmetry-breaking parameter z in the (8,8) model. In order to clarify our discussion we first present the similar analysis for the well-known $(3,\bar{3})\oplus(\bar{3},3)$ scheme. In this case our methods are analogous to those of Ref. 13 and we differ mainly in the point of view. Our arguments lead to the well-known result of GMOR, i.e., $c=-1.25$. However, we differ from them in that we do not make any SU(3) or smoothness assumptions for the matrix element $\langle P_\alpha|u_\beta|P_\gamma\rangle$. [We denote the scalar and pseudoscalar densities of the $(3,\bar{3})\oplus(\bar{3},3)$ model by u_α and v_α, respectively.] We rather directly assume that *either* of the matrix elements $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$ or $\langle\Omega|v_\alpha|P_\beta\rangle$ is SU(3)-symmetric. Any one of these assumptions first yields two possible values of c. In either of these cases the solutions are $c\approx-1.25$ and $c\approx 0$. For both values it follows that $\sigma_V(4)\approx 0$, i.e., the vacuum is approximately SU(3)-symmetric. However, we argue that the solutions $c\approx 0$ are unlikely in the real world since they yield a *large* amount of SU(3) breaking in $\langle\Omega|v_\alpha|P_\beta\rangle$ or $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$, respectively.[21] We are thus left with $c\approx-1.25$ as the physical solution to either of these SU(3) assumptions.

Our method to determine c for the $(\bar{3},3)\oplus(3,\bar{3})$ model consists in eliminating $\sigma_V(4)$ from Eqs. (3.1) and (3.2). As discussed above this yields as one of its consequences SU(3) invariance for the vacuum, i.e., $\sigma_V(4)\approx 0$. On the other hand, for (8,8) we only have *one* SFSR. Thus we will for this case further assume that the vacuum is approximately SU(3)-invariant in order to eliminate $\sigma_V(4)$. In this way the value $z=-0.565$ is obtained for the (8,8) model. This is in agreement with the result obtained in Ref. 5 by generalizing the GMOR arguments to the (8,8) model and assuming in addition that $\langle\Omega|S_a^{\alpha\beta}|P_\gamma\rangle = rf^{\alpha\beta\gamma}$. Furthermore, this value of z was also found in Ref. 1 at a Lagrangian level.

We would like to stress at this point that our assumptions are apparently not influenced by the possible contribution of ϵ poles to the three-point function [for both the $(3,\bar{3})\oplus(\bar{3},3)$ and the (8,8) model]. This may be of some value since the assumptions of GMOR have been recently challenged on the basis of such possible pole contributions.[14,15]

We next first turn our attention to the $(3,\bar{3})\oplus(\bar{3},3)$ model and proceed to obtain consequences of the assumption that the wave function renormalization constants of the pseudoscalar densities v_α obey SU(3) symmetry in the form ($\alpha=0,\ldots,8$, $\beta=1,\ldots,8$)

$$\langle\Omega|v_\alpha|P_\beta\rangle = s\,\delta_{\alpha\beta}. \qquad (4.1)$$

Using the definition in Eq. (3.11) it follows immediately from the above [and from the well-known expressions for the axial current divergences in the $(3,\bar{3})\oplus(\bar{3},3)$ model] that

$$\sqrt{Y_K} = \left(\frac{m_\pi}{m_K}\right)^2 \frac{2\sqrt{2}-c}{2(\sqrt{2}+c)} \qquad (4.2)$$

and

$$\sqrt{Y_\eta} = \left(\frac{m_\pi}{m_\eta}\right)^2 \frac{\sqrt{2}-c}{\sqrt{2}+c}\ . \qquad (4.3)$$

In the above we have defined

$$Y_K = \frac{f_K^{\;2}}{m_K^{\;4}}\,\frac{m_\pi^{\;4}}{f_\pi^{\;2}} \qquad (4.4)$$

and

$$Y_\eta = \frac{f_\eta^{\;2}}{m_\eta^{\;4}}\,\frac{m_\pi^{\;4}}{f_\pi^{\;2}}\ . \qquad (4.5)$$

Note that according to our definition one has experimentally that

$$\sqrt{Y_K} = 1.28\,, \qquad (4.6)$$

i.e., $\sqrt{Y_K}$ is near to its SU(3) value

$$Y_K = Y_\eta = 1\,. \qquad (4.7)$$

We next proceed by eliminating $\sigma_V(4)$ from Eqs. (3.1) and (3.2). The result is a quadratic equation in c which involves (after saturating) Y_K and Y_η. It reads[16] (assuming the Gell-Mann–Okubo mass formula)

$$\left[-5-16Y_K\left(\frac{m_K}{m_\pi}\right)^2+3Y_\eta\left(\frac{m_\eta}{m_\pi}\right)^2\right]c^2 + \sqrt{2}\left[11-8Y_K\left(\frac{m_K}{m_\pi}\right)^2-3Y_\eta\left(\frac{m_\eta}{m_\pi}\right)^2\right]c + 4\left[-1+4Y_K\left(\frac{m_K}{m_\pi}\right)^2-3Y_\eta\left(\frac{m_\eta}{m_\pi}\right)^2\right]=0\,. \qquad (4.8)$$

If we next assume SU(3) symmetry for $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$ we have $Y_K=Y_\eta=1$ and thus, upon solving for c in (4.8) it follows that

$$c\approx\begin{cases}-1.25\\ \;\;0.05\end{cases}\ . \qquad (4.9)$$

On the other hand, if we instead assume Eq. (4.1) we may eliminate Y_K and Y_η from (4.8) by making use of Eqs. (4.2) and (4.3) [which are consequences of (4.1)]. In this fashion another quadratic equation for c is obtained. It reads

$$\left[-5-4\left(\frac{m_\pi}{m_K}\right)^2+3\left(\frac{m_\pi}{m_\eta}\right)^2\right]c^2 +2\sqrt{2}\left[-2+5\left(\frac{m_\pi}{m_K}\right)^2-3\left(\frac{m_\pi}{m_\eta}\right)^2\right]c +2\left[1-4\left(\frac{m_\pi}{m_K}\right)^2+3\left(\frac{m_\pi}{m_\eta}\right)^2\right]=0\,. \tag{4.10}$$

The solutions of the above equations are

$$c\approx\begin{cases}-1.25\\ \ \ 0.15\end{cases}. \tag{4.11}$$

Our conclusions now follow from (4.2), (4.3), (4.9), and (4.11). First of all, the assumption $Y_K=1$ together with (4.2) numerically implies $c\approx-1.25$. From $Y_\eta=1$ we then obtain the Gell-Mann–Okubo mass formula. Thus, $c\approx-1.25$ in the solutions (4.9) and (4.11) [obtained there from either $Y_K=Y_\eta=1$ or (4.1)] is in agreement with SU(3) symmetry for both $\langle\Omega|v_\alpha|P_\beta\rangle$ and $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$. For the other value of c in (4.9) (i.e., $c\approx0.05$) it immediately follows from Eqs. (4.2) and (4.3) that SU(3) symmetry as written in Eq. (4.1) is badly violated. In fact, this value of c and $Y_K=Y_\eta=1$ approximately yields

$$\frac{\langle\Omega|v_\pi|\pi\rangle}{\langle\Omega|v_K|K\rangle}=\left(\frac{m_\pi}{m_K}\right)^2 \tag{4.12}$$

and

$$\frac{\langle\Omega|v_\pi|\pi\rangle}{\langle\Omega|v_\eta|\eta\rangle}=\left(\frac{m_\pi}{m_\eta}\right)^2. \tag{4.13}$$

On the other hand, the value $c\approx0.15$ in (4.11) badly violates SU(3) symmetry for $\langle\Omega|\partial^\mu A_\mu^\alpha|P_\beta\rangle$. With this value of c we approximately obtain

$$Y_K=\left(\frac{m_\pi}{m_K}\right)^2 \tag{4.14}$$

and

$$Y_\eta=\left(\frac{m_\pi}{m_\eta}\right)^2. \tag{4.15}$$

Finally, we return to Eq. (3.2) and note that for this value of c (as well as for $c\approx0$) it follows that

$$\sigma_V(4)\approx0\,. \tag{4.16}$$

After having illustrated our method in the well-known case of $(3,\overline{3})\oplus(\overline{3},3)$-symmetry breaking we next proceed to the (8,8). The SU(3) assumption analogous to (4.1) now reads

$$\langle\Omega|S_a^{\alpha\beta}|P_\gamma\rangle=rf^{\alpha\beta\gamma}\,. \tag{4.17}$$

(The preceding assumption has also been made in Ref. 5.) Using again the definition in (3.11) we immediately have, in analogy to (4.2) and (4.3),

$$\sqrt{Y_K}=\left(\frac{m_\pi}{m_K}\right)^2\frac{1}{2}\,\frac{4z-1}{2z+1} \tag{4.18}$$

and

$$\sqrt{Y_\eta}=\left(\frac{m_\pi}{m_\eta}\right)^2\frac{2z-1}{2z+1}. \tag{4.19}$$

In contrast to (3.1) and (3.2) we have only *one* spectral-function sum rule for the σ terms in the case of (8,8). It reads

$$\sigma_V(4)=\frac{1}{3-4z}\left[\sigma_A(4)+\frac{3(1-2z)}{4(1+z)}\sigma_A(3)+\frac{1+6z-4z^2}{4(1-z-2z^2)}\sigma_A(8)\right]. \tag{4.20}$$

Thus, since $\sigma_V(4)$ cannot be eliminated as in the $(\overline{3},3)\oplus(3,\overline{3})$ model, we assume in addition approximate SU(3) invariance of the vacuum, i.e.,

$$Q_V^\alpha|\Omega\rangle\approx0\,. \tag{4.21}$$

First of all, (4.21) has the obvious solution of approximate SU(3) symmetry for the Hamiltonian ($z^{-1}=0$). Excluding this solution for the moment we then obtain from (4.20) and (4.21) the (saturated) sum rule[22] (assuming the Gell-Mann–Okubo mass formula)

$$3(1-2z)^2+4(1+z)(1-2z)Y_K\left(\frac{m_K}{m_\pi}\right)^2 +(1+6z-4z^2)Y_\eta\left[\frac{4}{3}\left(\frac{m_K}{m_\pi}\right)^2-\frac{1}{3}\right]=0\,. \tag{4.22}$$

For $z^{-1}=0$ the relations (4.18) and (4.19) once again imply that either (4.17) is badly violated or that $Y_K=(m_\pi/m_K)^4$ and $Y_\eta=(m_\pi/m_\eta)^4$. Thus, as before, the solution with physical masses and $z^{-1}=0$ is unlikely. Next, assuming (4.17), (4.21), and $z^{-1}=0$, we may combine Eqs. (4.18), (4.19), and (4.22) in order to obtain a cubic relation for z which reads

$$16z^3\left[-6\left(\frac{m_K}{m_\pi}\right)^4+7\left(\frac{m_K}{m_\pi}\right)^2-1\right] +4z^2\left[-12\left(\frac{m_K}{m_\pi}\right)^4-\left(\frac{m_K}{m_\pi}\right)^2-2\right] +z\left[24\left(\frac{m_K}{m_\pi}\right)^4-22\left(\frac{m_K}{m_\pi}\right)^2+7\right] +12\left(\frac{m_K}{m_\pi}\right)^4+4\left(\frac{m_K}{m_\pi}\right)^2-1=0\,. \tag{4.23}$$

The solutions of Eq. (4.23) are

$$z_1 = -\frac{1}{4}\,\frac{2(m_K/m_\pi)^2+1}{(m_K/m_\pi)^2-1} \tag{4.24}$$

and

$$z_\pm = \frac{-b \pm (a^2+b^2)^{1/2}}{2a}\ , \tag{4.25}$$

with

$$a = 4\left(\frac{m_\pi}{m_K}\right)^2\left[-6\left(\frac{m_K}{m_\pi}\right)^8 + 19\left(\frac{m_K}{m_\pi}\right)^6 - 21\left(\frac{m_K}{m_\pi}\right)^4 + 9\left(\frac{m_K}{m_\pi}\right)^2 - 1\right] \tag{4.26}$$

and

$$b = 3\left(\frac{m_\pi}{m_K}\right)^2\left[\left(\frac{m_K}{m_\pi}\right)^6 - 3\left(\frac{m_K}{m_\pi}\right)^4 + 3\left(\frac{m_K}{m_\pi}\right)^2 - 1\right]. \tag{4.27}$$

Numerically, we have

$$z_1 \approx -0.565\ , \tag{4.28}$$

$$z_- \approx -0.49\ , \tag{4.29}$$

and

$$z_+ \approx 0.51\ . \tag{4.30}$$

Out of these, only z_1 appears to be physical. Namely, making use of (4.18) and (4.19) it yields $Y_K = Y_\eta = 1$. On the other hand, for z_- and z_+ one gets $Y_K = 121$ and $Y_K = 4\times10^{-4} \approx 0$, respectively. Thus these two solutions are ruled out. Incidentally, note also that as $m_\pi \to 0$, $z_1 = z_- = -z_+ = -\frac{1}{2}$.

In analogy to the $(3,\bar{3})\oplus(\bar{3},3)$ case we next use the facts derived above to obtain the value $z \approx -0.565$ from various assumptions. Combining SU(3) for $\langle\Omega|S^{\alpha\beta}|P_\gamma\rangle$ and $|\Omega\rangle$ and assuming, say, $0.01 < Y_K < 100$, we have the desired result. Secondly we may also assume $Y_K = 1$ together with (4.17) and obtain Eq. (1.1) from (4.18). Furthermore $Y_\eta = 1$ once again yields the Gell-Mann–Okubo mass formula. If one uses the physical value of Y_K in (4.18) one obtains

$$z = -0.55 \tag{4.31}$$

(together with $\sqrt{Y_\eta} = 1.29$). The reader should however notice that this solution yields a negative $\sigma_V(4)$ [Eq. (3.9)] and is therefore excluded. Finally, in case of SU(3) for the vacuum, $z^{-1} \neq 0$, and $Y_K = Y_\eta = 1$ we may use (4.22) in order to obtain the solutions $z = \frac{4}{5}$ or Eq. (4.24) (i.e., $z \approx -0.565$). The solution $z = \frac{4}{5}$ yields $Y_K \sim Y_\eta \sim 0.01$ if combined with (4.18) and (4.19), showing that for this value (4.17) [from which (4.18) and (4.19) were derived] is badly broken. We conclude again that this solution is less likely so that (4.21) together with $Y_K = Y_\eta = 1$ implies $z \approx -0.565$.

In summary, we have presented above three arguments which lead to $z \approx -0.565$. For a discussion of the implications of this result and comparison with GMOR we refer the reader to the beginning of this section.

V. COMMENTS ON PION σ TERMS AND THE KUO TRANSFORMATION IN THE (8, 8) MODEL

The pion σ terms are isoscalar at an operator level in the (8, 8) model if and only if (Appendix B)

$$z = -1. \tag{5.1}$$

On the other hand, recall that the SU(3) assumptions of Sec. IV led us to

$$z \approx -0.565\ . \tag{5.2}$$

The values for z in Eqs. (5.1) and (5.2) differ by about 40%. Depending on the point of view this difference might or might not *a priori* be considered acceptable.

We next analyze in more detail the value in Eq. (5.1) and show that the value $z = -1$ in fact implies strong violation of SU(3) symmetry.

To see this we first combine (5.1) with (3.3) (assuming only that the σ terms are finite) and obtain [noticing that (3.3) has a pole at $z = -1$]

$$3\sigma_A(3) = \sigma_A(8) \tag{5.3}$$

which when saturated yields

$$Y_\eta = 3\left(\frac{m_\pi}{m_\eta}\right)^2. \tag{5.4}$$

This result already implies a large violation of SU(3). Furthermore, it also implies that (4.17) is strongly violated since (4.19) [which is directly obtained from (4.17)] leads for $z = -1$ to

$$Y_\eta = 9\left(\frac{m_\pi}{m_\eta}\right)^4. \tag{5.5}$$

Furthermore, from (4.18) [another direct consequence of (4.17)] one obtains for $z = -1$ that

$$Y_K = \frac{25}{4}\left(\frac{m_\pi}{m_K}\right)^4\ , \tag{5.6}$$

in strong violation of the experimental value for Y_K. We therefore conclude that SU(3) symmetry as written in Eq. (4.17) is badly violated if the pion σ terms are isoscalar at an operator level in the (8, 8) model.

We conclude this section by discussing the value

$$z = \tfrac{1}{2}\ , \tag{5.7}$$

since only for this value of z the apparent paradox discovered by Kuo[3] is not present in the (8, 8) model. The pole at $z = \frac{1}{2}$ appears in Eqs. (3.8) and (3.9) only in the form $\sigma_A(8)(1-2z)^{-1}$ and thus Eq. (5.4) implies

$$\sigma_A(8)=0, \tag{5.8}$$

with the obvious consequence $f_\eta^2/m_\eta^4=0$ or, if finite, $m_\eta^2=0$. Thus, in contrast to the $(3,\bar{3})\oplus(\bar{3},3)$ case, a large amount of SU(3) breaking is present in the (8, 8) model if the Hamiltonian is invariant under Kuo transformations.

VI. DISCUSSION OF K_{l3} DECAYS IN THE (8, 8) MODEL

We start by writing the K_{l3} matrix element as

$$\langle\pi^0(\vec{q})|\partial^\mu V_\mu^{K^-}(0)|K^+(\vec{p})\rangle = -\frac{i}{\sqrt{2}}[f_+(q^2,t)(m_\pi^2-m_K^2)-tf_-(q^2,t)] \tag{6.1}$$

with $t=(p-q)^2$. Reducing out the pion and performing the limit $q_\mu\to 0$ one arrives in the standard fashion at

$$-\frac{im_\pi^2}{f_\pi}\langle\Omega|[Q_A^{\pi^0},\partial^\mu V_\mu^{K^-}(0)]|K^+(\vec{p})\rangle = -\frac{i}{\sqrt{2}}[f_+(0,m_K^2)(m_\pi^2-m_K^2)-m_K^2 f_-(0,m_K^2)]\,. \tag{6.2}$$

As is also well known, if one instead starts from the matrix element

$$\langle\pi^0(\vec{q})|V_\mu^{K^-}(0)|K^+(\vec{p})\rangle$$

one arrives by the same technique at the Callan-Treiman relation, which reads

$$-i\frac{m_\pi^2}{f_\pi}\langle\Omega|[Q_A^{\pi^0},V_\mu^{K^-}(0)]|K^+(\vec{p})\rangle = -\frac{1}{\sqrt{2}}[f_+(0,m_K^2)+f_-(0,m_K^2)]p_\mu\,. \tag{6.3}$$

[Comparing (6.2) and (6.3) we neglect the explicit m_π^2 in the right-hand side of Eq. (6.2).]

In the $(3,\bar{3})\oplus(\bar{3},3)$ model Eqs. (6.2) and (6.3) are equivalent[7,23] if and only if $c\approx-\sqrt{2}$. More generally, one may use the relation

$$[Q_A^\pi(0),\partial^\mu V_\mu^K(0)]-[Q_V^K(0),\partial^\mu A_\mu^\pi(0)]=\partial^\mu[Q_A(0),V_\mu^K(0)] \tag{6.4}$$

in order to see that this equivalence means

$$\langle\Omega|[Q_V^K,\partial^\mu A_\mu^\pi(0)]|K(\vec{q})\rangle\approx 0. \tag{6.5}$$

In the $(3,\bar{3})\oplus(\bar{3},3)$ model *each* of the terms in Eq. (6.4) is proportional to

$$\partial^\mu[Q_A^\pi(0),V_\mu^K(0)]\,\alpha\,\partial^\mu A_\mu^K$$

for all values of c. In contrast, this is never the case in the (8, 8) scheme. However, for $z=+1$ the matrix elements of the commutators in Eq. (6.4) may be solely expressed in terms of

$$\langle\Omega|\partial^\mu[Q_A^\pi(0),V_\mu^K(0)]|K^+\rangle\,\alpha\,\langle\Omega|\partial^\mu A^K|K^+\rangle\,.$$

Thus, only for this value of z can the soft-pion prediction for the K_{l3} form factors be completely worked out in the (8, 8) model without making any additional assumptions. It is worthwhile to emphasize at this point that Eq. (6.5) at most holds for certain values of the symmetry-breaking parameter. These, however, are expected to be approximately the same as those discussed in Sec. IV [i.e., $c\approx-\sqrt{2}$ for the $(\bar{3},3)\oplus(3,\bar{3})$ model, and $z\approx-0.565$ for the (8, 8) case]. In order to see this we note that by making use of the SU(3) assumptions of that section [i.e., $Q_V|\Omega\rangle\approx 0$, SU(3) for pseudoscalar-meson states, and $\langle\Omega|v_\alpha|P_\beta\rangle=s\delta_{\alpha\beta}$ or $\langle\Omega|S_{\alpha\beta}|P_\gamma\rangle=rf_{\alpha\beta\gamma}$] the left-hand side of Eq. (6.5) becomes essentially proportional to m_π^2 and may then be neglected. Thus, Eqs. (6.2) and (6.3) lead to physically different results only if there is a strong breaking of SU(3). This is, of course, well known for the $(3,\bar{3})\oplus(\bar{3},3)$ model and will be seen explicitly later on in this section for the (8, 8) chiral-symmetry-breaking model.

We next proceed to calculate the matrix element in Eq. (6.2) in the (8, 8) model. We start by noting that from $SU(2)_I\otimes Y$ invariance it follows that, e.g.,

$$\begin{aligned}\langle\Omega|S_a^{17}|K^+\rangle &= i\langle\Omega|S_a^{16}|K^+\rangle\\ &= \langle\Omega|S_a^{26}|K^+\rangle = -i\langle\Omega|S_a^{27}|K^+\rangle\\ &= \langle\Omega|S_a^{35}|K^+\rangle = i\langle\Omega|S_a^{34}|K^+\rangle\end{aligned} \tag{6.6}$$

and

$$\langle\Omega|S_a^{58}|K^+\rangle = i\langle\Omega|S_a^{48}|K^+\rangle\,. \tag{6.7}$$

The commutator of interest to us may be computed using the formulas of Sec. II and Appendix A. The result is

$$[Q_A^3,\partial^\mu V_\mu^4-i\partial^\mu V_\mu^5]=\tfrac{1}{8}B(\tfrac{3}{5})^{1/2}[3\sqrt{3}(-S_a^{17}-S_a^{26}-iS_a^{16}+iS_a^{27})+iS_a^{48}+S_a^{58}+\sqrt{3}(S_a^{35}+iS_a^{34})]\,. \tag{6.8}$$

For comparison we also note that (using again Appendix A)

$$\begin{aligned}\partial^\mu[Q_A^3,V_\mu^4-iV_\mu^5] &= -\tfrac{1}{2}(\partial^\mu A_\mu^4-i\partial^\mu A_\mu^5)\\ &= -\frac{1}{2}\left[\left(\frac{B}{4\sqrt{5}}+\frac{A}{\sqrt{8}}\right)(-S_a^{17}-S_a^{26}-iS_a^{16}+iS_a^{27}-S_a^{35}-iS_a^{34})+\left(-\frac{3B}{4\sqrt{5}}+\frac{A}{\sqrt{8}}\right)\sqrt{3}(S_a^{58}+iS_a^{48})\right]\,.\end{aligned} \tag{6.9}$$

Using Eqs. (6.6) and (6.7) we have therefore

$$2\langle\Omega|[Q_A^3,\, \partial^\mu V_\mu^4 - i\partial^\mu V_\mu^5]|K^+\rangle = -\tfrac{1}{2}B(\tfrac{3}{5})^{1/2}(5\sqrt{3}\,\langle\Omega|S_a^{17}|K^+\rangle - \langle\Omega|S_a^{58}|K^+\rangle) \tag{6.10}$$

and

$$\langle\Omega|\,\partial^\mu A_\mu^4 - i\partial^\mu A_\mu^5|K^+\rangle = -\frac{B\sqrt{3}}{2\sqrt{5}}\{\sqrt{3}\,(1+4z)\langle\Omega|S_a^{17}|K^+\rangle + (3-4z)\langle\Omega|S_a^{58}|K^+\rangle\}\,. \tag{6.11}$$

As discussed after Eq. (6.4), further assumptions are generally necessary in order to express the matrix element in Eq. (6.10) in terms of the expression in (6.11). However, it is amusing to note that there is a value of z, i.e., $z=1$, for which these two matrix elements do become proportional. Therefore, only in this case can the K_{l3} calculation be carried out without making any additional assumptions. Thus, although this value of z would imply a large amount of SU(3) symmetry breaking it is nevertheless interesting to also present our results in this case. We next proceed to discuss this value of z (i.e., $z=1$) and note that Eq. (6.11) then becomes

$$\begin{aligned}\langle\Omega|[Q_A^3,\, \partial^\mu V_\mu^4 - i\partial^\mu V_\mu^5]|K^+\rangle &= \tfrac{1}{2}\langle\Omega|\,\partial^\mu A_\mu^4 - i\partial^\mu A_\mu^5|K^+\rangle\\ &= \tfrac{1}{2}\sqrt{2}\,f_K\,.\end{aligned} \tag{6.12}$$

Thus we obtain in the (8, 8) model with $z=1$

$$\left\{\left[1-\left(\frac{m_\pi}{m_K}\right)^2\right]f_+(0,m_K^{\;2}) + f_-(0,m_K^{\;2})\right\} = -\left(\frac{f_K/m_K^{\;2}}{f_\pi/m_\pi^{\;2}}\right). \tag{6.13}$$

The above is to be compared with the usual Callan-Treiman relation which in our notation reads

$$f_+(0,m_K^{\;2}) + f_-(0,m_K^{\;2}) = \frac{f_K/m_K^{\;2}}{f_\pi/m_\pi^{\;2}}\quad . \tag{6.14}$$

The reader should notice that Eq. (6.13) is identical to the Callan-Treiman relation *except for the sign*. Thus it leads to

$$\lambda_0 = -0.18. \tag{6.15}$$

This is to be compared with the experimental value[17] of

$$\lambda_0 = -0.024 \pm 0.02, \tag{6.16}$$

in contrast to the Callan-Treiman value of

$$\lambda_0 = +0.02. \tag{6.17}$$

In order to compute the right-hand side of Eq. (6.10) for the physical value of z we assume SU(3) [as in Eq. (4.17)] in the form

$$\langle\Omega|S_a^{\alpha\beta}|P_\gamma\rangle = r f^{\alpha\beta\gamma}. \tag{6.18}$$

This formula implies that, using (6.2), (6.10), (6.11), and the definitions of f_K and f_π,

$$\left[1-\left(\frac{m_\pi}{m_K}\right)^2\right]f_+(0,m_K^{\;2}) + f_-(0,m_K^{\;2}) = -\sqrt{Y_K}\,\frac{3}{4z-1}\,. \tag{6.19}$$

Using the physical value of z [Eq. (1.1)] we then essentially obtain the Callan-Treiman relation.

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APPENDIX A

In this appendix we want to list the expressions one obtains for the current divergences. Defining τ, σ, ρ, and κ by

$$\tau = \frac{B}{\sqrt{5}}\left[\frac{1}{4}+\left(\frac{5}{8}\right)^{1/2}\frac{A}{B}\right], \tag{A1}$$

$$\sigma = \frac{B}{\sqrt{5}}\left[-\frac{1}{2}+\left(\frac{5}{8}\right)^{1/2}\frac{A}{B}\right], \tag{A2}$$

$$\rho = \frac{2B}{\sqrt{5}}\left[1+\left(\frac{5}{8}\right)^{1/2}\frac{A}{B}\right], \tag{A3}$$

and

$$\kappa = \frac{B\sqrt{3}}{\sqrt{5}}\left[-\frac{3}{4}+\left(\frac{5}{8}\right)^{1/2}\frac{A}{B}\right], \tag{A4}$$

we have

$$\partial^\mu V_\mu^a = 0 \quad \text{for } a = 1, 2, 3, \text{ and } 8,$$

$$\partial^\mu V_\mu^4 = \tfrac{1}{4}B(\tfrac{3}{5})^{1/2}[-\sqrt{3}\,(S_s^{17}+S_s^{26}+S_s^{35}) + S_s^{58}], \tag{A5}$$

$$\partial^\mu V_\mu^5 = \tfrac{1}{4}B(\tfrac{3}{5})^{1/2}[\sqrt{3}\,(S_s^{16}-S_s^{27}+S_s^{34}) - S_s^{48}], \tag{A6}$$

$$\partial^\mu V_\mu^6 = \tfrac{1}{4}B(\tfrac{3}{5})^{1/2}[\sqrt{3}\,(-S_s^{15}+S_s^{24}+S_s^{37}) + S_s^{78}], \tag{A7}$$

$$\partial^\mu V_\mu^7 = \tfrac{1}{4}B(\tfrac{3}{5})^{1/2}[\sqrt{3}\,(S_s^{14}+S_s^{25}-S_s^{36}) - S_s^{68}], \tag{A8}$$

$$\partial^\mu A_\mu^1 = \rho S_a^{23} + \sigma(S_a^{47} - S_a^{56}), \tag{A9}$$

$$\partial^\mu A_\mu^2 = -\rho S_a^{13} + \sigma(S_a^{46} + S_a^{57}), \tag{A10}$$

$$\partial^\mu A_\mu^3 = \rho S_a^{12} + \sigma(S_a^{45} - S_a^{67}), \tag{A11}$$

$$\partial^\mu A_\mu^4 = \tau(-S_a^{17} - S_a^{26} - S_a^{35}) + \kappa S_a^{58}, \tag{A12}$$

$$\partial^\mu A_\mu^5 = \tau(S_a^{16} - S_a^{27} + S_a^{34}) - \kappa S_a^{48}, \tag{A13}$$

$$\partial^\mu A_\mu^6 = \tau(-S_a^{15} + S_a^{24} + S_a^{37}) + \kappa S_a^{78}, \tag{A14}$$

$$\partial^\mu A_\mu^7 = \tau(S_a^{14} + S_a^{25} - S_a^{36}) - \kappa S_a^{68}, \tag{A15}$$

and

$$\partial^\mu A_\mu^8 = \sigma\sqrt{3}\,(S_a^{45} + S_a^{67}). \tag{A16}$$

APPENDIX B

In this appendix we wish to list those σ terms which have been used in this paper. These are the diagonal ones, i.e., $[Q^a, \partial^\mu J_\mu^a]$ and in addition the off-diagonal ones for $SU(2)\otimes SU(2)$. Even though these σ terms could be written in a more compact notation we find the completely explicit form given here much more instructive. One has (with τ, σ, ρ, and κ as defined in Appendix A)

$$[Q^4, \partial^\mu V_\mu^4] = [Q^5, \partial^\mu V_\mu^5] = -i\tfrac{1}{4}B(\tfrac{3}{5})^{1/2}\,[\tfrac{1}{2}\sqrt{3}\,(S_s^{11} + S_s^{22} + S_s^{33} - S_s^{66} - S_s^{77} - S_s^{88}) + S_s^{38}], \tag{B1}$$

$$[Q^6, \partial^\mu V_\mu^6] = [Q^7, \partial^\mu V_\mu^7] = -i\tfrac{1}{4}B\,(\tfrac{3}{5})^{1/2}[\tfrac{1}{2}\sqrt{3}\,(S_s^{11} + S_s^{22} + S_s^{33} - S_s^{44} - S_s^{55} - S_s^{88}) - S_s^{38}], \tag{B2}$$

$$[Q_A^1, \partial^\mu A_\mu^1] = i[-\rho(S_s^{22} + S_s^{33}) - \tfrac{1}{2}\sigma(S_s^{44} + S_s^{55} + S_s^{66} + S_s^{77})], \tag{B3}$$

$$[Q_A^1, \partial^\mu A_\mu^2] = [Q_A^2, \partial^\mu A_\mu^1] = i\rho S_s^{12}, \tag{B4}$$

$$[Q_A^1, \partial^\mu A_\mu^3] = [Q_A^3, \partial^\mu A_\mu^1] = i\rho S_s^{13}, \tag{B5}$$

$$[Q_A^2, \partial^\mu A_\mu^2] = i[-\rho(S_s^{11} + S_s^{33}) - \tfrac{1}{2}\sigma(S_s^{44} + S_s^{55} + S_s^{66} + S_s^{77})], \tag{B6}$$

$$[Q_A^2, \partial^\mu A_\mu^3] = [Q_A^3, \partial^\mu A_\mu^2] = i\rho S_s^{23}, \tag{B7}$$

$$[Q_A^3, \partial^\mu A_\mu^3] = i[-\rho(S_s^{11} + S_s^{22}) - \tfrac{1}{2}\sigma(S_s^{44} + S_s^{55} + S_s^{66} + S_s^{77})], \tag{B8}$$

$$[Q_A^4, \partial^\mu A_\mu^4] = -\tfrac{1}{2}i[\tau(S_s^{11} + S_s^{22} + S_s^{33} + S_s^{66} + S_s^{77}) + (\tau + \sqrt{3}\,\kappa)S_s^{55} + \sqrt{3}\,\kappa S_s^{88} + (\sqrt{3}\,\tau + \kappa)S_s^{38}], \tag{B9}$$

$$[Q_A^5, \partial_\mu A_\mu^5] = -\tfrac{1}{2}i[\tau(S_s^{11} + S_s^{22} + S_s^{33} + S_s^{66} + S_s^{77}) + (\tau + \sqrt{3}\,\kappa)S_s^{44} + \sqrt{3}\,\kappa S_s^{88} + (\sqrt{3}\,\tau + \kappa)S_s^{38}], \tag{B10}$$

$$[Q_A^6, \partial^\mu A_\mu^6] = -\tfrac{1}{2}i[\tau(S_s^{11} + S_s^{22} + S_s^{33} + S_s^{44} + S_s^{55}) + (\tau + \sqrt{3}\,\kappa)S_s^{77} + \sqrt{3}\,\kappa S_s^{88} - (\sqrt{3}\,\tau + \kappa)S_s^{38}], \tag{B11}$$

$$[Q_A^7, \partial^\mu A_\mu^7] = -\tfrac{1}{2}i[\tau(S_s^{11} + S_s^{22} + S_s^{33} + S_s^{44} + S_s^{55}) + (\tau + \sqrt{3}\,\kappa)S_s^{66} + \sqrt{3}\,\kappa S_s^{88} - (\sqrt{3}\,\tau + \kappa)S_s^{38}], \tag{B12}$$

and

$$[Q_A^8, \partial^\mu A_\mu^8] = -i\tfrac{3}{2}\sigma(S_s^{44} + S_s^{55} + S_s^{66} + S_s^{77}). \tag{B13}$$

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[21]If we permit a large amount of SU(3) breaking in $\langle\Omega|v_\alpha|P_\beta\rangle$, the solution $c\simeq 0$ is not excluded. However, the value for c given in Ref. 23 (i.e., $c\approx -0.28$) is acceptable only if we allow in addition a large amount

of SU(3) breaking for the vacuum state. In fact, substituting this value for c in Eq. (3.2) we obtain $\sigma_V(4) \approx 5.1$. Note also that use may be made of Eq. (4.8) to obtain $Y_\eta \approx 2.2$ for this c and the physical value for Y_K (i.e., $\sqrt{Y_K} \approx 1.28$).

[22]Note that Eq. (4.22) follows from (4.18) and (4.19) upon replacing $\sqrt{Y}$ by Y in the latter two equations.

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