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Scale Invariance, Fixed Poles, and Cuts in Compton and Deep-Inelastic Scattering*

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Finite-energy sum rules in Compton and deep-inelastic scattering are analyzed at $q^2=0$ and $-q^2=\infty$ on the basis of a Regge-pole model including cuts. It is found that the sum rules can be satisfied without the use of real fixed poles. Good fits are obtained to the data for the deep-inelastic structure functions in the Regge region, and to the data for the high-energy total and differential cross sections.

I. INTRODUCTION

One of the interesting features of Compton scattering of real or virtual photons on protons is that if one assumes simple Regge-pole dominance at high energies, the finite-energy sum rules which one writes down are not satisfied by the experimental data. This situation is clear for real photons, and there is strong evidence for the failure of simple Regge-pole dominance for virtual photons in the scale-invariance (deep-inelastic) region. The standard interpretation of this fact is that there exists a fixed pole at an angular momentum value $J=0$ in the full Compton amplitude.[1,2] Such real poles are forbidden by unitarity in strong-interaction processes, but may occur (to first order in the electromagnetic coupling constant α) in electromagnetic interactions involving hadrons. A $J=0$ fixed pole contributes an asymptotic behavior $(\text{energy})^{-1}$ to the real part of the amplitude; it does not contribute to the total cross section, but does contribute to the differential cross section.

The purpose of this paper is to propose an alternative mechanism for satisfying the finite-energy sum rules, namely a cut in the angular momentum plane.[3,4] This cut may be fixed or moving. The advantage of this approach is that the mechanism for producing a cut is familiar in strong-interaction physics, whereas the dynamical origin of a fixed pole is obscure. Numerically, of course, the two possibilities cannot yet be distinguished, as the precision of the experimental data is not sufficiently good. Unlike a fixed pole, however, a cut contributes to the absorptive part of the amplitude, i.e., the total cross section, so in principle the distinction can be made.

In Sec. II, we derive the appropriate finite-energy sum rules (FESR) from analyticity in the energy variable, and show how one can, with the appropriate assumptions, obtain the local form of the Bloom-Gilman sum rule.[5] It is pointed out that this sum rule is not a *necessary* consequence of analyticity and scale invariance. We further discuss the possibility of fixed poles at $J=0$ and at $J=1$. In Sec. III, we exhibit a Regge-pole-plus-cut model for Compton scattering of virtual and real photons. We constrain the model to satisfy the FESR's, both for real photons and in the deep-inelastic region. We then apply the model and fit the total γ-p cross section, the inelastic structure functions in the Regge region, and the differential cross section for Compton scattering up to $t=-1.3$ (GeV/c)^2. Although these results apply specifically to proton Compton scattering, the extension to neutrons and pions is straightforward. In Sec. IV, we end the paper with concluding remarks.

II. FINITE-ENERGY SUM RULES

A. Kinematics

Consider elastic scattering of a photon of four-momentum q off a proton of four-momentum P, with $P^2=M^2$. The barycentric energy squared is

$$s=(P+q)^2=M^2+q^2+2M\nu\,, \tag{2.1}$$

where $\nu=q\cdot P/M$ is the laboratory energy (also denoted by E) of the photon. The momentum transfer squared is

$$t=(q'-q)^2=(P'-P)^2\,, \tag{2.2}$$

where q' and P' are the photon and proton final four-momenta, respectively. Following the notation of our earlier work,[6] the forward Compton amplitude (averaged over nucleon spins) is given by

$$T(\nu,q^2)=4\pi\alpha\left(\frac{M}{E_p}\right)\epsilon_2^{\mu *}\epsilon_1^{\nu}T^*_{\mu\nu}\,, \tag{2.3}$$

where E_p is the proton energy and ϵ_1 and ϵ_2 are the polarization vectors of the incoming and outgoing photons, respectively. The tensor $T^*_{\mu\nu}$ can be expanded in terms of two invariant amplitudes,

$$\frac{1}{M}T^*_{\mu\nu}=\frac{1}{M^2}\left(P_\mu-\frac{P\cdot q}{q^2}q_\mu\right)\left(P_\nu-\frac{P\cdot q}{q^2}q_\nu\right)T_2(\nu,q^2) -\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)T_1(\nu,q^2)\,. \tag{2.4}$$

In this work, we shall be concerned with the amplitudes νT_2 and T_1. We shall write down FESR's for νT_2 particularly; the treatment for T_1 is similar. In fact, under the assumption that the matrix elements for transverse and longitudinal photons are equal, the imaginary parts of T_1 and of νT_2 are simply related[6]:

$$2MW_1(\nu,q^2)=\omega[\nu W_2(\nu,q^2)]\,, \tag{2.5}$$

where

$$W_i(\nu,q^2)=\frac{1}{\pi}\,\mathrm{Im}\,T_i(\nu,q^2)$$

and $\omega=2M\nu/(-q^2)$ is the scaling variable introduced by Bjorken.[7] W_1 and W_2 are the structure functions for deep-inelastic electroproduction. We may extend[8] (2.5) to T_1 and T_2:

$$2MT_1(\nu,q^2)=\omega[\nu T_2(\nu,q^2)]\,. \tag{2.6}$$

The amplitude T_i is an even function of ν (crossing symmetry):

$$T_i(\nu,q^2)=T_i(-\nu,q^2)\,. \tag{2.7}$$

For fixed q^2 it is cut in the ν plane from $\nu=(\mu^2+2M\mu-q^2)/2M$ to $\nu=+\infty$ along the positive real axis, and from $\nu=-(\mu^2+2M\mu-q^2)/2M$ to $\nu=-\infty$ along the negative real axis; μ is the pion mass. It has poles (the nucleon poles) at $\nu=\pm q^2/2M$.

B. Asymptotic Behavior

It was shown in our previous work[6] that T_1 (and νT_2) can be described in terms of a Sommerfeld-Watson representation of the t-channel process $\gamma+\gamma\to N+\bar{N}$ continued from the region $t>4M^2$, $q^2>0$ to the region $t<0$, $q^2<0$ with ν physical. We now define the *Regge region* as the region in the (ν,q^2) plane where $\nu>2$ GeV and $\omega>6$ (for $q^2<0$). That this is a reasonable definition can be seen from the success of Regge-pole fits to the structure function $\nu W_2(\nu,q^2)$.[6] In this region, we may express νT_2 as

$$\nu T_2(\nu,q^2,t) =\sum_i A_i(q^2,t)\nu^{\alpha_i(t)-1}\xi_i(t)\Gamma(\alpha_i(0)) +A_c(q^2,t)\nu\left[\frac{\nu^{\alpha_c(t)-2}}{\ln^\delta\nu+f(q^2)}+\frac{(-\nu)^{\alpha_c(t)-2}}{\ln^\delta(-\nu)+f(q^2)}\right]. \tag{2.8}$$

The first term is a standard Regge-pole expansion. The index i runs over the t-channel Regge poles; $\alpha_i(t)$ is the Regge trajectory; $\xi_i(t)$ is the signature factor, and $\Gamma(\alpha_i(0))$ is a convenient normalization factor. For proton Compton scattering, Regge poles of even signature only are allowed (i.e., the Pomeranchukon, the P', and the A_2), so that

$$\xi_i(t)=\frac{1+e^{-i\pi\alpha_i(t)}}{\sin\pi\alpha_i(t)\,\Gamma(\alpha_i(t))}. \tag{2.9}$$

The Γ function cancels the pole which would otherwise occur at $\alpha_i(t)=0$. $A_i(q^2, t)$ is the Regge residue.

The second term is a cut, of the usual form; it is real, analytic, and crossing-symmetric. The cut trajectory is $\alpha_c(t)$. The function $f(q^2)$ has been inserted so as to give the correct limiting behavior as $q^2\to 0$, and also the scale-invariance limit, as we shall discuss below. This q^2 dependence is not the most general we could have chosen, but it serves to illustrate the point. For simplicity, in what follows, we shall choose the power to which the logarithm has been raised (δ) to be unity.[3] We do not attempt to *derive* the cut from iterated Regge poles.

The structure function νW_2 is thus

$$\nu W_2=\frac{1}{\pi}\left[-\sum_i A_i(q^2,0)\nu^{\alpha_i-1}+A_c(q^2,0)\nu^{\alpha_c-1}\left(\frac{\pi\cos\pi\alpha_c-[\ln\nu+f(q^2)]\sin\pi\alpha_c}{[\ln\nu+f(q^2)]^2+\pi^2}\right)\right]. \tag{2.10}$$

where

$$\alpha_i=\alpha_i(0)\quad\text{and}\quad\alpha_c=\alpha_c(0).$$

Experimentally in the limit ν large, ω fixed, the function $\nu W_2(\nu, q^2)$ tends to a nontrivial scale-invariant limit[7]

$$\lim_{\nu\to\infty;\,\omega\,\text{fixed}}\nu W_2(\nu,q^2)=F_2(\omega)\neq 0\,. \tag{2.11}$$

This implies that

$$\begin{aligned}A_i(q^2,0)&\underset{-q^2\to\infty}{\sim}A_i\left(\frac{2M}{-q^2}\right)^{\alpha_i-1},\\ A_c(q^2,0)&\underset{-q^2\to\infty}{\sim}A_c\left(\frac{2M}{-q^2}\right)^{\alpha_c-1},\\ f(q^2)&\underset{-q^2\to\infty}{\sim}\ln\left(\frac{2M}{-q^2}\right),\end{aligned} \tag{2.12}$$

where A_i and A_c are constants. Thus, in the scale-invariance limit, we have

$$F_2(\omega)=\frac{1}{\pi}\left[-\sum_i A_i\omega^{\alpha_i-1}+A_c\omega^{\alpha_c-1}\left(\frac{\pi\cos\pi\alpha_c-\ln\omega\sin\pi\alpha_c}{(\ln\omega)^2+\pi^2}\right)\right]. \tag{2.13}$$

Since the Regge region ($\omega>6$) does not include the nucleon poles (which occur at $\omega=\pm1$), we have

$$\nu W_2\sim O(q^2)\quad\text{as}\quad q^2\to 0 \tag{2.14}$$

for fixed $\nu\geq 2$ GeV. This implies that

$$\begin{aligned}A_i(q^2,0)&\underset{q^2\to 0}{\sim}a_iq^2,\\ A_c(q^2,0)&\underset{q^2\to 0}{\sim}a_cq^2,\end{aligned} \tag{2.15}$$

where the a_i are constants. Furthermore, if we wish the cut to contribute to the total γp cross section for real photons, we require

$$\lim_{q^2\to 0}f(q^2)=f_c, \tag{2.15a}$$

where f_c is a finite constant.

C. Derivation of FESR

We shall now write down the appropriate sum rules for νT_2. Let q^2 be fixed and nonzero. Cauchy's theorem reads

$$\oint_C \nu T_2(\nu, q^2) d\nu = 2\pi i \sum(\text{residues}), \tag{2.16}$$

where C is a contour excluding the cuts but enclosing the poles. This becomes

$$\frac{q^2}{2M}[G(q^2)]^2 = \int_{(\mu^2+2M\mu-q^2)/2M}^{R} \nu W_2(\nu, q^2) d\nu + \frac{1}{\pi}\sum_i A_i(q^2, 0)\frac{R^{\alpha_i}}{\alpha_i}$$

$$+\frac{1}{\pi}A_c(q^2, 0)R^{\alpha_c}\int_0^\pi \frac{(\cos\alpha_c y)[\ln R + f(q^2)] + y\sin\alpha_c y}{[\ln R + f(q^2)]^2 + y^2} dy. \tag{2.17}$$

The upper limit R marks the beginning of the Regge region, and, by virtue of our earlier remarks, depends on q^2; thus $R > 2$ GeV *and* $R \gg -q^2/2M$. Moreover, $[G(q^2)]^2$ is the nucleon pole contribution, and involves the electric and magnetic form factors G_E and G_M:

$$[G(q^2)]^2 = \frac{[G_E(q^2)]^2 + (-q^2/4M^2)[G_M(q^2)]^2}{1 - q^2/4M^2}. \tag{2.18}$$

Now consider the scale-invariance limit of (2.17). The term $(q^2/2M)[G(q^2)]^2$ goes to zero, and if we change the variable of integration from ν to ω and use (2.11) and (2.12), we have

$$0 = \int_1^{\omega_R} F_2(\omega) d\omega + \frac{1}{\pi}\sum_i \frac{A_i \omega_R^{\alpha_i}}{\alpha_i} + \frac{1}{\pi}A_c \omega_R^{\alpha_c}\int_0^\pi \frac{(\cos\alpha_c y)\ln\omega_R + y\sin\alpha_c y}{(\ln\omega_R)^2 + y^2} dy, \tag{2.19}$$

with $\omega_R \equiv 2MR(q^2)/-q^2$. If we now subtract (2.17) from (2.19), again changing the variable of integration, we obtain

$$[G(q^2)]^2 = \int_1^{1+(\mu^2+2M\mu)/(-q^2)} F_2(\omega) d\omega + \int_{1+(\mu^2+2M\mu)/-q^2}^{\omega_R} [F_2(\omega) - \nu W_2(\omega, q^2)] d\omega$$

$$+\frac{1}{\pi}\sum_i \frac{\omega_R^{\alpha_i}}{\alpha_i}\left[A_i - A_i(q^2, 0)\left(\frac{-q^2}{2M}\right)^{\alpha_i - 1}\right]$$

$$+\frac{1}{\pi}\omega_R^{\alpha_c}\left[A_c\int_0^\pi \frac{(\cos\alpha_c y)\ln\omega_R + y\sin\alpha_c y}{(\ln\omega_R)^2 + y^2} dy\right.$$

$$\left. - A_c(q^2, 0)\left(\frac{-q^2}{2M}\right)^{\alpha_c - 1}\int_0^\pi \frac{(\cos\alpha_c y)[\ln\omega_R + f(q^2) + \ln(-q^2/2M)] + y\sin\alpha_c y}{[\ln\omega_R + f(q^2) + \ln(-q^2/2M)]^2 + y^2} dy\right]. \tag{2.20}$$

Now let us suppose that scale invariance is approached by a power-law behavior, specifically

$$F_2(\omega) - \nu W_2(\omega, q^2) < \frac{F(\omega)}{|q|^n} \quad \text{as } -q^2 \to \infty \quad (\text{all } \omega), \tag{2.21}$$

implying that

$$A_{i,c} - A_{i,c}(q^2, 0)\left(\frac{-q^2}{2M}\right)^{\alpha_i - 1} < \frac{\text{const}}{|q|^n} \quad \text{as } -q^2 \to \infty. \tag{2.22}$$

Then $n \geq 8$ to give the observed dipole falloff of the elastic nucleon form factor, barring cancellations with the first term of (2.20). If $n > 8$, or if the approach to scale invariance is faster than a power law, then for large $-q^2$ we have

$$[G(q^2)]^2 \simeq \int_1^{1+(\mu^2+2M\mu)/-q^2} F_2(\omega) d\omega. \tag{2.23}$$

This is the local version of the Bloom-Gilman[5] sum rule, but we note that the upper limit is predicted. Of course, the procedure of subtracting two sum rules may not be valid, since we *approximate* the high-energy behavior of νW_2, and so the sum rules are not locally valid. If, however, $n = 8$, then the local version of the Bloom-Gilman sum rule may not even be approximately true, as the other terms in (2.20) must all be considered. Experimentally, it is not yet possible to fix the value of n, but it is of interest to note that the local version of the Bloom-Gilman sum rules requires a much larger upper limit than is written in (2.23) to fit the data.[6] We observe that (2.23) cannot be tested with a purely Regge model, as the region of integration involves *small* ω, i.e., $\nu \sim -q^2/2M$.

Now let us consider the limit $q^2 = 0$. Using (2.15) and the relation

$$\sigma_T(\gamma N) = \lim_{-q^2 \to 0} \left(\frac{4\pi^2 \alpha}{-q^2}\right)[\nu W_2(\nu, q^2)], \tag{2.24}$$

we obtain from (2.17)

$$1 = -\left(\frac{M}{2\pi^2 \alpha}\right)\int_{\mu+\mu^2/2M}^{R} \sigma_T(\gamma p)\,d\nu + \frac{2M}{\pi}\sum_i a_i \frac{R^{\alpha_i}}{\alpha_i} + \frac{2M}{\pi} a_c R^{\alpha_c}\int_0^{\pi} \frac{(\cos\alpha_c y)(\ln R + f_c) + y \sin\alpha_c y}{(\ln R + f_c)^2 + y^2}\,dy\,. \tag{2.25}$$

So far, we have assumed that the asymptotic behavior is dominated by Regge poles plus a cut. What about fixed poles? The original interest in fixed poles arose when it was discovered that Regge poles alone were insufficient to satisfy the $q^2 = 0$ and $-q^2 = \infty$ FESR's. The failure of Regge-pole dominance, for the latter case, depends of course on the validity of our assumption about the Regge region; if Regge-pole dominance does not hold for $\omega < 100$, say, then one learns nothing from the $-q^2 = \infty$ sum rule. However, as noted, there is evidence for approximate Regge-pole dominance for $\omega > 6$.

At any rate, the solution has hitherto been to introduce real fixed poles at $J = 0$ to satisfy both sum rules.[2] Generally speaking, fixed poles can be of two types: wrong-signature fixed poles (occurring at odd J in an even-signatured amplitude and vice versa) and right-signature fixed poles [occurring for even (odd) J in an even- (odd-) signatured amplitude]. An example of a right-signature fixed pole is the pole at $J = 1$ implied by the Dashen–Fubini–Gell-Mann sum rule[9] in the amplitude for the scattering of charged photons off protons. Such a pole does *not* contribute to the asymptotic behaviour of neutral γ scattering, and so can be ignored. Unlike real fixed poles, cuts contribute to the total cross section.

III. REGGE-POLE-PLUS-CUT MODEL

We now turn our attention to a specific model for the functions $A_i(q^2, t)$, $A_c(q^2, t)$, and $f(q^2)$. Subject to the constraints (2.12) and (2.15), we choose the simple forms

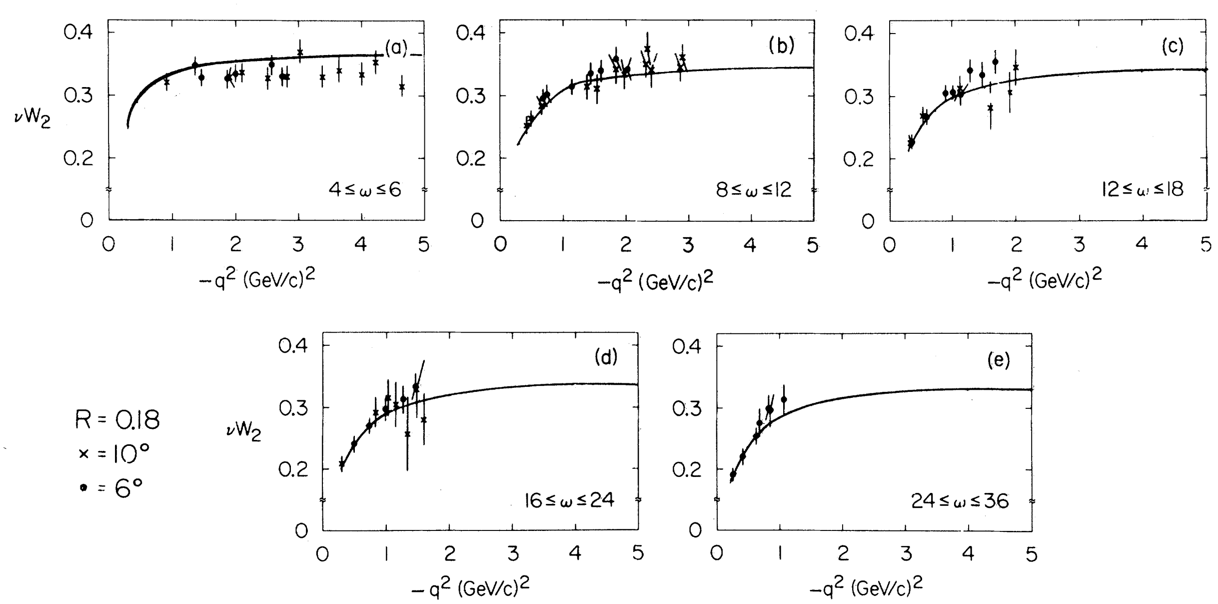

FIG. 1. The structure function νW_2 versus $-q^2$ for fixed values of ω. Our fits (solid lines) are calculated using the average value of ω in each case. The data (Ref. 11) show the breaking of scale invariance for small $-q^2$.

$$f(q^2)=\ln\left(\frac{2M}{-q^2+m_0{}^2}\right),$$

$$A_i(q^2,t)=\pi q^2\left(\frac{2M}{-q^2+m_0{}^2}\right)^{\alpha_i(t)}\left(\frac{1}{2M}\right)^{\alpha_i}\beta_i(t), \tag{3.1}$$

$$A_c(q^2,t)=-\pi q^2\left(\frac{2M}{-q^2+m_0{}^2}\right)^{\alpha_c(t)}\frac{1}{2M}\left(\frac{-q^2-\lambda^2}{-q^2+\lambda^2}\right)\beta_c(t).$$

In general, more stringent constraints on $A_c(q^2,t)$ and $A_i(q^2,t)$ are implied by (2.20): We should choose the q^2 dependence of $A_c(q^2,t)$ and $A_i(q^2,t)$ so as to reproduce the nucleon form factors. But as accurate data on $\nu W_2(\nu,q^2)$ do not exist for all q^2, and since there is no difficulty in principle in constructing functions with sufficiently complex q^2 behavior so as to satisfy (2.20) for all q^2, it was decided simply to test the sum rules at $q^2=0$ and $-q^2=\infty$. The mass m_0 is introduced as a scale-breaking parameter,[6,10] and to satisfy (2.15). At $q^2=0$, m_0 determines the normalization of the total cross section, and the choice $m_0=0.665$ GeV was found to be satisfactory. This value is also consistent with the violation of scale invariance exhibited by the data[11] for the structure function νW_2 plotted as a function of $-q^2$ and for fixed $\omega>6$, as shown in Fig. 1. The parameter λ is simply a mass which forces the cut contribution to change sign between $q^2=0$ and $-q^2=\infty$, an essential property if the model is to satisfy both sum rules simultaneously, and fit the data. In practice, the value $\lambda\sim0.8$ GeV was used. Of course at $q^2=0$ and $-q^2=\infty$, the sum rules do not depend on the *value* of λ chosen, but simply on the functional form; the value of λ quoted above was used to give the fits in Fig. 1.

Two of the four remaining parameters $\beta_P(0)$, $[\beta_{P'}(0)+\beta_{A_2}(0)]$, $\beta_c(0)$, and α_c are then determined by requiring the model to satisfy the sum rules (2.19) and (2.25). [Since the P' and the A_2 trajectories have the same intercept and slope, only the sum $(\beta_{P'}+\beta_{A_2})$ enters into this calculation.] The integral in (2.25) over the resonance region has been evaluated by Damashek and Gilman,[2] who find

$$\frac{M}{2\pi^2\alpha}\int_{\mu+\mu^2/2M}^{1.68\,\mathrm{GeV}}\sigma_T(\gamma p)d\nu=5.57, \tag{3.2}$$

with $\sigma_T(\nu=1.68\text{ GeV})=151\ \mu$b. Close and Gunion[2] evaluate the integral of $F_2(\omega)$ over the non-Regge region, finding

$$\int_1^{12}F_2(\omega)d\omega=3.32, \tag{3.3}$$

with $F_2(12)=0.35$. The last two parameters are fixed by requiring that the Regge model join on as smoothly as possible to the resonance region; for the sum rule (2.25), this is achieved to about 3%. We find

$$\begin{aligned}&\beta_P(0)\equiv\beta_P=0.365,\\&\beta_{P'}(0)+\beta_{A_2}(0)\equiv\beta_{P'}+\beta_{A_2}=0.203\text{ GeV}^{-1/2},\\&\beta_c(0)\equiv\beta_c=0.493,\\&\alpha_c=0.95,\end{aligned} \tag{3.4}$$

and we take the usual intercepts $\alpha_P=1$, $\alpha_{A_2}=\alpha_{P'}=\frac{1}{2}$. It was found that the results are quite insensitive to the value of α_c: The value in (3.4) gave the best fit, but any value of α_c from 0.8 to 1 could just as well be used. Of course, α_c cannot be greater than 1. In terms of our model, the total γp cross section is ($\nu>2$ GeV)

$$\sigma_T(\gamma p)=4\pi^2\alpha\left[\sum_i\beta_i\left(\frac{1}{m_0{}^2}\right)^{\alpha_i}\nu^{\alpha_i-1}-\frac{\beta_c}{m_0{}^2}\left(\frac{2M\nu}{m_0{}^2}\right)^{\alpha_c-1}\left(\frac{\pi\cos\pi\alpha_c-\ln(2M\nu/m_0{}^2)\sin\pi\alpha_c}{[\ln(2M\nu/m_0{}^2)]^2+\pi^2}\right)\right] \tag{3.5}$$

and the structure function $F_2(\omega)$ in the Regge region is given by

$$F_2(\omega)=\sum_i\beta_i\left(\frac{\omega}{2M}\right)^{\alpha_i-1}+\beta_c\omega^{\alpha_c-1}\left(\frac{\pi\cos\pi\alpha_c-\ln\omega\sin\pi\alpha_c}{(\ln\omega)^2+\pi^2}\right). \tag{3.6}$$

Using the parameters above, we fit the data[12] for $\sigma_T(\gamma p)$ for $\nu>2$ GeV and the data[11] for $F_2(\omega)$ for $\omega>4$. These fits are shown in Figs. 2, 3, and 4. It will be of great interest to see if the curve for $F_2(\omega)$ continues to be consistent with the Regge picture for values of $\omega>20$. Also shown is the prediction of our earlier work in Ref. 6.

Consider now the differential cross-section for Compton scattering of real photons. In general six invariant amplitudes describe this process. Four of these vanish at $t=0$, however, so we assume that T_1 and T_2 dominate and that the approximation of treating the nucleon as spinless is not important. In our model,

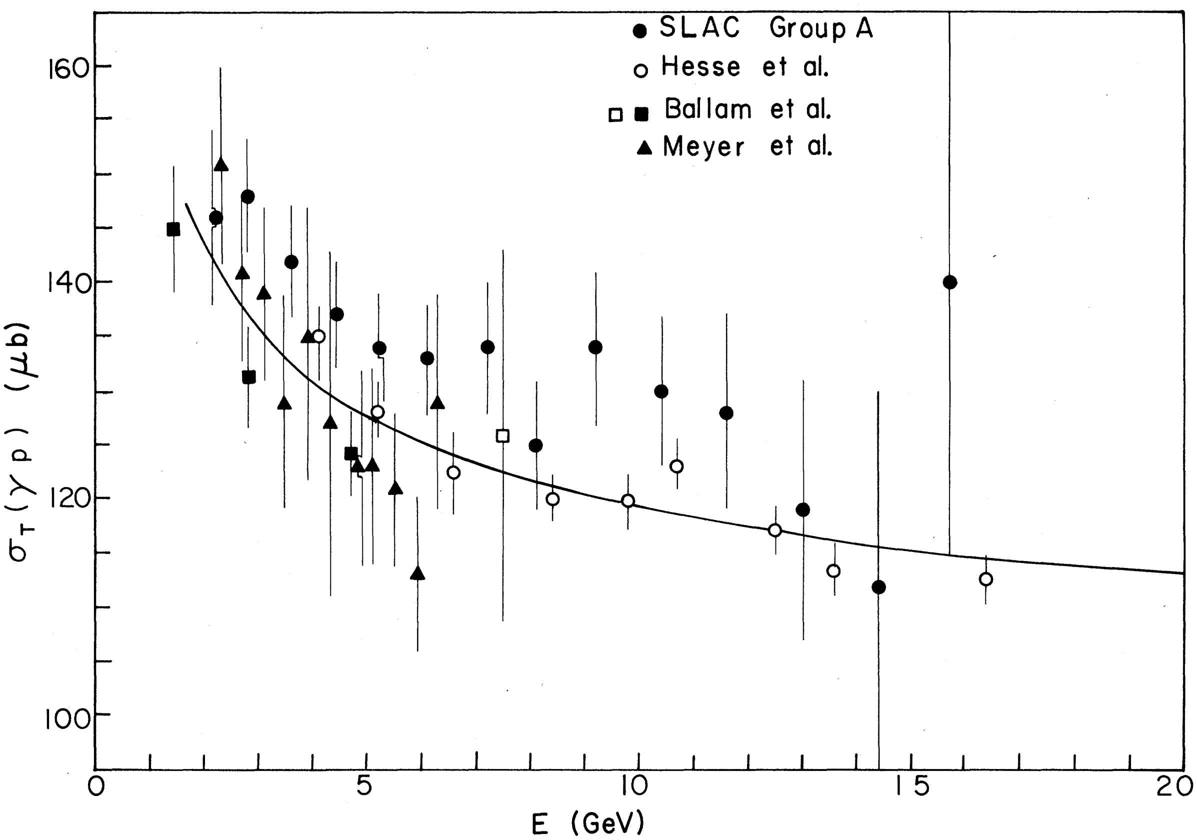

FIG. 2. The total photon-proton cross section for real photons, plotted in the Regge region, versus the photon energy E. The solid line is our fit. The data are from Ref. 12.

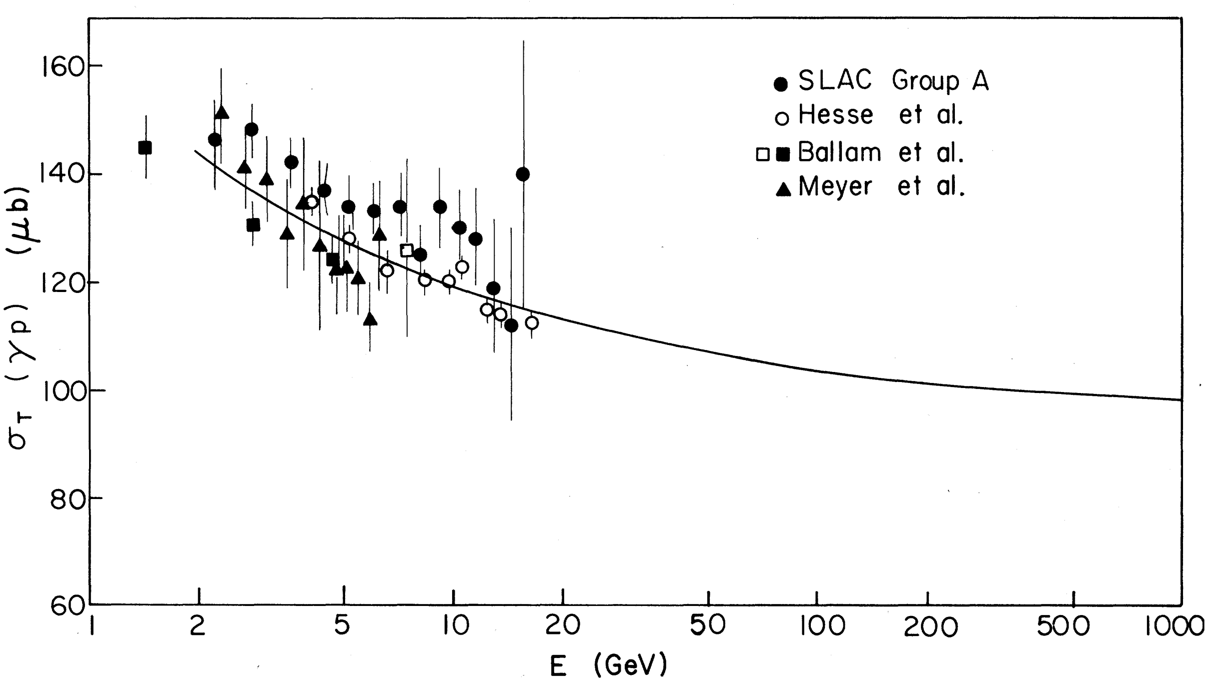

FIG. 3. The same data and the same fit as Fig. 2, plotted to show the predictions of our fit up to $E = 1000$ GeV.

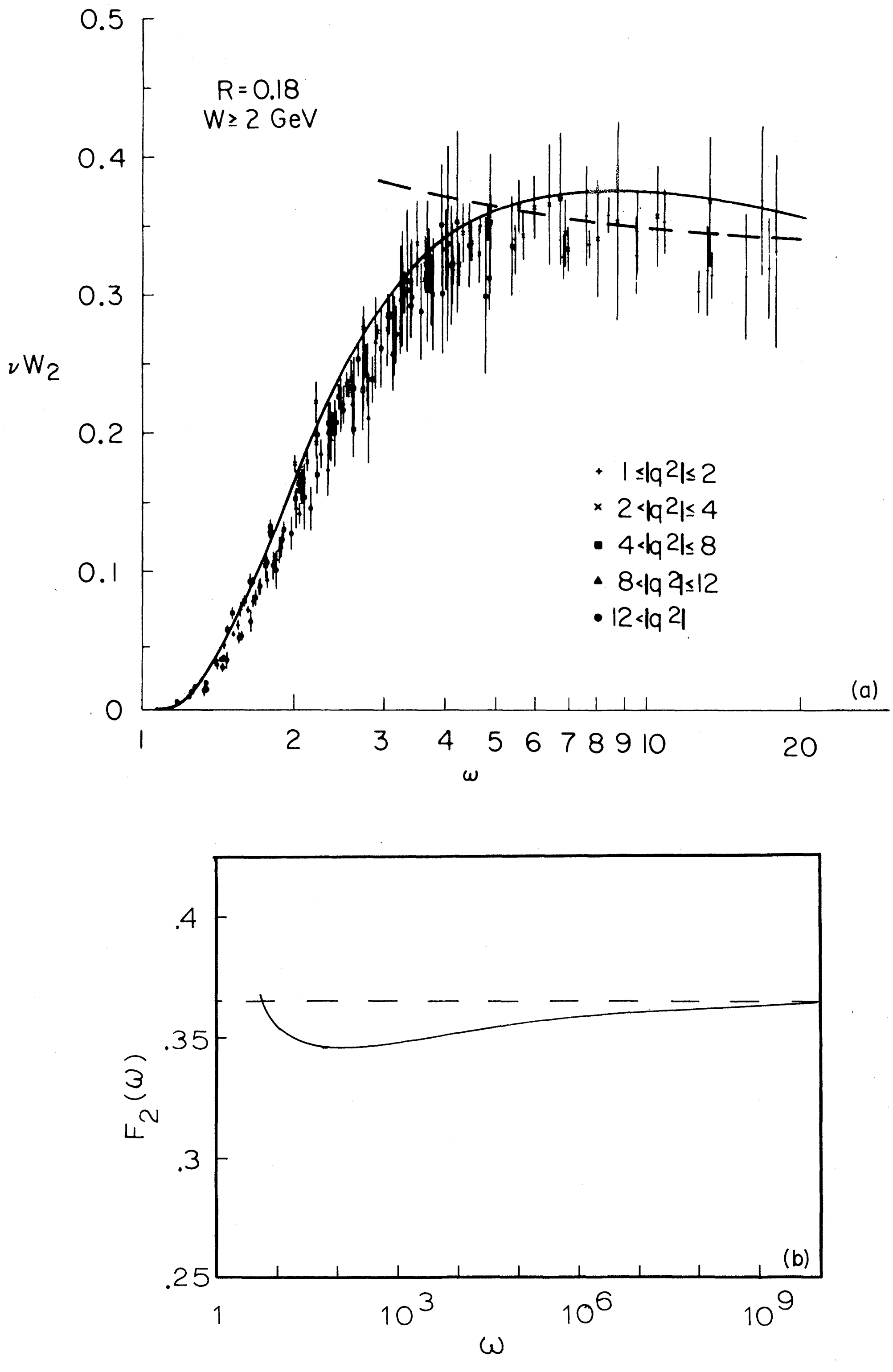

FIG. 4. (a) The proton structure function νW_2 versus ω. The data are from Ref. 11. The dashed line is the fit to νW_2 in the scale-invariance limit, in the Regge region, using the model of this paper. The solid line is the fit using the model of Ref. 6. (b) Our prediction for the structure function $F_2(\omega)$ for large values of ω, showing the slow approach to the asymptotic value. The dashed line shows $F_2(\infty) = 0.365$ in our model.

$$\nu T_2(\nu, q^2, t) = \pi q^2 \sum_i \left(\frac{2M}{-q^2+m_0^2}\right)^{\alpha_i(t)} \left(\frac{1}{2M}\right)^{\alpha_i} \beta_i(t) \nu^{\alpha_i(t)-1} \xi_i(t) \Gamma(\alpha_i)$$
$$- \pi q^2 \left(\frac{2M}{-q^2+m_0^2}\right)^{\alpha_c(t)} \frac{1}{2M} \beta_c(t) \left(\frac{-q^2-\lambda^2}{-q^2+\lambda^2}\right) \nu \left[\frac{\nu^{\alpha_c(t)-2}}{\ln[2M\nu/(-q^2+m_0^2)]} + \frac{(-\nu)^{\alpha_c(t)-2}}{\ln[-2M\nu/(-q^2+m_0^2)]}\right]. \quad (3.7)$$

If we use (2.6) and (3.7), we can evaluate the differential cross section[6]

$$\frac{d\sigma}{dt} = \frac{\pi\alpha^2}{\nu^2} |T_1(\nu, 0, t)|^2. \quad (3.8)$$

Let us consider the t dependence. For the Regge residues, we take the usual form

$$\beta_i(t) = \beta_i e^{At+Bt^2} \quad (i = P, P', A_2, \text{ cut}) \quad (3.9)$$

with A and B constants. The cut trajectory is taken to be flat,

$$\alpha_c(t) = \alpha_c. \quad (3.10)$$

Fairly large changes in the slope of the cut trajectory have no effect on the results, so for simplicity we choose a slope of zero. It should be emphasized that no special significance is to be attached to this choice. It is interesting to note that a cut generated by double Pomeranchukon exchange would have intercept 1 and slope $\frac{1}{2}\alpha_P'$; these values are quite consistent with the cut parameters we use, although this work is independent of the detailed model used to calculate the cut. Because cuts do not factorize, we cannot make any prediction as to whether this particular cut will also appear in the strong interactions, although cuts are of course present there as well.

The P' and the A_2 were given the usual Regge slope, 1 GeV^{-2}. A least-squares fit to the data[13] then determined the constants A, B, and the slope of the Pomeranchukon, α_P'. We obtain the results

$$A = 4.8 \text{ GeV}^{-2}, \; B = 1.2 \text{ GeV}^{-4}, \; \alpha_P' = 0.3 \text{ GeV}^{-2}. \quad (3.11)$$

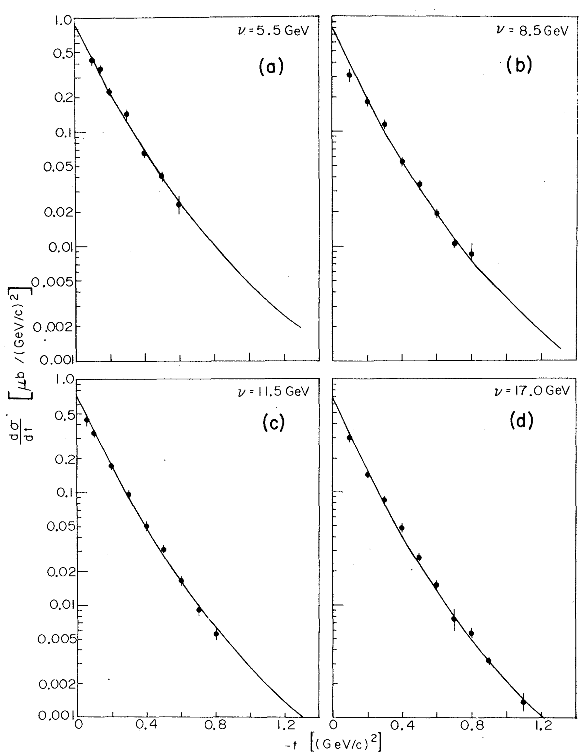

FIG. 5. The differential cross section $d\sigma/dt$ for proton Compton scattering plotted against $-t$. Four values of the photon laboratory energy ν are shown: (a) 5.5, (b) 8.5, (c) 11.5, (d) 17 GeV. The data are from Ref. 13; the solid line is our fit.

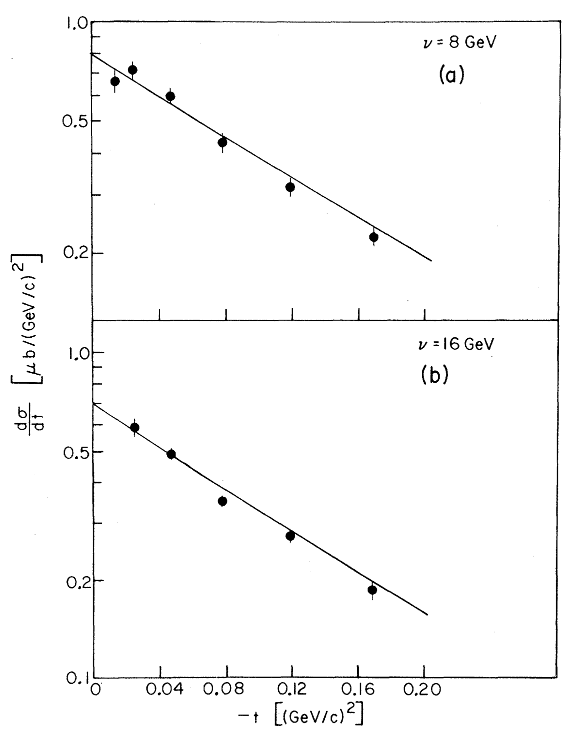

FIG. 6. The differential cross section $d\sigma/dt$ plotted against $-t$ for small values of $-t$. The data are from Ref. 13; the solid lines are our fit. The photon laboratory energy ν is (a) 8, (b) 16 GeV.

The Pomeranchukon slope is small. The explanation for this is that the data show remarkably little shrinkage. If we define Γ as the width of the diffraction peak,

$$\Gamma^{-1}=\frac{1}{2}\frac{d}{dt}\left(\ln\frac{d\sigma}{dt}\right), \tag{3.12}$$

then the shrinkage S is defined by

$$S=\left(\frac{M+2\nu}{2}\right)\frac{d}{d\nu}\Gamma^{-1}. \tag{3.13}$$

The experimental shrinkage[13] is consistent with zero, although the errors are large,

$$S_{\text{average}}=-0.07\pm 0.88\ \text{GeV}^{-2},\quad 5\leqslant\nu\leqslant 17\ \text{GeV}, \tag{3.14}$$

whereas our model gives $S\simeq 0.25\ \text{GeV}^{-2}$ in the same range of ν.

In Figs. 5 and 6, we present our fits to $d\sigma/dt$, and we see the agreement is good over the range $5.5\leqslant\nu\leqslant 17$ GeV and $0\leqslant -t\leqslant 1.2$ (GeV/c)^2.

IV. CONCLUSIONS

We have exhibited a Regge-pole-plus-cut model for proton Compton scattering for real and virtual photons. The model satisfies the FESR's at $q^2=0$ and $-q^2=\infty$ and gives good fits to the total cross section, the inelastic structure functions in the Regge region, and the differential cross section.

A similar treatment can be given to neutron Compton scattering. The data for $q^2=0$ are not sufficiently precise to draw conclusions as to the presence or absence of non-Regge-pole asymptotic behavior, although the deep-inelastic data are suggestive of the former. The model can also be extended to pion Compton scattering, as outlined previously.[6]

We conclude from our work that cuts in the angular momentum plane should be considered as viable alternatives to fixed poles in Compton and deep-inelastic scattering, for all the requirements of various sum rules, as well as fits to presently available data, can be accommodated by postulating only cuts, in addition to the Regge poles. This of course does not preclude the possibility of fixed poles, but it does remove the argument that fixed poles are a necessary feature of Regge-pole phenomenology, as applied to electromagnetic interactions with hadrons.

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