[1]M. D. Scadron, University of Arizona report (unpublished).
[2]M. D. Scadron, Phys. Rev. D 5, 188 (1972).
[3]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).
[4]S. L. Adler, Phys. Rev. 177, 2426 (1969).
[5]J. Steinberger, Phys. Rev. 76, 1180 (1949).
[6]The divergence is calculated by writing the triangle amplitude in Feynman parameter form, contracting with q, and evaluating without regard to possible contact terms due to translating the formally divergent but non-gauge-invariant parts. See R. Jackiw, MIT-CTP Report No. 183, 1970 (unpublished).
[7]N. Kroll and M. Ruderman, Phys. Rev. 93, 233 (1954).
[8]Note that for $V=\rho$ the coupling g_{VNN} is twice the standard coupling $g_{\rho NN}$ as given by Eq. (18). The isovector analog of Eq. (16) is $g_{\rho NN}=g_\rho$.
[9]W. Schmidt and D. R. Yennie, Phys. Rev. Letters 23, 623 (1969); 23, 946(E) (1969).
[10]J. LeFrancois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).
[11]Particle Data Group, Phys. Letters 33B, 1 (1970).
[12]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).
[13]S. Fubini, Nuovo Cimento 43A, 475 (1966); R. Dashen and M. Gell-Mann, Phys. Rev. Letters 17, 340 (1966).
[14]S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969).
[15]R. A. Brandt, Phys. Rev. 180, 1490 (1969).
[16]M. D. Scadron, Phys. Rev. D 2, 213 (1970).
[17]See, for example, D. G. Sutherland, Nucl. Phys. B2, 433 (1967).
[18]C. Michael and R. Odorico, Phys. Letters 34B, 422 (1971).
[19]J. Rosner, private communication.

PHYSICAL REVIEW D VOLUME 6, NUMBER 3 1 AUGUST 1972

Conformal Symmetry and Single-Meson Production*

P. J. O'Donnell and C. Ouellet
Department of Physics, University of Toronto, Toronto, Canada
(Received 24 March 1972)

The results of combining conformal invariance and chiral symmetry are given for the process $\pi N\to\pi\pi N$. It is shown that if the symmetries are broken spontaneously the data are insufficient to determine the scale of the breaking term. The $\pi\pi$ scattering lengths remain small although both a_0 and a_2 are altered substantially.

I. INTRODUCTION

In the past few years there has been considerable interest in two aspects of phenomenological Lagrangians. Initially, the interest lay in using such Lagrangians to understand better the results of current algebra and chiral symmetry.[1] More recently, with the possibility that conformal symmetry might also be a useful approximate symmetry, attempts have been made to combine the chiral and conformal symmetries. In particular, Ellis,[2] on the basis of the phenomenological Lagrangian method, has shown that both chiral and conformal symmetries must be realized in the same way when the symmetries are combined. Since the pion is normally considered as an example of a Goldstone particle, this leads one to view the dilaton, scalar-isoscalar σ, also as a Goldstone particle. With a probable mass of around 700 MeV, and possibly a broad width of ~300–400 MeV, it is not easy to find reliable evidence of the underlying conformal symmetry.

In this paper we wish to test these ideas in the process $\pi N\to\pi\pi N$. This is one of the few processes which have not as yet been looked at with a view to finding out more about the breaking of conformal and chiral symmetry. It is a process which appears to be able to answer in an independent way a different problem which arises in chiral symmetry. It is well known that there exists an arbitrariness in choosing a suitable chiral-symmetry-breaking mechanism. The single-pion production process has been used successfully in eliminating some of the existing ambiguity.[3] In these studies, however, contributions which would arise from the existence of a scalar-isoscalar resonance, such as the σ, have been omitted, and it is important to be sure that the results would not be affected too much by their inclusions. We shall show that there is a substantial change in a_0 and a_2, although these scattering lengths still remain small in magnitude, when such terms are introduced. It has, of course, been known for some time that the current-algebra scattering lengths would change[1,4] from the Weinberg solutions if there were a broad $\pi\pi$ resonance below 500 MeV. The present work is to some extent complementary to that of Carbone *et al.*[4] from the point of view of chiral symmetry in that we also look at the effect of introducing an S-wave $\pi\pi$ resonance characterized by the σ. In addition, we are able to study the problem of combining conformal and chiral symmetries. As in the calculation by Olsson and Turner,[3] we evaluate only the threshold amplitudes for $\pi N \to \pi\pi N$.

In Sec. II we set up the appropriate Lagrangians which enter into our calculation. Section III presents the results and conclusions.

II. SINGLE-PION PRODUCTION

A. Chiral- and Conformal-Invariant Lagrangian

The problem of combining conformal and chiral symmetry in a nonlinear phenomenological Lagrangian has been studied by Ellis[2] and Isham *et al.*[5]

We start with the chiral-invariant Lagrangian

$$L = \bar{\psi}(i\gamma_\mu D^\mu - M)\psi + \tfrac{1}{2}D^\mu\vec{\phi}\cdot D_\mu\vec{\phi} + (G/2M)\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\,\psi\cdot D^\mu\vec{\phi}\,, \tag{2.1}$$

where the chiral-covariant derivative of the baryon field is defined by

$$D_\mu\psi = \partial_\mu\psi + \tfrac{1}{2}i\frac{v(\phi^2)}{[f^2(\phi^2)+\phi^2]^{1/2}}\,\vec{\tau}\cdot\vec{\phi}\times\partial_\mu\vec{\phi}\,\psi$$

and (2.2)

$$v(\phi^2) = \{f(\phi^2) + [f^2(\phi^2)+\phi^2]^{1/2}\}^{-1}\,.$$

For the pion field the chiral-covariant derivative is given by

$$D_\mu\vec{\phi} = [(f^2+\phi^2)^{-1/2}\,\partial_\mu\vec{\phi} - (f^2+\phi^2)^{-1}(2f'+v)\vec{\phi}(\phi\partial_\mu\phi)]f_\pi\,. \tag{2.3}$$

We have, by the Goldberger-Treiman relation,

$$f(0) = f_\pi = -\frac{M}{G}\frac{g_A}{g_V} = \frac{M}{G}\left|\frac{g_A}{g_V}\right|\,. \tag{2.4}$$

We introduce a scalar field σ and make a canonical redefinition of $\psi \to \psi\exp(-l-\frac{3}{2})g\sigma$, with g a parameter and l the dimension of ψ.

With these definitions and the prescription

$$\partial_\mu\psi \to \partial_\mu\psi - g(lg_{\mu\nu} - iS_{\mu\nu})(\partial^\nu\sigma)\psi$$

the first term becomes

$$\bar{\psi}\gamma_\mu D^\mu\psi\,,$$

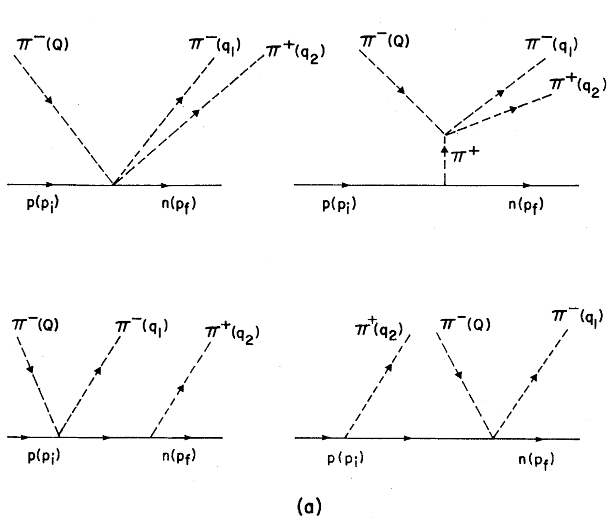
and is chiral- and conformal-invariant. A chiral-invariant and conformal-covariant mass term $-M\bar{\psi}\psi\,e^{-g\sigma}$ can be added to give a Lagrangian

$$L_0 = \bar{\psi}\gamma_\mu D^\mu\psi - M\bar{\psi}\psi\,e^{-g\sigma}\,. \tag{2.5}$$

The second term in Eq. (2.1) is made conformal-invariant on multiplication by $\exp(-2g\sigma)$. The last term is both chiral- and conformal-invariant when the canonical redefinition of ψ is used.

B. The Symmetry-Breaking Term

Within chiral $SU(2)\times SU(2)$ symmetry there is an ambiguity in breaking the chirality, which results in the ratio a_0/a_2 of the $\pi\pi$ scattering lengths depending on the specific breaking hypothesis. When we consider the breaking of conformal symmetry an additional complication arises in that, at the



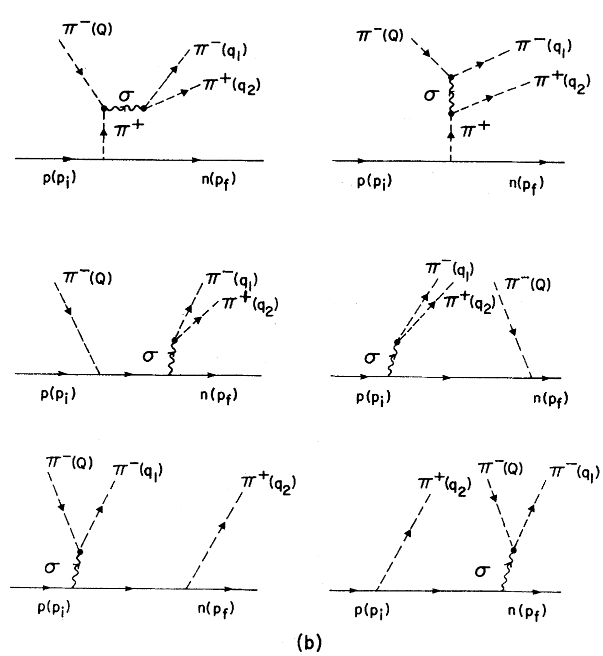


FIG. 1. (a) Diagrams previously considered in calculating $\pi^- p \to \pi^+\pi^- n$; (b) new diagrams involving the σ meson for the same process.

SU(2)×SU(2) level, it appears necessary[2] to have a δ term which breaks conformal symmetry, but not that of SU(2)×SU(2). Otherwise, we shall find $m_\sigma^2 \sim O(m_\pi^2)$. To avoid this we shall assume that $\delta = 0$ at the SU(3)×SU(3) level so that the same term will be responsible for breaking conformal and chiral SU(3)×SU(3) symmetries. We shall assume that the term is of conformal weight λ. That is,

$$L_B \to L_B e^{\lambda g \sigma}, \tag{2.6}$$

where L_B is the SU(3)×SU(3) chiral-symmetry-breaking term. We define a parameter[6] R by the nonlinear σ-model condition[2,5,7]

$$\pi^2 + \chi^2 = (R^2/g^2)\, e^{-2g\sigma}. \tag{2.7}$$

C. The Broken-Symmetry Lagrangians

Following the above discussion we can write down the Lagrangians appropriate to the single-pion-production problem.[2,3] The π-π interaction is given by

$$L_\pi = -\left(\frac{G}{2M}\right)^2\left(\frac{g_V}{g_A}\right)^2[\phi^2(\partial_\mu\phi)^2 + \tfrac{1}{4}(\xi-2)\mu^2(\phi^2)^2]. \tag{2.8}$$

Other interaction Lagrangians, in an obvious notation, are

$$L_{NN\pi} = \frac{G}{2M}\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\phi}, \tag{2.9}$$

$$L_{NN\pi\pi\pi} = -\left(\frac{G}{2M}\right)^3\left(\frac{g_V}{g_A}\right)^2\bar{\psi}\gamma_\mu\gamma_5\vec{\tau}\psi\cdot\partial^\mu\vec{\phi}\,\phi^2, \tag{2.10}$$

$$L_{NN\pi\pi} = -\left(\frac{G}{2M}\right)^2\left(\frac{g_V}{g_A}\right)^2\bar{\psi}\gamma_\mu\vec{\tau}\psi\cdot\vec{\phi}\times\partial^\mu\vec{\phi}, \tag{2.11}$$

$$L_{NN\sigma} = gM\bar{\psi}\psi\sigma, \tag{2.12}$$

$$L_{\pi\pi\sigma} = -g\sigma\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi} - \tfrac{1}{2}\lambda g\mu^2\phi^2\sigma. \tag{2.13}$$

The parameter ξ is a measure of breakdown of the usual "σ" commutator.[3] From (2.8)–(2.11) we reproduce the calculation of Olsson and Turner,[3] who concluded that $\xi = 0$ gave a best fit to the data which in turn picks out the original Weinberg results,[8] with $a_0/a_2 = -\frac{7}{2}$.

III. RESULTS AND CONCLUSIONS

The Lagrangians (2.12) and (2.13) lead us to consider six further diagrams in addition to those considered by Olsson and Turner.[3] These are shown in Fig. 1(b). We define the amplitude $A(\pi N \to \pi\pi N)$ by

$$\langle N(p_f)\pi^\alpha(q_1)\pi^\beta(q_2)|S|N(p_i)\pi^\gamma(Q)\rangle \equiv -\left(\frac{G}{2M}\right)^3\left(\frac{g_V}{g_A}\right)^2\frac{M\delta^{(4)}(p_f+q_1+q_2-p_i-Q)}{(2\pi)^{7/2}(E_iE_f\omega_1\omega_2\omega_Q)^{1/2}}\left(\frac{M}{2(E_i+M)}\right)^{1/2} \times \phi_f^\dagger\vec{\sigma}\cdot\vec{Q}\phi_i A(\pi^\gamma N\to\pi^\alpha\pi^\beta N). \tag{3.1}$$

From the graphs shown in Fig. 1 we arrive at

$$\begin{aligned} A(\pi^-p\to\pi^+\pi^-n) = {} & \left(\frac{2\omega_Q-\xi\mu}{\omega_Q-\mu}+\frac{2\mu}{M}-2-\frac{2\mu}{\mu+2M}+\frac{2\mu}{2E_i-\mu}\right) \\ & -\left(\frac{2M}{G}\right)^2\left(\frac{g_A}{g_V}\right)^2 g^2\left\{\frac{(4\mu\omega_Q-(\lambda+2)\mu^2)(2-\lambda)\mu}{4(\mu-\omega_Q)(4\mu^2-m_\sigma^2)}+\frac{(M+\mu)(2-\lambda)\mu}{2(4\mu^2-m_\sigma^2)}+\frac{(M^2-\mu^2+\mu\omega_Q)(2-\lambda)\mu}{2(4\mu^2-m_\sigma^2)(\mu-E_i)}\right. \\ & \left.+\frac{(\lambda\mu-2(M-E_i))(2\omega_Q+\lambda\mu)}{4(\mu-\omega_Q)(2\mu^2-m_\sigma^2-2\mu\omega_Q)}-\frac{\mu^2(\lambda\mu+2\omega_Q)}{2(\mu+2M)(2\mu^2-m_\sigma^2-2\mu\omega_Q)}+\frac{(2\mu E_i+2\mu M-\mu^2)(2\omega_Q+\lambda\mu)}{2(2\mu^2-m_\sigma^2-2\mu\omega_Q)(\mu-2E_i)}\right\}, \end{aligned} \tag{3.2}$$

where the second bracket contains the terms additional to the previous chiral-Lagrangian treatments. Here we have used

$$Q=(\omega_Q,\vec{Q}),\quad p_i=(E_i,-\vec{Q}),\quad q_1=q_2=(\mu,\vec{0}),\quad \text{and}\quad p_f=(M,\vec{0}).$$

Similarly, we can obtain

$$\begin{aligned} A(\pi^+p\to\pi^+\pi^+n) = {} & \left(\frac{(2+\xi)\mu}{\mu-\omega_Q}+\frac{4\mu}{\mu+2M}-\frac{4\mu}{2E_i-\mu}-\frac{4\mu}{M}\right) \\ & -\left(\frac{2M}{G}\right)^2\left(\frac{g_A}{g_V}\right)^2 g^2\left(\frac{2\omega_Q+\mu\lambda}{2\mu^2-2\mu\omega_Q-m_\sigma^2}\,\frac{\mu^2(4+\lambda)-2\mu\omega_Q}{2(\mu-\omega_Q)}-\frac{2\mu\omega_Q+\mu^2\lambda}{2\mu^2-2\mu\omega_Q-m_\sigma^2}\,\frac{\mu}{\mu+2M}\right. \\ & \left.+\frac{2\mu\omega_Q+\mu^2\lambda}{2\mu^2-2\mu\omega_Q-m_\sigma^2}\,\frac{2E_i+2M-\mu}{\mu-2E_i}\right). \end{aligned} \tag{3.3}$$

In Fig. 2 we show the differences arising from the insertion of a dilaton into the nonlinear chiral Lagrangian.[9] We see that the general result is a lowering of the cross sections. To determine a "best" solution we shall consider the $\pi^+p\to\pi^+\pi^+n$ cross sections. There are two values available[10]: at 300 MeV, $\sigma(\pi^+p)=25\pm18$ μb, and at 357 MeV, $\sigma(\pi^+p)=120\pm10$ μb. Table I shows our calculations. Together with our results on $\pi^-p\to\pi^+\pi^-n$, it would appear that a negative ξ, probably near -1, is favored.[11] Figure 3 shows, for $\xi=-1$ and $\lambda=-1$, the variation of our solutions with increasing R, and Fig. 4 shows the behavior of the solutions for $\lambda=-1, -2, -3$ when ξ is held fixed and the σ mass and width remain approximately constant.

From these we can conclude that, to within the present accuracy of the data, the single-pion production experiments do not allow a determination of λ. If we assume that the dilaton is the ϵ meson at ~700 MeV with a fairly large width then the "best" solution would appear to have $\xi\sim-1$. This would increase[12] a_0 by ~30% from its value of $0.23\mu^{-1}$ and decrease a_2 from $-0.045\mu^{-1}$ to $\sim-0.026\mu^{-1}$ (using $f_\pi\sim 82$ MeV). This would change the value of $2a_0+a_2\simeq 0.34\mu^{-1}$ found by Olsson and Turner to $2a_0+a_2=0.56\mu^{-1}$, which lies just outside the estimate[1,13] $0.4\mu^{-1}\pm0.1\mu^{-1}$. In terms of transformations of the pion field the value $\xi=-1$ does not have the simple behavior advocated by Weinberg. Table II shows the variation of the scattering lengths with R. These scattering lengths include the σ-exchange terms[12] and are given by

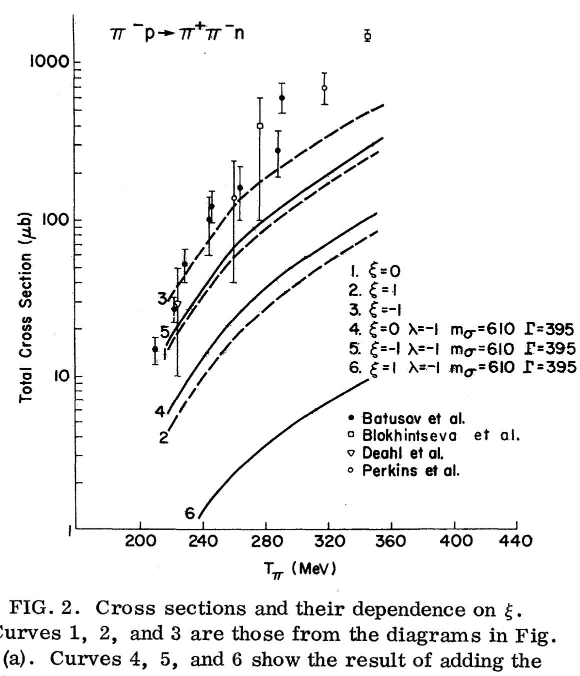


FIG. 2. Cross sections and their dependence on ξ. Curves 1, 2, and 3 are those from the diagrams in Fig. 1(a). Curves 4, 5, and 6 show the result of adding the diagrams in Fig. 1(b). The data are from Ref. 9.

TABLE I. For the process $\pi^+p\to\pi^+\pi^+n$, some values of the cross sections (in μb) predicted are given. The experimental values are[10] 120 ± 10 μb at $T_\pi=357$ MeV and 25 ± 18 μb at $T_\pi=300$ MeV, where T_π is the pion kinetic energy in the lab. m_σ and Γ_σ are given in MeV.

	$T_\pi=357$ MeV			$T_\pi=300$ MeV		
	$\xi=-1$	$\xi=0$	$\xi=1$	$\xi=-1$	$\xi=0$	$\xi=1$
$m_\sigma=610$, $\Gamma_\sigma=395$, $\lambda=-1$	77	180	326	44	102	185
$m_\sigma=610$, $\Gamma_\sigma=322$, $\lambda=-2$	58	149	285	33	85	163
$m_\sigma=610$, $\Gamma_\sigma=488$, $\lambda=-3$	52	140	272	30	80	156

$$32\pi\mu a_0=\frac{\mu^2}{f_\pi^{\,2}}\left(7-\tfrac{5}{2}\xi+\frac{R^2\mu^2[2(\lambda+2)^2+3(2-\lambda)^2(1-4\mu^2/m_\sigma^{\,2})^{-1}]}{m_\sigma^{\,2}}\right) \tag{3.4}$$

and

$$32\pi\mu a_2=-\frac{\mu^2}{f_\pi^{\,2}}\left(2+\xi-\frac{2R^2(\lambda+2)^2\mu^2}{m_\sigma^{\,2}}\right). \tag{3.5}$$

We note that, for $\xi=0$ and $m_\sigma\sim5\mu$, a_0 is increased by approximately 20%, in agreement with Carbone *et al.*[4] For $\xi=-1$ a fairly substantial change in both a_0 and a_2 is obtained, although the scattering lengths themselves still remain small.

In deriving the scattering length relations (3.4) and (3.5) we have put the pions on their mass shell. If instead we imposed the Adler condition on the π-π scattering amplitude then the relation

$$h_0=\frac{R^2\mu^2(\lambda+1)^2}{2f_\pi^{\,2}(m_\sigma^{\,2}-\mu^2)} \tag{3.6}$$

results, where h_0 is defined by[3] the partial conservation of axial-vector current (PCAC) assumption

$$\partial_\mu A^\mu=f_\pi\mu^2\phi(1+h_0\phi^2+\cdots). \tag{3.7}$$

Then $\lambda=-1$, or $h_0=0$, reproduces the previous results based on smoothness arguments.[14]

TABLE II. Pion scattering lengths for two values of ξ and different values of masses.

m_σ (MeV)	Γ_σ (MeV)	$\xi=0$ a_0 (μ^{-1})	$\lambda=-1$ a_2 (μ^{-1})	$\xi=-1$ a_0 (μ^{-1})	$\lambda=-1$ a_2 (μ^{-1})
436	57	0.238	−0.056	0.311	−0.027
523	167	0.232	−0.056	0.305	−0.027
610	395	0.230	−0.056	0.302	−0.027

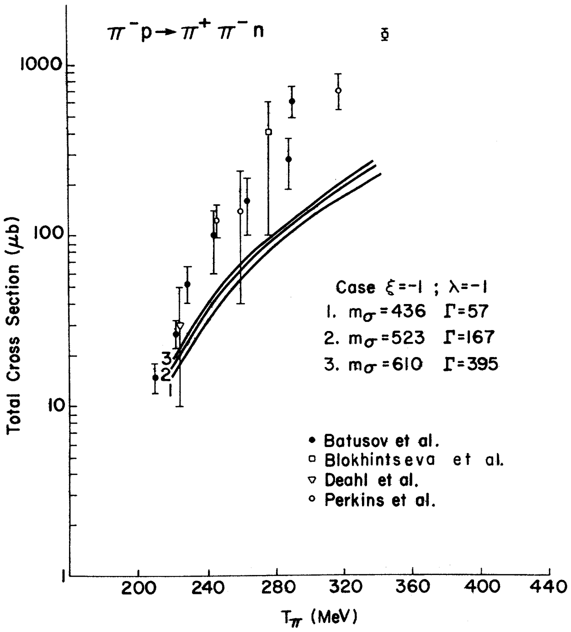


FIG. 3. This shows the variation in the cross sections with the σ mass and width.

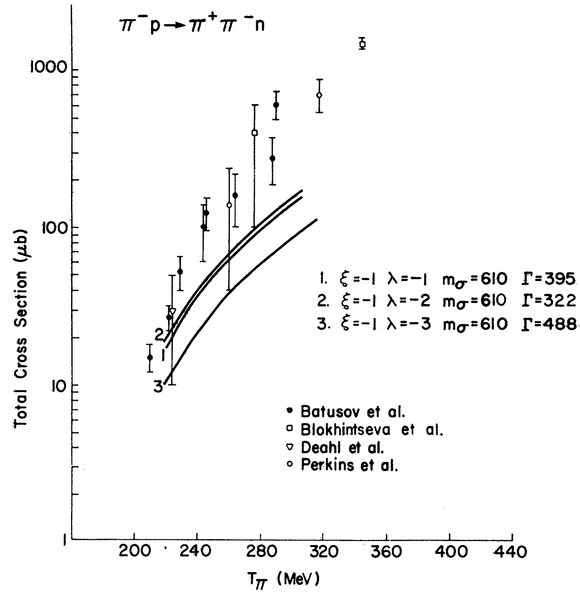


FIG. 4. For a fixed ξ and σ mass and width the variation in cross section with λ is shown.

---

*Research supported in part by the National Research Council of Canada.

[1]See S. Weinberg, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968).

[2]J. Ellis, Nucl. Phys. B22, 478 (1970).

[3]M. G. Olsson and L. Turner, Phys. Rev. Letters 20, 1127 (1968); J. L. Petersen, Phys. Reports 2C, 155 (1972).

[4]G. Carbone, E. Donini, S. Sciuto, and G. Furlan, Nuovo Cimento 58A, 668 (1968).

[5]C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D 2, 685 (1970).

[6]In terms of R, the on-mass-shell width is given by

$$\Gamma_{\sigma\pi\pi}=\frac{3}{16\pi}\,\frac{p_\pi}{m_\sigma^{\,2}}\,\frac{R^2}{f_\pi^{\,2}}\,[m_\sigma^{\,2}-(\lambda+2)\mu^2]^2$$

on using Eq. (2.13).

[7]Here $\pi=\phi\,\exp(-g\sigma)$.

[8]S. Weinberg, Phys. Rev. 166, 1568 (1968).

[9]Yu. Batusov *et al.*, Zh. Eksp. Teor. Fiz. 39, 1850 (1960) [Sov. Phys. – JETP 12, 1290 (1961)]; Y. A. Batusov, S. A. Bunyatov, V. M. Sidorov, and V. A. Yarba, Yad. Fiz. 1, 526 (1965) [Sov. J. Nucl. Phys. 1, 374 (1965)]; T. Blokhintseva, V. Grebinnik, V. Zhukov, G. Libman, L. Nemenov, G. Selivanov, and Y. Jung-Fang, Zh. Eksp. Teor. Fiz. 44, 498 (1963) [Sov. Phys. – JETP 17, 3450 (1963)]; J. Deahl, M. Derrick, J. Fetkovich, T. Fields, and G. Yodh, Phys. Rev. 124, 1987 (1961); W. Perkins, J. Caris, R. Kenney, and V. Perez-Mendez, *ibid.* 118, 1364 (1960); M. G. Olsson and G. B. Yodh, *ibid.* 145, 1309 (1966).

[10]J. Kirz *et al.*, Phys. Rev. 126, 763 (1962); V. Barnes *et al.*, CERN Report No. 63-67 (unpublished).

[11]We are not attempting a best fit, but instead looking for a representative solution. Much more accurate data near threshold are important in specifying ξ, λ, etc. more exactly.

[12]We are solely concerned here with the σ terms arising from introducing conformal-symmetry concepts. There have been other, hard-pion calculations involving σ, ρ, and A_1 exchanges in π-π scattering; see, e.g., R. Arnowitt *et al.*, Phys. Rev. Letters 20, 475 (1968).

[13]Both the value $0.56\mu^{-1}$ and the estimate $(0.4\pm0.1)\mu^{-1}$ assume the value $f_\pi\sim 82$ MeV, consistent with the Goldberger-Treiman relation. The later estimate of Olsson and Turner [Phys. Rev. 181, 2141 (1969)] indicates that their value of $2a_0+a_2$ is insensitive to the value of f_π in the range 82–95 MeV. This latter analysis is altered by the inclusion of the σ to read $a_0=0.18+(28/f_\pi)^2$ and $a_2=0.073-(24.5/f_\pi)^2$.

[14]R. Jackiw, Phys. Rev. D 3, 1347 (1971); L. Turner, *ibid.* 4, 398 (1971).