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Current Algebra, the Triangle Approximation, and Meson Decays

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Axial-vector-current conservation in a world of massless pions is used to investigate two-body decays of vector and tensor mesons involving at least one pion. The axial-vector channel is saturated with the single-pion and $B\bar{B}$ intermediate states, which leads to absolute predictions for vector-meson decay rates, with no adjustable parameters. Good agreement with experiment is found for $\omega\to\pi\gamma$ and $\rho\to\pi\pi$. Comparisons with the vector-meson-dominance model and ρ-meson universality are made. The model is extended to tensor-meson decays, and vector-tensor exchange degeneracy is found.

I. INTRODUCTION

In previous papers it has been shown how the usual Ward identities of current algebra have their origin in the "kinematical" principles of vector- and axial-vector-current conservation.[1,2] Furthermore, it has been demonstrated that one of the problems of current algebra, K_{l3} decay, can be resolved within the "dynamical" approximation of saturating the axial-vector channel by the π and $B\bar{B}$ (B = baryon) intermediate states (the triangle approximation).

In this paper we extend these ideas to $\pi^0\to 2\gamma$ and decays of vector and tensor mesons. In particular we treat the "abnormal" decay $V\to\pi\gamma$ in a fashion similar to $\pi^0\to\gamma\gamma$ (Sec. II), and the "normal" processes $\rho\to\pi\pi$, $f\to\pi\pi$, and SU(3)-related process similar to K_{l3} decay (Sec. III). Replacing a pion by the axial-vector current, using the axial Ward identities,[1] and saturating the axial-vector channel by the π and $N\bar{N}$ (triangle) intermediate states, we obtain rates for these decays which agree well with experiment. Our results further demonstrate the validity of the dynamical triangle approximation which we discuss in detail in Sec. IV.

Our approach complements the usual vector-dominance model (VDM) applied to vector-meson decays.[3] We saturate the *axial*-vector channel and obtain *absolute* rates; VDM saturates the *photon* vector channel and gives *relative* rates. Our extrapolation is in the *pion* mass variable with the vector-meson coupling *on*-shell; VDM extrapolates in the *photon* mass variable with the vector-meson couplings *off*-shell. The concept of universality of vector-meson couplings will be used to compare these two extrapolation procedures (Sec. III). The predictions of this model for SU(3)-symmetry breaking are also discussed. Vector-tensor exchange degeneracy is investigated and found to be valid in this model.

II. $\pi^0\to\gamma\gamma$ AND $V^0\to\pi^0\gamma$

The prototype for abnormal (intrinsic parity change) decays is $\pi^0\to\gamma\gamma$. It has recently been shown by Adler[4] how the perturbation-theory triangle graph for π^0 decay[5] has its origin in an anomaly of the axial-vector triangle graph $M^x_{\mu\nu\alpha}$ for axial-vector decay (coupling $\frac{1}{2}\gamma_\mu\gamma_5$) into two photons. The divergence of the latter is not proportional to $M_{\mu\nu}$ (coupling γ_5), but one must also include an anomalous term

$$iq^\alpha M^x_{\mu\nu\alpha}+mM_{\mu\nu}=\frac{\alpha}{\pi}\epsilon_{\mu\nu}(k'k). \tag{1}$$

Adler then proceeds to write down the Ward identity for total axial-vector decay which does not vanish, but is equal to the right-hand side of (1),

$$iq^\alpha M_{\mu\nu\alpha}=\frac{\alpha}{\pi g_A}\epsilon_{\mu\nu}(k'k), \tag{2}$$

where g_A^{-1} comes from renormalization in the σ model. Finally, the pion-pole dominates (2) to obtain a low-energy theorem for the pion decay amplitude F_π,

$$f_\pi F_\pi=-\frac{\alpha}{\pi g_A}. \tag{3}$$

Rather than use the anomalous Ward identity and a low-energy theorem, we formulate the problem as an expansion, keeping only the π and the "naive" $N\bar{N}$ intermediate states. By naive we mean that the divergence of the related triangle graph (Fig. 3) is[6]

$$q^\alpha M^x_{\mu\nu\alpha}(\text{naive})=\frac{i\alpha g_A}{\pi}\epsilon_{\mu\nu}(k'k), \tag{4}$$

equivalent to the vanishing of the right-hand side of (1). The Ward identity for the total naive amplitude is

$$q^\alpha M_{\mu\nu\alpha}(\text{naive})=0. \tag{5}$$

Cutting off the expansion after the π and $N\bar{N}$ intermediate states, we obtain

$$0 = if_\pi F_\pi \epsilon_{\mu\nu}(k'k) + q^\alpha M^x_{\mu\nu\alpha}(\text{naive}), \tag{6}$$

and using (4) we again obtain (3), aside from a factor of $g_A{}^2$. Both (3) and this procedure give F_π in agreement with measured values, depending on the value of f used (see Sec. IV E). Our procedure is, then, to ignore the integration ambiguity when using axial-vector current conservation, $\partial_\mu A^\mu = 0$. Without Adler's analysis, this procedure is *ad hoc* and analogous to Steinberger's original triangle calculation.[5] However, this procedure can be extended to processes where low-energy theorems do not exist and the axial Ward identity is normal in the sense of (5). We use the $\pi^0 \to \gamma\gamma$ analysis as a guide to our general triangle diagram approximation and justify its significance by the reasonable agreement with experiment in all known cases.

We begin by examining the process $V_\mu(k) \to \gamma_\nu(k') + A_\alpha(q)$ describing the decay of a neutral vector meson into a photon and an axial-vector current. The general form of the amplitude is $[\epsilon_{\mu\nu}(k'k) \equiv \epsilon_{\mu\nu\rho\sigma}k'^\rho k^\sigma]$

$$\begin{aligned} M_{\mu\nu\alpha} = {} & i(k-k')_\alpha \epsilon_{\mu\nu}(k'k)F_1 \\ & + i(k+k')_\alpha \epsilon_{\mu\nu}(k'k)F_2 + i m_V{}^2 \epsilon_{\mu\nu\alpha}(k')F_3, \end{aligned} \tag{7}$$

where m_V is the mass of the vector meson and the F_i are invariant functions. Axial-vector current conservation (no anomalous Ward identity[4,1]) requires

$$\begin{aligned} 0 &= q^\alpha M_{\mu\nu\alpha} \\ &= i\epsilon_{\mu\nu}(k'k)[q^2F_1 + m_V{}^2(F_2+F_3)] \equiv i\epsilon_{\mu\nu}(k'k)A(q^2). \end{aligned} \tag{8}$$

Since all of the covariants in $M_{\mu\nu\alpha}$ reduce to $\epsilon_{\mu\nu}(k'k)$ when contracted with q^α, we need not evaluate the F_i separately. Thus the procedure becomes exactly the same as for the $\pi^0 \to \gamma\gamma$ case, where there exists only one invariant amplitude in $M_{\mu\nu\alpha}$.

In the limit $q^2 \to 0$, Eq. (8) relates the pole part (q^{-2}) of F_1 to $m_V(F_2+F_3)$. Note that we do *not* take the limit $q_\alpha \to 0$ (as in the soft-pion theorem) which is obviously invalid for vector-meson decays. Our only kinematic approximation is exact axial-vector current conservation in a world of massless pions. We evaluate the *invariants* in Eq. (8) at $q^2 = 0$ and assume that they are slowly varying in the region $0 \le q^2 \le m_\pi{}^2$.

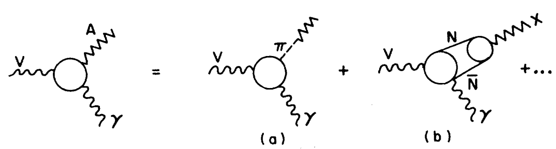

FIG. 1. $\omega \to A\gamma$ axial-vector channel saturation.

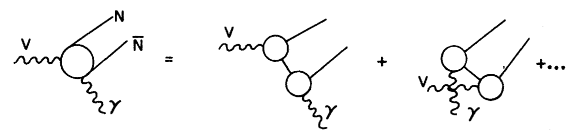

FIG. 2. Pole dominance of ω threshold photoproduction.

Evaluation of $A(q^2)$ is accomplished by saturating the axial-vector channel. Only the single-pion and nucleon-antinucleon states are kept in Fig. 1. The $N\bar{N}$ state is dominated by the nucleon pole term with electric coupling via the Kroll-Ruderman theorem,[7] since the diagram of Fig. 2 can be seen to represent threshold vector-meson photoproduction. This is a frame-dependent statement leading to the "triangle approximation" of Fig. 3, with corrections of order m_V/E_N, where $E_N > m_N = m$ is some average energy of the intermediate nucleons. Magnetic moment couplings in the nucleon triangle graphs are intimately connected with multiparticle states, and justification for neglecting both will be given in Sec. IV B.

The pion state, Fig. 1(a), gives the contribution

$$M^\pi_{\mu\nu\alpha} = if_\pi \frac{q_\alpha}{q^2} F_V(k^2, k'^2, q^2)\epsilon_{\mu\nu}(k'k), \tag{9}$$

where f_π is the pion–axial-vector-current coupling and $F_V(m_V{}^2, 0, m_\pi{}^2)$ is the amplitude for $V^0 \to \pi^0\gamma$ decay, defined by

$$\begin{aligned} S_{fi} = {} & -i(2\pi)^4\delta^4(k-k'-q) \\ & \times F_V \epsilon_\mu(k)\epsilon^*_\nu(k')\epsilon^{\mu\nu}(k'k). \end{aligned} \tag{10}$$

We calculate the $N\bar{N}$ triangle graph as in the $\pi^0 \to \gamma\gamma$ case,[6] and obtain[8]

$$A^{N\bar{N}}(0) = -\frac{eg_{VNN}g_A(0)}{(2\pi)^2}\left(1 + \tfrac{1}{3}x + \tfrac{1}{6}x^2 + \cdots\right), \tag{11}$$

where $x = m_V{}^2/4m_N{}^2$. Combining Eqs. (8), (9), and (11), we predict a decay amplitude

$$F_V(m_V{}^2, 0, 0) = -\frac{eg_Ag_{VNN}}{(2\pi)^2 f_\pi}\left(1 + \tfrac{1}{3}x + \tfrac{1}{6}x^2 + \cdots\right), \tag{12}$$

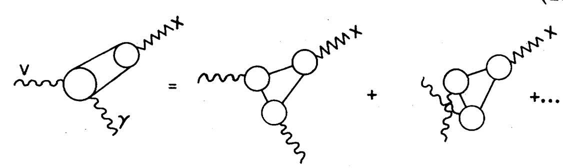

FIG. 3. ω triangle graphs.

with a decay rate

$$\Gamma(V^0 \to \pi^0\gamma) = |F_V|^2 \frac{(m_V^2 - m_\pi^2)^3}{96\pi m_V^3}. \tag{13}$$

Because the pion is far lighter than the vector mesons it is consistent to replace g_A/f_π in Eq. (12) by the Goldberger-Treiman value of g/m (see Sec. IV E).

The vector-meson–nucleon coupling constant g_{VNN} in Eq. (12) is to be evaluated on mass shell ($k^2 = m_V^2$). This region is not available in either scattering or decay experiments, so indirect means must be used. One method is vector-meson dominance of the nucleon electromagnetic form factors. This relates g_{VNN} to g_V, the direct photon–vector-meson coupling, which is measured in leptonic decay processes $V \to e^+e^-$. One has

$$eF_1(t) = \frac{e g_{VNN}(t) m_V^2}{g_V(t)(m_V^2 - t)}, \tag{14}$$

$$\Gamma(V \to e^+e^-) = \frac{\alpha^2}{3}\left[\frac{g_V(m_V^2)}{4\pi}\right]^{-1} m_V. \tag{15}$$

The constraint at $t=0$, $[g_{VNN}(0)/g_V(0)] = F_1(0)$, is assumed to hold in the timelike region also ($t = m_V^2$). Thus if the ω meson dominates the isoscalar form factor, we have

$$g_{\omega NN}(m_\omega^2) = \tfrac{1}{2} g_\omega(m_\omega^2), \tag{16}$$

and a similar relation for the ρ couplings in the isovector form factor.[8] Note that this is not the usual vector-meson dominance assumption $g_V(0) \simeq g_V(m_V^2)$, which is known not to be a reliable estimate.[9] All we require is that g_{VNN} and g_V have the same t dependence in the timelike region, so that the vector mesons give simple poles in the form factors for $0 \le t \le m_V^2$.

From $\Gamma(\omega \to e^+e^-) = (0.76 \pm 0.08)\times 10^{-3}$ MeV, one can extract $g_\omega^2/4\pi = 18.4 \pm 2.0$[10] and predict $\Gamma(\omega \to \pi\gamma) = 1.14 \pm 0.12$ MeV from (12) and (13), in excellent agreement with the experimental value of 1.13 ± 0.17 MeV.[11] Similarly, from $\Gamma(\rho^0 \to e^+e^-) = (6.1 \pm 0.7)\times 10^{-3}$ MeV, one extracts $g_\rho^2/4\pi = 2.56 \pm 0.27$,[10] and predicts $\Gamma(\rho^0 \to \pi^0\gamma) = 0.15 \pm 0.01$ MeV. The decay rate for $\rho^\pm \to \pi^\pm\gamma$ can also be predicted. The factor of $\sqrt{2}$ for charged-pion and vector-meson couplings to nucleons is compensated by the absence of an exchange graph, so the resulting number is the same, $\Gamma(\rho^\pm \to \pi^\pm\gamma) = 0.15 \pm 0.01$ MeV. This is consistent with the experimental upper bound of 240 keV.[11] The equality of decay rates for all charge states can be explained more simply by noting that isospin invariance in the strong interaction parts of the triangle diagram requires the photon to be purely isoscalar.

In the determination of g_ω, we have omitted the ϕ-meson contribution to the isoscalar form factor which we now include. The form-factor normalization becomes

$$\frac{g_{\omega NN}}{g_\omega} + \frac{g_{\phi NN}}{g_\phi} = \frac{1}{2}. \tag{17}$$

This can be expressed in terms of decay rates from (12), (13), and (15). The experimental values[10,11] $\Gamma(\phi \to e^+e^-) = (1.64 \pm 0.24)\times 10^{-3}$ MeV, $\Gamma(\omega \to \pi^0\gamma) = 1.1$ MeV, $\Gamma(\phi \to \pi^0\gamma) \le 13.6\times 10^{-3}$ MeV, and $\Gamma(\omega \to e^+e^-) = (0.76 \pm 0.08)\times 10^{-3}$ MeV, give

$$|0.50 \pm 0.14 \pm R| \le 0.5, \tag{18}$$

where $R \le 0.044 \pm 0.006$. The first term comes from the ω alone, and again shows the agreement between calculated and observed decay rates for $\omega \to \pi\gamma$. The last term is a correction due to the coupling of the ϕ meson to nucleons, and is consistent with ω dominance of the isoscalar form factor.

III. $\rho \to \pi\pi$ AND UNIVERSALITY

The prototype for normal decays is "axial"-vector K_{l3} decay, $K^k(K) \to A_\mu^i(q) + V_\nu^j(\Delta)$, where ijk are the isospin labels of the particles in question. We review the key points of Ref. 2. First, one solves the axial Ward identity

$$q^\mu M_{\mu\nu}^{ijk} = -if^{ijk}(-if_k K_\nu) \tag{19}$$

as

$$M_{\mu\nu}^{ijk} = if^{ijk}\frac{q_\mu}{q^2}(if_k K_\nu) + \cdots, \tag{20}$$

where $\cdots$ means only the $q_\mu K_\nu$ part of $M_{\mu\nu}$ can be determined. Then once again we expand the amplitude and keep only the π pole and $B\bar{B}$ intermediate states as in Fig. 2. From the $q_\mu K_\nu$ part of the π pole and $B\bar{B}$ triangle graphs, we find

$$M_{\mu\nu}^{ijk} = if_\pi \frac{q_\mu}{q^2}(if^{ijk})[f_+(\Delta^2) + f_-(\Delta^2)]K_\nu + M_{\mu\nu}^{B\bar{B}}. \tag{21}$$

Our triangle approximation is really only justified in the limit as $\Delta \to 0$, for then the intermediate photoproduction process is baryon pole dominated in the Kroll-Ruderman sense.[7] Furthermore, all possible members of the baryon octet should be included. It turns out that the $NN\Lambda$ and $\Sigma^+\Sigma^+\Xi^0$ triangles dominate and lead to

$$M_{\mu\nu}^{B\bar{B}} \approx if_\pi(if^{ijk})\frac{g^2}{4\pi^2}\frac{q_\mu K_\nu}{m_N^2}. \tag{22}$$

Now as $\Delta \to 0$, $q \to K$, and $q^2 \to m_K^2$, where we keep the kaon on shell. Combining (20), (21), and (22), we obtain

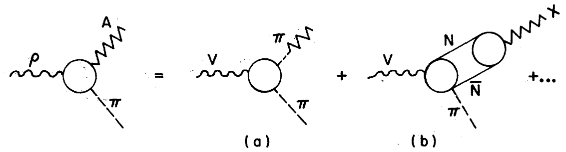

FIG. 4. $\rho\to A\pi$ axial-vector channel saturation.

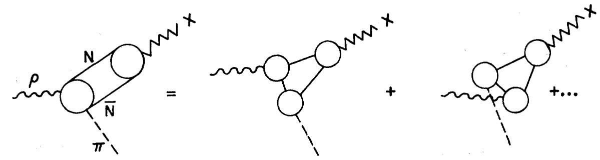

FIG. 5. ρ triangle graphs.

$$f_+(0)+f_-(0)=\frac{f_k}{f_\pi}-\frac{g^2}{4\pi^2}\frac{m_k^{\,2}}{m_N^{\,2}}\approx 0\,, \tag{23}$$

leading to the conclusion

$$\xi=\frac{f_-(0)}{f_+(0)}\approx -1\,. \tag{24}$$

It is important to stress that this approach is a model in that we have again cut off the intermediate states after the $B\overline{B}$, and also keep only $q_\mu K_\nu$ coefficients.

Now we consider the process $\rho_\nu^k(k)\to A_\mu^j(q)+\pi^i(q')$. Since this decay involves only the one conserved axial-vector current in a world of massless pions, the axial Ward identity for this amplitude $M_{\mu\nu}$ is similar to Eq. (8),

$$q^\mu M_{\mu\nu}^{ijk}=0\,. \tag{25}$$

Contrary to $A\to\gamma\gamma$ or $V\to A_\gamma$, this amplitude is "normal" in the sense that it does not depend upon the abnormal tensor $\epsilon_{\mu\nu\alpha\beta}$. Hence we analyze this decay by following the techniques developed for another "normal" process, K_{l3} decay.[2] To this end we partially "solve" Eq. (25) and determine the $q_\mu(q-q')_\nu$ coefficient of $M_{\mu\nu}$ to be

$$M_{\mu\nu}^{ijk}=0\times q_\mu(q-q')_\nu+\cdots\,, \tag{26}$$

which can be verified by examination of all possible current-conserving covariants, as in Ref. 1.

Now we relate this process to the physical decay $\rho_\nu^k(k)\to\pi^j(q)+\pi^i(q')$, defined by

$$S_{fi}=-i(2\pi)^4\delta^4(k-q-q')\times g_{\rho\pi\pi}i\epsilon^{ijk}(q-q')\cdot\epsilon(k)\,. \tag{27}$$

This definition of the $\rho\to\pi\pi$ coupling constant is consistent with the Hamiltonian

$$H=g_{\rho\pi\pi}\epsilon^{ijk}\pi^{*i}\partial^\nu\pi^j\rho_\nu^k+\tfrac{1}{2}g_{\rho NN}\overline{N}\tau^k\gamma^\nu N\rho_\nu^k+\cdots\,. \tag{28}$$

Then applying pion pole dominance to $M_{\mu\nu}$ according to Fig. 4(a), we find

$$\begin{aligned}M_{\mu\nu}^{ijk}(\pi)&=if_\pi\frac{q_\mu}{q^2}g_{\rho\pi\pi}(-i\epsilon^{ijk})(q-q')_\nu\\&=\left(\frac{f_\pi g_{\rho\pi\pi}}{q^2}\right)\epsilon^{ijk}q_\mu(q-q')_\nu\,.\end{aligned} \tag{29}$$

Next we include the $N\overline{N}$ intermediate state in the saturation of the axial-vector channel as in Fig. 4(b). Once again an important contribution to the $N\overline{N}$ intermediate state is the triangle graph of Fig. 5. Accounting for the direct and exchange triangle graphs, we use the Feynman rules to write

$$\begin{aligned}M_{\mu\nu}^{ijk}(N\overline{N})=&-\tfrac{1}{2}g_Ag_{\rho NN}g_{\pi NN}(\mathrm{Tr}\,\tau^i\tau^j\tau^k)(2\pi)^{-4}\\&\times\int d^4r\,\mathrm{Tr}\{\gamma_\mu\gamma_5[\gamma\cdot(r+k)-m]^{-1}\gamma_\nu(\gamma\cdot r-m)^{-1}\gamma_5[\gamma\cdot(r+q')-m]^{-1}\}+\cdots\,.\end{aligned} \tag{30}$$

As in the case of K_{l3} decay, this "normal" triangle graph diverges logarithmically; yet the coefficient of $q_\mu(q-q')_\nu$ is finite. Evaluating this finite part of Eq. (30) by field theory or dispersion techniques, we find (using $mg_A=f_\pi g$)

$$M_{\mu\nu}^{ijk}(N\overline{N})=-\frac{f_\pi g_{\rho NN}g_{\pi NN}^{\;2}}{(4\pi)^2}\left(1+\frac{k^2+q^2+q'^2}{12m_N^{\,2}}+\cdots\right)\epsilon^{ijk}q_\mu(q-q')_\nu+\cdots\,. \tag{31}$$

In order that the triangle graph be the dominant contribution to the $N\overline{N}$ intermediate state, we must take $q'\to0$ to enhance the nucleon pole. Even then we must neglect the magnetic ρNN coupling as well as other intermediate states; we postpone justification for this triangle approximation until Sec. IV. Since $k^2=m_\rho^{\,2}$, as $q'\to0$ we have $q^2\to m_\rho^{\,2}$ and we find from Eqs. (29) and (31)

$$\begin{aligned}M_{\mu\nu}^{ijk}&=M_{\mu\nu}^{ijk}(\pi)+M_{\mu\nu}^{ijk}(N\overline{N})+\cdots\\&\underset{q'\to0}{\sim}\epsilon^{ijk}f_\pi\left[\frac{g_{\rho\pi\pi}}{m_\rho^{\,2}}-\frac{g_{\rho NN}g_{\pi NN}^{\;2}}{(4\pi)^2m_N^{\,2}}\left(1+\frac{m_\rho^{\,2}}{6m_N^{\,2}}+\cdots\right)\right]q_\mu(q-q')_\nu+\cdots\,.\end{aligned} \tag{32}$$

Equating the coefficients of $q_\mu(q-q')_\nu$ in Eqs. (26) and (32), we obtain

$$g_{\rho\pi\pi} - 0.89\, g_{\rho NN} = 0 . \tag{33}$$

We interpret Eq. (33) as a dynamical statement of the universal coupling of vector mesons. Note that the arguments of the coupling constants in (33) are $g_{\rho\pi\pi}(m_\rho^2, m_\rho^2, 0)$ and $g_{\rho NN}(m_\rho^2, m_N^2, r^2)$ where $r^2 \leq m_N^2$. Taking for $g_{\rho NN}$ the value found from $\rho^0 \to e^+e^-$, $g_{\rho NN}(m_\rho^2, m_N^2, m_N^2) = 5.67 \pm 0.29$, we find from (33) the value $g_{\rho\pi\pi}(m_\rho^2, m_\rho^2, 0) = 5.05 \pm 0.26$. This result is in agreement with the on-shell value of $g_{\rho\pi\pi}(m_\rho^2, m_\pi^2, m_\pi^2) = 5.14$ found from $\rho \to \pi\pi$ decay, $g_{\rho\pi\pi}^2/4\pi = 2.1 \pm 0.11$, and, of course, 11% lower than the universal value $g_{\rho\pi\pi} = g_\rho (= g_{\rho NN})$ implied by fits to ρ decay with finite width correction.[10]

Furthermore, we remind the reader of the Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin (KSFR) relation,[12]

$$\frac{1}{2f_\pi^2} = \frac{g_{\rho\pi\pi}\, g_{\rho NN}}{m_\rho^2} , \tag{34}$$

which is a vector-dominance type of statement obtained by ρ dominating the Adler-Weisberger low-energy theorem $M^{ij}_{\pi N \to \pi N} \to (\frac{1}{2} f_\pi^2) i \epsilon^{ijk} \tau^k$ at $t = 0$. Equation (34) can also be used to test universality.[12] Note that the relevant variables in this case are $g_{\rho\pi\pi}(0, m_\pi^2, m_\pi^2)$ and $g_{\rho NN}(0, m_N^2, m_N^2)$. Taking $f_\pi = 94$ MeV and $g_{\rho NN} = 5.67$, we find $g_{\rho\pi\pi}(0, m_\pi^2, m_\pi^2) = 5.77$, which is in agreement with universality and 12% higher than the on-shell value quoted earlier. Note too that the relative signs of $g_{\rho\pi\pi}$ and $g_{\rho NN}$ in (33) and (34) are consistent.

There is yet another way we can probe the meaning of the universal coupling of vector mesons, and that is by replacing the vector ρ (V_ν) with a conserved vector-isovector current (J_ν). The axial Ward identity for the process $J^k_\nu(k) \to A^j_\mu(q) + \pi^i(q')$ can then be obtained from the local commutation relation, $[A, V] = A$, or from the "kinematic" approach of axial-vector current conservation similar to K_{l3} decay,[1] giving

$$q^\mu M^{ijk}_{\mu\nu} = -i\epsilon^{ijk}(-if_\pi K_\nu) . \tag{35}$$

The "formal" solution of Eq. (35) yields

$$M^{ijk}_{\mu\nu} = \frac{f_\pi}{q^2} \epsilon^{ijk} q_\mu (q-q')_\nu + \cdots , \tag{36}$$

as opposed to Eq. (26) for real ρ meson decay. Again saturating the axial-vector channel with the π and $N\bar{N}$ intermediate state triangle diagrams as $q' \to 0$, we obtain from Eqs. (29), (31), and (36) the universality statement

$$g_{J\pi\pi} - 0.89 \frac{q^2}{m_\rho^2} g_{JNN} = 1 . \tag{37}$$

This relation for the conserved vector current coupling to pions and nucleons is stronger than Eq. (33) because the factor one on the right-hand side of (37) determines the *sign* and *scale* of $g_{J\pi\pi}$ and g_{JNN}. Choosing $q^2 = m_\rho^2$ and $g_{JNN} = g_{\rho NN} = 5.67$, we obtain from (37) $g_{J\pi\pi} = 6.05$ – somewhat higher than the measured value of $g_{\rho\pi\pi} = 5.14$, or the universal value $g_{\rho\pi\pi} = g_\rho$.

Note that Eqs. (33) and (37) are not inconsistent and bear the same relationship that the $\rho\pi \to \rho\pi$ superconvergence relation has to the Dashen–Fubini–Gell-Mann[13] sum rule for $J\pi \to J\pi$. That is, using the current-field identity, one can ρ dominate (37), multiply by $m_\rho^2 - k^2$, and recover (33) in the limit $k^2 \to m_\rho^2$.

IV. THE TRIANGLE APPROXIMATION

Having successfully explained the decay processes $\pi^0 \to \gamma\gamma$, K_{l3}, $\omega \to \pi\gamma$, and $\rho \to \pi\pi$ by triangle graphs, we now synthesize our interpretation of the triangle approximation.

A. Abnormal vs Normal Decays

One treats the processes $\pi VV (\pi \to \gamma\gamma,\ \omega \to \pi\gamma)$ and $\pi PV (K_{l3}, \rho \to \pi\pi)$ by replacing the (massless) pion by a conserved axial-vector current $A_\mu(q)$, as AVV and APV. Abnormal (AVV) triangle graphs diverge linearly, but multiplication by the axial momentum (q_μ) renders them convergent due to the peculiar nature of the abnormal tensor $\epsilon_{\mu\nu\alpha\beta}$. On the other hand, normal (APV) triangle graphs (and their momentum contractions) diverge logarithmically and one obtains a finite number only by isolating the coefficient of a particular covariant also occurring in the Ward identity.

B. Axial-Vector Channel Saturation

One then equates the axial Ward identity to axial-vector channel saturation, singling out the single-particle pion and $N\bar{N}$ intermediate states. Magnetic moment couplings, multiparticle states, and resonance states are lumped together and ignored. For the case of $\pi^0 \to \gamma\gamma$, this approximation has been justified by Adler and Bardeen using the field theory σ model[14] and spinor electrodynamics.[14,15] The σ model embodies the content of PCAC (partially conserved axial-vector current) and current algebra, can simulate resonance states, and accounts for nucleon magnetic moments.[16] We invoke the Adler-Bardeen proof to justify neglect of these higher-order states for $\omega \to \pi\gamma$ and the normal decays K_{l3}, $\rho \to \pi\pi$ as well.

C. Triangle Enhancement

In order that the $N\bar{N}$ states be dominated by their triangle diagrams, one must assume a particular momentum configuration. For abnormal decays the covariant $\epsilon_{\mu\nu\alpha\beta}$ suppresses the importance of the pion pole because $\epsilon_{\mu\nu}(k'k) \sim q^2$, and taking the limit $q^2 \to 0$ keeps the triangle graph on the same footing with the pion pole. In the case of ω decay, the triangle graph is enhanced at $q^2=0$ (not $q \to 0$), but only in the threshold configuration of the rest frame of the decaying vector meson. For normal APV decays, the triangle graph is enhanced in the soft limit of the other final-state momentum [$q' \to 0$ for $\rho \to A(q)\pi(q')$ and $\Delta \to 0$ for $K \to A(q)V(\Delta)$]. This makes the pole graphs of $\rho \to N\bar{N}\pi(q')$ and $K \to N\bar{N}V(\Delta)$ dominant, leading to universal ρ couplings, and $\xi \approx -1$ in K_{l3} decay.[2]

D. Inclusion of Hyperon States

Our approximation of π plus $N\bar{N}$ intermediate states can be amended to include all $B\bar{B}$ octet baryon states. Such states do not appreciably alter the $\pi \to \gamma\gamma$ and $\omega \to \pi\gamma$ triangle contributions because of the SU(3) values of the couplings constants in conjunction with a D/F ratio of $\frac{3}{2}$. For K_{l3}, however, strangeness conservation rules out the exchange $N\bar{N}$ state, and substitutes in its place the $\Sigma^+\Sigma^+\Xi^0$ triangle graph to match the "direct" triangle graph $pp\Lambda$. So we see that $B\bar{B}$ states should be included in the triangle approximation. However, $\rho \to \pi\pi$ involves no strange particles, and hyperon triangle graphs should not play a significant role. One can, in fact, show this by comparing $\rho \to \pi\pi$ with $K^* \to K\pi$. (See Sec. V.)

E. Coupling-Constant Prescriptions

It is a subtle but important point to note that if the pion is the initial decay particle, as in $\pi^0 \to \gamma\gamma$, then one uses the value of f_π obtained from the charged decay process ($f_\pi = 94$ MeV) $\pi^- \to \mu^-\nu$. If instead, the pion is a final-state decay product, as in $\omega \to \pi\gamma$, K_{l3}, or $\rho \to \pi\pi$, then one uses the Goldberger-Treiman value of f_π ($f_\pi = 83$ MeV), $mg_A = f_\pi g$. The prescription for VNN couplings is to ignore off-mass-shell nucleon corrections, keep the V (or K) on shell, and use SU(3) on-shell couplings.

F. Importance of Axial-Vector Current Conservation

One might argue that Steinberger's original pion triangle graph,[5] which does not depend upon the Ward identities of current algebra, is just as valid an approximation as our procedure. Aside from the fact that the π^0 rate is 30% higher because g_A/f_π is replaced by g/m, such a prescription leads to a rate for $\eta \to \gamma\gamma$ which is an order of magnitude too small.[17] We have used axial-vector current conservation to justify the (axial-vector) triangle approximation for decays involving a pion. Less justification exists for using the triangle approximation for decays with no pions, such as, $\eta \to \gamma\gamma$.[4] Furthermore, "normal" pion triangle graphs are divergent and no covariance arguments can be used to extract a finite result.

V. SU(3) EXTENSIONS AND TENSOR-MESON DECAYS

We now examine the effect of inserting the entire baryon octet in the triangle diagram for vector-meson decay into two pseudoscalar mesons. The pseudoscalar SU(3) symmetric coupling to baryons is used with the usual $D/F = \frac{3}{2}$. The vector-meson coupling to baryons is the SU(3) generalization of the electric ρNN coupling used in Sec. III, with f_V, $d_V = 1 - f_V$ left as a free parameter and with over-all coupling strength $g_{VBB} = g_{\rho^0 PP}$. The vector-meson coupling to two pseudoscalar mesons is taken in the usual form with strength g_{VPP}, such that

$$\Gamma(V \to PP) = \frac{2}{3}\,\frac{g_{VPP}{}^2}{4\pi}\,\frac{K_0{}^3}{m_V{}^2}\,, \tag{38}$$

where K_0 is the momentum of the decay products. For a given charge state of the external particles, there are 12 triangle graph contributions. The integral is performed with the physical masses inserted, and the factor $(1/m_N{}^2)[1 + (m_\rho{}^2/6m_N{}^2)]$ in Eq. (32) is replaced by

$$\frac{1}{m_N M_1}\left(1 + \frac{m_V{}^2}{6M_1{}^2} - \frac{2}{3}\,\frac{M_2{}^2 - M_1{}^2}{M_1{}^2} - \frac{1}{3}\,\frac{M_3{}^2 - M_2{}^2}{M_1{}^2}\right),$$

where M_1, M_2, and M_3 are the masses of the baryons in the triangle, with M_2 and M_3 coupled at the axial vertex. The axial coupling g_{A23} is determined by an SU(3) generalization of the Goldberger-Treiman relation as

$$2f_\pi g_{\pi 23} = (M_2 + M_3) g_{A23}\,, \tag{39}$$

where $g_{\pi 23}$ is the SU(3) value for pion coupling to M_2 and M_3. The result for $\rho^\pm \to \pi^\pm\pi^0$ is

$$0 = g_{\rho\pi\pi} - g_{VBB}\,\frac{g_{\pi NN}{}^2}{4\pi}\,\frac{m_\rho{}^2}{4\pi m_N{}^2}(1.09 + 0.78 f_V)\,, \tag{40}$$

and for $K^*(890) \to K^+\pi^0$ is

$$0 = g_{K^*K\pi^0} - g_{VBB}\,\frac{g_{\pi NN}{}^2}{4\pi}\,\frac{m_{K^*}{}^2}{4\pi m_N{}^2}(0.56 + 0.37 f_V)\,. \tag{41}$$

Note that for no strange particles in the ρ triangle graphs [Eq. (32)], the factor $(1.09+0.78f_V)$ is unity. Using f_V as an adjustable parameter, we can satisfy (40) with the experimental values $g_{\rho\pi\pi} = g_{VBB} = g_{\rho NN} = 5.67$. Since the relative sign is not determined by experiment, two solutions exist. One is $f_V = 0.21$, which gives $g_{K^*K\pi^0} = 3.92$. The other is $f_V = -3.0$, giving $g_{K^*K\pi^0} = 3.36$. The experimental value, from $\Gamma_{K^*} = 50$ MeV, $\Gamma(K^{*+} \to K^+\pi^0)/\Gamma(K^+ \to \text{all}) = \frac{1}{3}$, and using Eq. (38), is $g_{K^*K\pi^0} = 3.23$. Thus we have the interesting results that the two f_V values which satisfy ρ universality relation (40) with experimental values of couplings both give fair agreement with the K^* decay value. Note also that these couplings are evaluated with the vector meson on the mass shell and the pseudoscalar mesons off the mass shell. Thus we should not be surprised that the f_V values are different than the pure-f coupling expected from universal ρ coupling (presumably valid at zero ρ mass) and the values for vector-meson Regge residues in meson-baryon scattering[18] (valid for $m_V^2 = t \leq 0$).

We now consider the tensor-meson decays into pseudoscalar mesons. The relevant quantities for the decay are

$$H_{TPP} = \frac{g_{TPP}}{m_T} \epsilon^{\lambda}_{\mu\nu}(k)(p_1 - p_2)^\mu (p_1 - p_2)^\nu \tag{42}$$

and

$$\Gamma(T \to PP) = \frac{16}{15} \frac{g_{TPP}^2}{4\pi} \frac{K_0^5}{m_T^4}, \tag{43}$$

where (43) is multiplied by $\frac{1}{2}$ if the pseudoscalar mesons are identical. For coupling to baryons, we use

$$H_{TBB} = \epsilon^{\lambda}_{\mu\nu}(k)\bar{\mu}(p_1)\Gamma^{\mu\nu}\mu(p_2), \tag{44}$$

$$\Gamma^{\mu\nu} = \frac{g_{TBB}}{m}[\gamma^\mu(p_1 - p_2)^\nu + \gamma^\nu(p_1 - p_2)^\mu], \tag{45}$$

in analogy to the electric vector-meson coupling. The other possible coupling is proportional to $(p_1 - p_2)^\mu (p_1 - p_2)^\nu$ and gives a divergent contribution to the triangle diagram, as does the magnetic coupling for vector mesons. It is interesting to note that the coupling (45) conserves s-channel helicity in a scattering diagram. To account for f, f' mixing, we include a term for singlet coupling as well as the SU(3) octet coupling,

$$\frac{g_{TBB}}{m}[d_T \operatorname{Tr}(\{\bar{B}, B\}T) - (1 - d_T)\operatorname{Tr}([\bar{B}, B]T)] + \frac{h_{TBB}}{m} T \operatorname{Tr}(\bar{B}B).$$

We use $m = m_f = 1264$ MeV as the scaling mass. The ratio h_{TBB}/g_{TBB} is fixed by requiring the absence of $f' \to \pi\pi$ decays. The results are

(a) $f^0 \to \pi^0\pi^0$, $(g_{f\pi\pi})_{\text{exp}} = 5.82 \pm 0.50$,

$$0 = g_{f\pi\pi} + \sin\theta(1 + \cot^2\theta) \times \frac{g_{TBB}}{m} \frac{g_{\pi NN}^2}{4\pi} \frac{m_f^3}{4\pi m_N^2}(8.84 - 9.71 d_T), \tag{46}$$

with $\theta = 27.7°$, determined from mass formula;

(b) $K^{**0}_{1420} \to K^0\pi^0$, $(g_{K^{**}K\pi})_{\text{exp}} = 2.94 \pm 0.21$,

$$0 = g_{K^{**}\pi} - \frac{g_{TBB}}{m} \frac{g_{\pi NN}^2}{4\pi} \frac{m_{K^{**}}^3}{4\pi m_N^2}(-5.40 + 4.82 d_T); \tag{47}$$

(c) $A_2^0 \to \eta^0\pi^0$, $(g_{A_2\eta\pi})_{\text{exp}} = 3.58 \pm 0.85$,

$$0 = g_{A_2\eta\pi} - \frac{g_{TBB}}{m} \frac{g_{\pi NN}^2}{4\pi} \frac{m_{A_2}^3}{4\pi m_N^2}(5.62 - 4.28 d_T). \tag{48}$$

There are two parameters g_{TBB} and d_T to fit three decay widths, so one prediction is possible. We choose the parameters to fit (a) and (b), and then predict (c). Since the sign of the couplings is not determined, we get two possibilities. One choice ($g_{f\pi\pi}$ and $g_{K^{**}K\pi}$ have opposite signs) gives $d_T = 0.984$, $g_{TBB} = 5.34 \pm 0.45$, and predicts $g_{A_2\eta\pi} = 5.86 \pm 0.50$ – not very good agreement. However, the other choice gives $d_T = 0.64$, $g_{TBB} = -1.47 \pm 0.13$, and predicts $g_{A_2\eta\pi} = -3.28 \pm 0.28$ in good agreement with the experimental value.

One can regard this as a model of SU(3) symmetry breaking, since ratios of Eqs. (46)–(48) are independent of g_{TBB}. This should be compared to the results of SU(3) symmetry for (g_{TPP}/m_T) with pure D-coupling in (42), which also gives satisfactory results for ratios of decay widths.[19]

It is interesting to note that the favored value of d_T gives a D/F ratio for TBB quite close to one of the solutions in the VBB case. This result is expected in the exchange-degenerate Regge trajectory model, so that one could consider d_T determined externally and increase the number of predictions by one. The residue function magnitudes are also predicted to be equal in the model. The corresponding quantities in this model for πp elastic scattering are

$$\frac{\beta_T}{\beta_V} = -\frac{8 g_{TBB} g_{f\pi\pi} s_0}{m_f^2 g_{\rho\pi\pi} g_{VBB}} = 1.23, \tag{49}$$

where we have used $s_0 = 1\ \text{GeV}^2$. Note that the residue functions are evaluated at the particle poles, i.e., $\beta_T(t = m_f^2)$ and $\beta_V(t = m_\rho^2)$, so that the deviation from equality could be just the t dependence. In fact, a form $\beta_{V,T}(t) = \beta_0 e^{At}$ with $A = 0.2\ (\text{GeV}/c)^{-2}$ will give exact exchange degeneracy.

[1]M. D. Scadron, University of Arizona report (unpublished).
[2]M. D. Scadron, Phys. Rev. D 5, 188 (1972).
[3]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).
[4]S. L. Adler, Phys. Rev. 177, 2426 (1969).
[5]J. Steinberger, Phys. Rev. 76, 1180 (1949).
[6]The divergence is calculated by writing the triangle amplitude in Feynman parameter form, contracting with q, and evaluating without regard to possible contact terms due to translating the formally divergent but non-gauge-invariant parts. See R. Jackiw, MIT-CTP Report No. 183, 1970 (unpublished).
[7]N. Kroll and M. Ruderman, Phys. Rev. 93, 233 (1954).
[8]Note that for $V=\rho$ the coupling g_{VNN} is twice the standard coupling $g_{\rho NN}$ as given by Eq. (18). The isovector analog of Eq. (16) is $g_{\rho NN}=g_{\rho}$.
[9]W. Schmidt and D. R. Yennie, Phys. Rev. Letters 23, 623 (1969); 23, 946(E) (1969).
[10]J. LeFrancois, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).
[11]Particle Data Group, Phys. Letters 33B, 1 (1970).
[12]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966); Riazuddin and Fayyazuddin, Phys. Rev. 147, 1071 (1966).
[13]S. Fubini, Nuovo Cimento 43A, 475 (1966); R. Dashen and M. Gell-Mann, Phys. Rev. Letters 17, 340 (1966).
[14]S. L. Adler and W. A. Bardeen, Phys. Rev. 182, 1517 (1969).
[15]R. A. Brandt, Phys. Rev. 180, 1490 (1969).
[16]M. D. Scadron, Phys. Rev. D 2, 213 (1970).
[17]See, for example, D. G. Sutherland, Nucl. Phys. B2, 433 (1967).
[18]C. Michael and R. Odorico, Phys. Letters 34B, 422 (1971).
[19]J. Rosner, private communication.

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Conformal Symmetry and Single-Meson Production*

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The results of combining conformal invariance and chiral symmetry are given for the process $\pi N \to \pi\pi N$. It is shown that if the symmetries are broken spontaneously the data are insufficient to determine the scale of the breaking term. The $\pi\pi$ scattering lengths remain small although both a_0 and a_2 are altered substantially.

I. INTRODUCTION

In the past few years there has been considerable interest in two aspects of phenomenological Lagrangians. Initially, the interest lay in using such Lagrangians to understand better the results of current algebra and chiral symmetry.[1] More recently, with the possibility that conformal symmetry might also be a useful approximate symmetry, attempts have been made to combine the chiral and conformal symmetries. In particular, Ellis,[2] on the basis of the phenomenological Lagrangian method, has shown that both chiral and conformal symmetries must be realized in the same way when the symmetries are combined. Since the pion is normally considered as an example of a Goldstone particle, this leads one to view the dilaton, scalar-isoscalar σ, also as a Goldstone particle. With a probable mass of around 700 MeV, and possibly a broad width of ~300–400 MeV, it is not easy to find reliable evidence of the underlying conformal symmetry.

In this paper we wish to test these ideas in the process $\pi N \to \pi\pi N$. This is one of the few processes which have not as yet been looked at with a view to finding out more about the breaking of conformal and chiral symmetry. It is a process which appears to be able to answer in an independent way a different problem which arises in chiral symmetry. It is well known that there exists an arbitrariness in choosing a suitable chiral-symmetry-breaking mechanism. The single-pion production process has been used successfully in eliminating some of the existing ambiguity.[3] In these studies, however, contributions which would arise from the existence of a scalar-isoscalar resonance, such as the σ, have been omitted, and it is important to be sure that the results would not be affected too much by their inclusions. We shall show that there is a substantial change in a_0 and a_2, although these scattering lengths still remain small in magnitude, when such terms are introduced. It has, of course, been known for some time that the current-algebra scattering lengths would change[1,4] from the Weinberg solutions if there were a broad $\pi\pi$ resonance below 500 MeV. The present work is to some extent complementary to that of Carbone *et al.*[4] from the point of view of chiral symmetry