36B, 615 (1971).
[4]See references given in S. Weinberg, in *Proceedings of the Fourteenth International Conference on High-Energy Physics, Vienna, 1968*, edited by J. Prentki and J. Steinberger (CERN, Geneva, 1968), Sec. 3; F. C. P. Chan, Lett. Nuovo Cimento 3, 108 (1972).
[5]A. Sirlin and M. Weinstein, Phys. Rev. D (to be published).
[6]See, for example, R. Brandt and G. Preparata, in Proceedings of the Summer Institute in Theoretical Physics, Hamburg, DESY, 1971 (unpublished); R. Brandt, in Proceedings of the Varenna Summer School, 1971 (unpublished).
[7]R. Brandt and G. Preparata, Phys. Rev. Letters 25, 1530 (1970); 26, 1605 (1971); Phys. Rev. D (to be published); G. Preparata, University of Rome, Nota Interna No. 299 (unpublished); G. Valenti, Nucl. Phys. B (to be published).
[8]R. Brandt and G. Preparata, Ann. Phys. (N.Y.) 61, 119 (1970).
[9]N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters 22, 336 (1966).

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Corrections to a Theorem on the K_{l3} Decay Form Factors

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It was pointed out in a recent series of papers that for theories possessing an approximate Goldstone symmetry some physical amplitudes are necessarily nonanalytic functions of the symmetry-breaking parameter, ϵ. In particular, this fact implies some corrections to a theorem on the K_{l3} form factors, which was derived under the assumption of an approximate SU(3) × SU(3) symmetry of the Goldstone type. In this paper we show how these effects arise, and prove that

$$\frac{d}{dt}[\langle\pi^0|\partial_\mu V^\mu_{K^-}(0)|K^+\rangle(t)]\Big|_{t=m_\pi^2+m_K^2}$$

is of the form $(\epsilon\ln\epsilon)A+\epsilon B+O(\epsilon^2,\epsilon^2\ln\epsilon)$. Furthermore, we prove that $A\epsilon\ln\epsilon=\frac{1}{2}(f_\pi/f_K-f_K/f_\pi)$ independently of any assumption about the structure of the symmetry-breaking term in the Hamiltonian. However the term ϵB is model-dependent. In the popular $(\overline{3},3)+(3,\overline{3})$ model this term is small as compared to the term $A\epsilon\ln\epsilon$. Also we note that the theorem is given at the unphysical point $t=m_\pi^2+m_K^2$ rather than at $t=0$ as was previously stated.

I. INTRODUCTION

In a recent series of papers[1] it was observed that for any theory possessing an approximate Goldstone symmetry, the approach to the symmetrical limit is a nonanalytic function of the parameter ϵ which sets the scale of symmetry breaking. The essential point in the argument leading to these conclusions is that as this parameter, ϵ, approaches zero so do the masses of the would-be Goldstone particles of the theory. It is then easy to show that if one is careful to include the effects of exchanging two would-be Goldstone bosons when proving low-energy theorems, then the amplitudes to which such exchanges contribute have terms going like $\epsilon\ln\epsilon$ in addition to terms like $\epsilon,\epsilon^2,\ldots,$ etc. Since it has become increasingly apparent that the PCAC (partially conserved axial-vector current) hypothesis can be discussed in terms of an approximate SU(3)×SU(3) Goldstone symmetry of the strong interactions, it is naturally interesting to see if the existence of the nonanalytic behavior seriously modifies results previously obtained ignoring these effects.

In this paper we rediscuss a theorem[2] concerning the form factors measured in the process

$K^+ \to \pi^0 + l^+ + \nu$ and attempt to answer this question. As we shall show, it is possible to derive an identity for the function

$$\tfrac{1}{2}D(t) = \langle \pi^0 | i\partial_\mu V^\mu_{K^-}(0) | K^+\rangle = \tfrac{1}{2}[(m_K{}^2 - m_\pi{}^2)f_+(t) + tf_-(t)] \qquad (1)$$

which allows us to explicitly exhibit these corrections. This identity allows us to show that

(i) $$\frac{d}{dt}D(t)\bigg|_{t=m_K{}^2+m_\pi{}^2} = \frac{1}{2}\left(\frac{f_\pi}{f_K} - \frac{f_K}{f_\pi}\right) + X + O(\epsilon^2, \epsilon^2 \ln\epsilon),$$

(ii) $$\frac{1}{2}\left(\frac{f_\pi}{f_K} - \frac{f_K}{f_\pi}\right)$$

is the entire term of order $\epsilon \ln\epsilon$, and

(iii) X is a model-dependent term of order ϵ, which can be determined by assuming the symmetry-breaking Hamiltonian belongs to a particular representation of SU(3)×SU(3).

Three interesting aspects of these results are worth emphasizing. First the fact that $(f_\pi/f_K - f_K/f_\pi)$ is of order $\epsilon \ln\epsilon$, and not ϵ, as one would expect from SU(3) argument alone, means it is quite insensitive to the actual value of ϵ. Second, any reasonable model for symmetry breaking gives values for X which are small. Last, the arguments leading to these results imply that one should not fit the K_{l3} data by a linear function of t over the entire Dalitz plot.

In Sec. II, we derive an identity which yields an expression for $(d/dt)D(t)|_{t=m_\pi{}^2+m_K{}^2}$ correct to order ϵ. Before concluding this section, we want to review briefly the basic physical ideas underlying our calculation. Our starting point is to assume we can write the strong-interaction Hamiltonian as $H = H_0 + \epsilon H'$, where H_0 is SU(3)×SU(3)-invariant, H' breaks *both* SU(3) and SU(3)×SU(3), and ϵ is small enough so that a discussion of small-ϵ behavior to lowest nonvanishing order in ϵ makes sense. The symmetric limit ($\epsilon = 0$) is to be understood to be one in which the octet of pseudoscalar mesons plays the role of eight massless Goldstone bosons, whose presence allows the axial-vector currents to be conserved without requiring the existence of SU(3)×SU(3) multiplets of particles. Of course, the vacuum of the symmetrical theory is assumed to be annihilated by the vector charges, requiring the existence of SU(3) multiplets of particles.

Some useful formulas follow from the general expression for the matrix element of an axial-vector current between a single pseudoscalar-meson state and vacuum,

$$\langle M_a(q) | A^\mu_b(0) | 0\rangle = -iq^\mu \delta_{ab}/2f_a(\epsilon), \qquad (2)$$

where, in the limit $\epsilon \to 0$, $f_a{}^{-1}(\epsilon) \to f_0{}^{-1} \neq 0$,[3]

$$\langle M_a(q) | \partial_\mu A^\mu_b(0) | 0\rangle = \frac{m_a{}^2(\epsilon)}{2f_a(\epsilon)}\delta_{ab}. \qquad (3)$$

Since in the limit $\epsilon \to 0$ the axial-vector current is conserved, our previous assumption implies $m_a{}^2(0) = 0$.

II. A BASIC IDENTITY

In this section we shall proceed to derive an identity for the function

$$D_{abc}(p\cdot p') = \langle M_a(p) | -i\partial_\nu V^\nu_b(0) | M_c(p')\rangle = if_{abc}D(t) \qquad (4)$$

which can be used to discuss the process $K^- \to \pi^0 + l + \nu$. Note that for this process $D_{abc}(t)$ becomes

$$D_{\pi^0K^+K^-}(p\cdot p') = \tfrac{1}{2}[(m_K{}^2 - m_\pi{}^2)f_+(p\cdot p') + (m_\pi{}^2 + m_K{}^2 - 2p\cdot p')f_-(p\cdot p')], \qquad (5)$$

where we have used the variable $(p\cdot p')$ instead of

$$t = (p - p'^2) = m_K{}^2 + m_\pi{}^2 - 2p\cdot p'$$

for reasons which will become apparent as we proceed. Note that to lowest order in SU(3) breaking,

$$f_+(t=0) = f_+(p\cdot p' = \tfrac{1}{2}(m_K{}^2 + m_\pi{}^2)) = 1\,.$$

We begin by studying the function

$$\langle 0 | T(\partial_\mu A^\mu_a(p)\partial_\sigma V^\sigma_b(0)\partial_\nu A^\nu_c(-p')) | 0\rangle(p^2, p'^2, p\cdot p') \equiv \int d^4x d^4y\, e^{-ip\cdot x} e^{-ip'\cdot y} \langle 0 | T(\partial_\mu A^\mu_a(x)\partial_\sigma V^\sigma_b(0)\partial_\nu A^\nu_c(y)) | 0\rangle\,. \qquad (6)$$

Straightforward application of the familiar techniques of current algebra allows us to pull the derivatives through the T-ordering instruction to yield

$$\begin{aligned}&\langle 0 | T(\partial A_a(p)\partial V_b(0)\partial A_c(-p')) | 0\rangle(p^2, p'^2, p\cdot p')\\
&= i(p-p')_\sigma p_\mu p'_\nu \langle 0 | T(A^\mu_a(p)V^\sigma_b(0)A^\nu_c(-p')) | 0\rangle(p^2, p'^2, p\cdot p') - ip_\mu p'_\nu f_{bal}\langle 0 | T(A^\mu_l(p')A^\nu_c(-p')) | 0\rangle(p'^2; p\cdot p')\\
&\quad - ip_\mu p'_\nu f_{bcl}\langle 0 | T(A^\mu_l(-p)A^\nu_a(p)) | 0\rangle(p^2; p\cdot p') - i\langle 0 | T(\partial_\sigma V^\sigma_b(0)\,\Sigma_{ac}(p-p')) | 0\rangle((p^2 + p'^2 - 2p\cdot p'))\end{aligned}$$

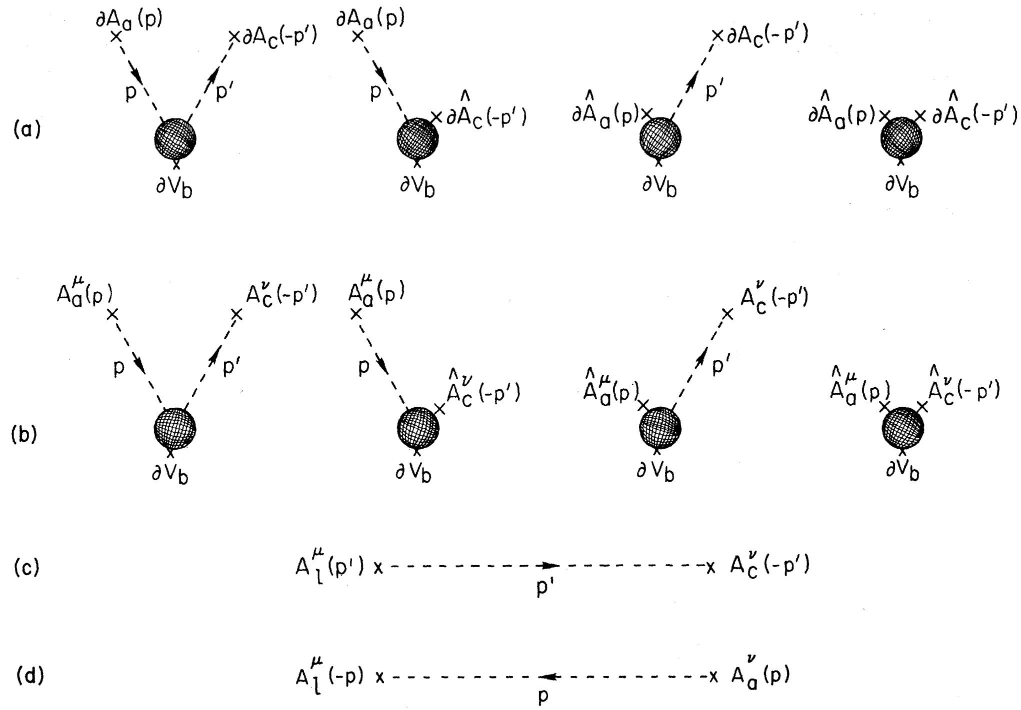

FIG. 1. Diagrams for the K_{l3} Ward identity.

$$+\tfrac{1}{2}(p+p')_\mu f_{acl}\langle 0|T(V_l^\mu(p-p')\partial_\sigma V_b^\sigma(0))|0\rangle(p^2,p'^2,p\cdot p')-\langle 0|T(\partial_\mu A_a^\mu(p)P_{cb}(0))|0\rangle$$
$$-\langle 0|T(\partial_\nu A_c^\nu(-p')P_{ab}(0))|0\rangle+\tfrac{1}{2}[\langle 0|W_{cab}(0)|0\rangle+\langle 0|W_{acb}(0)|0\rangle], \tag{7}$$

where we have adopted the obvious definitions

$$-i\Sigma_{ac}(p-p')\equiv\int d^4x d^4y\, e^{ip\cdot x}e^{-ip'\cdot y}\delta(x_0-y_0)\tfrac{1}{2}\{[A_c^0(x),\partial_\lambda A_a^\lambda(y)]+[A_a^0(x),\partial_\lambda A_c^\lambda(y)]\}, \tag{8}$$

$$-iP_{cb}(0)\equiv\int d^4x\, e^{-ip'\cdot x}\delta(x_0)[A_c^0(x),\partial_\sigma V_b^\sigma(0)], \tag{9}$$

$$W_{acb}\equiv\int d^4x d^4y\, e^{ip\cdot x}e^{-ip'\cdot y}\delta(x_0)\delta(y_0)[A_a^0(x),[A_c^0(y),\partial_\sigma V_b^\sigma(0)]]. \tag{10}$$

We have explicitly indicated the dependence on p^2, p'^2, and $p\cdot p'$ for each term in Eq. (7), and these assignments depend only on the locality of the equal-time commutators. From these explicit dependence on $p\cdot p'$, we see that the last three terms in Eq. (7) do not depend on variable $p\cdot p'$. Hence we can drop them in calculating the derivative with respect to $(p\cdot p')$. The next step in manipulating Eq. (7) into a useful form is to isolate the terms on both sides of Eq. (7) which have poles at $p^2=m_a^2$ and/or $p'^2=m_c^2$. Figure 1(a) shows the diagrams which contribute to the left-hand side of Eq. (7) and Figs. 1(b), 1(c), and 1(d) show the diagrams contributing to the three terms on the right-hand side of Eq. (7). We then multiply both sides of Eq. (7) by $(p^2-m_a^2)(p'^2-m_c^2)$ and differentiate with respect to $(p\cdot p')$ with p^2, p'^2 fixed, to get to the final form of the identity which we need in order to discuss K_{l3} decay, namely

$$\frac{d}{d(p\cdot p')}\bigg[\frac{m_a^2m_c^2}{(2f_a)(2f_c)}D_{abc}(p\cdot p')+\frac{m_a^2(p'^2-m_c^2)}{2f_a}\langle M_a|T(\partial V_b(0)\partial\hat{A}_c(-p'))|0\rangle(p'^2;p\cdot p')$$
$$+\frac{m_c^2}{2f_c}(p^2-m_a^2)\langle 0|T(\partial\hat{A}_a(p)\partial V_b(0))|M_c\rangle(p^2;p\cdot p')$$
$$+(p^2-m_a^2)(p'^2-m_c^2)\langle 0|T(\partial\hat{A}_a(p)\partial V_b(0)\partial\hat{A}_c(-p'))|0\rangle(p^2,p'^2,p\cdot p')\bigg]$$

$$= \frac{d}{d(p\cdot p')}\Bigg\{\frac{p^2p'^2}{(2f_a)(2f_c)}D_{abc}(p\cdot p') + \frac{ip^2}{(2f_a)}(p'^2-m_c^2)(p'-p)_\sigma p'_\nu\langle M_a|T(V_b^\sigma(0)\hat{A}_c^\nu(-p'))|0\rangle(p'^2;p\cdot p')$$
$$-\frac{ip'^2}{2f_c}(p^2-m_a^2)(p'-p)_\sigma p_\mu\langle 0|T(V_b^\sigma(0)\hat{A}_a^\mu(p))|M_c\rangle(p^2;p\cdot p')$$
$$-i(p^2-m_a^2)(p'^2-m_c^2)(p'-p)_\sigma p_\mu p'_\nu\langle 0|T(\hat{A}_a^\mu(p)V_b^\sigma(0)\hat{A}_c^\nu(-p'))|0\rangle(p^2,p'^2,p\cdot p')$$
$$-\frac{p'^2}{(2f_c)^2}(p^2-m_a^2)f_{bac}(p\cdot p')+(p^2-m_a^2)(p'^2-m_c^2)\left[\frac{f_{bac}(p\cdot p')}{(2f_c)^2}-p^2(p\cdot p')O_1(p^2;p\cdot p')\right]$$
$$-\frac{p^2}{(2f_a)^2}(p'^2-m_c^2)f_{bac}(p\cdot p')+(p^2-m_a^2)(p'^2-m_c^2)\left[\frac{f_{bca}(p\cdot p')}{(2f_a)^2}-p'^2(p\cdot p')O_2(p'^2;p\cdot p')\right]$$
$$+\tfrac{1}{2}i(p^2-m_a^2)(p'^2-m_c^2)[\langle 0|T(\partial_\sigma V_b^\sigma(0)\Sigma_{ac}(p-p'))|0\rangle((p-p')^2)]$$
$$-\tfrac{1}{2}if_{acb}(p^2-m_a^2)(p'^2-m_c^2)(p'^2-p^2)V_2((p-p')^2)]\Bigg\}, \qquad (11)$$

where the caret denotes the fact that meson poles have been removed.

We have used the fact that[4]

$$-ip_\mu p'_\nu f_{bal}\langle 0|T(\hat{A}_l^\mu(p')\hat{A}_c^\nu(-p'))|0\rangle(p'^2;p\cdot p') = -\frac{1}{(2f_c)^2}(p\cdot p')f_{bca}+p'^2(p\cdot p')O_2(p'^2;p\cdot p') \qquad (12)$$

and V_2 is defined by

$$f_{acl}\langle 0|T(V_l^\mu(p-p')\partial_\sigma V_b^\sigma(0))|0\rangle = f_{acb}(p-p')_\mu V_2((p-p')^2). \qquad (13)$$

If we choose $p'^2=0$, $p^2=0$, $p\cdot p'=0$ [this corresponds to letting $p_\mu=0$ on the right-hand side of Eq. (11)], we obtain

$$\frac{d}{d(p\cdot p')}[D_{abc}(p\cdot p')-(2f_c)\langle M_a|T(\partial V_b(0)\partial\hat{A}_c(-p'))|0\rangle(0;p\cdot p')-(2f_a)\langle 0|T(\partial\hat{A}_a(p)\partial V_b(0))|M_c\rangle(0;p\cdot p')$$
$$+(2f_a)(2f_c)\langle 0|T(\partial\hat{A}_a(p)\partial V_b(0)\partial\hat{A}_c(-p'))|0\rangle(0,0,p\cdot p')]\Big|_{(p\cdot p')=0}$$
$$=\frac{d}{d(p\cdot p')}\Bigg[-i(2f_a)(2f_c)(p'-p)_\sigma p_\mu p'_\nu\langle 0|T(\hat{A}_a^\mu(p)V_b^\sigma(0)\hat{A}_c^\nu(-p'))|0\rangle(0,0,p\cdot p')$$
$$+f_{abc}\left(\frac{f_c}{f_a}-\frac{f_a}{f_c}\right)(p\cdot p')+\tfrac{1}{2}i(2f_a)(2f_c)\langle 0|T(\partial V_b^\sigma(0)\Sigma_{ac}(p-p'))|0\rangle((p\cdot p'))\Bigg]\Bigg|_{p\cdot p'=0}. \qquad (14)$$

Counting explicit powers of ϵ we find that the first term on the left-hand side of Eq. (14) is of order ϵ, the next two are of order ϵ^2, and the last is of order ϵ^3. Since it is our purpose to prove a theorem correct to order ϵ, if the formal power counting was sufficient the only term we would have to keep on the left-hand side of Eq. (14) would be the first term. However, as we shall see, each of the terms of order ϵ^2 and ϵ^3 contains terms of order $\epsilon^2\ln(p\cdot p'-\epsilon)$ and $\epsilon^3\ln(p\cdot p'-\epsilon)$; clearly differentiating these with respect to $p\cdot p'$ and setting $p\cdot p'$ equal to zero gives us contributions of order ϵ and ϵ^2. Therefore it is necessary for us to examine all ϵ^2 terms for logarithms. On the right-hand side of Eq. (14) we see that the first term is of the form

$$\frac{d}{d(p\cdot p')}[(p^2)^2A(p^2,p'^2;p\cdot p')+p^2p'^2B(p^2,p'^2;p'\cdot p)+p^2(p\cdot p')C(p^2,p'^2;p\cdot p')$$
$$+p'^2(p\cdot p')D(p^2,p'^2;p\cdot p')+(p\cdot p')^2E(p^2,p'^2;p\cdot p')],$$

which clearly vanishes at $p^2=p'^2=p\cdot p'=0$; thus we may restrict our attention to the last three terms.

As we will show in Sec. III, all these terms give contribution of order ϵ, instead of ϵ^2, because of the nonanalyticity in ϵ. It is pointed out in Ref. 1 that the leading term in

$$\frac{d}{d(p\cdot p')}D_{abc}(p\cdot p')\Big|_{p\cdot p'=0}$$

is of order $\epsilon \ln\epsilon$. Therefore by inspecting Eq. (14), we conclude that the term $\frac{1}{2}(f_a/f_c - f_c/f_a)$ is the only term of order $\epsilon \ln\epsilon$; all the other terms contribute to order ϵ. This result can be stated in the form of a theorem.

Theorem. Under the assumptions listed in the Introduction, we can show that

$$\frac{d}{dt} D_{abc}(t)\bigg|_{t=m_a^2+m_c^2} = -\tfrac{1}{2}(if^{abc})\left(\frac{f_c}{f_a} - \frac{f_a}{f_c}\right) + O(\epsilon, \epsilon^2 \ln\epsilon, \epsilon^2, \ldots).$$

In the next section, we will discuss how the terms of order ϵ come about.

III. THE CONTRIBUTIONS OF TWO-MESON INTERMEDIATE STATES

Let us, for convenience, rewrite Eq. (14) as

$$\begin{aligned}\frac{d}{d(p\cdot p')} D_{abc}(p\cdot p')\bigg|_{p'\cdot p=0} = f_{abc}\left(\frac{f_c}{f_a} - \frac{f_a}{f_c}\right) + \frac{d}{d(p\cdot p')}[\tfrac{1}{2}i(2f_a)(2f_c)\langle 0|T(\partial V_b(0)\Sigma_{ac}(p-p'))|0\rangle((p-p')^2) \\ +(2f_c)\langle M_a|T(\partial V_b(0)\partial \hat{A}_c(-p'))|0\rangle(0, p\cdot p') \\ +(2f_a)\langle 0|T(\partial \hat{A}_a(p)\partial V_b(0))|M_c\rangle(0, p\cdot p')]\bigg|_{p\cdot p'=0}. \quad (15)\end{aligned}$$

The easiest way to see how the terms of the form $\epsilon^2 \ln(p\cdot p' - \epsilon)$ come about is to consider the dispersion relation for $\langle 0|T(\partial V_b(0)\Sigma_{ac}(p-p'))|0\rangle$. As usual we write

$$\langle 0|T(\partial V_b(0)\Sigma_{ac}(p-p'))|0\rangle((p-p')^2) = \frac{1}{2\pi i}\int \frac{dt'\rho_{AB}(t')}{(p-p')^2 - t'}, \quad (16)$$

where $\rho_{AB}(t)$ is the spectral function obtained by inserting a complete set of intermediate states in the T product. Now we can write for its derivative with respect to $(p\cdot p')$ as

$$\frac{d}{d(p\cdot p')}[\langle 0|T(\partial V_b(0)\Sigma_{ac}(p-p'))|0\rangle]\bigg|_{p'^2=p^2=p\cdot p'=0} = \frac{1}{\pi i}\int \frac{dt'}{t'^2}\rho_{AB}(t'). \quad (17)$$

The contributions of two pseudoscalar-meson intermediate states are of the form

$$\frac{-i}{16\pi^2}\int_{(m_e+m_d)^2}^{\infty} \frac{dt}{t^2} D_{bde}(t)\Sigma_{ac}^{de}(t)\frac{1}{t}\{[t-(m_e+m_d)^2][t-(m_d-m_e)^2]\}^{1/2},$$

where

$$\Sigma_{ac}^{de}(t) = \langle M_d M_e|\Sigma_{ac}|0\rangle, \quad t = (p-p')^2.$$

As we can see in the limit $m_d^2 = m_e^2 = 0$, the two-body phase space is independent of t and the integral diverges linearly in t. To get the leading divergent behavior, it is sufficient to replace D_{bde} and Σ_{ac}^{de} by their values at $t=0$; we get

$$-\frac{i}{16\pi^2} D_{bde}(0)\Sigma_{ac}^{de}(0)\frac{1}{m_d^2}\left[\frac{1+x}{2(1-x)^2} + \frac{x}{(1-x)^3}\ln x\right],$$

where

$$x = m_e^2/m_d^2.$$

Since $D_{bde}\times\Sigma_{ac}^{de}$ is of order ϵ^2, m_d^2 is of order ϵ, and x is of order unity, we see that the two-meson contribution is of order ϵ, not ϵ^2. The contributions coming from four or more meson intermediate states would give terms of order $\epsilon^2 \ln\epsilon$ or higher.

One can apply the same analysis to all of the terms in Eq. (15). For the particular process $K^+ \to \pi^0 + e^+ + \nu$, we get the contribution of the π-K intermediate state to the derivative

$$\begin{aligned}Y = -\frac{i}{16\pi^2}\frac{m_K^2 - m_\pi^2}{m_K^2} f_+(0)[\tfrac{1}{2}i(2f_K)(2f_\pi)\Sigma_{\pi K}^{\pi K}(0) + 2f_K\langle\pi|\partial A_K|\pi K\rangle + (2f_\pi)\langle\pi K|\partial A_\pi|K\rangle] \\ \times\left[\frac{1+x}{2(1-x)^2} + \frac{x}{(1-x)^3}\ln x\right], \quad (18)\end{aligned}$$

where Y is defined as

$$Y=\frac{d}{dt}D(t)_{\pi KK}\bigg|_{t=m_K{}^2+m_\pi{}^2}-\frac{1}{2}\left(\frac{f_\pi}{f_K}-\frac{f_K}{f_\pi}\right).$$

The quantity in the first square bracket can be related, via the generalization of Weinberg's analysis of $\pi\pi$ scattering[5] (to the order in which we are working), to the part of the π-K scattering amplitude which is independent of t.

In addition to the πK intermediate states, we also have ηK intermediate states contributing to the dispersion integral. Exactly the same analysis goes through and a similar result can be obtained.

In each of these cases [in, for example, the $(3,\overline{3})+(\overline{3},3)$ model of symmetry breaking], we can calculate explicitly $\Sigma_{\pi K}^{\pi K}(0)$, $\langle\pi|\partial A_K|\pi K\rangle$, and $\langle\pi K|\partial A_\pi|K\rangle$ in terms of the symmetry-breaking parameters, which can be expressed in terms of pseudoscalar-meson masses and their decay constants. The result in this model for the πK intermediate states is

$$Y_{K\pi}=\frac{3I(m_\pi,m_K)}{32\pi^2}(m_K{}^2-m_\pi{}^2)(f_\pi{}^2m_K{}^2+f_K{}^2m_\pi{}^2), \tag{19}$$

where

$$I(m_\pi,m_K)=\int_{(m_\pi+m_K)^2}^{\infty}\frac{dt}{t^2}\left(\frac{[t-(m_K+m_\pi)^2][t-(m_K-m_\pi)^2]}{t^2}\right)^{1/2}.$$

Numerically, this is small by one order of magnitude compared to the leading term $\frac{1}{2}(f_\pi/f_K-f_K/f_\pi)$. A similar result can be obtained for the contribution coming from the $\eta\pi$ intermediate state, which is also small.

IV. CONCLUSION

In the preceding sections we discussed how to prove a theorem about $(d/dt)D_{\pi KK}(t)$ at the point $t=m_\pi{}^2+m_K{}^2$, which is outside the physical region for the decay $K\to\pi+l+\nu$. It is important to notice that our arguments can only be expected to yield the correct slope for $D_{\pi KK}(t)$ at $t=0$ if one assumes $D_{\pi KK}(t)$ is essentially a linear function of t over the entire Dalitz plot. If, however, our arguments about the contribution of two-pseudoscalar-meson states are correct, the most illuminating way to discuss experimental data is to fit the function $D_{\pi KK}(t)$ to the sum of a term linear in t plus a term of the form $\ln(-t/t_0)$ coming from the πK intermediate state. Certainly, if there is no deviation from a linear dependence in t over the entire region of the Dalitz plot for $K\to\pi+l+\nu$ and if the slope of $D_{\pi KK}(t)$ at $t=0$ is inconsistent with our formula, then one would be forced to reconsider either the whole idea of SU(3)×SU(3) as an approximate symmetry of the strong interactions, or the implicit assumption that the weak current is by Gell-Mann–Cabibbo theory related to the almost conserved SU(3)×SU(3) currents.

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[1]L. F. Li and H. Pagels, Phys. Rev. Letters 26, 1204 (1971); 27, 1089 (1971).

[2]R. Dashen and M. Weinstein, Phys. Rev. Letters 22, 1337 (1969).

[3]With this definition $\langle\pi^+(q)|A_\mu^{\pi^+}(0)|0\rangle=-iq^\mu\sqrt{2}/2f_\pi$ and so $1/2f_\pi\sim(0.96/\sqrt{2})m_\pi\approx0.7m_\pi$ as determined from $\pi_{\mu2}$ decay.

[4]See Ref. 2 for the details of the proof.

[5]S. Weinberg, Phys. Rev. Letters 17, 616 (1966).