

Resonance Decays Involving Two Partial Waves

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Resonance decays involving two partial waves, such as $B \rightarrow \omega\pi$, can test for basic violations of $SU(6)_W$. A formalism is presented for carrying out these tests using available data on $\pi N \rightarrow \pi\Delta(1236)$ and $K^-p \rightarrow \pi Y_1^*(1385)$.

When a meson or baryon resonance decays into two particles, at least one of which has sufficiently high intrinsic spin, these two particles can appear in more than one partial wave. The helicity distribution of the final particles will then depend on the relative phase and magnitude of the amplitudes in these different partial waves. The most familiar case is the decay $B \rightarrow \omega\pi$, in which the helicity distribution of the final ω indicates that both S and D waves are present.¹

The ratio of such amplitudes is of interest for several reasons.

(1) A definite value for it is predicted by $SU(6)_W \times O(2)_{L_z}$.^{2,3} This symmetry is found to be violated in a way not explicable by centrifugal-barrier effects alone.¹

(2) It is the only free parameter by which $SU(6)_W \times O(2)_{L_z}$ is modified in certain recent pictures of hadron decays.^{1,4-7}

(3) In relativistic quark models it is parametrized by "quark recoil effects." These effects have been shown essential to a correct description of hadronic decays and photoproduction.^{6,7} Models not incorporating such effects^{8,9} fix this ratio at the value predicted by $SU(6)_W \times O(2)_{L_z}$.

(4) It is now experimentally accessible in a number of cases involving baryon decays, particularly into $\Delta(1236)\pi$ and $Y_1^*(1385)\pi$.

In Ref. 1 it was found that data on $B \rightarrow \omega\pi$ (and to some extent on $A_1 \rightarrow \rho\pi$) suggested a sign for the two partial waves *opposite* to that predicted by

$SU(6)_W$.¹⁰ In the present work we extend similar considerations to baryon decays.

We shall show how baryon decays tell the sign of the amplitude ratio, and what progress has been made to date. The subject is of present experimental interest because of recent studies of $\pi N \rightarrow \pi\pi N$ (Ref. 11) and $K^-N \rightarrow \pi\pi\Lambda$.^{12,13} Details of more theoretical interest will be presented elsewhere.¹⁴

Consider decays involving a pion or kaon and a "ground state" baryon:

$$A(J^A, L, S^A; \lambda) \rightarrow B(J^B, L=0, S^B=J^B; \lambda) + C(J^C=0). \quad (1)$$

Here L and S refer to quark orbital and spin angular momenta.

Let the z axis be the direction of B , and λ the projection of J^A along this axis. Assign A , B , and C to multiplets of *static* $SU(6) \times O(3)$. Then take the decay to proceed via the creation of a $q\bar{q}$ pair, one of whose members emerges in each final hadron. The pair is assumed to have the quantum numbers of the vacuum: $SU(3)$ singlet, $J^{PC} = 0^{++}$ (i.e., 3P_0).^{1,3-5} This language, motivated by duality graphs,^{15,4} may be more valid than the quark model on which it is based.¹⁶ It was first used by Micu,⁵ and is equivalent to a particular limit of the model in Ref. 1.

The decay helicity amplitudes are then written in our model as

$$M^{(\lambda)}(A \rightarrow BC) = \sum_i \left(\begin{array}{c} A \\ \alpha, a \end{array} \middle| \begin{array}{c} B \\ \beta, b \end{array} \begin{array}{c} 35 \\ 8, 3 \end{array} \right)_i \left(\begin{array}{c} \alpha \\ A \end{array} \middle| \begin{array}{c} \beta \\ B \end{array} \begin{array}{c} 8 \\ C \end{array} \right)_i X_\lambda(A \rightarrow BC), \quad (2)$$

where

$$X_\lambda(A \rightarrow BC) = \sum_{L_z} (J^B \lambda 1 - L_z | S^A \lambda - L_z) (S^A \lambda - L_z L L_z | J \lambda) a_{L_z}^{(L)} \quad (3)$$

In Eq. (2) the first term is an SU(3) scalar factor,¹⁷ with \underline{A} , \underline{B} , and $\underline{35}$ labeling SU(6) representations and (α, a) , (β, b) , and $(8, 3)$ labeling SU(3)×SU(2) representations. The second term is an isoscalar factor¹⁸ with A , B , and C labeling specific isomultiplets. The sum over i corresponds to d and f coupling when α and β are octets. For decays into specific charge states $M^{(\lambda)}$ is to be multiplied by an appropriate Clebsch-Gordan coefficient.

In Eq. (2) and 3P_0 , the "spurion" has been combined with the pseudoscalar meson to form an effective $(8, 3)$ member of the $\underline{35}$ of SU(6). For baryon decays the formalism is thus manifestly equivalent to SU(6)_w×O(2)_{L_z} when one takes $a_L^{(\pm 1)} = 0$. This is so for meson decays as well.^{3,14}

The first Clebsch-Gordan coefficient in Eq. (3) describes quark spin conservation while the second describes the coupling of quark spin and quark L to $J \equiv J^A$. The reduced matrix elements $a_L^{(L, z)}$ are assumed to depend only on L_z and on the specific SU(6)×O(3) multiplets involved in the decay $\underline{A}(L) \rightarrow \underline{BC}$.¹⁹ The rule derived in Ref. 1 relating the decays $A_1 \rightarrow \rho\pi$ and $B \rightarrow \omega\pi$, namely, $2(X_1/X_0)_A = (X_0/X_1)_B + 1$, then follows directly from Eq. (3).

In our normalization the partial width is

$$\Gamma(A \rightarrow BC) = \frac{1}{2J^A + 1} \frac{p}{M_A^2} \sum_{\lambda} |M^{(\lambda)}(A \rightarrow BC)|^2, \quad (4)$$

where p is the magnitude of the final c.m. 3-momentum.

Symmetry breaking due to centrifugal-barrier terms is best described in terms of reduced matrix elements labeled by the partial wave $l = L \pm 1$ of the final state. We shall define amplitudes $a_L^{(l)}$ normalized so that when $a_L^{(\pm 1)} = 0$, $a_L^{(L+1)} = a_L^{(L-1)} = a_L^{(0)}$. Then, since L and the spurion angular momentum make up the final orbital angular momentum $l = L \pm 1$, we have

$$a_L^{(l)} = (L \ 0 \ 1 \ 0 | l \ 0)^{-1} \sum_{L_z} (L \ L_z \ 1 \ -L_z | l \ 0) a_L^{(L, z)}. \quad (5)$$

The inverse of (5) allows one to write Eq. (3) as

$$X_{\lambda} = \sum_{l=L \pm 1} (2l+1)^{1/2} (J^B \ \lambda \ l \ 0 | J \ \lambda) \gamma_l a_L^{(l)}, \quad (6)$$

where

$$\gamma_l \equiv (2S+1)^{1/2} (L \ 0 \ 1 \ 0 | l \ 0) W(J^B \ 1 \ J \ L; S^A \ l). \quad (7)$$

The normalization in Eq. (6) is appropriate since in calculating partial widths we use the quantity

$$\frac{1}{2J+1} \sum_{\lambda} |X_{\lambda}|^2 = \sum_{l=L \pm 1} \gamma_l^2 |a_L^{(l)}|^2. \quad (8)$$

In Eq. (7) $W(J^B \ 1 \ J \ L; S^A \ l)$ is a Racah coefficient as defined, for example, in Edmonds.²⁰

The coefficients $a_L^{(l)}$ are taken to be universal for decays within a given multiplet, aside from a centrifugal-barrier term. For this term we take

TABLE I. Comparison of predicted partial widths of $(70, L=1)$ baryons with experiment. Data from Refs. 18 and 26.

J^P	Process	Γ (MeV)	$M^2 \Gamma / p$ (predicted)	$ a^{(l)} $ (GeV)
$\frac{5}{2}^-$	$N(1670) \rightarrow N\pi$	56	$D^2/360$	$ \bar{D} = 8.0$
	$\rightarrow \Delta\pi$	35	$7D^2/180$	5.1
	$\Sigma(1765) \rightarrow N\bar{K}$	53	$D^2/135$	6.8
	$\rightarrow \Lambda\pi$	18	$D^2/360$	5.8
	$\rightarrow Y_1^*\pi$	16	$7D^2/1080$	12.3
	$\Lambda(1830) \rightarrow N\bar{K}$	11	0	...
	$\rightarrow \Sigma\pi$	34	$D^2/120$	4.8
	$\frac{3}{2}^-$	$\Delta(1670) \rightarrow N\pi$	36	$D^2/432$
$N(1520) \rightarrow N\pi$		64	$D^2/54$ ($S_q = \frac{1}{2}$)	5.0
$\rightarrow N\eta$		~ 0.7	$D^2/216$	17.1
$N(?) \rightarrow N\pi$		~ 0	$D^2/2160$ ($S_q = \frac{3}{2}$)	...
$\Lambda(1520) \rightarrow N\bar{K}$		7.4	$\left\{ \begin{array}{l} \text{Sum } (S_q = \frac{1}{2}): \\ D^2/24 \end{array} \right\}$	6.2
$\Lambda(1690) \rightarrow N\bar{K}$		17		
$\Lambda(1520) \rightarrow \Sigma\pi$		6.6	$\left\{ \begin{array}{l} \text{Sum } (S_q = \frac{1}{2}): \\ 5D^2/144 \end{array} \right\}$	5.7
$\Lambda(1690) \rightarrow \Sigma\pi$		22		
$N(1520) \rightarrow \Delta\pi$		64	$(S^2 + D^2)/54$	$(\bar{S}^2 + 0.04\bar{D}^2)^{1/2} = 6.0$
$\frac{1}{2}^-$		$N(1535) \rightarrow N\pi$	37	Sum: $5S^2/216$
	$N(1700) \rightarrow N\pi$	160		
	$N(1535) \rightarrow N\eta$	58	Sum: $S^2/108$	9.0
	$N(1700) \rightarrow N\eta$	~ 0		
	$\Delta(1630) \rightarrow N\pi$	46	$S^2/432$	10.0

TABLE II. Approximate mixing scheme for $\Lambda(\frac{1}{2}^-)$.

Process	Γ (MeV)	$M^2\Gamma/p$ (GeV ²)	$M^2\Gamma/p$ (predicted)	$ \tilde{S} $ (GeV)
$\Lambda(1405) \rightarrow \Sigma\pi$	40	0.56	$S^2/144$	8.9
$\rightarrow N\bar{K}$...	4.4 ^a	$S^2/24$	10.3
$\rightarrow \Lambda\eta$...	?	$S^2/108$...
$\Lambda(1670) \rightarrow \Sigma\pi$	12	0.09	set ≈ 0	...
$\rightarrow N\bar{K}$	5	0.04	set ≈ 0	...
$\rightarrow \Lambda\eta$	9	0.38	$S^2/144$	7.4
$\Lambda(?) \rightarrow \Sigma\pi$?	?	$S^2/24$?
$\rightarrow N\bar{K}$	small	small	set ≈ 0	...
$\rightarrow \Lambda\eta$?	?	0	...

^aC. Weil, Phys. Rev. **161**, 1682 (1967).

a zero-radius form,^{1,5}

$$a_L^{(l)} = \tilde{a}_L^{(l)}(p|p_0)^l, \quad (9)$$

and define a scale for $\tilde{a}_L^{(l)}$ by setting $p_0 = 0.5$ GeV as in Ref. 1.

Preliminary fits to the decays of (35, $L=1$) mesons and (70, $L=1$) and (56, $L=2$) baryons yield reasonably self-consistent values of $|\tilde{a}_L^{(l)}|$ in all three cases. (See Tables I-III.) These cases share the property that

$$|\tilde{a}_L^{(L+1)}| \approx |\tilde{a}_L^{(L-1)}|. \quad (10)$$

Since a large number of the decays considered have $p \approx 0.5$ GeV, this indicates (modulo symmetry breaking due to mass differences) that $|a_L^{(L+1)}| \approx |a_L^{(L-1)}|$. The $SU(6)_W \times O(2)_{L_z}$ symmetric case is $a_L^{(L+1)} = a_L^{(L-1)}$. On the other hand, for $L=1$ meson

decays one finds $\tilde{a}_L^{(L+1)} \approx -\tilde{a}_L^{(L-1)}$, as a result of the dominantly transverse nature of the $B \rightarrow \omega\pi$ decay.¹

It is clearly of interest to determine when such a sign change occurs, and hence when $SU(6)_W \times O(2)_{L_z}$ is breaking down in a manner more fundamental than would follow from barrier effects alone. We list some examples whereby this may be learned from baryon decays.

1. *Reaction $\pi N \rightarrow \pi\Delta$.* Partial-wave analyses of $\pi^-p \rightarrow \pi^+\Delta^-$ in the range $E_{c.m.} = 1.5-1.7$ GeV can extract the relative signs of various amplitudes for decay into a ground-state baryon and meson. Let us call

$$S \equiv a_{L=1}^{(l=0)}(70 \rightarrow 56 \ 35), \quad D \equiv a_{L=1}^{(l=2)}(70 \rightarrow 56 \ 35), \quad (11)$$

$$P \equiv a_{L=2}^{(l=1)}(56 \rightarrow 56 \ 35), \quad F \equiv a_{L=2}^{(l=3)}(56 \rightarrow 56 \ 35).$$

The dominant $\Delta\pi$ resonances in this region are the $D_{13}(1520)$, $D_{15}(1670)$, and $F_{15}(1690)$. Their decays are given in the present model by

$$M^{(\lambda=1/2)}(D_{13} \rightarrow N\pi) = D/3\sqrt{3}, \quad (12a)$$

$$M^{(\lambda=1/2)}(D_{13} \rightarrow \Delta\pi) = -(S+D)/3\sqrt{6}, \quad (12b)$$

$$M^{(\lambda=3/2)}(D_{13} \rightarrow \Delta\pi) = -(S-D)/3\sqrt{6}, \quad (12c)$$

$$M^{(\lambda=1/2)}(D_{15} \rightarrow N\pi) = -D/2\sqrt{30}, \quad (13a)$$

$$M^{(\lambda=1/2)}(D_{15} \rightarrow \Delta\pi) = D/2\sqrt{15}, \quad (13b)$$

$$M^{(\lambda=3/2)}(D_{15} \rightarrow \Delta\pi) = D/\sqrt{10}, \quad (13c)$$

$$M^{(\lambda=1/2)}(F_{15} \rightarrow N\pi) = -F/3\sqrt{2}, \quad (14a)$$

TABLE III. (56, $L=2$) baryons. Data from Refs. 18, 26.

J^P	Process	Γ (MeV)	$M^2\Gamma/p$ (predicted)	$ \tilde{a}^{(l)} $ (GeV)
$\frac{1}{2}^+$	$\Delta(1950) \rightarrow N\pi$	80	$4F^2/525$	$ \tilde{F} = 2.3$
	$\rightarrow \Sigma K$	4	$4F^2/525$	2.7
	$\rightarrow \Delta\pi$	115	$F^2/70$	4.9
	$\Sigma(2030) \rightarrow N\bar{K}$	25	$4F^2/1575$	2.8
	$\rightarrow \Lambda\pi$	35	$2F^2/525$	2.7
$\frac{3}{2}^+$	$\rightarrow \Sigma\pi$	5	$4F^2/1575$	1.5
	$N(1690) \rightarrow N\pi$	91	$F^2/54$	3.3
	$\Lambda(1815) \rightarrow \Sigma\pi$	9	$2F^2/225$	2.5
	$\rightarrow N\bar{K}$	51	$F^2/75$	3.9
	$\Sigma(1915) \rightarrow N\bar{K}$	8	$F^2/2025$	5.2
	$\Delta(1890) \rightarrow N\pi$	45	$8F^2/4725$	4.2
	$N(1690) \rightarrow \Delta\pi$	18 ^a	$(16/3375)(2F^2+3P^2)$	$(\tilde{P}^2+0.20\tilde{F}^2)^{1/2} = 4.1$
$\Lambda(1815) \rightarrow Y_1^+\pi$	14	$(4/1125)(2F^2+3P^2)$	$(\tilde{P}^2+0.17\tilde{F}^2)^{1/2} = 4.8$	
$\frac{3}{2}^+$	$N(1860) \rightarrow N\pi$	75	$P^2/54$	$ \tilde{P} = 3.3$
	$\rightarrow \Lambda K$	18	$P^2/150$	5.3
$\frac{1}{2}^+$	$\Delta(1910) \rightarrow N\pi$	66	$8P^2/675$	3.7

^a See Ref. 21.

$$M^{(\lambda=1/2)}(F_{15} \rightarrow \Delta\pi) = -\frac{4}{75}(3P + 2F), \quad (14b)$$

$$M^{(\lambda=3/2)}(F_{15} \rightarrow \Delta\pi) = -\frac{4}{25}\left(\frac{2}{3}\right)^{1/2}(P - F), \quad (14c)$$

where we have taken the D_{13} to belong to an (8, 2) of (70, $L=1$), the D_{15} to be (8, 4) of (70, $L=1$), and the F_{15} to be (8, 2) of (56, $L=2$).⁶ Note that only $l=2$ is predicted for $D_{15} \rightarrow \Delta\pi$; the model forbids $l=4$.

To a first approximation we neglect D in comparison with S in Eqs. (12b) and (12c), and F in comparison with P in Eqs. (13b) and (13c), because of the large centrifugal barrier. Then one may write amplitudes due to these resonances as, for example,

$$M_{1/2, 1/2}(\pi N \rightarrow \pi\Delta) = [-D^2 B_D^{5/2}(s) + FP B_F^{5/2}(s)] d_{1/2, 1/2}^{5/2}(\theta) - SD B_D^{3/2}(s) d_{1/2, 1/2}^{3/2}(\theta), \quad (15)$$

where $B_D^{5/2}(s)$, $B_F^{5/2}(s)$, and $B_D^{3/2}(s)$ are appropriately normalized Breit-Wigner factors with positive imaginary parts.

Equation (15) shows that for $FP < 0$, the D_{15} and F_{15} interfere constructively in $M_{1/2, 1/2}$, while for $FP > 0$ they interfere destructively. Published phase-shift solutions allow for both cases: $FP > 0$ ("solution A") and $FP < 0$ ("solution B"). [Solution A is that of $SU(6)_w \times O(2)_{L_z}$]. Recent modifications of the analysis favor solution A.²¹

The relative sign of D_{13} and F_{15} or D_{15} contributions may also be compared with experiment. An early fit²² contained such information but has been superseded¹¹; the analysis is still in progress.

2. *Reaction $K^-N \rightarrow \pi Y_1^*$.* One can obtain the relative sign of the contributions of $\Sigma(1765)$ ($\frac{5}{2}^-$) and $\Lambda(1815)$ ($\frac{5}{2}^+$) by similar means.²³ In $M_{1/2, 1/2}$, we predict that for $FP > 0$, $\Sigma(1765)$ and $\Lambda(1815)$ will interfere constructively in $K^-p \rightarrow \pi^- Y_1^{*+}$ and destructively in $K^-p \rightarrow \pi^+ Y_1^{*-}$, and vice versa for $FP < 0$. Experimentally the situation is not resolved as yet. An older fit¹² supports $FP > 0$, while a more recent one favors $FP < 0$.¹³ This more recent work has the disturbing feature that the branching ratio for $\Lambda(1815) \rightarrow \pi Y_1^*$ is far too low to be consistent via $SU(3)$ with even the lowest possible estimates¹¹ of $N(1690) \rightarrow \pi\Delta$.

3. *Helicity structure in decays.* Equations (12b), (12c), (14b), and (14c) lead to the ratios

$$\frac{M^{(\lambda=3/2)}(D_{13} \rightarrow \Delta\pi)}{M^{(\lambda=1/2)}(D_{13} \rightarrow \Delta\pi)} = \frac{1 - D/S}{1 + D/S} \quad (16)$$

and

$$\frac{M^{(\lambda=3/2)}(F_{15} \rightarrow \Delta\pi)}{M^{(\lambda=1/2)}(F_{15} \rightarrow \Delta\pi)} = \left(\frac{2}{3}\right)^{1/2} \frac{1 - F/P}{1 + 2F/3P}. \quad (17)$$

These ratios are quite sensitive to the sign of D/S and F/P , even where these quantities are small.

The values of \bar{D} and S obtained in our fit to total decay rates of (70, $L=1$) baryons are roughly equal, implying $|D/S| \approx \frac{1}{5}$ when the centrifugal barrier is taken into account. In $SU(6)_w$, one would expect $D/S = +1$. We thus refer to the solution $D/S \approx +\frac{1}{5}$ as the "SU(6)-like" solution. It predicts a value of $\frac{2}{3}$ for the ratio in Eq. (16). On the other hand, the "anti-SU(6)" solution $D/S \approx -\frac{1}{5}$ [so called since the sign of D/S is opposite to that predicted by $SU(6)_w$] predicts

$$\frac{M^{(\lambda=3/2)}(D_{13} \rightarrow \Delta\pi)}{M^{(\lambda=1/2)}(D_{13} \rightarrow \Delta\pi)} = \frac{3}{2}.$$

For the decay $F_{15} \rightarrow \Delta\pi$ the fit again gives roughly equal values of F and \bar{P} , so that $|F/P| \approx 0.55$. If $F/P = +0.55$ [the same sign as predicted by $SU(6)_w$] then $M^{(\lambda=3/2)}/M^{(\lambda=1/2)}$ for $F_{15} \rightarrow \Delta\pi$ is predicted by Eq. (17) to be $\approx \frac{1}{4}$. If $F/P = -0.55$, this ratio is predicted to be 2.

Note that whichever sign of D/S or F/P holds, the fitted values of these ratios are too small to change the sign of any helicity amplitude, so that the assumption of dominance by the lower partial waves made in Secs. 1 and 2 is indeed valid.

Some theoretical support, based on duality, exists for the enhancement of the $\lambda = \frac{3}{2}$ couplings. In Ref. 24 it was found that in $\pi\Delta$ charge-exchange scattering an effective ρ trajectory exchange with desirable angular momentum properties (such as wrong-signature nonsense zeros) was "built" by direct-channel resonances only if these resonances coupled to $\pi\Delta$ mainly via the $\lambda = \frac{3}{2}$ state. Such a circumstance is in sharp contrast to $SU(6)_w$, for which the $\lambda = \frac{3}{2}$ couplings of *all* the $S_q = \frac{1}{2}$ resonances (like D_{13} and F_{15}) to $\pi\Delta$ are expected to be suppressed.²⁵ [This result follows most directly from S_z conservation, a property of $SU(6)_w$ but not of our model.]

In our model, any abnormal-parity resonance on the leading trajectory ($J^P = \frac{5}{2}^-, \frac{7}{2}^+, \frac{9}{2}^-, \dots$) is predicted to decay to $\Delta\pi$ only via $l = J - \frac{1}{2}$, i.e., $l = L + 1$. The decay via $l = J + \frac{3}{2} = L + 3$ is forbidden since the 3P_0 "spurion" only allows $|l - L| \leq 1$. In such cases the $\lambda = \frac{3}{2}$ couplings are automatically enhanced: $M^{(3/2)}/M^{(1/2)} = [3(l+2)/l]^{1/2}$. For example, for $D_{15} \rightarrow \Delta\pi$, $M^{(3/2)}/M^{(1/2)} = \sqrt{6}$, and for the $\Delta(1950) (F_{37}) \rightarrow \Delta\pi$, $M^{(3/2)}/M^{(1/2)} = \sqrt{5}$.

The helicity ratios just predicted, particularly those for the decays $\frac{5}{2}^+ \rightarrow \frac{3}{2}^+ 0^-$, should be testable. In particular, one should be able to test for the sign of F/P in Eq. (17) if this quantity has a magnitude as large ($\gtrsim \frac{1}{2}$) as our fits predict. Some analyses¹² fix F/P *a priori*; others neglect it altogether.¹³ We suspect this practice to be unsound.

Mixing. When comparing our formalism with the data, one must allow for mixing among nearby

states of different S^A or $SU(3)$ or $SU(6)$ representation. One hint that the $D_{13}(1520)$ is not strongly mixed is that (here) the $S^A = \frac{3}{2} D_{13}\pi N$ resonance has $\frac{1}{40}$ the partial width into πN (modulo phase space) of its $S^A = \frac{1}{2}$ partner. The experimental absence of a second D_{13} resonance in πN phase shifts²⁶ then suggests that the two states are relatively pure in S^A . Our discussion (Sec. 1) regarding SD/D^2 is thus likely to be reliable. On the other hand, the $F_{15}(1690)$ may not be a pure member of the $(56, L=2)$. The existence of a $(70, L=2)$ degenerate with the $(56, L=2)$ is suggested by the harmonic-oscillator quark model,^{8,9} by duality,²⁷ and to some extent by the data.²⁸ If the two $F_{15} N^*$ states in the $(70, L=2)$ were to cause the $F_{15}(1690)$ to be appreciably mixed, its $\Delta\pi$ decay would probably not have the simple significance noted here.

Note added in proof. We thank D. Faiman for

pointing out that the S -wave and D -wave decays of $D_{33}(1670) \rightarrow \Delta\pi$ could provide information similar to that discussed above. $SU(6)_W$ predicts $M^{(3/2)} \sim S-D=0$, while if $D/S < 0$, $M^{(3/2)}$ will dominate. Mixing should not affect this state.

Summary. We have introduced a scheme for describing hadron decays which avoids the link between two different partial waves characteristic of certain symmetry approaches^{2,3,8,9} but preserves all other relations of such approaches. We have described several crucial tests involving the helicity structure of baryon decays that would indicate whether avoiding this link is in fact important.

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