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[7]We note in passing that the paragraph following Eq. (2) of Ref. 5 is incorrect: Even if $g_S(t)$ had a pole at t_0, this pole would fall well outside the physical region $m_l^2 \le t \le (m-\mu)^2$ and would in fact be not too distant from the (known) K^* pole. Hence, a pole at $t=(m+\mu)^2$ would no more prevent $g_S(t)$ from being expanded about $t=0$ than does a pole at M_*^2.

[8]We stress that the entire discussion of Ref. 5 concerning the zero in $\hat{f}_0(t)$ is relevant only in the presence of explicit symmetry breaking, since $\hat{f}_0(t)$ vanishes identically in the symmetry limit. The reader can verify that in the symmetry limit ($m=\mu$) our results coincide with the conventional K-G formulation, as expected.

[9]We again emphasize that although a K-G interaction can presumably be rigged up to simulate a result obtained by use of the Kemmer formalism, the resulting interaction might have some bizarre characteristics. The reader is invited to construct a K-G interaction (or a Feynman diagram) which reproduces the factor $\delta=(\mu-m)/(\mu+m)$ in the expression $\hat{\xi}=\delta-\rho$.

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Comment on the Preceding Paper of Fischbach, Nieto, and Scott

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In answer to Fischbach, Nieto, and Scott, we amplify our argument for the equivalence of the Klein-Gordon and Kemmer formulations of K_{l3} decay.

The preceding paper – hereafter referred to as FNS – is the latest in a series of articles where the authors propose and investigate a model for the theoretical description of K_{l3} decays, based on the use of spin-0 Kemmer wave functions to describe the kaon and pion.

There are two aspects on the work given in these papers:

(a) The authors present a model – including in the model the prescription that the form factor $g_S(t)$ should be smooth – which is more restricted, and hence has a higher predictive power, than the ordinary phenomenological description in terms of the form factors $f_\pm(t)$.

(b) The authors try to show that this model is a necessary consequence of the use of Kemmer wave functions, thus implying that the descriptions in terms of Kemmer wave functions or Klein-Gordon (KG) wave functions are inequivalent.

In the authors' presentation, (b) comes before and motivates (a).

It is aspect (b) that was criticized by us in Ref.

5. (We use the list of references of FNS.) The purpose of FNS is to show that our criticism was unjustified.

Before we take up this point, let us say a word about (a). The investigation of a particular model is of course a legitimate thing to do, although in our opinion the present model is rather awkward and unnatural. We also think that the authors are far too optimistic in describing the success of their model in the comparison with experimental results. In view of the fact that different types of experiments give incompatible sets of parameters describing the K_{l3} decays, and also considering the various other assumptions, not directly related to their model, which are used in the authors' analysis, it is at present highly premature to assess the success – or, for that matter, the failure – of their model.

We now turn to a discussion of point (b), which is the main issue in the present context. To make the situation clear, it should be recalled that the spin-0 Kemmer equation is a set of coupled first-order equations for five field components; these equations express the fact that one component is a scalar field satisfying the KG equation, and the other four components are the space-time derivatives of this scalar field. In the quantization of the Kemmer equation the equal-time commutation relations for the field components are exactly the KG-field commutation relations written in the new notation. Since evidently any local (e.g., bilinear) covariant expressed in KG fields can be rewritten in terms of the corresponding Kemmer fields (and vice versa), the result of a perturbation calculation up to any given order must be the same in both formulations, provided the corresponding local interactions are used. In particular this holds for form factors describing, e.g., the K_{l3} decays. Hence it is clear that descriptions in terms of KG or Kemmer wave functions cannot possibly be inequivalent. The statement made in FNS that one might have to resort to a nonlocal KG Lagrangian to reproduce the result of the particular Kemmer interaction the authors have chosen in their K^*-pole model is of course incorrect; it is very easy to give the corresponding local KG interaction.

The authors state: "To our knowledge there has never been a field theoretic proof of the equivalence of the Kemmer and KG equations in the presence of symmetry breaking" (meaning interacting fields describing two particles with different masses). This statement might be correct; the reason – as we have seen above – is that for anyone who has studied and understood the Kemmer equation and its quantization, this equivalence is evident. From a remark in Ref. 1, p. 1200, first paragraph, one gets the impression that there is an equivalence proof in the case of quantum electrodynamics (QED) of a spin-0 particle in Ref. 2 of Ref. 1 (Akhiezer and Berestetskii); we have not been able to find any such explicit proof in this reference, which uses the Kemmer formulation throughout to treat this case. The reason for using Kemmer fields instead of KG fields in this connection is that in this way one avoids the appearance of two terms, one linear and one quadratic in the electromagnetic potential, in the interaction with the electromagnetic field. One thus gets – in the Kemmer formulation – only one type of vertex, and obtains Feynman rules formally very similar to those in QED of spin-$\frac{1}{2}$ particles. The formal manipulations become more involved, however, due to the more involved algebra of the β matrices, as compared with the γ matrices, and also due to the fact that the five-component Kemmer wave functions for plane-wave states – as distinguished from the Dirac four-component spinors – carry no information in addition to that given by the four-momentum p. This last fact makes it rather awkward to base a phenomenological analysis on these redundant five-component functions.

We have here insisted at some length on these matters, since we suspect that ignorance of some of these basic facts in the quantization of the spin-0 Kemmer equation underlies the authors' hope that the KG and Kemmer formulations might be shown to be inequivalent. Our suspicion is strengthened by the discussion given in Ref. 1, p. 1201, of the scaling properties of the Kemmer fields. Since the Kemmer components are a scalar field and its gradient, they transform under scaling as $(\text{mass})^1$ and $(\text{mass})^2$, respectively, and not $(\text{mass})^{3/2}$ and $(\text{mass})^2$, as the authors erroneously believe. The first term on the right-hand side of commutation relation (2) in Ref. 1 – this term has dimension $(\text{mass})^3$ – comes from the commutation of the scalar field component and its time derivative.

Since, as we have seen, the KG and Kemmer formulations are equivalent, one might wonder how the authors arrive at their formulation, which – with the assumption of a smooth $g_S(t)$ – is more restricted than the usual formulation. The basic parametrization is given by formula (6) in FNS and it is here that the authors make their essential mistake: they say that if "the pion and kaon are described by the Kemmer equations" (6) follows, thus implying that (6) is *the* Kemmer parametrization. This is not correct; (6) is only a possible Kemmer parametrization, and, as we shall see, not a very natural one. The parametrization (6) corresponds to a combination of the local vector covariants (a) and (b) of Ref. 5; thus, in particular, the form factor $g_S(t)$ is related to the derivative coupling $\partial_\lambda[\bar{\psi}_\pi(x)\psi_K(x)]$. As discussed above, the main reason for introducing the Kemmer description in the case of the electrodynamics of spin-0 particles is exactly to avoid a current containing derivatives of the field, since such a current gives both a linear and a quadratic interaction term in the electromagnetic potential. So the vector covariants (a) and (c) of Ref. 5, not containing derivatives of the Kemmer fields, are the most natural ones to use, and they lead directly to a linear combination with constant (i.e., t-independent) coefficients of the ordinary K_{l3} form factors. It might be added that in the electromagnetic case, with one Kemmer field coupled bilinearly to itself to form a conserved vector current, (c) must be absent because of the conservation condition.

In short: The authors' parametrization (6) in FNS, the basis of their analysis, is not implied by the Kemmer description.

The same sort of error is made by the authors in their example with a world containing only π's, K's, K^*'s, and the lepton current (see, e.g., their figure); since the masses of π and K are different, and the current is not conserved, the assumptions that the basic vertex interactions are

$$\bar{\psi}_\pi(x)\beta_\lambda\psi_K(x)$$

and

$$\varphi_\pi^*(x)\partial_\lambda\varphi_K(x) - [\partial_\lambda\varphi_\pi^*(x)]\,\varphi_K(x)\,,$$

respectively, are completely *ad hoc* and arbitrary.

It should also be pointed out that in the argumentation against Ref. 5 the equivalence proof of that reference is misconstrued. It is claimed that the proof of Ref. 5 follows from a proof of absence of a zero in $f_0(t)$. This is not so; in the paragraph following the equations (a)–(c) in Ref. 5 it is shown that a linear combination of (a) and (c) corresponds to the ordinary parametrization; the equivalence follows from this. We agree with the authors' statement that the question whether $f_0(t)$ has a zero at $t=t_0$ is a dynamical one; only it is the presence of a zero that is accidental, not the absence of it.

To summarize, we have seen that it is meaningless to talk about inequivalent formulations of K_{l3} decays in terms of Klein-Gordon or Kemmer wave functions. The model for K_{l3} decays presented by the authors of FNS is not a consequence of describing the spin-0 mesons by Kemmer wave functions. Hence it is completely misleading to say that agreement of their model with experiments would mean that the pion and kaon are actually better described by the Kemmer equation than by the Klein-Gordon equation.