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Inequivalence of the Klein-Gordon and Kemmer Formulations of K_{l3} Decay*

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We answer the criticism that was raised against our Kemmer K_{l3} theory by exhibiting in explicit calculational detail the inequivalence of the Kemmer and Klein-Gordon formulations of K_{l3} decay, taking care to clearly define "inequivalence." It is shown that our published results are correct as stated, including the prediction of a zero in the scalar form factor.

In a recent series of papers[1-3] a treatment of K_{l3} decays has been developed in which the pion and kaon are assumed to be described by the spin-0 Duffin-Kemmer-Petiau equation rather than by the usual Klein-Gordon (K-G) equation. Among other results it was shown[1] that our theory gives rise to an effective symmetry-breaking parameter $\hat{\xi}$ whose magnitude $\hat{\xi}_{\text{th}}=-0.77\pm0.10$ is in excellent agreement with the experimental value[4] $\hat{\xi}_{\text{exp}}=-0.85\pm0.20$. It was further shown[2,3] that our formulation of K_{l3} decays resolves a long-standing discrepancy between the values of the polar vector Cabibbo angle θ_V as determined from K_{e3} and nuclear β decays, respectively. Finally we predicted a zero in the effective scalar form factor $\hat{f}_0(t)$ at $t=(m+\mu)^2\equiv t_0$, where m and μ are the kaon and pion masses, respectively.

Subsequent to the appearance of Refs. 1–3, a criticism of our Kemmer formulation of K_{l3} decays was raised.[5] This paper[5] asserted that (a) our prediction of a zero in the scalar form factor was incorrect and therefore that (b) our claim of the inequivalence of the Kemmer and K-G formulations was incorrect. We will show, however, that even if assertion (a) were correct (which it is not), this would not imply assertion (b). The reason is that our remaining predictions are all independent of the existence of a zero in the scalar form factor.

It is thus the purpose of the present note to answer in detail the above two assertions, and to thereby dispel the misunderstandings that they have generated. We will show that the conclusions of Refs. 1–3 are indeed correct as stated. In the process we shall also clarify what is meant by the "inequivalence" of the K-G and Kemmer formulations[6] of K_{l3} decay, since this is admittedly a very subtle, albeit important, question.

We start by considering the matrix element in question, that of the hadronic $\Delta S=1$ current $V_\lambda(x)$, which is given by

$$\langle\pi(p')|V_\lambda(0)|K(p)\rangle\equiv\overline{\Psi}_\pi(p')\Gamma_\lambda(p',p,t)\Psi_K(p). \quad (1)$$

$\Psi_K(p)$ and $\overline{\Psi}_\pi(p')$ are the initial and final wave functions, respectively, $\Gamma_\lambda(p,p',t)$ is a covariant 4-vector function of p and p', and $t=-q^2=-(p-p')^2$. In the conventional treatment of K_{l3} decays the kaon and pion are assumed to be described by the usual K-G equations $(-\Box+m^2)|K\rangle=0$ and $\langle\pi|(-\Box+\mu^2)=0$. In that case $\Psi_K(p)$, $\overline{\Psi}_\pi(p')$, and $\Gamma_\lambda(p,p',t)$ are given by

$$\Psi_K^{\text{K-G}}(p)=(2p_0V)^{-1/2},\quad \overline{\Psi}_\pi^{\text{K-G}}(p')=(2p_0'V)^{-1/2}, \quad (2)$$

$$\Gamma_\lambda^{\text{K-G}}(p,p',t)=(p+p')_\lambda f_+(t)+q_\lambda f_-(t), \quad (3)$$

$$f_0(t)\equiv f_+(t)+[t/(m^2-\mu^2)]f_-(t), \quad (4)$$

where $f_\pm(t)$ are unknown form factors, $f_0(t)$ is the scalar form factor, and V is the normalization volume. By contrast the development of Refs. 1–3 begins with the assumption that the pion and kaon are described by the Kemmer equations $(\beta\cdot\partial+m)|K\rangle=0$ and $\langle\pi|(-\beta\cdot\partial+\mu)=0$. In this case $\Psi_K(p)$, $\overline{\Psi}_\pi(p')$, and $\Gamma_\lambda(p,p',t)$ are given by

$$\begin{aligned}\Psi_K^{\text{K}}(p)&=(m/p_0V)^{1/2}u_K(p),\\ \overline{\Psi}_\pi^{\text{K}}(p')&=(\mu/p_0'V)^{1/2}\overline{u}_\pi(p'),\end{aligned} \quad (5)$$

$$\Gamma_\lambda^{\text{K}}(p,p',t)=i\{\beta_\lambda g_V(t)+[iq_\lambda/(m+\mu)]g_S(t)\}, \quad (6)$$

$$g_0(t)\equiv g_V(t)-[t/(m^2-\mu^2)]g_S(t), \quad (7)$$

where $u_K(p)$ and $\overline{u}_\pi(p')$ are 5-component spinors.[1]

It is a straightforward matter to show that the Kemmer scalar form factor $g_0(t)$ is given by

$$\langle\pi(p')|\partial_\lambda V_\lambda(0)|K(p)\rangle$$
$$=-i(m-\mu)\left(\frac{m\mu}{p_0p_0'V^2}\right)^{1/2}\{g_0(t)\}[\bar{u}_\pi(p')u_K(p)]$$
$$=+i(m-\mu)\left(\frac{m\mu}{p_0p_0'V^2}\right)^{1/2}g_0(t)\left(\frac{t-t_0}{4m\mu}\right)$$
$$\equiv -i\hat{f}_0(t)(4p_0p_0'V^2)^{-1/2}(m^2-\mu^2). \qquad (8)$$

Thus, $\langle\partial_\lambda V_\lambda\rangle$ [or equivalently, the effective K-G scalar form factor $\hat{f}_0(t)$] was predicted[1] to have a zero at t_0, barring the possibility of a dynamical pole in $g_0(t)$ at t_0. In Ref. 5 it was claimed, however, that this zero could be made to disappear if, in place of the Kemmer current of Eq. (6), we use a new combination of invariants given by

$$\Gamma_\lambda^X(p,p',t)=i[\beta_\lambda h_V(t)+X_\lambda h_X(t)], \qquad (9)$$

$$X_\lambda=\tfrac{1}{3}[\beta_\lambda\beta_\mu\beta_\mu-\beta_\mu\beta_\mu\beta_\lambda]. \qquad (10)$$

(To clarify confusion that has arisen, we wish to emphasize that the matrix element can have only two independent form factors, irrespective of how they are obtained or chosen.) With this form of the current the scalar form factor is given by

$$i(m\mu/p_0p_0'V^2)^{-1/2}\langle\pi(p')|\partial_\lambda V_\lambda(0)|K(p)\rangle$$
$$=(4m\mu)^{-1}\bar{u}_\pi(p')[(m-\mu)h_V(t)+t(m+\mu)h_X(t)\beta\cdot\beta]u_K(p) \qquad (11a)$$
$$=(4m\mu)^{-1}\{[(m-\mu)h_V(t)+(m+\mu)h_X(t)](-t+t_0)+6m\mu(m+\mu)h_X(t)\}. \qquad (11b)$$

On the basis of Eq. (11) the claims were made[5] that (a) there is no zero in the scalar form factor at $t=t_0$ and that (b) as a consequence the Kemmer and K-G formulations of K_{l3} decay were "equivalent." To start with we reemphasize that (as we demonstrate below) our new predictions for $\hat{\xi}$ and θ_V are completely independent of the existence of a zero in $\langle\partial_\lambda V_\lambda\rangle$. Thus the conclusion (b) of Ref. 5 concerning the equivalence of the Kemmer and K-G formulations of K_{l3} decay is false as the inequivalence does not depend on the zero discussed in (a). Specifically, we note from Eq. (11b) that the mere introduction of the operator X_λ in no way "proves" that a zero is absent, since $h_X(t)$ can itself have a zero at $t=t_0$.[7] The question of whether $\langle\partial_\lambda V_\lambda\rangle$ has a zero is thus a combined one of kinematics and dynamics, as was stated clearly in Ref. 1. The only effect of introducing the parametrization of Ref. 5 in place of Eq. (6) is to trivially shift the dynamical question from whether or not $g_0(t)$ has a pole at t_0 to whether or not $h_X(t)$ has a zero at t_0. Ultimately, of course, this dynamical question must be settled by future experiments.

The situation with respect to the Kemmer formulation of K_{l3} decays is, in this regard, similar to that for the matrix element of the hadronic current in e-p scattering, which can also be written with two different sets of form factors:

$$\langle p'|J_\lambda(0)|p\rangle$$
$$=ie\bar{u}(p')[\gamma_\lambda F_1(t)-(\sigma_{\lambda\nu}q_\nu/2M)F_2(t)]u(p) \qquad (12a)$$
$$=e\bar{u}(p')[(P_\lambda/2M)G_E(t)-(r_\lambda/4M^2)G_M(t)]u(p), \qquad (12b)$$

where $P_\lambda=(p_\lambda+p_\lambda')$ and $r_\lambda=(-i/2)(\gamma_\lambda\not{P}\not{q}-\not{q}\not{P}\gamma_\lambda)$. Since the two form factor bases are related via the equations

$$\begin{aligned}(1-t/t_0')G_E(t)&=F_1(t)-tF_2(t)/t_0',\\ (1-t/t_0')G_M(t)&=F_1(t)-F_2(t),\end{aligned} \qquad (13)$$

where $t_0'=4M^2$, seemingly one can make a zero appear or disappear simply by choosing the appropriate parametrization of $\langle J_\lambda\rangle$ in Eqs. (12). This is of course not true here any more than in the Kemmer matrix element. If one starts from a fundamental Lagrangian theory, different interactions will predict different K_{l3} physical matrix elements (independent of what parametrization one uses), and these physical matrix elements will not be the same as the K-G matrix elements (see below).

To try to obtain some insight into the correct dynamics (or the best parametrization to use), we can resort to elementary Feynman diagrams, such as the K^*-pole diagram shown in Fig. 1. We leave it as an elementary exercise to the reader to show from Fig. 1(a) that $g_0(t)$ does *not* have a pole at t_0 [or equivalently that $h_X(t_0)=0$], from which we conclude that $\langle\partial_\lambda V_\lambda\rangle|_{t_0}=0$ as asserted in Ref. 1. The electromagnetic form factors also provide a clue as to the best parametrization to choose in the absence of a detailed dynamical theory, namely the one which maximizes the number of form factors which are angular momentum eigenfunctions. The reader can easily verify[1] that $g_V(t)$ in Eq. (6) is pure $J=1$, while $g_S(t)$ is a linear combination of $J=1$ and $J=0$. By contrast *both* $h_V(t)$ and $h_X(t)$ in Eq. (8) receive contributions from both $J=1$ and $J=0$ intermediate states. It is

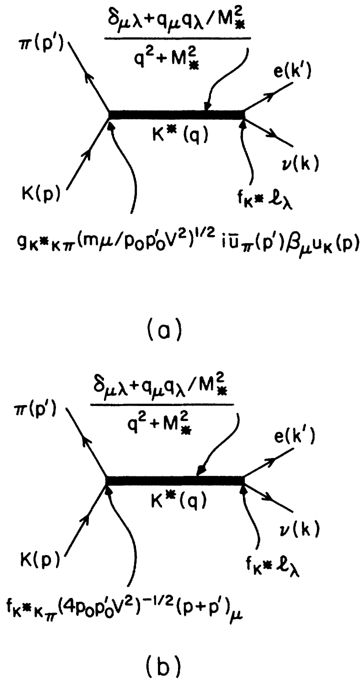

FIG. 1. K_{l3} Feynman diagrams for (a) the Kemmer K^*-pole dominance model and (b) the Klein-Gordon K^*-pole dominance model. $l_\lambda = i\bar{u}(k')\gamma_\lambda(1+\gamma_5)v(k)$ is the usual lepton current and f_{K^*} is the $K^* \to$ vacuum coupling constant. $f_{K^*K\pi}$ and $g_{K^*K\pi}$ are the strong $K^*K\pi$ coupling constants in the K-G and Kemmer formulations, respectively.

for this reason that there are no elementary pole diagrams which directly give either of the form factors $h_{V,X}(t)$, whereas the K^*-pole diagram of Fig. 1 gives $g_V(t)$ directly. We have thus shown that an appeal to elementary Feynman diagrams substantiates the prediction of a zero in $\langle\partial_\lambda V_\lambda\rangle$, as claimed in Ref. 1, irrespective of which Kemmer parametrization of the K_{l3} matrix element is used.

Finally, we wish to point out that

$$\bar{u}_\pi(p')q_\lambda X_\lambda u_K(p) = -i(m+\mu)\bar{u}_\pi(p')v_K(p). \tag{14}$$

Thus, this new current X_λ has the unusual property of changing the particle spinor $[-u_K(p)]$ into the antiparticle spinor $v_K(p)$ obtained from the adjoint equation (i.e., $m \to -m$ in $[-u_K(p)]$), but it does not change the sign in the exponential $\exp(ip\cdot x)$ (which is important in a first-order wave equation). So, in addition to not arising from an elementary Feynman diagram, the current X_λ would have to come from some very unusual Lagrangian theory that has never been used to describe K_{l3} decays.

We turn now to the question of "equivalence" and emphasize again that the parametrizations [above and in Eq. (7) of Ref. 1] of the K-G and Kemmer form factors should not be misconstrued as implying the "equivalence" of the two parametrizations. To understand this, we must define what "equivalence" means, in physical terms.[6] Given a knowledge of the Kemmer form factors, a set of effective K-G form factors $\hat{f}_\pm(t)$ can always be constructed *a posteriori*. However, in general there is no way of guaranteeing that the effective form factors so obtained could have been derived *a priori* from an acceptable K-G Lagrangian theory. For example, in order to reproduce the zero in $\hat{f}_0(t)$ which followed directly from the use of the Kemmer formalism in a simple K^*-pole model (see above), one might have to construct a (nonlocal) Lagrangian model of the K-G form factor $f_0(t)$ containing an infinite number of derivatives.

This is the sense in which the Kemmer and K-G K_{l3} formulations are "inequivalent." They require different dynamics to obtain the same physical matrix element. Contrariwise, if the same dynamical assumptions (e.g., minimal couplings, pole dominance, etc.) are made in the Kemmer and K-G formalisms, respectively, one ends up with different physical matrix elements.

Again, an analogous situation can be found elsewhere, the nonrelativistic hydrogen atom. If one uses the minimal electromagnetic substitution $p_\mu \to p_\mu - eA_\mu$ in both the Schrödinger and Dirac equations, one gets different energy levels. However, if one adds new dynamics (the Pauli spin interaction) into *only* the Schrödinger equation one can get the same energy levels, but one certainly does not say that this implies the Schrödinger and Dirac equations are "equivalent."

We illustrate these remarks with two further specific examples from the K_{l3} problem, which are both independent of the prediction of a zero in the scalar form factor. (Remember, just one example proves "inequivalence.")

Consider the problem of calculating the symmetry-breaking parameters $\xi \equiv f_-(0)/f_+(0)$ and $\rho \equiv g_S(0)/g_V(0)$ in the usual Kemmer and K-G K^*-dominance models shown in Figs. 1(a) and 1(b). Straightforward algebra gives

$$\xi = -\rho = \frac{\mu^2 - m^2}{M_*^{\,2}} = -0.28, \tag{15}$$

where M_* is the K^* mass. We note, however, [see Eq. (7) of Ref. 1] that the Kemmer matrix element with $\rho = 0.28$ is *not* equal to a K-G matrix element with $\xi = -0.28$; it is equal instead to a K-G matrix element with an *effective* symmetry-breaking parameter

$$\hat{\xi} = -0.28 + \frac{\mu - m}{\mu + m} = -0.85.$$

Thus, to obtain this (experimentally suggested) value of the symmetry-breaking parameter one would have to introduce *different* dynamics into the Kemmer and K-G formulations, respectively. (This prediction is independent of the zero in the scalar form factor since it is evaluated at $t=0$.)

As a second example[2,3] consider the problem of extracting the polar vector Cabibbo angle θ_V from the observed width for K_{e3} decay. The matrix element T_{e3} for this decay is given by[1,3]

$$T_{e3}=(G/\sqrt{2})\sin\theta_V\bar{\Psi}(p')\Gamma_\lambda(p,p',t)\Psi(p)l_\lambda, \tag{16}$$

$$\begin{aligned}\Gamma_\lambda^{\text{K-G}}(p,p',t)&=(p+p')_\lambda f_+(t),\\ \Gamma_\lambda^{\text{K}}(p,p',t)&=i\beta_\lambda g_V(t),\end{aligned} \tag{17}$$

where G is the Fermi constant and l_λ is the lepton current. In the usual development we expand $f_+(t)$ or $g_V(t)$ linearly in t,

$$f_+(t)=f_+(0)(1+\lambda_+t/\mu^2),\quad g_V(t)=g_V(0)(1+\gamma_Vt/\mu^2)$$

and then, after taking account of radiative and SU(3)-symmetry-breaking corrections, inserting for $f_+(0)$ or $g_V(0)$ their SU(3) symmetric values. Note that *only* Γ_λ is treated in the SU(3) limit, while the physical masses $(m\neq\mu)$ are used for $\Psi(p)$ and $\bar{\Psi}(p')$. The net result is that the Kemmer matrix element with

$$g_V(0)=1=g_V(0)|_{\text{SU(3)}}$$

is not equal to the K-G matrix element with $f_+(0)=1=f_+(0)|_{\text{SU(3)}}$; it is equal instead to a K-G matrix element with an effective form factor $f_+(0)(m+\mu)/2\sqrt{m\mu}$. This last factor is the origin of the difference in the K-G and Kemmer values for the Cabibbo angle, which are (after inserting experimental and theoretical corrections and errors)

$$\theta_V^{\text{K-G}}=0.235_-\pm0.019,\quad \theta_V^{\text{K}}=0.192_+\pm0.016. \tag{18}$$

Note that both values of θ_V in Eq. (18) are completely independent of the zero in the scalar form factor, since $f_-(t)$ and $g_S(t)$ do not contribute to the K_{e3} matrix elements.

One concluding comment on "equivalence." To our knowledge (which is contrary to much belief), there has never been a field-theoretic proof of the equivalence of the Kemmer and K-G equations in the presence of symmetry breaking.[8] In fact, our K^*-pole model of K_{l3} decays is enough to prove the "inequivalence." Consider a world with only K^*'s, π's, K's, and the lepton current, and where $g_{K^*K\pi}\ll1$. Then the K^*-pole term will dominate over all other diagrams, no matter what they are. As only one case is needed to prove the "inequivalence" of the Kemmer and K-G equations, this does it.

In summary, we have demonstrated all the things we said we would and the conclusions in Refs. 1–3 have been shown to be correct as stated. The Kemmer formulation of K_{l3} decays has been clearly and explicitly demonstrated to be "inequivalent" to the Klein-Gordon theory of K_{l3} decays.[9]

Note Added in Proof: In the following comment (hereafter called NS-2) Nagel and Snellman reformulate their criticism of our work in light of the present paper. Although we are pleased that NS-2 retreats somewhat from the more critical stance of NS-1 (Ref. 5), we still insist that our position is correct as originally stated, and so will comment on NS-2. (We note in passing that NS-2 does not dispute several serious criticisms of NS-1 that we have made in the main text, such as those contained in Ref. 7 and in our discussion of θ_V.)

Point by point (rearranged for continuity), our reply to NS-2 is the following:

(1) The Klein-Gordon (K-G) and Duffin-Kemmer-Petiau (D-K-P) equations are only equivalent in the symmetry case. This equivalence, however, is not as obvious as NS-2 seems to think. If it were, then the classic and long paper of Harish-Chandra[10] (let alone that of Klein[11]) would never have needed to prove this point.

(2) In the symmetry-breaking case, the algebra of Harish-Chandra[10] (as modified by Fujiwara[12]) does not yield the same result. Using this algebra we have explicitly shown[6] that the D-K-P current, when written in terms of K-G wave functions, is not the usual K-G current, but rather is

$$\begin{aligned}j_\lambda^{\text{DKP}}&=i\bar{\psi}_2\beta_\lambda\psi_1\\ &=-i\left\{(m_2{}^{1/2}\phi_2^*)\left[\frac{\partial_\lambda^-}{m_1}(m_1{}^{1/2}\phi_1)\right]\right.\\ &\qquad\left.-\left[\frac{\partial_\lambda^+}{m_2}(m_2{}^{1/2}\phi_2^*)\right](m_1{}^{1/2}\phi_1)\right\},\end{aligned} \tag{19}$$

$\partial_\lambda^\mp=\partial_\lambda\mp ieA_\lambda$. [The detailed proof of Eq. (19) is in press elsewhere.[6]] If it will help, we can restate this point in the language of nuclear physics: Different basis states (K-G and D-K-P) can imply different residual interactions.

(3) From Eq. (19) one sees that various mass operators would be needed in the K-G current to make it equivalent to the D-K-P current. This is the reason why the two fields have different scale dimensions. Recall that masses are considered as constants in scaling and not as operators. To obtain the scaling dimensions of the D-K-P field, all components of ψ must be consistently treated together. (See Ref. 6 for a detailed discussion.)

(4) The above points are related to the fact that the D-K-P and K-G equations are first-order and second-order differential equations, respectively. Because of this, their solutions require the knowledge of one or two boundary conditions, respectively. The boundary condition in the D-K-P formalism can be given by specifying the wave function at a specific time. In the K-G formulation both the wave function and its first derivative must be specified. In certain formal theories, such as ordinary quantum electrodynamics, when the appropriate boundary conditions are specified the results obtained by use of the D-K-P equation exactly coincide with those obtained from the K-G equation.[13] What we have shown[6] is that this is no longer necessarily true in an interacting field theory in which explicit symmetry-breaking effects are present.

(5) In a phenomenological theory (as distinguished from a formal field theory), one does not attempt to actually solve the equations of motion for the fields; instead one calculates appropriate matrix elements in some simplified manner. The starting point in such a treatment is the covariant form of the matrix elements, such as in Eqs. (1)–(7) for K_{l3} decays. Note that in Eqs. (1)–(7) only the wave functions are specified, but not their derivatives. Since this limited information is all that is needed to specify the D-K-P boundary conditions (but not to specify the K-G boundary conditions), one sees that a phenomenological theory based on the D-K-P equation does indeed have greater predictive power than one based on the K-G equation. To the extent that our results agree with experiment, we can conclude that the D-K-P wave functions have indeed been correctly specified. The objection of NS-2 that we have chosen "unnatural" form factors in Eqs. (1)–(7) is answered in the main text. We give explicit calculational motivation for our form factors, which seems not to have been understood.

(6) NS-2 states that our K^*-pole model results could very easily be duplicated by a K-G Lagrangian. However, if it is so simple, why have they not written it? Remember, they must obtain the residues

$$\begin{aligned}\hat{\xi}(K^*\text{ pole}) &= \frac{\operatorname{Res} f_-(t)}{\operatorname{Res} f_+(t)}\\ &= -\left(\frac{m-\mu}{m+\mu}\right)-\left(\frac{m^2-\mu^2}{{M_*}^2}\right)\\ &= -0.85. \end{aligned} \tag{20}$$

Such residues do not resemble any of those obtained from the usual K-G theories.[14] Thus, it would indeed be of interest if NS would exhibit the "simple" K-G Lagrangian that yields the above result.

(7) There is further evidence that NS-2 does not understand our K^*-pole calculation. The $K\pi K^*$ vertices in both the K-G and D-K-P Feynman diagrams are correct as given since they are obtained from the K-G and D-K-P 4-currents in the usual way. To call these vertices "*ad hoc*" not only attacks our work, but also calls the standard K-G K^*-pole model "*ad hoc*." In other words, NS-2 is calling everybody's K^*-pole model "*ad hoc*." Surely something must be definite. In any event, details of the predictions and differences between the K-G and D-K-P K^*-pole models will appear elsewhere.[15] In addition to the results which we have already quoted [such as in Eq. (20)] we mention one other interesting result, which is

$$\hat{\lambda}_0(K^*\text{ pole}) = \frac{-\mu^2}{(\mu+m)^2} = -0.046. \tag{21}$$

(8) NS-2 still confuses the zero in $f_0(t)$ and the question of equivalence. In the main text we have explicitly shown in calculational detail how K-G and D-K-P give different results in two cases $(\hat{\xi},\, \theta_V)$ where the zero has nothing at all to do with the results. NS-2 has in no way faulted these arguments.

(9) With respect to all the above, we make the following point, which should be obvious: Since neither NS nor anyone else has faulted our calculations (and they have been checked by a number of people), we can presume that our calculations are correct. Hence, if the conventional K-G formulation of K_{l3} decays were equivalent to ours (as NS asserts), then the K-G formulation would have obtained our results long ago. Thus, the K-G formulation would have yielded our large negative value for ξ, our large negative value for λ_0, and our agreement between the determinations of the Cabibbo angle θ_V from K_{e3} decay and $0^+ \to 0^+$ nuclear β decay (all of which agree with experiment), and there would never have been any problem of disagreement with experiment. But the K-G formulation did not do this. Therefore, how can K-G and D-K-P be equivalent if they yield different *numerical* results? In the end, the simplest answer to NS-1 and NS-2 is *res ipsa loquitur* ("the thing speaks for itself").

(10) We conclude that K-G and D-K-P are indeed inequivalent in the symmetry-breaking case. It remains for experiment to determine which is the better equation to use to describe mesons. In addition to our previous results, we are encouraged by new work which extends our theory to other processes where again we obtain better agreement with experiment. We shall report on these results shortly.[16]

We wish to express our deep appreciation to P. Carruthers, H. Primakoff, and S. P. Rosen for numerous valuable discussions and for their continued interest in our work. We also wish to thank S.-J. Chang, whose work on the Schwinger-Chang[17] formalism of multispinor fields led to a stimulating discussion in support of our philosophy of inequivalence.

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[7]We note in passing that the paragraph following Eq. (2) of Ref. 5 is incorrect: Even if $g_S(t)$ had a pole at t_0, this pole would fall well outside the physical region $m_l^2 \leq t \leq (m-\mu)^2$ and would in fact be not too distant from the (known) K^* pole. Hence, a pole at $t=(m+\mu)^2$ would no more prevent $g_S(t)$ from being expanded about $t=0$ than does a pole at M_*^2.

[8]We stress that the entire discussion of Ref. 5 concerning the zero in $\hat{f}_0(t)$ is relevant only in the presence of explicit symmetry breaking, since $\hat{f}_0(t)$ vanishes identically in the symmetry limit. The reader can verify that in the symmetry limit ($m=\mu$) our results coincide with the conventional K-G formulation, as expected.

[9]We again emphasize that although a K-G interaction can presumably be rigged up to simulate a result obtained by use of the Kemmer formalism, the resulting interaction might have some bizarre characteristics. The reader is invited to construct a K-G interaction (or a Feynman diagram) which reproduces the factor $\delta=(\mu-m)/(\mu+m)$ in the expression $\hat{\xi}=\delta-\rho$.

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Comment on the Preceding Paper of Fischbach, Nieto, and Scott

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In answer to Fischbach, Nieto, and Scott, we amplify our argument for the equivalence of the Klein-Gordon and Kemmer formulations of K_{l3} decay.

The preceding paper – hereafter referred to as FNS – is the latest in a series of articles where the authors propose and investigate a model for the theoretical description of K_{l3} decays, based on the use of spin-0 Kemmer wave functions to describe the kaon and pion.

There are two aspects on the work given in these papers:

(a) The authors present a model – including in the model the prescription that the form factor $g_S(t)$ should be smooth – which is more restricted, and hence has a higher predictive power, than the ordinary phenomenological description in terms of the form factors $f_\pm(t)$.

(b) The authors try to show that this model is a necessary consequence of the use of Kemmer wave functions, thus implying that the descriptions in terms of Kemmer wave functions or Klein-Gordon (KG) wave functions are inequivalent.

In the authors' presentation, (b) comes before and motivates (a).

It is aspect (b) that was criticized by us in Ref.