the sum of all proper vertex diagrams. For a discussion of this and related matters, see J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), Chap. 19. Also, we shall be following their notational conventions throughout this paper.

[7]This general form for the vertex function is derived in J. Bernstein, *Elementary Particles and Their Currents* (Freeman, San Francisco, 1968).

[8]The validity of this approach will be confirmed in Sec. III C when the integral equations for the form factors will be expanded in terms of α and compared with known perturbation results.

[9]Y. Nambu, Nuovo Cimento 9, 610 (1958).

[10]The location of this branch cut can be determined by means of the Landau conditions. See L. Landau, Nucl. Phys. 13, 181 (1959).

[11]J. Schwinger, Phys. Rev. 73, 416 (1948).

[12]R. Karplus and N. Kroll, Phys. Rev. 77, 536 (1950).

[13]Depending upon the indices and the differential equation involved, the general solution may contain logarithmic terms. For a complete discussion of the series solution about a regular singular point, see E. L. Ince, *Ordinary Differential Equations* (Dover, New York, 1956).

[14]The solution of any homogeneous, second-order, linear differential equation whose only singularities are three regular singular points can be expressed in terms of a hypergeometric function. See P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1957).

[15]Again, depending upon the indices, there may be logarithmic solutions present. The various cases are discussed in R. E. Langer, *Ordinary Differential Equations* (Wiley, New York, 1956); and H. Wayland, *Differential Equations; Applied in Science and Engineering* (Van Nostrand, New York, 1957).

[16]
$$ {}_2F_1(a,b;c|x)=\frac{\Gamma(c)}{\Gamma(a)\,\Gamma(b)}\sum_{n=0}^{\infty}\frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{x^n}{n!} $$

[17]D. R. Yennie, S. C. Frautschi, and H. Suura, Ann. Phys. (N.Y.) 13, 379 (1961).

[18]A. A. Abrikosov, Zh. Eksperim. i Teor. Fiz. 30, 96 (1956) [Sov. Phys. JETP 3, 71 (1956)].

[19]*Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. I.

[20]J. K. Baird *et al.*, Phys. Rev. 179, 1285 (1969); W. Dress *et al.*, *ibid.* 170, 1200 (1968).

[21]E. W. Barnes, Proc. London Math. Soc. 4, 291 (1906).

[22]Reference 19, pp. 27–31.

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Spontaneous Breakdown and Hadronic Symmetries*

K. Bardakci and M. B. Halpern
Department of Physics, University of California, Berkeley, California 94720
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Using the ideas of Higgs and Weinberg about spontaneous symmetry breakdown, we construct possibly renormalizable models of low-spin hadrons, including massive non-Abelian vector mesons. An intriguing new view of hadron symmetries and symmetry breaking emerges.

I. INTRODUCTION

Some time ago, Higgs[1] pointed out that gauge vector mesons can acquire a mass through spontaneous symmetry breakdown (SSB) *without* the appearance of corresponding Goldstone mesons. Shortly thereafter, Weinberg[2] proposed a fascinating model of the leptons based on these ideas. The interest in this model has further increased with the recent claims by 't Hooft[3] and Lee[4] that such theories may be renormalizable.

Our purpose in this paper is to extend such ideas to hadron physics. We have succeeded in constructing models in which, *after* SSB, we are left with an unbroken global U(n) [or U(n)$\otimes$U(n)] realized through equal-mass vector mesons (and certain scalars). The global U(n)$\otimes$U(n) can further be spontaneously broken by adding ordinary Goldstone pions. There is also an intriguing mechanism for extrinsic symmetry breakdown (ESB) – i.e., symmetry breaking occurs explicitly in the Lagrangian – in the models.

The plan of the paper is as follows: In Sec. II, we establish the minimum number of scalar mesons needed to raise (by SSB) a U(n) multiplet of gauge vector mesons leaving either (a) no zero-mass "photons" at all, or (b) just one "photon." The answers are: For case (a), n complex fundamental representations (CFR) of scalar mesons (the so-called "n scheme"); for case (b), $n-1$ (CFR) (the so-called "$n-1$ scheme"). Though we will concentrate mainly on the n scheme, the two schemes will eventually be shown to be closely related.

Interestingly enough, the n scheme is large enough so that, if we choose, we may maintain an *exact* global U(n) *after* SSB. We choose then these symmetric models for our first Lagrangian realization in Sec. III leaving various modes of further symmetry breaking until later. The mechanism is as follows. Before SSB, the models exhibit both a local *and* a global U(n), say, $[\mathrm{U}(n)]_L \otimes [\mathrm{U}(n)]_G$. After SSB, only one U($n$), the final global "product" group $[\mathrm{U}(n)]_F = [\mathrm{U}(n)]_L \oplus [\mathrm{U}(n)]_G$ survives as a symmetry.

Section IV contains an equivalent "Hermitian" formalism for the same models, and in that language, we discuss the extension to a final (after SSB) U(n)⊗ U(n) [U(3)⊗ U(3) is discussed explicitly]. Partial conservation of axial-vector current (PCAC) is also discussed here. Section V deals with extrinsic symmetry breakdown, and returns finally to the models of the $n-1$ scheme. Section VI contains our summarizing remarks and observations for future directions.

II. COUNTING SCALARS AND "CLASHING SCHEMES"

In this section, we want to discover how many complex fundamental representations (CFR) are necessary to raise the n^2 gauge mesons of a U(n) multiplet. As a by-product, we establish the criterion for models with just one "photon" left behind at zero mass.

We begin by counting the number of "photons" (zero-mass vector mesons) left behind in U(n) using just one CFR. Let $\xi = (\xi_1 \cdots \xi_n)$ be the most general vacuum expectation value of the CFR of U(n). Now, it is clear[5] that "photons" are $n \times n$ Hermitian matrices γ satisfying

$$\gamma\xi = 0, \quad \xi^\dagger\gamma = 0\,. \tag{II.1}$$

We construct solutions for γ as follows: Let $\xi_{(i)}$, $i = 2, 3, \ldots, n$ be the $n-1$ vectors in the n-dimensional vector space, which, together with $\xi \equiv \xi_{(1)}$, form an orthogonal set,

$$\xi_{(i)}^\dagger \cdot \xi_{(j)} = 0 \quad \text{for} \quad i \neq j\,.$$

Then the $(n-1)^2$ matrices $\gamma^{(i,j)}$, $i, j = 2, 3, \ldots, n$, formed as follows,

$$\gamma^{(i,j)} \equiv \xi_{(i)}\xi_{(j)}^\dagger\,, \tag{II.2}$$

clearly satisfy Eq. (II.1). Though they are not Hermitian as they stand, they can easily be made so by symmetrizing and antisymmetrizing with respect to i and j. Therefore, there are $(n-1)^2$ photons in general.

With only one CFR, then, the scheme is workable at all only for SU(2) – being Weinberg's scheme – and for larger groups it fails since it results in several photons.

Thus, we turn to "clashing schemes," involving interference between the vacuum expectation values of two or more CFR's. Suppose we introduce m CFR's for U(n), with $m \leq n$, and let $\xi_{(i)}$ be the vacuum expectation value of ith CSR, with $1 \leq i \leq m$. The "photons" are the solutions of the following simultaneous equations:

$$\gamma\xi_{(i)} = 0\,, \quad 1 \leq i \leq m\,. \tag{II.3}$$

Taking the $\xi_{(i)}$'s linearly independent, the vectors that are orthogonal to each $\xi_{(i)}$ form an $(n-m)$-dimensional vector space, and we end up with $(n-m)^2$ photons. The construction and counting are simple generalizations of the special case $m = 1$ given earlier. To raise the masses of all n^2 vector mesons (no "photons" at all), we have to take $m = n$. We call this scheme the "n scheme."

Notice that, with n CFR's we have the option of beginning with a larger symmetry group than the minimal local U(n). In fact, in Sec. III, we construct models with a U(n)⊗U(n) group – one local and one global. Further, as mentioned in the Introduction, we will be able to choose our vacuum expectation values to maintain a single final global U(n).

As a by-product of our counting, we find that a *scheme with n – 1 CFR's* may be interesting in its own right. Evidently, only one photon is left behind in this case. Alternately, if the U(1) (nonzero-trace) vector meson is decoupled from the start in this scheme, then the remaining $n^2 - 1$ gauge mesons can all be raised. We call this scheme the "$n-1$ scheme," and note that Weinberg's theory is the case $n = 2$. Except for $n = 2$, these models must necessarily emerge (after SSB) *broken* SU(n) symmetric, and as we shall see in Sec. V, are quite closely related to the models of the n scheme.

III. REALIZATION OF $[\mathrm{U}(n)]_F$ MODELS

We introduce our n^2 gauge vector mesons in an $n \times n$ matrix notation

$$V_\mu \equiv \sum_{\alpha=1}^{n^2-1} \lambda^\alpha V_\mu^\alpha\,, \quad B_\mu \equiv \lambda^{(n^2)} V_\mu^{(n^2)}\,, \tag{III.1}$$

where the λ^α matrices transform according to the adjoint representation of SU(n), while

$$\lambda^{(n^2)} = \left(\frac{2}{n}\right)^{1/2} 1\,,$$
$$\mathrm{Tr}(\lambda^\alpha\lambda^\beta) = 2\delta^{\alpha\beta} \quad (\alpha, \beta \text{ run over } 1, \ldots, n^2)\,. \tag{III.2}$$

In this space the local gauge transformation matrices may be taken as

$$S(x) = \exp\left(i \sum_{\alpha=1}^{n^2-1} \kappa^\alpha(x)\lambda^\alpha\right)\,,$$
$$S_B(x) = \exp[i\kappa^{(n^2)}(x)]\,. \tag{III.3}$$

Under the gauge transformations

$$V_\mu \to S^{-1}V_\mu S - \frac{i}{g}S^{-1}\partial_\mu S\,,$$
$$B_\mu \to B_\mu - \frac{i}{g'}S_B{}^{-1}\partial_\mu S_B\,, \tag{III.4a}$$

the field tensors

$$F_{\mu\nu} \equiv \partial_\mu V_\nu - \partial_\nu V_\mu + ig[V_\mu, V_\nu]\,,$$
$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \tag{III.4b}$$

transform very simply:

$$F_{\mu\nu} \to S^{-1}F_{\mu\nu}S\,,$$
$$B_{\mu\nu} \to S_B{}^{-1}B_{\mu\nu}S_B\,.$$

Thus we form gauge-invariant structures via traces. For the pure vector-meson part of the Lagrangian, we have

$$\mathfrak{L}_V = -\tfrac{1}{8}\,\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \tfrac{1}{8}\,\mathrm{Tr}(B_{\mu\nu}B^{\mu\nu})\,. \tag{III.5}$$

Further, we introduce n CFR's of scalar mesons, also in an $n\times n$ matrix form

$$\phi_{ij} \equiv \phi_i^{(j)}\,, \quad i=1,\ldots,n\,; \quad j=1,\ldots,n \tag{III.6}$$

where (j) is the "name" or "number" of the CFR, and i is the internal-symmetry label within the multiplet. We emphasize this by recording that ϕ transforms *from the left* under the local gauge transformations

$$\phi \to (SS_B)^{-1}\phi, \quad \phi^\dagger \to \phi^\dagger(SS_B)\,. \tag{III.7}$$

Further, the covariant derivatives transform simply,

$$\Delta_\mu\phi \equiv (\vec{\partial}_\mu + igV_\mu + ig'B_\mu)\phi\,,$$
$$\Delta_\mu\phi^\dagger \equiv \phi^\dagger(\overleftarrow{\partial}_\mu - igV_\mu - ig'B_\mu)\,,$$
$$\Delta_\mu\phi \to (SS_B)^{-1}\Delta_\mu\phi\,,$$
$$\Delta_\mu\phi^\dagger \to \Delta_\mu\phi^\dagger(SS_B)\,. \tag{III.8}$$

Thus we *may* take the scalar-vector interaction simply as

$$\mathfrak{L}_{SV} = \tfrac{1}{2}\,\mathrm{Tr}(\Delta_\mu\phi^\dagger\Delta^\mu\phi)\,. \tag{III.9}$$

Finally, for reasons to be discussed, we choose the particular scalar-mass and self-interaction terms

$$\mathfrak{L}_S = +\tfrac{1}{2}m_0{}^2\,\mathrm{Tr}(\phi^\dagger\phi) - h_1[\mathrm{Tr}(\phi^\dagger\phi)]^2 - h_2\sum_{\alpha=1}^{n^2-1}\mathrm{Tr}(\phi^\dagger\phi\lambda^\alpha)\,\mathrm{Tr}(\lambda^\alpha\phi^\dagger\phi)\,, \tag{III.10}$$

thus completing our total Lagrangian $\mathfrak{L} = \mathfrak{L}_V + \mathfrak{L}_{SV} + \mathfrak{L}_S$.

$\mathfrak{L}$ has been constructed with a U(n) local gauge invariance, defined via (III.4a) and (III.7). We shall call this local group $[\mathrm{U}(n)]_L$. *Further,* however, we have chosen couplings to preserve an exact *global* U(n), called $[\mathrm{U}(n)]_G$, defined as *right-handed* transformations on ϕ:

$$\phi \to \phi(SS_B)\,, \quad \phi^\dagger \to (SS_B)^{-1}\phi^\dagger\,,$$
$$V_\mu \to V_\mu\,, \quad B_\mu \to B_\mu\,. \tag{III.11}$$

As remarked in the Introduction, it is the presence of this "extra" $[\mathrm{U}(n)]_G$ that allows us a "final" U(n) symmetry after spontaneous breakdown.

We choose our vacuum expectation values simply as

$$\phi(x) \to \phi(x) + \eta 1\,, \tag{III.12}$$

where η is a real number. In terms of CFR's, we are assigning a nonzero vacuum expectation value to one entry of each, namely, $\phi_i^{(j)} \to \phi_i^{(j)} + \delta_{ij}\eta$. The effect is obvious: Both $[\mathrm{U}(n)]_L$ and $[\mathrm{U}(n)]_G$ are broken, leaving only a *global* invariance under $[\mathrm{U}(n)]_F \equiv [\mathrm{U}(n)]_L \oplus [\mathrm{U}(n)]_G$, realized via *two-sided* transformations

$$V_\mu \to S^{-1}V_\mu S\,,$$
$$B \to B\,,$$
$$\phi \to (SS_B)^{-1}\phi(SS_B)\,,$$
$$\phi^\dagger \to (SS_B)^{-1}\phi^\dagger(SS_B)\,. \tag{III.13}$$

The resulting masses of the vector mesons reflect the $[\mathrm{U}(n)]_F$ symmetry: We find

$$m_V{}^2 = g^2\eta^2\,, \quad m_B{}^2 = (g')^2\eta^2\,. \tag{III.14}$$

Turning then to the scalar mesons, we find first the consistency relation $m_0{}^2 = 8\lambda^2 h_1$. The n^2 degrees of freedom $\frac{1}{2}i(\phi^\dagger - \phi)$ decouple completely, leaving the other n^2 degrees of freedom

$$M^\alpha \equiv \mathrm{Tr}[\lambda^\alpha\tfrac{1}{2}(\phi + \phi^\dagger)]\,, \quad \alpha = 1,\ldots,n^2\,. \tag{III.15}$$

The masses of these scalars come out

$$m_\alpha{}^2 = 16h_2\eta^2 \quad (\alpha = 1,\ldots,n^2-1)\,,$$
$$m_{(n^2)}{}^2 = 16h_1\eta^2\,, \tag{III.16}$$

and it is easy to verify that in fact, the $\alpha = 1,\ldots, n^2-1$ scalars transform as an adjoint representation of $[\mathrm{U}(n)]_F$ while $\alpha = n^2$ is a scalar.

Evidently, our final symmetry $[\mathrm{U}(n)]_F$ (after spontaneous breakdown) was only obtainable because we chose the extra symmetry $[\mathrm{U}(n)]_G$ to start. In fact, we can find other theories with a *broken* $[\mathrm{U}(n)]_F$ by beginning with a *broken* $[\mathrm{U}(n)]_G$. We shall return to this subject in Sec. V, after introducing a Hermitian formalism to describe our effect in the following Sec. IV.

IV. THE HERMITIAN REPRESENTATION

In Sec. III we saw that the $2n^2$ degrees of freedom in the CFR's broke up into two adjoint representations under $[\mathrm{U}(n)]_F$, only one of which couples. This suggests strongly that a more conventional Hermitian formalism exists, parallel to the ordinary σ model. We now proceed to develop such a formalism. For definiteness, we work with SU(3), and later discuss SU(3)⊗SU(3) in this formalism, but generalization to SU(n) etc. will be evident.

In this approach, we need enlarge the space of the λ^α matrices by the introduction of an additional two-dimensional space spanned by the Pauli matrices $\vec{\sigma}$, which commute with λ^α. In this larger space we represent $[\mathrm{U}(3)]_L \otimes [\mathrm{U}(3)]_G$ via

$$\begin{aligned} S_L &= \exp[i\kappa(x)\cdot\lambda\tfrac{1}{2}(1+\sigma_3)]\,,\\ S_G &= \exp[i\bar{\kappa}\cdot\lambda\tfrac{1}{2}(1-\sigma_3)]\,, \end{aligned} \tag{IV.1}$$

and similarly for the "ninth" transformations. We have called the (+) group "local" and the (−) group "global" because we intend having the vector mesons transform locally under (+), while being scalars under (−), as in Sec. III. Thus, we write

$$V_+^\mu \equiv \tfrac{1}{2}(1+\sigma_3)V^\mu\,,\quad B_+^\mu = \tfrac{1}{2}(1+\sigma_3)B^\mu\,, \tag{IV.2}$$

where V, B are defined as in Eq. (III.4a). For the scalar mesons, we take

$$M = M^{(1)}\sigma_1 + M^{(2)}\sigma_2\,, \tag{IV.3}$$

$$M^{(1),(2)} = \sum_{\alpha=1}^{9} \lambda^\alpha M_\alpha^{(1),(2)}\,, \tag{IV.4}$$

which in fact is the same 18 scalar degrees of freedom as in Sec. III. M transforms in a two-sided manner under both groups ($M \to S^{-1}MS$).

The Lagrangian can also be constructed along the lines of Sec. III:

$$\begin{aligned} L_V &= -\tfrac{1}{8}\,\mathrm{Tr}(F_{\mu\nu}^+ F_+^{\mu\nu}) - \tfrac{1}{8}\,\mathrm{Tr}(B_{\mu\nu}^+ B_+^{\mu\nu})\,,\\ L_{VS} &= \tfrac{1}{8}\,\mathrm{Tr}(\Delta_\mu M \Delta^\mu M)\,,\\ \Delta_\mu M &\equiv \partial_\mu M + ig[V_\mu^+, M] + ig'[B_\mu^+, M]\,,\\ L_{SS} &= +\tfrac{1}{8}m_0^2\,\mathrm{Tr}(M^2) - g_1\,\mathrm{Tr}(M^4)\,. \end{aligned} \tag{IV.5}$$

As in Sec. III, other terms can be added to L_{SS}. The total $L = L_V + L_{VS} + L_{SS}$ has the second-kind gauge invariance under $[\mathrm{U}(3)]_L$ and a global $[\mathrm{U}(3)]_G$.

Because of the local gauge freedom in $[\mathrm{U}(3)]_L$, we may now go to a gauge in which (effectively) either $M^{(1)}$ or $M^{(2)}$ can be set to zero. We will *choose* to eliminate $M^{(2)}$, and then let $M^{(1)}$ have the vacuum expectation value,

$$M \to M + \sigma_1\eta\,. \tag{IV.6}$$

Eliminating linear terms forces the usual relation

$$m_0^2 = 8g_1\eta^2 \tag{IV.7}$$

together with

$$m_V^2 = g^2\eta^2\,,\quad m_B^2 = (g')^2\eta^2 \tag{IV.8}$$

and so on for the scalar mesons, the details of which will be omitted for the "sake of *brevity*." The "final" global symmetry group is clearly $[\mathrm{U}(3)]_F = [\mathrm{U}(3)]_L \oplus [\mathrm{U}(3)]_G$. Thus, we have all the results of Sec. III in this more conventional notation.

It is instructive to notice in this formalism the relation of our vector-meson "lifting" mechanism to what goes on in the ordinary σ model[6]: If we imagine the correspondence, $\gamma_0' \sim \sigma_1$, $i\gamma_0'\gamma_5' \sim \sigma_2$, $\gamma_5' \sim \sigma_3$, where the "prime" indicates that these γ' matrices have nothing to do with ordinary γ matrices (or parity), then our model can be thought of as the SU(3)⊗SU(3) σ model in which a vacuum expectation value of σ is used to raise the $(1+\gamma_5')$ vector mesons, while the coupling to the $(1-\gamma_5')$ vector mesons is taken zero.

Now, we want to discuss extending our model to the case of a (more physical) *final* U(n)⊗U(n). The extension is perfectly straightforward, and can be done either in the CFR or the Hermitian formalism, via the introduction of a further doubling – associated with the actual (fermionic) γ_5. For definiteness, we discuss only the case of SU(3)⊗SU(3) in the Hermitian formalism.

For the vector mesons we need only write

$$V_\mu \to V_\mu + \gamma_5 A_\mu\,, \tag{IV.9}$$

where V_μ is the usual 3×3 vector matrix and A_μ is the corresponding 3×3 axial-vector matrix. Similarly, for the scalar mesons

$$M^{(1),(2)} \to M_S^{(1),(2)} + \gamma_5 M_P^{(1),(2)}\,, \tag{IV.10}$$

where S and P subscripts denote scalar and pseudoscalar. With this doubling in mind, we may keep the same form of the Lagrangian, now with traces over the full space including γ_5. We thus maintain a local $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_L$,

$$S_L = \exp[i\tfrac{1}{2}(1\pm\gamma_5)\kappa_\pm(x)\cdot\lambda\tfrac{1}{2}(1+\sigma_3)]\,, \tag{IV.11a}$$

and a global $[\mathrm{U}(3)\otimes\mathrm{U}(3)]_G$,

$$S_G = \exp[i\tfrac{1}{2}(1\pm\gamma_5)\bar{\kappa}_\pm\cdot\lambda\tfrac{1}{2}(1-\sigma_3)]\,. \tag{IV.11b}$$

In the special gauge we can eliminate (say) $M_S^{(2)}$ and $M_P^{(2)}$. After the vacuum expectation value

$$M \to M + \sigma_1\eta\,, \tag{IV.12}$$

we reach a model with degenerate vector and axial vectors $(1\cdots8)$, though the ninth vector and ninth axial vector can be split from the $\underline{8}$ and from each other. The remaining nine scalars and nine pseudoscalars can be taken as massive as we choose by adjusting g_1 (etc.). The final symmetry is in fact an exact (global) SU(3)⊗SU(3),

$$S_F = \exp[i\tfrac{1}{2}(1 \pm \gamma_5)\kappa_{\pm}\cdot\lambda], \qquad \text{(IV.13)}$$

and *not* a Goldstone realization.

We can spontaneously break the SU(3)⊗SU(3) by introducing another set of nine scalars and nine pseudoscalars, being exactly the $(3,\bar{3})\oplus(\bar{3},3)$ representation of the SU(3)⊗SU(3) model.[6] Thus, to M as above, we add an additional term,

$$M^{(3)} = \gamma_0\tfrac{1}{2}(1+\sigma_3)(M_S^{(3)} + i\gamma_5 M_P^{(3)}). \qquad \text{(IV.14)}$$

Now allowing $\langle M_S^{(3)}\rangle = \eta'$ as well, we can (in just the usual fashion) split vector and axial vectors etc., and $M_P^{(3)}$ end up the zero-mass Goldstone[7] pseudoscalars. Baryons can be given a mass in the usual way via the coupling $\psi^\dagger M^{(3)}\psi$ [where the baryons need transform only under $\frac{1}{2}(1+\sigma_3)$].

This mechanism leaves us then with a perfectly good Goldstone realization of SU(3)⊗SU(3), but with the extra nine scalars and nine pseudoscalars with (high) mass. One time, we had hoped that we could find a more economical scheme in which the only scalars and pseudoscalars remaining were the Goldstone multiplet, but to this date, we have not been able to do so, and now believe that the "extra" scalars are necessary for the approach to renormalizability.

V. SYMMETRY BREAKING

In this section we want to briefly discuss models with a *final broken* symmetry group. Suppose, e.g., we wanted a model of broken SU(3) symmetry. In the language of Sec. III, we could try taking a different set of vacuum expectation values: $\phi \to \phi + \eta(1+\epsilon\lambda_8)$. Certainly this (further) spontaneous breakdown induces SU(3) breaking among the vector mesons. Unfortunately, for *arbitrary* ϵ (Ref. 8), we find zero-mass scalar mesons (which couple). This is easy to understand physically: In raising the n^2 vector mesons via $\eta 1$, we have used all available "Higgs's phenomenon." Any further (perturbative) spontaneous breakdown around the symmetric solution proceeds via ordinary Goldstone particles. By itself, spontaneous breakdown is then an unsatisfactory approach to hadron symmetry breaking. In any case, we know that, in nature, hadron symmetry breaking in fact involves some extrinsic symmetry breaking (ESB) ($\partial_\mu J^\mu \neq 0$). Thus we turn to this topic.

The structure of our models allows very naturally for ESB through the "extra" global groups (here $[\mathrm{U}(3)]_G$). Still in the language of Sec. III, we could take, e.g., instead of (3.9), the vector-scalar interaction as

$$L_{SV} = \tfrac{1}{2}\,\mathrm{Tr}[(1+\epsilon\lambda_8)\partial_\mu\phi^\dagger\partial^\mu\phi], \qquad \text{(V.1)}$$

which keeps the local gauge invariance but extrinsically breaks the $[\mathrm{U}(3)]_G$. After $\phi\to\phi+\kappa 1$, the resulting $[\mathrm{U}(3)]_F = [\mathrm{U}(3)]_L \oplus [\mathrm{U}(3)]_G$ is thus also extrinsically broken. This mechanism can, if desired, be *combined* with similar "insertions" in L_S, and even with SSB $[\phi\to\phi+\kappa(1+\epsilon'\lambda_8)]$; some relations emerge[9] between various ϵ's used, but the scheme appears quite flexible.[10]

Once we recognize the need for symmetry breakdown to proceed at least in part extrinsically via the extra global groups, we are in a position to return to the "$n-1$ schemes" of Sec. II, and to see their relation with the "n schemes" explicitly discussed: These schemes can be realized with (say) an $(n)\times(n-1)$ matrix formalism[11] "$\phi_i^{(j)}$, $i=1,\ldots,n$, $j=1,\ldots,n-1$," quite analogous to Sec. III. Evidently they correspond to an extrinsic breaking of the global group, because the most natural starting point is $[\mathrm{U}(n)]_L\otimes[\mathrm{U}(n-1)]_G$. On top of this, a vacuum expectation value scheme such as

$$\langle 0|\phi_i^{(j)}(x)|0\rangle = \kappa\delta_{ij}, \quad j=1,\ldots,n-1$$

leaves $[\mathrm{U}(n)]_F$ extrinsically broken down to a global $\mathrm{U}(n-1)$.[12] The particle spectrum either has one photon or not depending on whether $g'\neq 0$ or $g'=0$. Unfortunately (for $n\geq 3$), the adjoint representations of $\mathrm{U}(n-1)$ come out higher in mass than the spinorial [e.g., K^* vector mesons below ρ mesons in SU(3)]. So, at least in these simple models, the $n-1$ schemes are not satisfactory.

VI. CONCLUDING REMARKS

In the preceding sections we have written down Lagrangians for vector mesons coupled to other hadrons. If the current criteria[3,4] for renormalizability are correct, then these models are renormalizable. Even if this is so, however, it is not clear that the models have immediate value for any numerical calculation. Using them as effective Lagrangians (tree graphs) is suspect because strong couplings are large. An interesting additional defect of the tree graphs of these models is the absence of a ρ-π-ω coupling (e.g., $K\bar{K}\to 3\pi$ vanishes).

This problem is familiar to gauge-field–effective-Lagrangian theorists. Such a coupling is of course generated through closed nuclear loops, or one can add explicit ($\rho\pi\omega$) terms to the Lagrangian. In our case, however, the renormalizability of the theory may be endangered. It is amusing to note that this problem is exactly that presently afflicting dual models with spin; and in the light of certain known analogies[13] between dual models and renormalizable field theories, one might even conjecture that the $\rho\pi\omega$ problem has a common origin in these diverse approaches.

In any case, for these reasons, and because we have no hadrons with spin greater than unity, we do not take our models seriously as complete hadron theories; we are, however, interested in the structure of the models, and, particularly, we are intrigued with the view of hadron symmetries that has emerged. It will be interesting, for example, to know the algebra of currents for our various models.

In a more ambitious line, we are struck by the appearance of our extra "global" groups, and are tempted to speculate about a "unified" theory of strong, electromagnetic, and weak interactions: For example, the baryons of our models transform as $\frac{1}{2}(1+\sigma_3)$; we are presently investigating the possibility that the starting lepton group (e.g., Weinberg's $[\mathrm{SU}(2)]_L \otimes Y$) may be a subgroup of our $\frac{1}{2}(1-\sigma_3)$ group. In this case, of course, the "global" group would be taken partly local, via the introduction of the W photon system.

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[1]P. W. Higgs, Phys. Letters 12, 132 (1964); Phys. Rev. Letters 13, 508 (1964); Phys. Rev. 145, 1156 (1966).

[2]S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

[3]G. 't Hooft, Phys. Letters 39B, 195 (1971).

[4]B. Lee, Phys. Rev. D 5, 823 (1972).

[5]G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble (unpublished).

[6]M. Gell-Mann and M. Lévy, Nuovo Cimento 16, 705 (1960); M. Lévy, *ibid.* 52A, 23 (1967).

[7]J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[8]The zero-mass scalar mesons can be eliminated, it turns out, for certain discrete values of ϵ, being, in this simple case, $\epsilon=\sqrt{3}\,(\frac{1}{2}, -1)$. Unfortunately, these solutions for $1+\epsilon\lambda_8$ are essentially projection operators in SU(3) space, and lead to one "photon" and one or more scalar tachyons, which are not acceptable. A moment's thought suffices to notice that, in fact, the vector-meson spectrum is that of the "$n-1$ scheme" for U(3), which, involving fewer scalars, avoids the tachyons. This is quite generally the case for "discretely broken" n schemes and their relation to "$n-1$ schemes."

[9]In the process of eliminating the linear terms induced by the spontaneous breakdown, certain relations emerge among the various ϵ's. In general, the situation is not overdetermined.

[10]It is by no means clear, however, whether we can get the symmetry-breaking results of certain popular schemes [e.g., M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968); see also, S. L. Glashow and S. Weinberg, Phys. Rev. Letters 20, 224 (1968)]. In the case of extrinsically broken SU(3)⊗SU(3), for example, we *cannot* add a term linear in σ_9, but must instead go through SSB *plus* "insertions" in the quartics, such as $\mathrm{Tr}(P_-M^2P_-M^2)$, $P_-\equiv 1+\epsilon\gamma_0\frac{1}{2}(1-\sigma_3)$.

[11]Alternately, the $n-1$ schemes can be realized via certain projection operators in the notation of the n schemes.

[12]This is true only for $n\geq 3$. In the special case of SU(2), i.e., Weinberg's scheme with $g'=0$, the resulting model (say of ρ mesons) is exact SU(2) symmetric. This case was also observed by G. 't Hooft, Nucl. Phys. B35, 167 (1971).

[13]J. Scherk (unpublished).