(to be published); V. N. Gribov and L. N. Lipatov, Ref. 15.

[17]Since there exists no nonperturbative way to solve field theory, one is forced always to identify leading terms before carrying out the sum over orders of perturbation theory.

[18]This multiplicity growth will not be altered by including subleading logarithms.

[19]That is to say, we sum also a selected class of nonleading logarithmic contributions. Gribov and Lipatov, Ref. 15, have by first summing over n_i covertly obtained the cancellation of the $\lambda \ln^2(Q^2/\mu^2)$ terms and have then gone on to sum (all) terms proportional to $\lambda \ln(Q^2/\mu^2)$. These latter (next-to-leading in our language) logarithms, aside from the $\lambda \ln(Q^2/\mu^2)\ln(1-x)$ terms which we keep, come from "ultraviolet" regions of integration and thus have a very different physical origin from the "infrared" logarithms which we consider. Summation of ultraviolet logarithms in both γ_μ and γ_5 field theories [for a discussion of this in the context of the elastic form factor see T. Appelquist and J. Primack, Phys. Rev. D 1, 1144 (1970)] seems to lead always to results which have a malevolent analytic behavior as in Ref. 15. On the contrary, infrared logarithms correspond to simple paths of momentum flow in diagrams and are well behaved when summed. See Ref. 1 for a discussion of momentum flow.

[20]See S. L. Adler and W. K. Tung, Phys. Rev. Letters 22, 978 (1969); R. Jackiw and G. Preparata, *ibid.* 22, 975 (1969).

[21]For a discussion of this point see Ref. 14.

[22]S. D. Drell, D. Levy, and T.-M. Yan, Phys. Rev. Letters 22, 744 (1969); Phys. Rev. 187, 2159 (1969); Phys. Rev. D 1, 1035 (1970). For a summary of this work see S. D. Drell and T.-M. Yan, Ann. Phys. (N.Y.) 66, 578 (1971).

[23]S. D. Drell and T. D. Lee, Phys. Rev. D 5, 1738 (1972).

[24]J. Stack, Phys. Rev. 164, 1904 (1967). See also J. Harte, *ibid.* 165, 1557 (1968); 171, 1825 (1968).

[25]K. Johnson, M. Baker, and R. Willey, Phys. Rev. 136, B1111 (1964); 163, 1699 (1967). For an efficient derivation of their results see S. L. Adler and W. A. Bardeen, Phys. Rev. D 4, 3045 (1971); 6, 734(E) (1972). The original formulation of this program is contained in M. Gell-Mann and F. E. Low, Phys. Rev. 95, 1300 (1954).

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Relation Between Nonlinear and Linear Realizations of SU(3)×SU(3): Theory and Applications*

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In an earlier paper we took the most general, nonlinear form for the action $[K_a, \pi_b]$ of chiral operators upon an octet of pseudoscalar-meson fields, and we developed functions z_α and $\bar{z}_\beta$ ($\alpha, \beta = 0, 1, 2, \ldots, 8$) which span the $(3,\bar{3})$ and $(\bar{3},3)$ representations of chiral SU(3) symmetry. Here we use the nonlinear properties of z_α to show that the 3×3 matrix $(\sum_0^8 \lambda_\alpha z_\alpha)$ is proportional to a unitary, unimodular matrix $\exp(i\sum_1^8 \omega_k \lambda_k)$. We then find that the matrix Λ which converts an arbitrary nonlinear field into a linear realization of SU(3)×SU(3) is a pure chiral transformation with ω_k as its parameters, $\Lambda = \exp(i\sum_1^8 \omega_k X_k)$. This result enables us to demonstrate the equivalence of different approaches to the theory of nonlinear realizations, and to construct a model for meson-baryon scattering. In the model, chiral symmetry is broken by a mass term which transforms as an admixture of singlet and octet members of the $(m,\bar{m})$ representation of SU(3)×SU(3). There are thirty parameters in the most general symmetry-breaking Lagrangian, but this number can be reduced to eight with reasonable assumptions. Unfortunately this is still too large a number for us to learn anything definitive about the manner of chiral symmetry breaking.

I. INTRODUCTION

In an earlier paper[1] we developed an SU(3) version of the σ model from pseudoscalar mesons which transform nonlinearly under chiral SU(3). That is, knowing the action of chiral operators upon an octet of meson fields, we constructed a set of functions of the fields so as to span the linear representations $(3,\bar{3})$ and $(\bar{3},3)$ of SU(3)×SU(3). From products of these functions we built up all representations of the type $(l,\bar{l})$, and then applied our results to the study of meson-meson scattering. In this paper we turn our attention to nonlinear fields other than pseudoscalar mesons, and to the associated problem of meson-baryon scattering.

Our main concern is to show how any field which transforms nonlinearly under chiral SU(3) can be converted to a linear realization of this group. For, once we know how to do this, we can construct effective Lagrangians with well-defined transformation properties and thereby study the general question of symmetry breaking. We took this approach to meson-meson scattering in our earlier paper,[1] and we were able to place definite limits upon the set of representations to which the symmetry-breaking Lagrangian may belong. In the case of meson-baryon scattering, however, we shall find the outcome less propitious.

To convert a nonlinear field into a linear realization, we use the fact that the functions[1] z_α and $\bar{z}_\alpha$ ($\alpha = 0, 1, 2, \ldots, 8$), which span the $(3, \bar{3})$ and $(\bar{3}, 3)$ representations, respectively, are not independent of one another. Instead they obey a set of bilinear equations of which[1]

$$z_0\bar{z}_0 + \sum_1^8 z_\alpha \bar{z}_\alpha = n$$

and

$$\sum_1^8 d_{\alpha\beta\gamma} z_\beta \bar{z}_\gamma + (\tfrac{2}{3})^{1/2}(z_0\bar{z}_\alpha + \bar{z}_0 z_\alpha) = 0 \tag{1.1}$$

are but two examples. One consequence of these equations is that the only representations of SU(3) $\times$SU(3) that we can construct from products of z_α and $\bar{z}_\beta$ are of the type $(l, \bar{l})$. Another consequence is that the 3×3 matrix

$$U = -\left(\frac{3}{2n}\right)^{1/2}\left(\sum_0^8 \lambda_\alpha z_\alpha\right) \tag{1.2}$$

is both unitary and unimodular.

These latter properties enable us to write U in exponential form:

$$U = \exp\left(i\sum_1^8 \omega_k \gamma_k\right) \tag{1.3}$$

and to prove a more general result about the real quantities ω_k ($k = 1, 2, \ldots, 8$), namely, that if the matrices B_i ($i = 1, 2, \ldots, 8$) form a representation (m) of SU(3), then the matrix elements of

$$U(B) = \exp\left(2i\sum_1^8 \omega_k B_k\right) \tag{1.4}$$

span the representation $(m, \bar{m})$ of SU(3)$\times$SU(3). Obviously U is a special case of $U(B)$ with $m = 3$ and $B_i = \frac{1}{2}\lambda_i$. With the aid of this result, we shall then demonstrate that the matrix which converts a nonlinear field into a linear realization is of the form

$$\Lambda = \exp\left(i\sum_1^8 \omega_k X_k\right), \tag{1.5}$$

where the X_k ($k = 1, 2, \ldots, 8$) are matrices representing the chiral operators of SU(3)$\times$SU(3).

Equations (1.2) and (1.3) serve as an implicit definition of the octet ω_k in terms of z_α, and hence in terms of the original pseudoscalar-meson fields π_i ($i = 1, 2, \ldots, 8$). From them we can develop all the results we need without knowing the explicit relation (if it exists) for ω_k in terms of π_i. For example, there is the property of covariance with respect to redefinitions of the meson field; we proved in our earlier paper[1] that z_α and $\bar{z}_\beta$ are covariant with respect to redefinitions of the meson field, and so it follows from Eqs. (1.2) and (1.3), that ω_k must behave in the same way.

The covariance of ω_k is important not only because it ensures that results derived from effective Lagrangians in the tree approximation are independent of the definition of the meson field, but also because it provides the link between different approaches to nonlinear realizations of chiral symmetry. Weinberg[2] develops the theory from the viewpoint of infinitesimal transformations, and he describes the action of chiral operators upon the meson field as a function of the meson field itself. Coleman, Wess, and Zumino,[3] on the other hand, use finite transformations as their starting point. Now it so happens that, in the Coleman-Wess-Zumino (CWZ) version of the theory, the operator which converts a nonlinear field into a linear one is a pure chiral transformation with parameters proportional to the meson fields:

$$\Lambda(\text{CWZ}) = \exp\left(i\sum_1^8 \xi_k X_k\right), \qquad \xi_k = \pi_k/F_\pi, \tag{1.6}$$

where F_π is the meson decay constant. Comparing the expression for Λ(CWZ) in Eq. (1.6) with the general result of Eq. (1.5), we see that the CWZ version can be obtained from the Weinberg version by choosing ω_k to be the redefined meson field.

Another approach to the nonlinear theory is that of Gürsey.[4] In it he starts with a 3×3 matrix which is some function of the meson field, and which transforms according to the $(3, \bar{3})$ representation of SU(3)$\times$SU(3). Thus Gürsey's matrix is equivalent to our matrix U in Eqs. (1.2) and (1.3), and the exact relationship between his meson field and our z_α or ω_k depends upon the way in which his matrix is parametrized. Again it is a matter of redefinition.

Given the linearizing matrix Λ of Eq. (1.5), we can now construct effective Lagrangians for the meson-baryon system using the standard techniques of linear representation theory. We divide the Lagrangian into two parts, one which preserves the chiral symmetry and another which breaks it. The chirally symmetric part consists of the kinetic terms for mesons and baryons, a meson-baryon interaction with derivative coupling, and an invariant baryon mass term; the symmetry-breaking part is assumed to consist only of noninvariant mass terms. With these assumptions the strength of the meson-baryon interaction can be expressed in terms of the meson decay constant via the Goldberger-Treiman relation.

The question of most interest concerns the symmetry-breaking part of the Lagrangian: Does it transform according to one single representation of SU(3)×SU(3), or is it an admixture of several different representations? Now, if we expand the noninvariant baryon mass term in powers of the meson field and assume the tree approximation to be valid, then the first term in the expansion will contribute only to baryon mass differences, and the next to meson-baryon scattering. Therefore we can, at least in principle, answer the question raised above by comparing the known mass differences and scattering lengths with the predictions of each representation.

This is the approach we applied to meson-meson scattering,[1] and it allowed us to say that the mass term could not belong to certain classes of representation. In the case of meson-baryon scattering, however, we cannot draw the same conclusion. The amount of data is small, and the number of different ways of forming a given representation from products of baryon, antibaryon, and meson fields is quite large; as a result we have more parameters to fit than conditions to impose on them. Under these circumstances, the most useful things we can do are to construct the most general Lagrangian which transforms according to the $(n, \bar{n})$ representation, and to indicate ways in which simple models might be developed.

The properties of the matrices U, $U(B)$, and Λ are derived in the second section, and meson-baryon scattering is analyzed in the third.

II. LINEARIZING THE NONLINEAR FIELD

We begin this section by developing the properties of the matrix U, and of the functions ω_k. Then, with the aid of these properties, we analyze the behavior of nonlinear fields under chiral transformations, and show that the linearizing matrix Λ is as given in Eq. (1.5). In conclusion we compare the approaches to nonlinear realizations of Weinberg,[2] and Coleman, Wess, and Zumino[3]; we also discuss the properties of covariant derivatives.

A. Properties of U and ω_k

From Eq. (1.2), we can write the product of U and its Hermitian adjoint as

$$UU^\dagger = \frac{3}{2n}\left(\sum_0^8 \lambda_\alpha z_\alpha\right)\left(\sum_0^8 \lambda_\beta \bar{z}_\beta\right)$$
$$= \frac{3}{2n}\left\{\sum_1^8 z_\alpha \bar{z}_\beta[(d_{\alpha\beta\gamma} + if_{\alpha\beta\gamma})\lambda_\gamma + \tfrac{2}{3}\delta_{\alpha\beta}I] + \tfrac{2}{3} z_0\bar{z}_0 I + (\tfrac{2}{3})^{1/2}\sum_1^8 (z_0\bar{z}_\alpha + \bar{z}_0 z_\alpha)\lambda_\alpha\right\}. \tag{2.1}$$

Using Eq. (1.1) together with the relation[1]

$$if_{\alpha\beta\gamma} z_\beta \bar{z}_\gamma = 0, \tag{2.2}$$

we then obtain

$$UU^\dagger = I. \tag{2.3}$$

The characteristic equation for any matrix $\sum_1^8 \lambda_k a_k$ is well known to be[5]

$$\det\left(\sum_1^8 \lambda_k a_k - \alpha I\right) = \tfrac{2}{3} d_{ijk} a_i a_j a_k + \alpha(a_k a_k) - \alpha^3 = 0. \tag{2.4}$$

With the particular choice

$$a_k = -\left(\frac{3}{2n}\right)^{1/2} z_k, \quad \alpha = \frac{z_0}{\sqrt{n}}, \tag{2.5}$$

we find that

$$\det U = \left[\left(\frac{3}{2n}\right)^{1/2}\right]^3 \{-\tfrac{2}{3} d_{ijk} z_i z_j z_k + (\tfrac{2}{3})^{1/2} z_0 (z_k z_k) - [(\tfrac{2}{3})^{1/2} z_0]^3\}. \tag{2.6}$$

Now, as we have shown elsewhere,[1] the products of the z_i satisfy the relations

$$d_{ijk} z_i z_j z_k = (\tfrac{2}{3})^{1/2}[2z_0^3 + 3\sqrt{n}\, z_0 \bar{z}_0 - (\sqrt{n})^3],$$
$$z_k z_k = 2(z_0^2 + \sqrt{n}\,\bar{z}_0), \tag{2.7}$$

and hence

$$\det U = 1. \tag{2.8}$$

Thus U is a unitary unimodular matrix and so can always be written in exponential form [see Eq. (1.3)].

Suppose that the chiral SU(3) group is generated by operators T_a and K_b ($a, b = 1, 2, \ldots, 8$) such that

$$[T_a, T_b] = [K_a, K_b] = if_{abc} T_c,$$
$$[T_a, K_b] = if_{abc} K_c. \tag{2.9}$$

The quantities z_α transform according to the $(3, \bar{3})$ representation, i.e.,

$$[T_a, z_b] = if_{abc} z_c,$$
$$[T_a, z_0] = 0,$$
$$[K_a, z_b] = d_{abc} z_c + (\tfrac{2}{3})^{1/2}\delta_{ab} z_0, \tag{2.10}$$
$$[K_a, z_0] = (\tfrac{2}{3})^{1/2} z_a,$$

and so U transforms according to the rules

$$[T_a, U] = [U, \tfrac{1}{2}\lambda_a],$$
$$[K_a, U] = \{U, \tfrac{1}{2}\lambda_a\}. \tag{2.11}$$

Now, because we can write U in the form of a finite SU(3) transformation [see Eq. (1.3)],

$$U = \exp[i2\omega_k(\tfrac{1}{2}\lambda_k)], \tag{2.12}$$

and because λ_a behaves as an octet tensor with respect to SU(3), we find that

$$U\lambda_a U^{-1} = \lambda_b[e^{2i\omega_k F_k}]_{ba}, \tag{2.13}$$

where F_k are the 8×8 matrices

$$[F_k]_{ij} = -if_{kij}. \tag{2.14}$$

Substituting Eq. (2.13) into Eq. (2.11), we obtain the expressions

$$[T_a, U] = (\tfrac{1}{2}\lambda_b)[e^{2i\omega\cdot F} - I]_{ba} U,$$
$$[K_a, U] = (\tfrac{1}{2}\lambda_b)[e^{2i\omega\cdot F} + I]_{ba} U \tag{2.15}$$

for the action of SU(3)×SU(3) operators upon the matrix U.

We can obtain other expressions for $[T_a, U]$ and $[K_a, U]$ from the Baker-Hausdorff lemma for any operator J_a:

$$e^{i\varphi\cdot B} J_a e^{-i\varphi\cdot B} = J_a + [i\varphi\cdot B, J_a] + \frac{1}{2!}[i\varphi\cdot B, [i\varphi\cdot B, J_a]] + \cdots, \tag{2.16}$$

where the matrices B_k ($k = 1, 2, \ldots, 8$) form a representation of SU(3), φ_k ($k = 1, 2, \ldots, 8$) are a set of functions, and $\varphi\cdot B \equiv \sum_1^8 \varphi_k B_k$. When J_a acts on the functions φ_k but not on the matrices B_k, we can use the commutation rule

$$[\varphi\cdot B, B_\beta] = if_{\alpha\beta\gamma}\varphi_\alpha B_\gamma = B_\gamma(\varphi\cdot F)_{\gamma\beta} \tag{2.17}$$

to rewrite Eq. (2.16) as

$$e^{i\varphi\cdot B} J_a e^{-i\varphi\cdot B} = J_a + B_\gamma \left(\frac{e^{i\varphi\cdot F} - I}{\varphi\cdot F}\right)_{\gamma\beta} [\varphi_\beta, J_a], \tag{2.18}$$

where

$$\left(\frac{e^{i\varphi\cdot F} - I}{\varphi\cdot F}\right)_{\gamma\beta} = i\delta_{\gamma\beta} + \frac{(i)^2}{2!}(\varphi\cdot F)_{\gamma\beta} + \frac{(i)^3}{3!}(\varphi\cdot F)^2{}_{\gamma\beta} + \cdots. \tag{2.19}$$

Multiplying both sides of Eq. (2.18) by $e^{i\varphi\cdot B}$ we obtain a general expression for the commutator of $e^{i\varphi\cdot B}$ with J_a:

$$[J_a, e^{i\varphi\cdot B}] = B_\gamma \left(\frac{e^{i\varphi\cdot F} - I}{\varphi\cdot F}\right)_{\gamma\beta} [J_a, \varphi_\beta] e^{i\varphi\cdot B}. \tag{2.20}$$

If we set $\varphi_k = 2\omega_k$ and $B_\gamma = \frac{1}{2}\lambda_\gamma$ in Eq. (2.20) and then take J_a to be successively T_a and K_a, we obtain alternative expressions for the commutators of U with the generators of SU(3)×SU(3). Comparing these expressions with Eq. (2.15), we conclude that

$$\left(\frac{e^{2i\omega\cdot F} - I}{2\omega\cdot F}\right)_{\gamma\beta} [T_a, 2\omega_\beta] = (e^{2i\omega\cdot F} - I)_{\gamma a}, \tag{2.21a}$$

$$\left(\frac{e^{2i\omega\cdot F} - I}{2\omega\cdot F}\right)_{\gamma\beta} [K_a, 2\omega_\beta] = (e^{2i\omega\cdot F} + I)_{\gamma a}. \tag{2.21b}$$

The first of these equations is equivalent to

$$[T_a, \omega_\beta] = (\omega\cdot F)_{\beta a} = if_{a\beta\gamma}\omega_\gamma \tag{2.22}$$

and it means that ω_β behaves as an octet with respect to SU(3). The second equation gives us an expression

$$[K_a, \omega_\beta] = \left(\frac{\omega\cdot F}{e^{2i\omega\cdot F} - I}\right)_{\beta\gamma} (e^{2i\omega\cdot F} + I)_{\gamma a} \tag{2.23}$$

for the action of the chiral operator upon ω_k; we shall analyze this expression in a later subsection, but for the moment we leave it in its symbolic form.

Although Eq. (2.21) has been derived from the $(3, \bar{3})$ properties of U in Eq. (2.15) together with the particular choice $B_\gamma = \frac{1}{2}\lambda_\gamma$ in Eq. (2.20), the result it describes does not depend on any particular SU(3) representation. We may therefore apply it to the general matrix $U(B) = e^{2i\omega\cdot B}$ in which the B_γ form an arbitrary representation (m) of SU(3) [see Eq. (2.17)]. We compute the action of T_a and K_a upon $U(B)$ by means of Eq. (2.20), and then, using Eq. (2.21) plus the octet transformation properties of B_γ [compare Eq. (2.13)], we find that

$$[T_a, U(B)] = [U(B), B_a],$$
$$[K_a, U(B)] = \{U(B), B_a\}. \tag{2.24}$$

Therefore the matrix elements of $U(B)$ belong to the $(m, \bar{m})$ representation of SU(3)×SU(3).

B. The Matrix Λ

Let us now consider a field Ψ which transforms linearly under SU(3) according to the representation C_a ($a = 1, 2, \ldots, 8$), and which is nonlinear under chiral transformations. In keeping with the work of Macfarlane, Sudbery, and Weisz (MSW),[6] we describe the action of SU(3)×SU(3) generators upon the components of Ψ as being

$$[T_a, \Psi_\alpha] = \Psi_\beta (C_a)_{\beta\alpha}, \qquad [K_a, \Psi_\alpha] = \Psi_\beta (v_{ab} C_b)_{\beta\alpha}, \tag{2.25}$$

where v_{ab} is a function of the meson field π_k such that[6]

$$\pi_a v_{ab} = 0, \qquad \Pi_a v_{ab} \equiv d_{akl}\pi_k\pi_l v_{ab} = 0. \tag{2.26}$$

From the Jacobi identity for T_a, K_b, and Ψ_α, we find that v_{ab} is a second-rank tensor with respect to SU(3):

$$[T_a, v_{bc}] = if_{abg}v_{gc} + if_{acg}v_{bg} \tag{2.27}$$

and from the Jacobi identity for K_a, K_b, and Ψ_α we learn that

$$[K_a, v_{bc}] - [K_b, v_{ac}] = if_{abc} - v_{ag} if_{cgk} v_{bk}. \tag{2.28}$$

To convert Ψ to the linear realization of SU(3) $\times$SU(3) spanned by matrices C_a and X_b, we look for a matrix Λ such that

$$[T_a, (\Psi\Lambda)_\alpha] = (\Psi\Lambda)_\beta (C_a)_{\beta\alpha}, \qquad [K_a, (\Psi\Lambda)_\alpha] = (\Psi\Lambda)_\beta (X_a)_{\beta\alpha}, \tag{2.29}$$

where

$$[C_a, C_b] = [X_a, X_b] = if_{abc} C_c, \qquad [C_a, X_b] = if_{abc} X_c. \tag{2.30}$$

From Eqs. (2.25) and (2.29) we see that Λ must satisfy the commutation rules

$$[T_a, \Lambda] = [\Lambda, C_a], \qquad [K_a, \Lambda] = \Lambda X_a - v_{ab} C_b \Lambda. \tag{2.31}$$

If we multiply Eq. (2.31) by π_a and by Π_a, and then make use of Eq. (2.26), we get

$$[\pi_a K_a, \Lambda] = \Lambda(\pi_a X_a), \qquad [\Pi_a K_a, \Lambda] = \Lambda(\Pi_a X_a). \tag{2.32}$$

This suggests that Λ should be of the form

$$\Lambda = \exp[i\eta_a X_a], \tag{2.33}$$

where η_a is some linear combination of π_a and Π_a:

$$\eta_a = h_1\pi_a + h_2\Pi_a. \tag{2.34}$$

Because π_a and Π_a are octet vectors, we can show from the Baker-Hausdorff lemma [see Eq. (2.16)] that this form for Λ satisfies the T_a commutator of Eq. (2.31) for all choices of the functions h_1 and h_2. Our problem therefore is to find the choice that also satisfies the K_a commutation rule.

Since Eq. (2.31) must hold for all representations of SU(3)$\times$SU(3), we consider the special case in which C_a and X_b span the $(m, \overline{m})$ representation. This means that

$$C_a^+ \equiv \tfrac{1}{2}(C_a + X_a) = B_a \otimes I, \qquad C_a^- \equiv \tfrac{1}{2}(C_a - X_a) = I \otimes B_a, \tag{2.35}$$

where I is a unit matrix, B_a are the matrices of Eq. (2.17), and $\overline{B}_a$ is the negative transpose of B_a:

$$\overline{B}_a = -B_a^t. \tag{2.36}$$

The appropriate basis vectors are $\chi_i^{(+)} \otimes \chi_j^{(-)}$ where

$$C_a^+\chi_i^{(+)} = \chi_j^{(+)}(B_a)_{ji}, \quad C_a^-\chi_i^{(+)} = 0, \qquad C_a^+\chi_i^{(-)} = 0, \quad C_a^-\chi_i^{(-)} = \chi_j^{(-)}(\overline{B}_a)_{ji}. \tag{2.37}$$

Now the reason for choosing this representation is that it contains a state

$$|S\rangle = \sum_i \chi_i^{(+)} \otimes \chi_i^{(-)} \tag{2.38}$$

which behaves as a singlet with respect to SU(3), i.e.,

$$C_a|S\rangle = 0 = \langle S|C_a. \tag{2.39}$$

Therefore, if we take matrix elements of Eq. (2.31) between $\langle S|$ and $|j,i\rangle \equiv \chi_i^{(+)} \otimes \chi_j^{(-)}$, and observe that the operators T_a and K_a do not affect these states, we find from Eqs. (2.37) and (2.39) that

$$\begin{aligned}[T_a, \langle S|\Lambda|j,i\rangle] &= \langle S|\Lambda|j,k\rangle (B_a)_{ki} - (B_a)_{jk}\langle S|\Lambda|k,i\rangle, \\ [K_a, \langle S|\Lambda|j,i\rangle] &= \langle S|\Lambda|j,k\rangle (B_a)_{ki} + (B_a)_{jk}\langle S|\Lambda|k,i\rangle.\end{aligned} \tag{2.40}$$

In other words $\langle S|\Lambda|j,i\rangle$ transforms according to the $(m, \overline{m})$ representation of SU(3)$\times$SU(3).

By direct computation, we get from Eqs. (2.33), (2.35), and (2.37) the result

$$\begin{aligned}\langle S|\Lambda|j,i\rangle &= \Big(\sum_k \chi_k^{(+)}\otimes\chi_k^{(-)}, \exp[i\eta_a(C_a^+ - C_a^-)]\chi_i^{(+)}\otimes\chi_j^{(-)}\Big) \\ &= \Big(\sum_k \chi_k^{(+)}\otimes\chi_k^{(-)}, \exp(i\eta_a C_a^+)\chi_i^{(+)}\otimes\exp(-i\eta_a C_a^-)\chi_j^{(-)}\Big) \\ &= \sum_k \Big(\chi_k^{(+)}, \chi_l^{(+)}[\exp i\eta_a B_a]_{li}\Big)\otimes\Big(\chi_k^{(-)}, \chi_n^{(-)}[\exp(-i\eta_a\overline{B}_a)]_{nj}\Big)\end{aligned}$$

$$= [\exp(i\eta_a B_a)]_{ki} \otimes [\exp(-i\eta_a \overline{B}_a)]_{kj}$$

$$= [\exp(2i\eta_a B_a)]_{ji} , \qquad (2.41)$$

where we have used Eq. (2.36) for $\overline{B}_a$. Comparing Eqs. (2.40) and (2.41) with (2.24) we see that

$$\eta_a = \omega_a . \qquad (2.42)$$

Thus the matrix Λ is

$$\Lambda = \exp(i\omega_a X_a) . \qquad (2.43)$$

Knowing the general form for Λ in Eq. (2.43), we can find the quantities v_{ab} by considering the special case of Eqs. (2.31) and (2.33) in which C_a and X_b correspond to the $(m, 0)$ representation of SU(3) $\times$SU(3), i.e.,

$$C_a \equiv B_a , \quad X_b = B_b ,$$
$$C_a^+ = B_a , \quad C_a^- = 0 . \qquad (2.44)$$

From these equations we get

$$[K_a, e^{i\omega \cdot B}] = e^{i\omega \cdot B} B_a - v_{ab} B_b e^{i\omega \cdot B} . \qquad (2.45)$$

We evaluate the commutator on the left-hand side of Eq. (2.45) by means of Eqs. (2.20), (2.23), and the octet transformation properties of B_γ [compare Eq. (2.13)]. Comparing the result with the right-hand side of Eq. (2.45), we find that

$$v_{ab} = \left(\frac{e^{i\omega\cdot F} - I}{e^{i\omega\cdot F} + I} \right)_{ba} = -v_{ba} . \qquad (2.46)$$

The antisymmetry of v_{ab} is a consequence of the antisymmetry of the matrices F_k.

To show that v_{ab} satisfies the conditions of Eqs. (2.27) and (2.28), we treat it as an element of a matrix v and rewrite Eq. (2.46) as

$$(I + e^{i\omega\cdot F}) v = I - e^{i\omega\cdot F} . \qquad (2.47)$$

From this expression we find that

$$[J_a, v] = -\tfrac{1}{2}(I + v)[J_a, e^{i\omega\cdot F}](I + v) \qquad (2.48)$$

for any operator J_a. When $J_a \equiv T_a$, we have

$$[T_a, e^{i\omega\cdot F}] = [e^{i\omega\cdot F}, F_a] \qquad (2.49)$$

[see Eq. (2.24)] and so, since

$$(I + v) e^{i\omega\cdot F} = I - v , \qquad (2.50)$$

it follows that

$$[T_a, v] = [v, F_a] . \qquad (2.51)$$

When $J_a \equiv K_a$, we use Eq. (2.45) with B_a set equal to F_a in order to obtain one expression for the commutator $[K_a, e^{i\omega\cdot F}]$. We then use the octet transformation properties of F_a [compare Eq. (2.13)] and the definition of v to recast this expression in the form

$$[K_a, e^{i\omega\cdot F}] = \tfrac{1}{2}\{e^{i\omega\cdot F}, F_a\} - \tfrac{1}{2}[e^{i\omega\cdot F}, F_\beta v_{\beta a}] . \qquad (2.52)$$

Substituting Eq. (2.52) into Eq. (2.48) and applying Eq. (2.50), we obtain

$$[K_a, v] = -\tfrac{1}{2}\{[v, F_\beta v_{\beta a}] + F_a - v F_a v\} . \qquad (2.53)$$

Equation (2.28) now follows immediately from this result.

C. Relation Between Different Approaches to Nonlinear Realizations

We observed in the Introduction that the linearizing matrix for the CWZ[3] theory of nonlinear SU(3) is a pure chiral transformation with parameters proportional to the meson fields [see Eq. (1.6)]. In the Weinberg[2] theory, as we have just shown [see Eq. (2.43)], these parameters are given by the octet vector ω_k of Eqs. (1.5), (2.21), and (2.22). Since the latter theory is covariant with respect to redefinitions of the meson field, we can study the relation between it and the CWZ theory by defining $F_\pi \omega_k$ to be the pseudoscalar meson field (F_π is the meson decay constant).

The starting point of the Weinberg theory is an expression for the action of chiral operators upon the meson field[6]:

$$[K_a, \pi_b] = i[f\delta_{ab} + b d_{abc}\pi_c + c d_{abc}\Pi_c + G\pi_a\pi_b + S\pi_a\Pi_b + T\pi_b\Pi_a + J\Pi_a\Pi_b] . \qquad (2.54)$$

Of the seven functions $f, b, \ldots, J$ of the variables $X = \pi_a\pi_a$ and $Y = d_{abc}\pi_a\pi_b\pi_c$, two are independent and the remaining five depend on them through either algebraic equations or differential ones. In our earlier work[1] we expressed the $(3, \overline{3})$ and $(\overline{3}, 3)$ quantities z_α and $\overline{z}_\beta$ in terms of the functions f, b, and c, and so our results apply to all forms of $[K_a, \pi_b]$. If we now identify $F_\pi\omega_k$ as the meson field, then by virtue of Eq. (2.23) for $[K_a, \omega_b]$, we are restricting ourselves to a particular choice for the right-hand side of Eq. (2.54).

To see exactly what this choice is we express $[K_a, \omega_\beta]$ in terms of the projection operators P_k associated with the seven distinct eigenvalues β_k of the matrix $(\omega \cdot F)$.[7] Six of these eigenvalues are nonzero, and their contribution to the commutator can be obtained directly from the right-hand side of Eq. (2.23): It is

$$-i\sum_{k=1}^{6}\beta_k\cot\beta_k P_k\,. \tag{2.55}$$

The contribution from the eigenvalue zero ($\beta_0=0$) is obtained by multiplying both sides of Eq. (2.23) by $(e^{2i\omega\cdot F}-I)/(\omega\cdot F)$ and using the known expression for this matrix as a fifth-order polynomial in $(\omega\cdot F)$. The resulting coefficient for P_0 is

$$W_0=-2ix\Big/\sum_{l=1}^{3}\frac{(\sin 2x\alpha_l)\cos\zeta}{3\alpha_l(2\alpha_l+\cos\zeta)}, \tag{2.56}$$

where[7]

$$\begin{aligned} &x^2=\omega_a\omega_a,\quad y=d_{abc}\omega_a\omega_b\omega_c\,,\\ &\sin\zeta=\sqrt{3}\,y/x^3,\\ &\alpha_l=\cos\tfrac{1}{3}[\zeta+2l\pi],\quad l=1,2,3\,. \end{aligned} \tag{2.57}$$

Since the projection operators are themselves polynomials in $(\omega\cdot F)$, and since there are a number of identities relating powers of $(\omega\cdot F)$ to the kinds of tensor appearing in Eq. (2.54), we can now determine the expressions we want. In particular the functions f, b, c which correspond to the identification $\omega_\beta=\pi_\beta/F_\pi$ are

$$\begin{aligned} f&=-F_\pi\sum_{l=1}^{3}\frac{\alpha_l x(4\alpha_l+\cos\zeta)\cot(2\alpha_l x)}{6(2\alpha_l+\cos\zeta)},\\ b&=\sum_{l=1}^{3}\frac{y(\cos\zeta+3\alpha_l)\cot(2\alpha_l x)}{x^3\cos\zeta(2\alpha_l+\cos\zeta)},\\ c&=-\sum_{l=1}^{3}\frac{(4\alpha_l^2\cos\zeta+3\alpha_l-\frac{1}{2}\cos\zeta)\cot(2\alpha_l x)}{F_\pi x\cos\zeta(2\alpha_l+\cos\zeta)}\,. \end{aligned} \tag{2.58}$$

The other functions in Eq. (2.54) depend upon these three, and are of much less interest.

We can express the matrix v of Eqs. (2.46) and (2.47) in much the same way:

$$v=-i\sum_{k=1}^{6}\tan(\tfrac{1}{2}\beta_k)P_k\,. \tag{2.59}$$

The tensor form of this expression is

$$v_{ab}=-i\sum_{l=1}^{3}\frac{\tan(\frac{1}{2}x\alpha_l)}{3\cos\zeta(2\alpha_l+\cos\zeta)}\left\{\frac{1}{x}\left(4\alpha_l+2\cos\zeta+\frac{\cos^2\zeta}{\alpha_l}\right)(-if_{abc}\omega_c)+\frac{6}{x^3}(4\alpha_l+\cos\zeta)(-if_{amc}\omega_c d_{bmn}\phi_n)+\frac{6y\alpha_l}{x^5}(if_{abc}\phi_c)\right\}. \tag{2.60}$$

From the properties of f and d coefficients we can easily show that

$$\omega_a v_{ab}=\phi_a v_{ab}=0\,, \tag{2.61}$$

where $\phi_a=d_{abc}\omega_b\omega_c$.

Further insight into the relation between different approaches to nonlinear theory can be gained by studying the covariant derivative. It is well known that the space-time derivative $\partial_\mu\Psi$ of a nonlinear field does not follow the same chiral transformation law as does the original field itself; and so we have to find a function of $\partial_\mu\Psi$ which does follow the same law as Ψ. One way of doing this is to introduce the covariant derivative $D_\mu\Psi$, and another is to convert Ψ to a linear realization. We now show that they both lead to the same result.

Let us suppose that Ψ transforms according to the representation (l) of SU(3) which is spanned by matrices C_a [see Eqs. (2.25) and (2.30)]. We can then convert Ψ to the linear realization $(l,0)$ of SU(3) $\times$SU(3) by taking $X_a\equiv C_a$ and multiplying Ψ by $e^{i\omega\cdot C}$; similarly, we can convert Ψ to the realization $(0,l)$ by taking $X_a\equiv -C_a$ and multiplying by $e^{-i\omega\cdot C}$. Consequently the space-time derivatives $\partial_\mu(\Psi e^{i\omega\cdot C})$ and $\partial_\mu(\Psi e^{-i\omega\cdot C})$ transform according to the linear realizations $(l,0)$ and $(0,l)$, respectively.

Using this result together with Eqs. (2.29), (2.31), (2.43), and (2.46), we can show in a straightforward way that the quantities

$$\begin{aligned} D_\mu\Psi&=\tfrac{1}{2}\{[\partial_\mu(\Psi e^{i\omega\cdot C})]e^{-i\omega\cdot C}+[\partial_\mu(\Psi e^{-i\omega\cdot C})]e^{+i\omega\cdot C}\},\\ E_\mu\Psi&=\tfrac{1}{2}\{[\partial_\mu(\Psi e^{i\omega\cdot C})]e^{-i\omega\cdot C}-[\partial_\mu(\Psi e^{-i\omega\cdot C})]e^{+i\omega\cdot C}\} \end{aligned} \tag{2.62}$$

both obey the same chiral transformation law as Ψ [see Eq. (2.25)]. Furthermore, if we determine the derivatives of the exponential functions from the Baker-Hausdorff lemma [see Eq. (2.18)] we find that

$$\begin{aligned} D_\mu\Psi&=\partial_\mu\Psi+\Psi C_\gamma\left(\frac{\cos\omega\cdot C-I}{\omega\cdot C}\right)_{\gamma\beta}\partial_\mu\omega_\beta\,,\\ E_\mu\Psi&=\Psi(2i)C_\gamma\left(\frac{\sin\omega\cdot C}{\omega\cdot C}\right)_{\gamma\beta}\partial_\mu\omega_\beta\,, \end{aligned} \tag{2.63}$$

where the sine and cosine functions are determined in terms of their power series. A comparison of Eq. (2.63) with the results of Callan, Coleman, Wess, and Zumino[8] indicates that the $D_\mu\Psi$ defined above is exactly the same as their covariant derivative of Ψ provided that ω_β is identified with the meson field.

Since both Ψ and $E_\mu\Psi$ follow the same chiral transformation law, it follows that the expression multiplying Ψ in Eq. (2.63) must do likewise. If we consider the special case in which $C_a \equiv \frac{1}{2}\lambda_a$, then we can define the covariant derivative of ω_β through the equation

$$\tfrac{1}{2}D_\mu(\lambda_a\,\omega_a) = \tfrac{1}{2}\lambda_\gamma\left(\frac{\sin\frac{1}{2}\omega\cdot\lambda}{\frac{1}{2}\omega\cdot\lambda}\right)_{\gamma\beta}\partial_\mu\omega_\beta\,. \tag{2.64}$$

Multiplying by λ_a and taking the trace of both sides, we obtain

$$D_\mu\,\omega_a = \left(\frac{\sin\frac{1}{2}\omega\cdot\lambda}{\frac{1}{2}\omega\cdot\lambda}\right)_{a\beta}\partial_\mu\omega_\beta\,. \tag{2.65}$$

Again the result is equivalent to that of Callan, Coleman, Wess, and Zumino.[8]

The reason for this equivalence is not hard to find. In the standard theory we construct covariant Lagrangians and currents by coupling covariant derivatives with other fields through the appropriate SU(3) Clebsch-Gordan coefficients. In the linearized theory we first convert Ψ to a linear realization, and then couple its space-time derivatives to other fields using the standard techniques of linear representation theory. The outcome is that $\partial_\mu\Psi$ always appears in some combination of the $D_\mu\Psi$ and $E_\mu\Psi$ of Eq. (2.62).

III. THE MESON-BARYON LAGRANGIAN

We are now in a position to construct meson-baryon Lagrangians with well-defined properties under chiral transformations. As discussed in the Introduction, we assume that both the baryon kinetic and the meson-baryon interaction terms are symmetric, and that the symmetry is broken only by mass terms. We further assume that the mass term belongs to an $(m,\bar{m})$ representation of SU(3) $\times$SU(3) and consists of unitary singlets and octets. This ensures that the Gell-Mann–Okubo mass formula will be satisfied.

Since we have an octet of nonlinear baryon fields, the basic linear realizations to which we can convert them are the (8, 1) and the (1, 8). In the first case the SU(3)$\times$SU(3) matrices of Eq. (2.30) are given by $C_a = X_a = F_a$, and the linearized fields are

$$\begin{aligned}(8,1):\quad \psi_i &= \Psi_j(e^{i\omega\cdot F})_{ji}\\ &= (e^{-i\omega\cdot F})_{ij}\Psi_j\,,\\ \bar{\psi}_i &= \bar{\Psi}_j(e^{i\omega\cdot F})_{ji}\,.\end{aligned} \tag{3.1}$$

In the second case the appropriate matrices are $C_a = -X_a = F_a$, and the linearized fields are

$$\begin{aligned}(1,8):\quad \varphi_i &= \Psi_j(e^{-i\omega\cdot F})_{ji}\\ &= (e^{i\omega\cdot F})_{ij}\Psi_j\,,\\ \bar{\varphi}_i &= \bar{\Psi}_j(e^{-i\omega\cdot F})_{ji}\,.\end{aligned} \tag{3.2}$$

The second expressions for ψ_i and φ_i in these equations follow from the antisymmetry of the F matrices.

A. The Symmetric Lagrangian

It is a simple matter to construct chiral invariants from ψ_i and φ_i alone, but to construct them from ψ_i, φ_i, and the meson field, we require meson currents which belong to the (8, 1) and (1, 8) representations of the chiral group. In terms of the $(3,\bar{3})$ matrix U of Eqs. (1.2) and (1.3), these currents are

$$\begin{aligned}J_{\mu k}(8,1) &= \tfrac{1}{2}\,\mathrm{Tr}[U^\dagger\overleftrightarrow{\partial}_\mu U\lambda_k]\,,\\ J_{\mu k}(1,8) &= \tfrac{1}{2}\,\mathrm{Tr}[U\overleftrightarrow{\partial}_\mu U^\dagger\lambda_k]\,,\end{aligned} \tag{3.3}$$

where $A\overleftrightarrow{\partial}_\mu B$ stands for $A(\partial_\mu B) - (\partial_\mu A)B$. If we use Eq. (1.2) for U, we can express the currents in terms of z_α and $\bar{z}_\beta$; and if we use Eq. (1.3), we can express them in terms of ω_k. The latter expressions are very much like the operator E_μ of Eq. (2.63) with $C_a = F_a$.

In order for the chirally invariant Lagrangian to conserve parity, it must be symmetric with respect to the linearized fields ψ_i and φ_i. It therefore takes the form

$$-\mathcal{L}_{\rm INV} = \tfrac{1}{2}[\bar{\psi}_i(\gamma_\mu\partial_\mu + m_0)\psi_i + \bar{\varphi}_i(\gamma_\mu\partial_\mu + m_0)\varphi_i] + \tfrac{1}{2}(if_{ijk}\hat{F} + d_{ijk}\hat{D})[\bar{\psi}_i\gamma_\mu\gamma_5\psi_j\,J_{\mu k}(8,1) + \bar{\varphi}_i\gamma_\mu\gamma_5\varphi_j\,J_{\mu k}(1,8)]\,, \tag{3.4}$$

where m_0 denotes the symmetric contribution to the mass of the baryon octet, and $\hat{F}$ and $\hat{D}$ are the coupling constants associated with the usual F- and D-type couplings of mesons and baryons. From Eqs. (3.1) and (3.2), we see that the kinetic terms in the Lagrangian are of the form $\bar{\Psi}\gamma_\mu D_\mu\Psi$ where D_μ is the covariant derivative of Eq. (2.63) with $C_a = F_a$. Similarly, the meson-baryon interaction is proportional to $\bar{\Psi}\gamma_\mu E_\mu\Psi$.

The right-hand side of Eq. (3.4) is a complicated function of the meson field, but it is sufficient for our purposes to expand it as a power series in π_i and retain only terms of second or lower order. Since the Lagrangian is covariant with respect to redefinitions of π_i, a simple way of carrying out the expansion is to identify ω_k with the meson field:

$$\omega_k = \pi_k/F_\pi\,. \tag{3.5}$$

We then find that up to second order the symmetric Lagrangian is

$$-\mathcal{L}_{\rm INV}=\overline{\Psi}(\gamma_\mu\partial_\mu+m_0)\Psi+\frac{i}{2F_\pi^2}\overline{\Psi}_i\gamma_\mu(if_{iml})\Psi_m if_{ijk}\pi_j\partial_\mu\pi_k+\frac{1}{F_0}(if_{ijk}\hat{F}+d_{ijk}\hat{D})\overline{\Psi}_i\gamma_\mu\gamma_5\Psi_j\partial_\mu\pi_k\,. \quad (3.6)$$

B. The Symmetry-Breaking Mass Term

In order to construct the symmetry-breaking mass term, we find it convenient to express the SU(3) scalar contained in the $(m,\overline{m})$ representation [see Eqs. (2.24), (2.40), and (2.41)] as

$$\mathcal{S}(m,\overline{m})=\mathrm{Tr}[e^{2i\omega\cdot B}]\,. \quad (3.7)$$

The form of $\mathcal{S}(m,\overline{m})$ as a function of z_0 and $\overline{z}_0$, and the leading terms of its expansion as a power series in π_i have been given in our earlier paper.[1] From $\mathcal{S}(m,\overline{m})$ we can generate octets by commuting it with chiral operators:

$$M_a=[K_a,\mathcal{S}]=2\,\mathrm{Tr}[e^{2i\omega\cdot B}B_a],$$
$$\tilde{M}_a=d_{abc}[K_b,M_c]=4\,\mathrm{Tr}[e^{2i\omega\cdot B}d_{abc}B_bB_c], \quad (3.8)$$

and from these octets we obtain second-rank SU(3) tensors by commuting once or twice again:

$$N_{ab}=[K_a,M_b],$$
$$\tilde{N}_{ab}=[K_a,\tilde{M}_b], \quad (3.9)$$
$$L_{ab}=d_{amn}[K_m,\tilde{N}_{nb}]\,.$$

Further commutation gives rise to higher-rank tensors, but we shall not need them here.

Suppose now that we couple the meson tensors of Eqs. (3.7)–(3.9) with the linearized (8, 1) fields $\overline{\psi}_i$ and ψ_i of Eq. (3.1) in such a way as to form SU(3) scalars. Altogether there are ten such couplings:

$$S_0^{(1)}=\overline{\psi}_i\psi_i\,\mathcal{S}(m,\overline{m}),$$
$$S_0^{(2)}=\overline{\psi}_i(R_k)_{ij}\psi_jM_k, \quad (3.10a)$$
$$S_0^{(3)}=\overline{\psi}_i(R_k)_{ij}\psi_j\tilde{M}_k$$

and

$$\overline{\psi}_i[27_{ab}]_{ij}\psi_jN_{ab},$$
$$\overline{\psi}_i[27_{ab}]_{ij}\psi_jL_{ab}, \quad (3.10b)$$
$$\overline{\psi}_i[A_{ab}]_{ij}\psi_j\tilde{N}_{ab},$$

where R_k can be either F_k or D_k, and $[A_{ab}]$ represents the projection operators for the $\underline{10}$, $\overline{\underline{10}}$, and $\underline{27}$ representations contained in $\underline{8}\otimes\underline{8}$:

$$[27_{ab}]=R_{ab}-\tfrac{6}{5}d_{abc}D_c-\tfrac{1}{4}\delta_{ab}I,$$

$$[10_{ab}]=\{F_a,D_b\}-d_{abc}F_c+T_{ab}-\tfrac{2}{3}if_{abc}F_c,$$

$$[\overline{10}_{ab}]=\{F_a,D_b\}-d_{abc}F_c-T_{ab}+\tfrac{2}{3}if_{abc}F_c, \quad (3.11)$$
$$(R_{ab})_{ij}=\delta_{ai}\delta_{bj}+\delta_{bi}\delta_{aj},$$
$$(T_{ab})_{ij}=\delta_{ai}\delta_{bj}-\delta_{bi}\delta_{aj}\,.$$

Because they are SU(3) scalars, the terms in Eq. (3.10) must belong to an SU(3)×SU(3) representation of the type $(l,\overline{l})$; however, they are already contained in the Kronecker product

$$(8,1)\otimes(8,1)\otimes(m,\overline{m})\equiv(8\otimes8\otimes m,\overline{m}),$$

and so the only representation open to them is $(m,\overline{m})$. The fact that there are ten such scalars corresponds to the general result that the representation $\underline{m}$ occurs ten times in $\underline{8}\otimes\underline{8}\otimes\underline{m}$.

We can form another set of SU(3) scalars by replacing the (8, 1) fields $\overline{\psi}_i$ and ψ_j in Eq. (3.10) by the (1, 8) fields $\overline{\varphi}_i$ and φ_j of Eq. (3.2). The same argument as above shows that these φ-type scalars also belong to $(m,\overline{m})$. However, since they differ from the ψ-type scalars only in the relative signs of even- and odd-parity terms, they provide us with no new possibilities for the symmetry-breaking Lagrangian.

The ten terms of Eq. (3.10) give rise to ten possible SU(3)-preserving terms in the Lagrangian, and from each of them we can obtain two octets by using the commutators of Eq. (3.8). Thus the most general symmetry-breaking Lagrangian in our model contains thirty independent terms. This is much too large a number for us to make meaningful comparisons between theory and experiment, and so we must reduce it by making further assumptions. We therefore suppose that all baryon currents and their four divergences belong to SU(3) octets or singlets. Since these quantities are derivable from the Lagrangian, our assumption implies that only the five terms of Eq. (3.10a) and their associated octets can appear in the symmetry-breaking Lagrangian.

We denote the eighth components of the octets derived from the SU(3) singlets of Eq. (3.10a) by

$$S_8^{(i)}=[K_8,S_0^{(i)}],$$
$$\tilde{S}_8^{(i)}=d_{8ab}[K_a,[K_b,S_0^{(i)}]], \quad (3.12)$$

for $i=1,2,3$. Although the chiral operators

$$T_a^{\pm}=\tfrac{1}{2}(T_a\pm K_a) \quad (3.13)$$

act in general upon both the baryon and meson parts of $S_0^{(i)}$, the structure of these terms is such that the over-all action is upon the meson parts alone. In $S_0^{(1)}$ the $\bar{\psi}_i$ and ψ_j fields are combined into a chiral singlet, and in $S_0^{(2)}$ we have

$$[T_a^-, (\bar{\psi}R_n\psi)M_n] = (\bar{\psi}R_n\psi)[T_a^-, M_n], \tag{3.14}$$

simply because $\bar{\psi}_i$ and ψ_j both belong to the (8, 1) representation; furthermore, we also have

$$\begin{aligned}[T_a^+, (\bar{\psi}R_n\psi)M_n] &= -[T_a^-, (\bar{\psi}R_n\psi)M_n] \\ &= -(\bar{\psi}R_n\psi)[T_a^-, M_n],\end{aligned} \tag{3.15}$$

because $S_0^{(2)}$ is an SU(3) scalar. The same argument holds for $S_0^{(3)}$, and so in all three cases we obtain the octets of Eq. (3.12) merely by commuting the meson factors $\mathcal{S}$, M_n, and $\tilde{M}$ with the appropriate chiral operators.

Another partial simplification occurs when we reexpress the (8, 1) fields $\bar{\psi}_i$ and ψ_j in terms of the original baryon field Ψ_i [see Eq. (1)]. The factor $\bar{\psi}\psi$ of $S_0^{(1)}$ reduces to $\bar{\Psi}\Psi$, and $\bar{\psi}R_n\psi$ becomes

$$\begin{aligned}\bar{\psi}R_n\psi &= \bar{\Psi}e^{i\omega\cdot F}R_n e^{-i\omega\cdot F}\Psi \\ &= \bar{\Psi}R_l\Psi(e^{i\omega\cdot F})_{ln},\end{aligned} \tag{3.16}$$

because R_n transforms as an octet vector. When we combine this result with the expression for M_a in Eq. (3.8), we find that

$$\begin{aligned}(\bar{\psi}R_n\psi)M_n &= (\bar{\Psi}R_l\Psi)(e^{i\omega\cdot F})_{ln}2\,\mathrm{Tr}(e^{2i\omega\cdot B}B_n) \\ &= (\bar{\Psi}R_l\Psi)2\,\mathrm{Tr}[e^{2i\omega\cdot B}B_n(e^{-i\omega\cdot F})_{nl}] \\ &= (\bar{\Psi}R_l\Psi)2\,\mathrm{Tr}[e^{2i\omega\cdot B}e^{-i\omega\cdot B}B_l e^{i\omega\cdot B}] \\ &= (\bar{\Psi}R_l\Psi)2\,\mathrm{Tr}[e^{2i\omega\cdot B}B_l] \\ &= (\bar{\Psi}R_l\Psi)M_l,\end{aligned} \tag{3.17}$$

where we have used the octet transformation properties of B_n to reach the third line of Eq. (3.17). This result also holds for $(\bar{\psi}R_n\psi)\tilde{M}_n$, and it has the effect of letting us substitute Ψ for ψ throughout Eq. (3.10a).

Equation (3.16) can also be applied to the octets $S_8^{(i)}$ and $\tilde{S}_8^{(i)}$, but it does not lead to as simple a result as above. For example, in the case of $S_8^{(2)}$, we have

$$\begin{aligned}S_8^{(2)} &= \bar{\Psi}R_l\Psi(e^{i\omega\cdot F})_{ln}[T_8^+ - T_8^-, M_n] \\ &= \bar{\Psi}R_l\Psi(e^{i\omega\cdot F})_{ln}4\,\mathrm{Tr}(B_8 e^{2i\omega\cdot B}B_n) \\ &= (\bar{\Psi}R_l\Psi)4\,\mathrm{Tr}(B_8 e^{i\omega\cdot B}B_l e^{i\omega\cdot B}).\end{aligned} \tag{3.18}$$

The corresponding expressions for the other octets are

$$\begin{aligned}\tilde{S}_8^{(2)} &= (\bar{\Psi}R_l\Psi)8\,\mathrm{Tr}[\tilde{B}_8 e^{i\omega\cdot B}B_l e^{i\omega\cdot B}], \\ S_8^{(3)} &= (\bar{\Psi}R_l\Psi)8\,\mathrm{Tr}[B_8 e^{i\omega\cdot B}\tilde{B}_l e^{i\omega\cdot B}], \\ \tilde{S}_8^{(3)} &= (\bar{\Psi}R_l\Psi)16\,\mathrm{Tr}[\tilde{B}_8 e^{i\omega\cdot B}\tilde{B}_l e^{i\omega\cdot B}],\end{aligned} \tag{3.19}$$

where $\tilde{B}_l = d_{lbc}B_bB_c$.

As in the case of the symmetric Lagrangian so in the case of the symmetry-breaking one, we must expand up to second order in the meson field. We can do this by identifying ω_k with the meson field [see Eq. (3.5)] and evaluating the traces in Eqs. (3.7), (3.8), (3.18), and (3.19). Alternatively we can follow the approach of our previous paper[1] in which we made use of the differential equations for $\mathcal{S}(m, \bar{m})$ and its commutators. We find that the even-parity terms are proportional to

$$\begin{aligned}S_0^{(1)} &= \bar{\Psi}\Psi\left(1 - \tfrac{1}{8}m_2\frac{X}{F_\pi^2}\right), \\ S_0^{(2)} &= +\tfrac{3}{20}m_3\,\bar{\Psi}R_k\Psi\frac{\Pi_k}{F_\pi^2}, \\ S_0^{(3)} &= -\frac{m_2(2m_2+3)}{6m_3}S_0^{(2)},\end{aligned} \tag{3.20a}$$

$$\begin{aligned}S_8^{(1)} &= +\tfrac{3}{20}m_3\bar{\Psi}\Psi\frac{\Pi_8}{F_\pi^2}, \\ S_8^{(2)} &= \tfrac{1}{4}m_2\bar{\Psi}R_k\Psi\left\{\delta_{8k}\left[1 + \frac{X}{3F_\pi^2}(\tfrac{11}{20} - \tfrac{3}{10}m_2)\right] + \tfrac{1}{4}d_{8ka}\frac{\Pi_a}{F_\pi^2} + \frac{\pi_8\pi_k}{10F_\pi^2}(-2m_2 + \tfrac{1}{3})\right\}, \\ S_8^{(3)} &= -\frac{2m_3}{m_2}S_8^{(2)}\end{aligned} \tag{3.20b}$$

and

$$\tilde{S}_8^{(1)} = \frac{-m_2(2m_2+3)}{6m_3} S_8^{(1)},$$

$$\tilde{S}_8^{(2)} = S_8^{(3)}, \tag{3.20c}$$

$$\tilde{S}_8^{(3)} = \bar{\Psi} R_k \Psi \Big\{ \tfrac{1}{12} m_2(2m_2+3)\Big[\delta_{8k} + \frac{11}{180}\frac{X}{F_\pi^2}(-2m_2+3)\delta_{8k} + \frac{\pi_8\pi_k}{90F_\pi^2}(-2m_2+3) + d_{8kc}\frac{\Pi_c}{F_\pi^2}(\tfrac{2}{15}m_2+\tfrac{1}{4})\Big]$$
$$+ \tfrac{16}{30} m_3^2 \left(\frac{X}{24F_\pi^2}\delta_{8k} - \frac{\pi_8\pi_k}{3F_\pi^2} - \tfrac{1}{4} d_{8kc}\frac{\Pi_c}{F_\pi^2}\right)\Big\} .$$

In these expressions the quantities m_2 and m_3 are the Casimir eigenvalues

$$m_2 = \tfrac{1}{3}[\mu_1^2 + \mu_2^2 + (\mu_1+\mu_2)^2 + 6(\mu_1+\mu_2)],$$
$$m_3 = \tfrac{1}{9}(\mu_2-\mu_1)[(\mu_1+2\mu_2)(\mu_2+2\mu_1) + 9(\mu_1+\mu_2+1)], \tag{3.21}$$

associated with the representation $(m) \equiv (\mu_1, \mu_2)$ of SU(3).

C. Meson-Baryon Scattering

Notice that up to second order there are eight independent terms in Eq. (3.20), three singlets and five octets. This enables us to write the symmetry-breaking Lagrangian as

$$-\mathcal{L}_B = m_0 \Big\{ \bar{\Psi}\Psi\Big[\gamma\Big(1 - \frac{m_2 X}{8F_\pi^2}\Big) + \gamma'\frac{\Pi_8}{F_\pi^2}\Big] + \bar{\Psi}(\mu D_k + \nu F_k)\Psi\frac{\Pi_k}{F_\pi^2}$$
$$+ \bar{\Psi}(\mu' D_k + \nu' F_k)\Psi\Big[\delta_{8k}\Big(1 + \frac{X}{60F_\pi^2}(11-6m_2)\Big) + \frac{1}{4F_\pi^2}d_{8kc}\Pi_c + \frac{1}{10F_\pi^2}\pi_8\pi_k(-2m_2+\tfrac{1}{3})\Big]$$
$$+ \bar{\Psi}(\mu'' D_k + \nu'' F_k)\Psi\Big[\delta_{8k}\Big(1 + \frac{11X}{180F_\pi^2}(-2m_2+3)\Big) + \frac{\pi_8\pi_k}{90F_\pi^2}(-2m_2+3) + d_{8kc}\frac{\Pi_c}{F_\pi^2}(\tfrac{2}{15}m_2+\tfrac{1}{4})$$
$$+ \frac{8m_3^2}{5m_2(2m_2+3)}\Big(\frac{X\delta_{8k}}{6F_\pi^2} - \frac{4\pi_8\pi_k}{3F_\pi^2} - d_{8kc}\frac{\Pi_c}{F_\pi^2}\Big)\Big]\Big\} . \tag{3.22}$$

The mass term, namely,

$$m_0[\gamma\bar{\Psi}\Psi + \bar{\Psi}(\mu' D_8 + \nu' F_8)\Psi + \bar{\Psi}(\mu'' D_8 + \nu'' F_8)\Psi] \tag{3.23}$$

contains two independent octet contributions, and so we determine only the combinations $(\mu' + \mu'')$ and $(\nu' + \nu'')$ from the known mass differences in the $J^P = \frac{1}{2}^+$ octet. The coefficient γ of the singlet term is not determined because the symmetric Lagrangian also contains a mass term [see Eq. (3.6)].

Picking out the terms corresponding to πN and KN elastic scattering we find that

$$-\mathcal{L}_B(\pi N) = \frac{m_0}{F_\pi^2}(\bar{N}\cdot N)(\pi\cdot\pi)\Big\{ -\tfrac{1}{8}m_2\gamma + \frac{1}{\sqrt{3}}\gamma' - \tfrac{1}{6}(\mu - 3\nu) - \frac{1}{20\sqrt{3}}(-m_2+1)(\mu' - 3\nu')$$
$$- \frac{1}{20\sqrt{3}}\Big[1 - \tfrac{5}{3}m_2 + \frac{8m_3^2}{m_2(2m_2+3)}\Big](\mu'' - 3\nu'')\Big\} \tag{3.24}$$

and that

$$-\mathcal{L}_B(KN) = \frac{m_0}{F_\pi^2}(\bar{N}N)(K^\dagger K)\Big[-\tfrac{1}{4}m_2\gamma - \frac{1}{\sqrt{3}}\gamma' + \tfrac{1}{6}(\mu - 3\nu) + \frac{1}{10\sqrt{3}}(m_2 - \tfrac{9}{4})(\mu' - 3\nu') + \frac{1}{10\sqrt{3}}(m_2 - \tfrac{9}{4})(\mu'' - 3\nu'')\Big]$$
$$+ \frac{m_0}{F_\pi^2}(\bar{N}\vec{\tau}N)\cdot(K^\dagger\vec{\tau}K)\Big\{\tfrac{1}{2}(\mu+\nu) + \frac{1}{8\sqrt{3}}(\mu'+\nu') + \frac{1}{2\sqrt{3}}\Big[\frac{2m_2}{15} + \frac{1}{4} - \frac{8m_3^2}{5m_2(2m_2+3)}\Big](\mu''+\nu'')\Big\} . \tag{3.25}$$

These expressions depend upon three more parameters, γ', μ, and ν, in addition to those already appearing in the mass term. Measurements of the S-wave baryon-meson scattering lengths would provide us with four constraints – not enough both to determine the additional parameters, *and* to remove the uncertainties in the mass parameters. Thus, even with the maximum amount of data at hand, we would not be able to determine the precise form of the symmetry-breaking Lagrangian. Add to this the facts that KN scattering lengths are not well known[9] and that the $N^*(1238)$ resonance also contributes to πN scattering,[10,11] and it becomes readily apparent that we can say very little about the way in which chiral symmetry is broken without making even more assumptions than we made already.

One such assumption might be that the breaking occurs in $(m,\bar{m})$ with m a triangular representation. In this case either $\mu_1=0$, or $\mu_2=0$, and the Casimir eigenvalues of Eq. (3.21) are related by

$$\frac{8m_3^2}{m_2(2m_2+3)}=\tfrac{2}{3}m_2\,. \tag{3.26}$$

As a result, the octets $S_8^{(2)}$ and $\tilde{S}_8^{(3)}$ of Eqs. (3.20b) and (3.20c) are proportional to one another, and the parameters μ'' and ν'' in the Lagrangian of Eq. (3.22) become redundant. The particular choice in which (m) is the (3) representation ($\mu_1=1$, $\mu_2=0$) has been explored by several authors[11,12]; our general expression for $\mathcal{L}_{\mathrm{INV}}+\mathcal{L}_B$ reduces to theirs, but the parameters in it cannot be fixed until the experimental scattering lengths are more certain.

Another approach is to assume that the symmetry-breaking term is even under the Kuo[13] transformation. This serves to fix the ratio of SU(3) singlet to octet in the symmetry-breaking Lagrangian; in the $(3,\bar{3})$ case, for example, it leads to the famous combination $u_0-\sqrt{2}\,u_8$ of Gell-Mann, Oakes, and Renner.[14] We hope to explore this possibility in a subsequent paper.

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[1]S. P. Rosen and A. McDonald, Phys. Rev. D 4, 1833 (1971).

[2]S. Weinberg, Phys. Rev. 166, 1568 (1968).

[3]S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2239 (1969).

[4]F. Gürsey, Nuovo Cimento 16, 230 (1960); Ann. Phys. (N.Y.) 12, 91 (1961); P. Chang and F. Gürsey, Phys. Rev. 164, 1752 (1967).

[5]S. MacLane and G. Birkhoff, *Algebra* (MacMillan, New York, 1967), p. 316.

[6]A. J. Macfarlane, A. Sudbery, and P. Weisz, Proc. Roy. Soc. (London) A314, 217 (1970).

[7]S. P. Rosen, J. Math. Phys. 12, 673 (1971).

[8]C. G. Callan, S. Coleman, J. Wess, and B. Zumino, Phys. Rev. 177, 2247 (1969).

[9]S. Goldhaber *et al.*, Phys. Rev. Letters 9, 135 (1962); V. J. Stenger *et al.*, Phys. Rev. 134, B1111 (1964).

[10]For experimental evaluations of πN scattering lengths, see J. Hamilton and W. Woolcock, Rev. Mod. Phys. 35, 737 (1963); V. Samaranayake and W. Woolcock, Phys. Rev. Letters 15, 936 (1965); L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965); K. Raman, *ibid.* 164, 1736 (1967).

[11]For attempts to evaluate the N^* contribution to πN scattering see R. D. Peccei, Phys. Rev. 176, 1812 (1968); A. M. Harun-ar Rashid, Trieste Report No. IC/69/84, 1969 (unpublished); C. F. A. Pantin, Cambridge Report No. DAMTP 71/3 (unpublished).

[12]J. A. Cronin, Phys. Rev. 161, 1483 (1967); B. W. Lee, *ibid.* 170, 1359 (1968); A. J. Macfarlane and P. H. Weisz, Lett. Nuovo Cimento 2, 25 (1969); L. Turner, Nucl. Phys. B11, 355 (1969); J. Schechter, Y. Ueda, and G. Venturi, Phys. Rev. 177, 2311 (1969); J. Šoln, Phys. Rev. D 2, 2404 (1970); Y -M. P. Lam and Y. T. Lee, *ibid.* 2, 2976 (1970); K. Yoshida, University of Durham report (unpublished).

[13]T. K. Kuo, Lett. Nuovo Cimento 3, 803 (1970); Phys. Rev. D 2, 342 (1970).

[14]M. Gell-Mann, R. J. Oakes, and B. Renner, Phys. Rev. 175, 2195 (1968). It is interesting to note that Lee (Ref. 12) also considered the possibility of chiral SU(2) as a residual symmetry.