$$s\hat{f}_0(\alpha a) \equiv \int d\lambda\, e^{-i\lambda\alpha} f_0(\lambda/s)$$
$$= \hat{f}(\alpha,a) - \hat{f}_1(\alpha s) - s^{-1}\hat{f}_2(\alpha s)\cdots,$$

so that

$$\int_0^1 d\alpha\, e^{i\alpha\lambda} s\hat{f}_0(\alpha s) = f(\lambda,s) + O(e^{-s})$$
$$= f_0(\lambda/s) + O(s^{-1}g(\lambda/s)).$$

There is no problem from $\alpha = 0$ since small α's will be seen to be excluded from our integrals.

[10]We expect from smooth threshold behavior (Ref. 1) that $l \gtrsim 3$.

[11]We might also comment, in this connection, on the recent paper by R. Jaffe [Phys. Letters 37B, 517 (1971)], who claims that the matrix element for massive μ-pair production is not LC dominated in the parton model. Although we are certainly not using the parton model, we would like to point out that, even though no LC singularity is present in the parton-model matrix element, $\langle J(x)\, J(0)\rangle_P$, the LC does dominate even here in the sense that substitution of $x^2\langle J(x)\, J(0)\rangle_P$ for $\langle J(x)\, J(0)\rangle_P$ gives a less leading contribution. Our approach does not, in fact, differ from that of the parton model in the question of LC dominance (Factors like $e^{-ix^2 s}$ would be necessary to ruin LC dominance. The parton model gives no such factor), but rather because the parton model does not exhibit the Regge behavior (19) at the five-point function level.

[12]The result is somewhat dependent on the specific form (31) chosen for ψ. Writing $\psi(\beta,\beta') = \Phi(\beta+\beta', (\beta\beta')^{1/2})$, the general statement is $d\bar{\sigma}/dx \sim \Phi((\kappa)^{1/2}, (\kappa)^{1/2})$ and $d\bar{\sigma}/d\kappa \sim \Phi(\kappa, (\kappa)^{1/2})$. Thus the requirement that the sum variable $\beta+\beta'$ be at least as important as the product variable $(\beta\beta')^{1/2}$ gives the result stated in the text. More generally the powers of κ in the exponentials can be left as free parameters and fit to the data.

[13]R. Brandt, A. Kaufman, and G. Preparata (unpublished).

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Scale and Conformal Transformations of Currents and Tensor-Meson Dominance

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We use the infinite-momentum limit and single-particle saturation to investigate consequences for baryon matrix elements of equal-time commutators of the generators of scale and conformal transformations with currents and their divergences. We show that the root-mean-square tensor mass radii are the same for all members of the baryon octet. We use Regge theory to show the validity of the procedure in this case. The result implies that some of the baryon gravitational form factors $F_1^B(q^2)$ or $F_2^B(q^2)$ must be subtracted. On demanding the subtractions to be SU(3)-symmetric we obtain $G_2(fNN)/G_1(fNN) = -1$, and find the f/d ratios to be the same for the two couplings. This is in agreement with the phenomenological analyses of Schlaile and of Strauss.

I. INTRODUCTION

In this paper we investigate consequences for baryon matrix elements of the behavior of currents and their divergences under scale and conformal transformations.[1] We make use of the infinite-momentum limit[2] (IML) and single-particle saturation of the commutation relations of scale and conformal generators Q_D and K_0 with currents and their divergences to show that the root-mean-square tensor mass radii are the same for all members of the nucleon octet. When combined with the usual tensor-meson-dominance (TMD) assumptions[3-5] these relations require some of the baryon gravitational form factors $F_1^B(q^2)$ and $F_2^B(q^2)$ to be subtracted. On demanding the subtractions to be SU(3)-symmetric we obtain $G_2(fNN)/G_1(fNN) = -1$, and find the f/d ratios in the two couplings to be the same.

In deriving our results we make use of the fact that the dimension of the time component of the currents J_μ^a, which are the vector V_μ^a or the axial-vector currents A_μ^a, is three. As is well known this follows from Gell-Mann's charge algebra if J_0^a has a dimension. Alternatively the same result holds if a state $|A\rangle$ exists such that $\langle A|J|A\rangle \neq 0$ and if under conformal transformations $[K_0(0), \partial_\mu J_\mu(0)] = 0$. The proof[6] makes use of the connection[7] between the first-order Schwinger term in the equal-time commutator $[iT_{0m}(x), J_0(0)]$ and the second-order Schwinger term in $[iT_{00}(x), \partial_\mu J_\mu(0)]$ for any symmetric energy-momentum tensor $T_{\mu\nu}$, i.e.,

$$-\int d^3x\, x_n [iT_{0m}(x), J_0(0)] = -\tfrac{1}{2}\int d^3x\, x_m x_n [iT_{00}(x), \partial_\mu J_\mu(0)] + \delta_{mn} J_0(0) + \partial_\mu S_{\mu, mn}(0)\,. \quad (1.1)$$

In the above $S_{\mu, mn}$ denotes possible second-order Schwinger terms in $i[T_{00}(x), J_\mu(0)]$. The result now follows on choosing $T_{\mu\nu}$ to be the new and improved energy-momentum tensor[8] and on using the definition of Q_D and K_0 given in Eqs. (2.5) and (2.6).

With resonance saturation alone our method implies that the dimension of the space components of the currents is 3 and that of $\partial_\mu J_\mu$ is 2. In contrast to our other results, Regge arguments do indicate the invalidity of our procedure in this case. Indeed, even in free-field theories Z diagrams give a contribution[9] in the evaluation of baryon matrix elements of $[i Q_D, \partial_\mu J_\mu]$ in the IML.

In Sec. II we evaluate Eqs. (2.1)–(2.4) for baryon matrix elements in the IML. We use Regge theory to show the validity of the IML applied to the baryon matrix element of Eqs. (2.3), (2.4), and (2.1) for $\mu = 0$. In other words, Regge arguments confirm our result regarding the tensor mass radii of the baryon octet mentioned above, whereas the conclusions that the dimensions of J_k and of $\partial_\mu J_\mu$ are 3 and 2, respectively, are less reliable. In Sec. III we study the consequences for TMD of the tensor mass radii being the same for all members of the baryon octet. Section IV is devoted to discussion and conclusions.

II. SCALE AND CONFORMAL TRANSFORMATIONS OF CURRENTS AND THEIR DIVERGENCES

The transformation properties of currents and their divergences under scale and conformal transformations are given[1] by the equal-time commutators

$$i[Q_D(0), J^a_\mu(0)] = d_{(\mu)} J^a_\mu\,, \quad (2.1)$$

$$i[Q_D(0), \partial_\mu J^a_\mu(0)] = d\partial_\mu J^a_\mu\,, \quad (2.2)$$

$$i[K_0(0), J^a_\mu(0)] = 0\,, \quad (2.3)$$

$$i[K_0(0), \partial_\mu J^a_\mu(0)] = 0\,. \quad (2.4)$$

In the above equations Q_D and K_μ denote the scale and conformal charges defined in terms of the new and improved energy-momentum tensor[8] $T_{\mu\nu}$ by

$$Q_D = -\int d^3x\, x_\mu T_{0\mu}(x) \quad (2.5)$$

and

$$K_\mu = \int d^3x\, [x^2 T_{0\mu}(x) - 2x_\mu x_\nu T_{0\nu}(x)]\,. \quad (2.6)$$

The dimensions of J_μ and $\partial_\mu J_\mu$ are denoted by $d_{(\mu)}$ and d, respectively. Here, $d_{(0)} = 3$ as required by Gell-Mann's charge algebra or the considerations of Sec. I.

The baryon matrix elements of the currents J_μ may be written as

$$\langle B(\vec{p}\,') | V^a_\mu | B(\vec{p})\rangle = \bar{u}(\vec{p}\,')\left(i\gamma_\mu f^a_1(q^2) + \frac{P_\mu}{2M} f^a_2(q^2)\right) u(\vec{p}) \quad (2.7)$$

and

$$\langle B(\vec{p}\,') | A^a_\mu | B(\vec{p})\rangle = \bar{u}(\vec{p}\,')[i\gamma_\mu \gamma_5 g^a_1(q^2) + q_\mu \gamma_5 g^a_2(q^2)] u(\vec{p})\,, \quad (2.8)$$

where $P_\mu = (p' + p)_\mu$, $q_\mu = (p' - p)_\mu$, and a is an SU(3) index.

Using covariant normalization for the states we define

$$\langle B_i(\vec{p}\,') | T_{\mu\nu}(0) | B_i(\vec{p})\rangle = \bar{u}(\vec{p}\,')\left(\tfrac{1}{4}(i\gamma_\mu P_\nu + i\gamma_\nu P_\mu) F^{(i)}_1(q^2) + \frac{P_\mu P_\nu}{4M_B} F^{(i)}_2(q^2) + (q_\mu q_\nu - \delta_{\mu\nu} q^2) F^{(i)}_3(q^2)\right) u(\vec{p})\,, \quad (2.9)$$

where the gravitational mass form factors are normalized, by the energy and the angular momentum of the states, to be

$$F^{(i)}_1(0) = 1 \quad (2.10a)$$

and

$$F^{(i)}_2(0) = 0\,. \quad (2.10b)$$

We evaluate Eqs. (1)–(4) in the IML between baryon states $|B_i\rangle$ and $|B_j\rangle$. We write

$$C^{(1)}_\mu = \langle B_j | \{i[Q_D, J^a_\mu] - d_{(\mu)} J^a_\mu\} | B_i\rangle\,, \quad (2.11)$$

$$C^{(2)}=\langle B_j|\{i[Q_D,\partial_\mu J_\mu^a]-d\partial_\mu J_\mu^a\}|B_i\rangle \ , \tag{2.12}$$

$$C_\mu^{(3)}=\langle B_j|[K_0,J_\mu^a]|B_i\rangle \ , \tag{2.13}$$

$$C^{(4)}=\langle B_j|[K_0,\partial_\mu J_\mu^a]|B_i\rangle \ . \tag{2.14}$$

With single-particle saturation in the IML we obtain

$$i\langle B_j|[Q_D,\Omega(0)]|B_i\rangle=-{\sum_n}'\left[\frac{\partial}{\partial k_m}\left(\frac{1}{2k_0}\langle B_j(\vec{p}\,')|T_{0m}(0)|B_n(\vec{k})\rangle\langle B_n(\vec{k})|\Omega(0)|B_i(\vec{p})\rangle\right)_{\vec{k}=\vec{p}\,'}+\frac{\partial}{\partial k_m}\left(\frac{1}{2k_0}\langle B_j(\vec{p}\,')|\Omega(0)|B_n(\vec{k})\rangle\langle B_n(\vec{k})|T_{0m}(0)|B_i(\vec{p})\rangle\right)_{\vec{k}=\vec{p}}\right] \tag{2.15}$$

and

$$\langle B_j|[K_0(0),\Omega(0)]|B_i\rangle=-{\sum_n}'\left[\frac{\partial^2}{\partial k_m{}^2}\left(\frac{1}{2k_0}\langle B_j(\vec{p}\,')|T_{00}(0)|B_n(\vec{k})\rangle\langle B_n(\vec{k})|\Omega(0)|B_i(\vec{p})\rangle\right)_{\vec{k}=\vec{p}\,'}-\frac{\partial^2}{\partial k_m{}^2}\left(\frac{1}{2k_0}\langle B_j(\vec{p}\,')|\Omega(0)|B_n(\vec{k})\rangle\langle B_n(\vec{k})|T_{00}(0)|B_i(\vec{p})\rangle\right)_{\vec{k}=\vec{p}}\right] , \tag{2.16}$$

where $\Omega(0)$ is either $J_\mu^a(0)$ or $\partial_\mu J_\mu^a(0)$ and the prime on the summation over n indicates that the integration over the momentum $\vec{k}$ has already been performed.

With baryon octet intermediate states the commutators can be easily evaluated in the IML and we obtain

$$[\langle B_j(\vec{p}\,')|[iQ_D,J_\mu^a(0)]|B_i(\vec{p})\rangle]_{B_n}\underset{|\vec{p}|\to\infty}{\sim}3\langle B_j(\vec{p})|J_\mu^a(0)|B_i(\vec{p})\rangle \ , \tag{2.17}$$

$$[\langle B_j(\vec{p}\,')|[iQ_D,\partial_\mu J_\mu^a(0)]|B_i(\vec{p})\rangle]_{B_n}\underset{|\vec{p}|\to\infty}{\sim}2\langle B_j(\vec{p})|\partial_\mu J_\mu^a(0)|B_i(\vec{p})\rangle \ , \tag{2.18}$$

$$[\langle B_j(\vec{p}\,')|[K_0,J_\mu^a(0)]|B_i(\vec{p})\rangle]_{B_n}\underset{|\vec{p}|\to\infty}{\sim}8|\vec{p}|\langle B_j|J_\mu^a(0)|B_i\rangle[F_1'^{(i)}(0)-F_1'^{(j)}(0)+F_2'^{(i)}(0)-F_2'^{(j)}(0)] \ , \tag{2.19}$$

$$[\langle B_j(\vec{p}\,')|[K_0,\partial_\mu J_\mu^a(0)]|B_i(\vec{p})\rangle]_{B_n}\underset{|\vec{p}|\to\infty}{\sim}8|\vec{p}|\langle B_j|\partial_\mu J_\mu^a(0)|B_i\rangle[F_1'^{(i)}(0)-F_1'^{(j)}(0)+F_2'^{(i)}(0)-F_2'^{(j)}(0)] \ . \tag{2.20}$$

For estimating the contribution of higher intermediate states we use the manipulations suggested in Refs. 2, 6, and 10 to write

$$\langle B_j(\vec{k})|Q_D|B_i(\vec{p})\rangle=\frac{i}{k_0-p_0}\langle B_j(\vec{k})|T_{\mu\mu}(0)|B_i(\vec{p})\rangle\,\delta^3(\vec{k}-\vec{p}) \ , \tag{2.21}$$

$$\langle B_j(\vec{k})|K_0|B_i(\vec{p})\rangle=\frac{-2}{(k_0-p_0)^2}\langle B_j(\vec{k})|T_{\mu\mu}(0)|B_i(\vec{p})\rangle\,\delta^3(\vec{k}-\vec{p}) \ , \tag{2.22}$$

$$\langle B_j(\vec{k})|J_4^a|B_i(\vec{p})\rangle=\frac{-1}{k_0-p_0}\langle B_j(\vec{k})|\partial_\rho J_\rho^a(0)|B_i(\vec{p})\rangle\,\delta^3(\vec{k}-\vec{p}) \ . \tag{2.23}$$

We note that $\partial_\mu A_\mu^a$, $T_{\mu\mu}$, and $\partial_\mu V_\mu^b$ (for $b\neq 1,2,3,8$) are suitable interpolating fields for the pseudoscalar mesons M^a, the scalar meson $\epsilon(750)$ and the scalar κ mesons, respectively. It is now straightforward to relate the contributions from higher-mass states to the scattering amplitudes for the reactions

$$B_i+\begin{pmatrix}M^a\\ \kappa^b\end{pmatrix}\to B_j+\epsilon \ . \tag{2.24}$$

We write

$$\rho^+(W^2,q^2)=\sum_{n\neq i,j}\delta(W^2-M_n{}^2)\langle B_j(\vec{p})|T_{\mu\mu}(0)|B_n(\vec{p})\rangle\langle B_n(\vec{p})|\partial_\mu J_\mu^a(0)|B_i(\vec{p})\rangle \tag{2.25}$$

and

$$\rho^-(W^2,q^2)=\sum_{n\neq i,j}\delta(W^2-M_n{}^2)\langle B_j(\vec{p})|\partial_\mu J_\mu^a(0)|B_n(\vec{p})\rangle\langle B_n(\vec{p})|T_{\mu\mu}(0)|B_i(\vec{p})\rangle \ , \tag{2.26}$$

where $\rho^\pm$ are proportional to the discontinuities in the forward amplitudes for massless mesons at total c.m. energy W in the reactions (2.24) and the corresponding u-channel reactions.

Using Regge arguments for the reactions (2.24) it is possible to state whether the contribution to the commutators (2.11)–(2.14) from intermediate states with masses higher than M_i or M_j are finite in the IML. With $E=(W^2+p^2)^{1/2}$ we have

$$C_0^{*(1)}(p) = -\int dW^2 \frac{\rho^+(W^2,q^2)-\rho^-(W^2,q^2)}{2E(E-p_0)(E-p_0')}$$

$$\underset{|\vec{p}|\to\infty}{\sim} 2|\vec{p}|\int dW^2 \frac{\rho^+(W^2)-\rho^-(W^2)}{(W^2-M_i^2)(W^2-M_j^2)}\ , \tag{2.27}$$

$$C^{*(2)}(p) = -i\int \frac{dW^2}{2E}\left(\frac{\rho^-(W^2,q^2)}{E-p_0'}+\frac{\rho^-(W^2,q^2)}{E-p_0}\right)$$

$$\underset{|\vec{p}|\to\infty}{\sim} -i\int dW^2\left(\frac{\rho^+(W^2)}{W^2-M_j^2}+\frac{\rho^-(W^2)}{W^2-M_i^2}\right)\ , \tag{2.28}$$

$$C_0^{*(3)}(p) = 2\int \frac{dW^2}{2E}\left(\frac{\rho^+(W^2,q^2)}{(E-p_0')^2(E-p_0)}+\frac{\rho^-(W^2,q^2)}{(E-p_0')(E-p_0)^2}\right)$$

$$\underset{|\vec{p}|\to\infty}{\sim} 8|\vec{p}|^2\int \frac{dW^2}{(W^2-M_i^2)(W^2-M_j^2)}\left(\frac{\rho^+(W^2)}{W^2-M_j^2}+\frac{\rho^-(W^2)}{W^2-M_i^2}\right), \tag{2.29}$$

$$C^{*(4)}(p) = -2\int \frac{dW^2}{2E}\left(\frac{\rho^+(W^2,q^2)}{(E-p_0')^2}-\frac{\rho^-(W^2,q^2)}{(E-p_0)^2}\right)$$

$$\underset{|\vec{p}|\to\infty}{\sim} -4|\vec{p}|\int dW^2\left(\frac{\rho^+(W^2)}{(W^2-M_j^2)^2}-\frac{\rho^-(W^2)}{(W^2-M_i^2)^2}\right). \tag{2.30}$$

In Eqs. (2.27)–(2.30) the C^* are the contributions of higher-mass states to the commutators in Eqs. (2.11)–(2.14). Also we have interchanged the limit $|\vec{p}|\to\infty$ with the integral over W^2 in obtaining the final expressions. Now Regge theory indicates that in the limit $W^2\to\infty$ the asymptotic behavior of $\rho^\pm(W^2)$ is $W^{2\alpha_\pm(0)}$ where $\alpha_\pm(0)$ are the intercepts of the highest Regge trajectories contributing to the t channel in the reactions (2.24). The experimentally determined[11] values of $\alpha(0)$ for the pseudoscalar, the vector-meson and the axial-vector-meson octet trajectories which contribute in the t channel in these reactions range from $0\leq\alpha(0)\leq 0.5$. In the limit $W^2\to\infty$, with $\alpha(0)\leq 0.5$, we have

$$C_0^{*(1)}\sim\int dW^2(W^2)^{-1.5}\ , \tag{2.31}$$

$$C^{*(2)}\sim\int dW^2(W^2)^{-0.5}\ , \tag{2.32}$$

$$C_0^{*(3)}\sim\int dW^2(W^2)^{-2.5}\ , \tag{2.33}$$

$$C^{*(4)}\sim\int dW^2(W^2)^{-1.5}\ . \tag{2.34}$$

Thus the interchange of the limit $|\vec{p}|\to\infty$ with the integral over W^2 is justified for $C_0^{*(1)}$, $C_0^{*(3)}$, and $C^{*(4)}$. The fact that $C^{*(2)}$ does not converge for $W^2\to\infty$ indicates that conclusions we may draw about the dimension d from the saturation procedure for the commutator $[iQ_D, \partial_\mu J_\mu]=d\partial_\mu J_\mu$ would be unreliable. In fact an evaluation of z diagrams[9] shows that they do contribute in the IML to this matrix element.

For the space components of currents, $C_k^{*(1)}$ is expected[12] to have the asymptotic behavior of $C^{*(2)}$, and $C_k^{*(3)}$ that of $C^{*(4)}$; hence conclusions on $d_{(k)}$ which we may draw from the saturation procedure for $[iQ_D, J_k]=d_{(k)}J_k$ would be unreliable.

The most important contributions to the C^*'s will come from states with masses nearest to the nucleon octet.[13] Furthermore, the conformal and dilatation charges are isoscalars. Thus the Roper resonance $N'(1470)$ with $J^P=\frac{1}{2}^+$, the $Y_0^*(1405)$ with $J^P=\frac{1}{2}^-$, the $Y_1^*(1385)$ and the $\Xi^*(1530)$ belonging to the $J^P=\frac{3}{2}^+$ decuplet could give important contributions. In other words, we allow for one resonance in each channel of definite isospin and strangeness. While this is an acceptable approximation in the case of the strange-particle channels due to the next resonance being ~300 MeV higher in mass, it is less so in the nucleon channel since $N(1520)$ with $J^P=\frac{3}{2}^-$ and $N(1535)$ with $J^P=\frac{1}{2}^-$ are close in mass to the Roper resonance $N(1470)$. We shall later return to this point.

Consider the commutators $[K_0, A_0^a]=0$ and $[K_0, \partial_\mu A_\mu^a]=0$ for $a=1$, 2, 3, and 8. For external baryon states $|B_i\rangle=|B_j\rangle$ we note from Eqs. (2.19) and (2.20) that the baryon octet contributions to these commutators vanish. This implies that the sum of resonance contributions to the integrals $C_0^{*(3)}$ and $C^{*(4)}$ should be zero.

We thus obtain with one resonance in each isospin-strangeness channel

$$\frac{\rho^+(W^2)+\rho^-(W^2)}{(M_W^2-M_i^2)^3}=0 \tag{2.35}$$

and

$$\frac{\rho^+(W^2)-\rho^-(W^2)}{(M_W^2-M_i^2)^2}=0 \ . \tag{2.36}$$

Hence[14]

$$\rho^\pm(W^2)=0 \tag{2.37}$$

or

$$\langle B_i|T_{\mu\mu}(0)|B_i^*\rangle\langle B_i^*|\partial_\mu A_\mu^a|B_i\rangle=0 \ . \tag{2.38}$$

Since the resonances B_i^* are experimentally[13] observed to decay into πB_i and $G(\pi B_i B^*)$ are nonvanishing we conclude that the off-diagonal matrix elements of $T_{\mu\mu}$ are zero in our saturation scheme. Thus

$$G(N'N\epsilon)=0 \ , \tag{2.39}$$

$$G(\epsilon Y_0^*\Lambda)=0 \ , \tag{2.40}$$

$$G(\epsilon Y_1^*\Sigma)=0 \ , \tag{2.41}$$

$$G(\epsilon \Xi^*\Xi)=0 \ . \tag{2.42}$$

The vanishing of the baryon off-diagonal matrix elements of $T_{\mu\mu}$ is not unexpected since, for example, it also usually follows in Lagrangian models. The same result also follows if we consider a saturation scheme in which only states belonging to the 56-dimensional representation (viz. the $J^P=\frac{1}{2}^+$ octet and the $J^P=\frac{3}{2}^+$ decuplet) are included. Thus the resonance contribution to Eqs. (2.27)–(2.30) is expected to be negligible for all currents J_μ^a.

In the above saturation scheme the resonance contributions to the integral $C_0^{*(1)}$ vanish and the commutation relation $[iQ_D, J_0^a]=3J_0^a$ is verified for all currents.

An alternative way of deriving Eqs. (2.39)–(2.42) would be to use commutation relations (2.1) and (2.3).

We now wish to discuss the possibility of including not only $N_1=N(1470)$ but also $N_2=N(1520)$ and $N_3=N(1535)$ resonances in the saturation scheme. Since the mass differences among these resonances are less than their individual widths we may consider them to be degenerate in mass with the effective mass at ~1510 MeV. With this assumption, for $|B_i\rangle=|B_j\rangle$, from Eqs. (2.13) and (2.14) we obtain

$$\sum_{i,\,\mathrm{spin}}\langle N|T_{\mu\mu}|N_i\rangle\langle N_i|\partial_\mu A_\mu^a|N\rangle=0 \quad \text{for } a=1,2,3,8 \tag{2.43}$$

together with Eqs. (2.40), (2.41), and (2.42) as before. We can also make use of the commutator $[iQ_D, J_0^a]=3J_0^a$ for $|B_i\rangle=|N\rangle$ and $|B_j\rangle=|\Sigma\rangle$ or $|\Lambda\rangle$ to obtain

$$\sum_{i,\,\mathrm{spin}}\langle N|T_{\mu\mu}|N_i\rangle\left\langle N_i\left|\partial_\mu A_\mu^K\right|\binom{\Sigma}{\Lambda}\right\rangle=0 \tag{2.44}$$

and

$$\sum_{i,\,\mathrm{spin}}\langle N|T_{\mu\mu}|N_i\rangle\left\langle N_i\left|\partial_\mu V_\mu^K\right|\binom{\Sigma}{\Lambda}\right\rangle=0 \ . \tag{2.45}$$

The Eqs. (2.43)–(2.45) evidently have the solution $\langle N|T_{\mu\mu}|N_i\rangle=0$. In order to argue that this solution is unique we note that Eqs. (2.43)–(2.45) are 6 linear relations for the three unknowns $\langle N|T_{\mu\mu}|N_i\rangle$ with nonvanishing and otherwise unknown coefficients $\langle N_i|\partial_\mu J_\mu^a|B\rangle$. Excluding the possibility that all the relevant determinants vanish we then obtain the desired result, i.e., $\langle N|T_{\mu\mu}|N_i\rangle=0$.

From Eqs. (2.17), (2.18), (2.27), and (2.28), with $\rho^\pm=0$ in our saturation scheme, we have for the dimension of J_k^a and of $\partial_\mu J_\mu^a$

$$d_{(k)}=3 \tag{2.46}$$

and

$$d=2 \ , \tag{2.47}$$

respectively. Furthermore from Eqs. (2.19), (2.20), (2.29), and (2.30) we obtain

$$F_1'^{(i)}(0)+F_2'^{(i)}(0)=F_1'^{(j)}(0)+F_2'^{(j)}(0) \ . \tag{2.48}$$

This result states that the mean square tensor mass radii,

$$\langle r_i^2\rangle=-6\left(\frac{F_1'(0)+F_2'(0)}{F_1(0)+F_2(0)}\right) , \tag{2.49}$$

are the same for all members of the baryon octet. It should be noted that Regge theory supports our derivation of Eq. (2.48), but not of Eqs. (2.46) and (2.47).

III. CONSEQUENCES FOR TENSOR-MESON DOMINANCE

Equation (2.48) provides restrictions on tensor-meson dominance[3-5] of the tensor mass form factors $F_1(q^2)$ and $F_2(q^2)$ of Eq. (2.9).

We define

$$\langle f|T_{\mu\nu}|0\rangle=z_f\,\epsilon_{\mu\nu} \ , \tag{3.1}$$

$$\langle f'|T_{\mu\nu}|0\rangle=z_{f'}\epsilon_{\mu\nu} \ , \tag{3.2}$$

where the $f(1260)$ and $f'(1514)$ are the tensor mesons with singlet and octet components given by

$$f_0 = f\cos\theta - f'\sin\theta\,, \tag{3.3}$$

$$f_8 = f\sin\theta + f'\cos\theta\,. \tag{3.4}$$

Here θ is the mixing angle and

$$\tan\theta \cong 1/\sqrt{2}\,. \tag{3.5}$$

We define the tensor-meson coupling constants $G_1^{f\bar{B}B}$ and $G_2^{f\bar{B}B}$ by

$$\lim_{-q^2\to m_f^2}(m_f^2+q^2)\left\langle B(\vec{p}\,')\left|\frac{1}{z_f}T_{\mu\nu}\epsilon^{\mu\nu}\right|B(\vec{p})\right\rangle = \epsilon_{\mu\nu}\bar{u}_B(\vec{p}\,')\left(i(\gamma_\mu P_\nu+\gamma_\nu P_\mu)G_1^{f\bar{B}B}+\frac{P_\mu P_\nu}{4M_B}G_2^{f\bar{B}B}\right)u_B(\vec{p}) \tag{3.6}$$

with a similar expression for the f' meson coupling. For the coupling constants $G_i^{f\bar{B}B}$, for $i=1,2$, we assume SU(3) symmetry in the form

$$\begin{aligned}\sum_{B,T} m_T G_i^{T\bar{B}B}\bar{B}BT &\equiv G_i\sum_{B,T}C_i^T\bar{B}BT\\ &= G_i\{f[(2\phi_i)\bar{\Sigma}\Sigma+(2\phi_i-\tfrac{4}{3}\delta_i)\bar{\Lambda}^0\Lambda^0+(3\phi_i-\delta_i)\bar{N}N+(\phi_i-\delta_i)\bar{\Xi}\Xi]\\ &\quad -\sqrt{2}f'[(\phi_i-\delta_i)\bar{\Sigma}\Sigma+(\phi_i+\tfrac{1}{3}\delta_i)\bar{\Lambda}^0\Lambda^0+(2\phi_i)\bar{\Xi}\Xi]\}\,,\end{aligned} \tag{3.7}$$

where ϕ_i and $\delta_i = 1-\phi_i$ are the SU(3)-antisymmetric and -symmetric coupling parameters.

The f-f' mixing has been chosen such that, as required by experiment and in agreement with the quark model, the f' decouples from the nucleons.

In the TMD approximation, allowing for at most constant subtractions we write

$$F_i^B(q^2) = A_i^B + G_i z_f\left(\frac{C_i^B}{m_f(m_f^2+q^2)}+\frac{z_{f'}}{z_f m_{f'}}\,\frac{C_i'^B}{m_{f'}^2+q^2}\right)\,. \tag{3.8}$$

The normalization conditions $F_1(0)=1$ and $F_2(0)=0$ now read

$$1 = A_1^B+\frac{G_1 z_f}{m_f^3}\left[C_1^B+\frac{z_{f'}}{z_f}\left(\frac{m_f}{m_{f'}}\right)^3 C_1'^B\right] \tag{3.9}$$

and

$$0 = A_2^B+\frac{G_2 z_f}{m_f^3}\left[C_2^B+\frac{z_{f'}}{z_f}\left(\frac{m_f}{m_{f'}}\right)^3 C_2'^B\right]\,. \tag{3.10}$$

In addition, we have our results in Eq. (2.48):

$$\begin{aligned}\frac{G_1 z_f}{m_f^5}C_1^N+\frac{G_2 z_f}{m_f^5}C_2^N &= \frac{G_1 z_f}{m_f^5}\left[C_1^\Sigma+\frac{z_{f'}}{z_f}\left(\frac{m_f}{m_{f'}}\right)^5 C_1'^\Sigma\right]+(1\to 2)\\ &= \frac{G_1 z_f}{m_f^5}\left[C_1^{\Lambda^0}+\frac{z_{f'}}{z_f}\left(\frac{m_f}{m_{f'}}\right)^5 C_1'^{\Lambda^0}\right]+(1\to 2)\\ &= \frac{G_1 z_f}{m_f^5}\left[C_1^{\Xi}+\frac{z_{f'}}{z_f}\left(\frac{m_f}{m_{f'}}\right)^5 C_1'^{\Xi}\right]+(1\to 2)\,.\end{aligned} \tag{3.11}$$

Using Eqs. (3.7)–(3.11) we first show that not *both* $F_1^B(q^2)$ and $F_2^B(q^2)$ can obey unsubtracted dispersion relations for all B. Suppose both A_1^B and A_2^B are zero. Then from Eq. (3.9) we obtain

$$\frac{z_{f'}}{z_f} = -\left(\frac{m_{f'}}{m_f}\right)^3\tan\theta\,, \tag{3.12}$$

and when (3.12) is substituted in Eq. (3.11) we obtain

$$G_1 = -G_2 \tag{3.13}$$

and

$$\phi_1 = \phi_2\,. \tag{3.14}$$

On the other hand, from Eq. (3.10) with $A_2^B=0$, we note that either

$$G_2 = 0 \tag{3.15}$$

or

$$\phi_2 = \tfrac{1}{4}\,. \tag{3.16}$$

When the relation (3.15) or (3.16) is combined with Eqs. (3.13) and (3.14) it implies that $G_1(fNN)=0$, i.e., that the f meson decouples from the nucleons. This is however not acceptable and $A_1^B = A_2^B = 0$

should be excluded.

We now investigate the possibilities that either (i) $A_2^B=0$ or (ii) $A_1^B=0$.

(i) With $A_2^B=0$ we obtain from Eq. (3.10) that either $G_2=0$ or alternatively Eq. (3.12) holds and $\phi_2=\frac{1}{4}$. The latter possibility is again not allowed since together with Eqs. (3.11) it implies that the f meson decouples from the nucleons. If $G_2=0$ we get from Eqs. (3.11) that

$$\frac{z_{f'}}{z_f}=-\left(\frac{m_{f'}}{m_f}\right)^5\tan\theta\,, \tag{3.17}$$

and hence Eq. (3.9) implies that A_1^B must be present and that it depends on the SU(3) index B.

(ii) When $A_1^B=0$ we first note that Eq. (3.12) holds and hence Eqs. (3.11) can be solved to obtain $G_1=-G_2$ and $\phi_1=\phi_2$. In order to avoid the earlier contradiction that the f decouples from the nucleons, these relations imply that A_2^B is present and is SU(3)-symmetric. In particular, we have

$$G_2^{f\bar{N}N}=-G_1^{f\bar{N}N}\ . \tag{3.18}$$

Actually Eqs. (3.13), (3.14), and (3.18) follow from the weaker assumption that A_1^B is SU(3)-symmetric. They also follow if we allow for arbitrary polynomial subtraction in $F_1^B(q^2)$, whose first two coefficients are SU(3)-symmetric.

In comparing the two alternatives the one leading to Eq. (3.18) is the more attractive one since in this case the subtraction constants for both $F_1^B(q^2)$ and $F_2^B(q^2)$ are SU(3)-symmetric whereas the first possibility requires A_1^B to be SU(3)-dependent. The phenomenological analysis[15] for the ratio of the coupling constants $G_2^{f\bar{N}N}/G_1^{f\bar{N}N}$ is inconclusive. It however partially supports the result [Eq. (3.18)] that this ratio is -1. In the literature, an unsubtracted form of F_2^N has been assumed to obtain $G_2^{f\bar{N}N}=0$. Some authors have viewed this formula as a possible explanation of s-channel helicity conservation.[16] It is clear that the data by themselves neither completely exclude nor support $G_2^{f\bar{N}N}=0$. The reader should also notice that our present results in Eq. (3.11) allow for both F_1^N and F_2^N to be unsubtracted if SU(3)-dependent subtractions are present in *both* F_1^B and F_2^B.

We should also compare our present results with the analogous ones for mesons.[6] To this end notice that the power of $(m_{f'}/m_f)$ in the ratio $(z_{f'}/z_f)$ in Eqs. (3.12) and (3.17) depends on the SU(3) assumptions made in Eq. (3.7). With our choice [Eq. (3.7)] for the definition of the SU(3)-symmetric coupling constants we obtain Eq. (3.12). This together with the results of Ref. 6 implies a particular form of SU(3) for the meson coupling constants or SU(3)-dependent subtractions of second order in the meson mass form factor $F_1^M(q^2)$. If these subtractions are absent, we need G_{TMM}/m_T^2, in the notation of Ref. 6, to be SU(3)-symmetric.

IV. CONCLUSIONS

We have shown that single-particle saturation of the equal-time commutators of Q_D and K_0 with J_μ and $\partial_\mu J_\mu$ in the IML leads to relations for the tensor mass form factors. The analogy with the traditional calculations of Adler and of Weisberger for current algebra is apparent.[17]

We have used the standard relations that the time components of currents have dimension $d_{(0)}=3$ and the result that baryon octet intermediate states alone saturate the matrix elements of the commutator $[iQ_D, J_0]=3J_0$, to estimate the contributions of the higher resonances. With the inclusion of only the next highest resonances we show that the couplings of the nucleon octet through the ϵ meson to these higher resonances should vanish. It may be noted that this result is not unexpected since, for example, it also follows in usual generalizations of the σ model.[18]

We have shown that the IML is valid for the commutators (2.3), (2.4), and (2.1) for $\mu=0$. We have done this by using Regge theory for the reactions $B_i+\binom{M^a}{K^a}\to B_j+\epsilon$, to show that the forward "scattering amplitudes" for $B_i+J_0\to B_j+Q_D$ and $B_i+\binom{J_0}{\partial_\mu J_\mu}\to B_j+K_0$ are convergent in the Regge limit. This is not the case for the commutators (2.2) and (2.1) for $\mu=i=1,2,3$. Thus we expect the results that $d_{(i)}=3$ and $d=2$ to be less reliable than the one for the tensor mass form factors (2.48).[19] Within our saturation scheme we have shown that Eq. (2.48) follows from either $[K_0, J_0]=0$ or $[K_0, \partial_\mu J_\mu]=0$. The Eq. (2.48) states that all members of the baryon octet have the same tensor mass radius. In the TMD approximation this then requires the presence of subtractions in some of the form factors $F_1^B(q^2)$ or $F_2^B(q^2)$. With the assumption that these subtractions are SU(3)-symmetric we have then obtained $G_2(fNN)/G_1(fNN)=-1$, in agreement with the phenomenological analyses of Schlaile and of Strauss, and have found the f/d ratios in the two couplings to be the same.

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Inequalities for the Pion-Pion Partial Waves: General Considerations and New Inequalities*

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A class of inequalities for the pion-pion s and p waves has been discussed in a series of recent papers. The present work attempts to provide a systematic method for writing such inequalities. An infinite number of new inequalities for the pion-pion s and p waves are also derived.

I. INTRODUCTION

In previous work,[1-6] several inequalities were derived for the pion-pion s and p waves using the analyticity and positivity properties of the π-π scattering amplitude. These derivations were not all very systematic; in particular, no attempt was made in Refs. 1–3 to show that the inequalities were complete and independent or to suggest a methodical approach to the problem.[7] Their merit consisted in their simplicity. In the present work, we attempt to develop a general framework for a systematic derivation of all the independent inequalities. The point of view we adopt is in a certain sense complementary to that of Pennington.[6] While we find many useful results, we also feel that they are far from complete.

In Sec. II, we recall some of the positivity properties of the π-π partial waves proved by Martin,[8] Common,[9] and Yndurain.[10] The use of these positivity properties in conjunction with the crossing symmetry of the system leads to the partial-wave inequalities of our interest. The general discussion of these inequalities is facilitated by the two-