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Inclusive Processes at Large Mass*

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The combined assumptions of strongly convergent operator-product expansions near the light cone (LC) and pure multi-Regge theory are used to study the inclusive process $\sigma(p)+\sigma(p')\to\gamma(q)+$anything, where σ and γ are scalar particles, in the limit of large $s=(p+p')^2$, $\nu=p\cdot q$, $\nu'=p'\cdot q$, and $\kappa=q^2$. The leading LC singularity is used to obtain the behavior for $\kappa\to\infty$, with s/κ, ν/κ, and ν'/κ fixed. The results are not changed by including nonleading LC contributions. The result for large s/κ, ν/κ, ν'/κ with fixed ratio $\eta=\nu\nu'/s\kappa$ is made to agree with the large-κ fixed-η behavior of the Regge (pionization) limit of large s, ν, ν' and fixed η and κ. We find that the cross section $d\sigma/d\kappa$ is a sum of two different exponentially falling terms, one being the pionization contribution and the other being the fragmentation contribution.

The kinematical relevance of the light cone (LC) to the description of a class of inclusive processes at large mass follows from the general analysis of LC dominance we have previously given.[1] The processes in question are of the form $I\to J(q)+$anything, where I is some (in general multiparticle) initial state and $J(q)$ is a current of mass $\kappa\equiv q^2$ ($\kappa>0$ when the current is in the final state). Choosing at most three (and at least one) independent momenta p_i from I to define the energy variables $\nu_i=q\cdot p_i$ and $s_{ij}=(p_i+p_j)^2$, LC dominance is kinematically present in the A-limit $\kappa\to\infty$; ν_i/κ, s_{ij}/κ fixed.[1] In order to conclude from this kinematical dominance an actual "physical" dominance in the multiparticle cases, a certain amount of "smoothness" in the many-variable functional dependencies is required. It is our purpose here to study such questions for the simplest two-particle case by incorporating some conventional dynamical assumptions (mainly multi-Regge theory) into the LC formalism. Our analysis supports the existence of the necessary smoothness and therefore the physical relevance of the LC. The present work extends and corrects a similar investigation[2] carried out last year.

For simplicity, we will consider here in detail only the spinless (or spin-averaged) case. We thus take $J(x)$ to be a scalar current of dimension (assumed canonical) two. The amplitude for the above general process is then

$$W(\kappa, s_{ij}, \nu_i)=\int d^4x\, e^{iq\cdot x}\ {}_{\rm in}\langle I|J(x)J(0)|I\rangle_{\rm in}\,. \qquad (1)$$

Kinematically, in the A limit, we have $W(\kappa, s_{ij}, \nu)\to W_L(\kappa, s_{ij}, \nu)$, where W_L is obtained by substituting in (1) the matrix element of the leading LC singularity

$$J(x)J(0)\underset{x^2\to 0}{\sim}\frac{1}{x^2}\sum_n x^{\alpha_1}\cdots x^{\alpha_n}O_{\alpha_1\cdots\alpha_n}(0). \qquad (2)$$

Notice that W_L need not have the scale-invariant form $(1/\kappa)F(\kappa/s, \nu/s)$. Light-cone dominance is more subtle and the actual form of W_L depends on more involved dynamics, including non-scale-invariant purely hadronic effects.

The simplest, and first measured,[3] inclusive process of the above type is deep-inelastic electron-proton scattering: $p+\gamma(q)\to$ anything. Here the only energy variable is $\nu=q\cdot p$ and the canonical LC analysis[1] implies the simple Bjorken[4] scaling behavior

$$W_2(\kappa,\nu)\underset{A}{\sim}\frac{1}{\nu}F_2(2\nu/\kappa). \qquad (3)$$

The Regge limit (R-limit: $\nu\to\infty$, κ fixed) is also expected to be simple:

$$W_2(\kappa,\nu)\underset{R}{\sim}\nu^{\alpha-2}\beta(\kappa). \qquad (4)$$

There are strong theoretical reasons[1] to expect these two limits to commute in the sense that the A limit for large ν/κ is the same as the R limit for large κ:

$$\lim_{\nu/\kappa\to\infty}\frac{1}{\nu}F_2(2\nu/\kappa)=\lim_{\kappa\to\infty}\nu^{\alpha-2}\beta(\kappa). \qquad (5)$$

This seems to be confirmed experimentally.[3] Similar commutativity relations are expected to hold in the many-variable cases.[1] In this paper, α will always be the Pomeranchukon intercept at $t=0$

(presumably $\alpha = 1$).

The amplitude for the process $[p + p' \to \gamma(q) + \text{anything}]$ of interest to us is

$$W(\kappa, s, \nu, \nu') = \int d^4x\, e^{iq\cdot x}\ {}_{\text{in}}\langle pp'|J(x)J(0)|pp'\rangle_{\text{in}}. \qquad (6)$$

The variables are

$$\kappa = q^2, \quad s = (p+p')^2, \qquad \nu = p\cdot q, \quad \nu' = p'\cdot q, \qquad (7)$$

and we have taken $p^2 = p'^2 = 1$. This is the simplest off-shell multiparticle inclusive process and there is even some experimental information[5] [for protons into (spin-1) photons] which we shall comment on later. The physical region is given by the inequalities

$$\nu \geq 0, \quad \nu' \geq 0, \qquad \nu + \nu' \leq \tfrac{1}{2}(s+\kappa), \quad \nu\nu' \geq \tfrac{1}{4}s\kappa, \qquad (8)$$

the last one following from the relation

$$\nu\nu' = \tfrac{1}{4}s(\kappa + \kappa_\perp), \quad \kappa_\perp = q_1{}^2 + q_2{}^2, \qquad (9)$$

valid in the c.m. (or lab) system. We will also use the scaling variables

$$\omega = \kappa/2\nu, \quad \omega' = \kappa/2\nu', \quad \eta = \nu\nu'/s\kappa. \qquad (10)$$

Let us first consider the Regge limits of (6). Last year[2] we suggested an analysis in terms of double-Regge-pole exchange. Motivated by this, Mueller[6] has recently given a double O(2, 1) analysis of (6). To state his results, we must distinguish between two Regge limits:

Pionization Limit (P): $\nu, \nu', s \to \infty$; $\nu\nu'/s$ fixed,

Fragmentation Limit (F): $\nu, s \to \infty$; ν/s, ν' fixed.

There is also the F' limit, which is the same as the F limit but with ν and ν' interchanged. The behavior of (6) in these limits is

$$W \underset{P}{\sim} s^\alpha \beta(\nu\nu'/s\kappa, \kappa) \qquad (11)$$

and

$$W \underset{F}{\sim} s^\alpha \beta(\nu/s, \nu', \kappa). \qquad (12)$$

These will be our R limits which generalize (4) to the present process. Note the exhibited scaling in the sense that in (11) β depends on only the ratio $\nu\nu'/s$ and in (12) β depends on only ν/s. In (11) we have used the dimensionless variable $\nu\nu'/s\kappa = \eta$ which is constant in both the P and A limits. The power α is expected to be the same as occurs in the 2-body process.[6]

We consider next the A limit.

Asymptotic Limit (A): $\kappa, \nu, \nu', s \to \infty$; $\nu/\kappa, \nu'/\kappa, s/\kappa$ fixed.

The behavior of (6) in this limit is determined by (3) to be

$$W \underset{A}{\sim} \int d^4x\, e^{iq\cdot x} \frac{1}{x^2} f(x\cdot p, x\cdot p'; s), \qquad (13)$$

where f is given by the power series

$$F(\lambda, \lambda'; s) = \sum_n \sum_{i=1}^n f_{in}(s)\lambda^i \lambda'^{n-i} \qquad (14)$$

determined by the matrix elements

$${}_{\text{in}}\langle pp'|O_{\alpha_1\cdots\alpha_n}(0)|pp'\rangle_{\text{in}} = \sum_{i=1}^n f_{in}(s) p_{\alpha_1}\cdots p_{\alpha_i} p'_{\alpha_{i+1}}\cdots p'_{\alpha_n} + g_{\alpha\beta} \text{ terms}. \qquad (15)$$

Taking the A limit of the right-hand side of (13) gives the behavior W_A of W in this limit:

$$W \underset{A}{\sim} W_A(\kappa, s, \nu, \nu'). \qquad (16)$$

The form of W_A will be determined below.

We can now state our commutativity assumption which connects the limits (11) and (16). We assume that the limit of (11) for large κ and fixed $\eta = \nu\nu'/s\kappa$ is the same as the limit of (16) for large s/κ, ν/κ, and ν'/κ and fixed ratio η:

$$\lim_{\substack{\kappa\to\infty \\ \eta \text{ fixed}}} s^\alpha \beta(\eta, \kappa) = \lim_{\substack{s/\kappa, \nu/\kappa, \nu'/\kappa \to\infty \\ \eta \text{ fixed}}} W_A(\kappa, s, \nu, \nu'). \qquad (17)$$

Notice that, because of the multiple-variable dependence of W_A, many different asymptotic limits can be taken. It is the particular form of (11) which dictates which such limit is to be taken on W_A. If the scaling behavior of (11) were not present, then the appropriate commutativity relation would be more complicated than (17). As justification for (17), we note the following: (i) Equation (17) follows from the integral representation for W in which the behaviors (11) and (16) are implemented by nice spectral functions. (ii) We will see that $W_A \sim s^\alpha$. Thus the contribution of the leading LC singularity to W for large s has the same behavior as does the complete W for large s and so should dominate for large κ since it is the piece of W which falls least fast for large κ.[7] (iii) The especially simple (scaling) form of (11) suggests the connection with the LC. (iv) The precocious-asymptopia principle[1] implies that the LC is relevant already for $\kappa \sim 2.5$ GeV^2 and so the P and A limits are really not very different. For these reasons, we believe (17) to be a natural assumption. As we shall see, the importance of (17) is that it greatly simplifies and makes physically relevant the A limit.

Let us now return to analyze the A limit in detail. Our main dynamical assumption is that the in-out production amplitudes ${}_{\text{in}}\langle pp'|O_{\alpha_1\cdots\alpha_n}(0)|pp'\rangle_{\text{out}}$ have pure Regge-pole behavior for large s.[8] It then follows that our in-in amplitudes have the same behavior. To see this, one simply considers these amplitudes slightly away from the forward direction by giving to O a small momentum k (to be taken to zero in the end). Then one can define two energy variables s_1 and s_2 in terms of which the in-in amplitude is the boundary value $A(s_1+i\epsilon, s_2-i\epsilon)$ and the in-out amplitude is the boundary value $A(s_1+i\epsilon, s_2+i\epsilon)$ of an analytic function $A(z_1, z_2)$. Thus the in-in and in-out amplitudes only differ by a discontinuity and so, if Regge behavior is correct, have the same asymptotic behavior. It follows from the usual helicity formalism that this behavior is (we ignore all possible logs factors)

$$f_{in}(s) \underset{s\to\infty}{\sim} s^{\alpha}\int_0^1 da\,\sigma_{in}(a)\left(\frac{1}{s^a}\right)^i\left(\frac{1}{s^{1-a}}\right)^{n-i}. \tag{18}$$

We note that here, the integration over a corresponds to the various ways of having the subenergies $E=p\cdot k/(k^2)^{1/2}\sim s^a$ and $E'=p'\cdot k/(k^2)^{1/2}\sim s^{1-a}$ comprise the total energy $s\sim EE'$.

It follows from (18) that (14) satisfies

$$f(\lambda,\lambda';s) \underset{s\to\infty}{\sim} s^{\alpha}\int_0^1 da\,F(\lambda/s^a, \lambda'/s^{1-a};a), \tag{19}$$

where

$$F(\zeta,\zeta';a)\equiv\sum_n\sum_{i=1}^n\sigma_{in}(a)\zeta^i\zeta'^{n-i}. \tag{20}$$

Clearly F exists at ζ or $\zeta'=0$. Last year[2] we took this to imply that only $a=0$ or 1 contributes in (14). We will see below, however, that, because of spectral restrictions, all a's can be important. Nonleading LC contributions are still negligible since they behave like, e.g.,

$$\kappa^{-1}s^{\alpha}\int_0^1 da\,G(\lambda/s^a, \lambda'/s^{1-a};a).$$

The principle of precocious asymptopia tells us that we need only keep the leading LC contribution even at $\kappa\sim 2.5\ \mathrm{GeV}^2$.

The next matter to be discussed is the support properties of the Fourier transform

$$\hat{f}(\alpha,\alpha';s)=\int\frac{d\lambda}{2\pi}\,\frac{d\lambda'}{2\pi}\,e^{i(\lambda\alpha+\lambda'\alpha')}f(\lambda,\lambda';s)$$

of f. The usual analyticity requires that $\hat{f}$ vanishes outside of the square $S=\{\alpha,\alpha':|\alpha|\le 1,\ |\alpha'|\le 1\}$. This exact support property is not possible for the Fourier transform $s\psi(\alpha s^a, \alpha' s^{1-a};a)$ of $F(\lambda/s^a, \lambda'/s'^a;a)$ but it follows from the asymptotic nature of the limit (19) and the analyticity of (14) that F is the Fourier transform of ψ over S apart from nonleading terms *and* that $\psi(\beta,\beta';a)$ is of fast decrease in β and β'.[9] We can therefore write

$$F(\lambda/s^a, \lambda'/s^{1-a};a)=\int_S d\alpha\, d\alpha'\, e^{-i(\lambda\alpha+\lambda'\alpha')}s^a s^{1-a}\times\psi(\alpha s^a, \alpha' s^{1-a};a). \tag{21}$$

In order to have W symmetric under $\nu\leftrightarrow\nu'$, we take

$$\psi(\beta,\beta';a)=\psi(\beta',\beta;1-a). \tag{22}$$

We will further take

$$\psi(\beta,\beta';a)=\rho(a)\psi(\beta,\beta'), \tag{23}$$

with

$$\psi(\beta,\beta')=\psi(\beta',\beta) \tag{24}$$

and

$$\rho(a)=\rho(1-a). \tag{25}$$

The factorization (23) is made for notational simplicity. It is true in models and, in any case, is unimportant since ψ will turn out to be independent of a.

With (19) and (21), (13) becomes

$$\begin{aligned}W \underset{A}{\sim} (\mathrm{const})s^{\alpha}\int_0^1 da\,\rho(a)\int_D d\alpha\, d\alpha'\, s^{1-a}\psi(\alpha s^a, \alpha' s^{1-a})\\ \times\delta(\kappa-2\alpha\nu-2\alpha'\nu'+\alpha\alpha' s)\,\theta(\alpha\alpha'-\kappa/s)\\ \equiv W_L(\kappa, s, \nu, \nu'),\end{aligned} \tag{26}$$

where the θ function comes from writing $\theta(-q_0+\alpha p_0+\alpha' p_0')\equiv\theta(v_0)$ in a covariant way $\theta(v\cdot(\alpha p+\alpha' p'))$. We have changed integration regions from the square S to the diamond $D\equiv\{\alpha,\alpha': |\alpha+\alpha'|\le 1\}$ since $\theta(v\cdot(p+p'))$ can be inserted in the integrand and, in view of (8), this gives $|\alpha+\alpha'|\le 1$. We decompose W_L into a "pionization" piece $\tilde{W}$ and a "fragmentation" piece $\bar{W}$:

$$W_L=\tilde{W}+\bar{W}. \tag{27}$$

The pionization piece is from a's satisfying $\epsilon(s)\le a\le 1-\epsilon(s)$, with $\epsilon(s)$ such that $s^{\epsilon(s)}=N\simeq 2\ \mathrm{GeV}^2$ is the energy at which Regge behavior is expected. Thus $\epsilon(s)=(\ln N)/(\ln s)$. The fragmentation piece comes from $0\le a\le\epsilon(s)$ and $1-\epsilon(s)\le a\le 1$. We write the corresponding decomposition of $\rho(a)$ as

$$\rho(a)=\tilde{\rho}(a)+\bar{\rho}(a). \tag{28}$$

We consider first $\tilde{W}$. We use the δ function in (26) to do the α' integration and then change integration variables from α to $\beta\equiv s^a\alpha$. Thus

$$\alpha'=\frac{\kappa-2\nu s^{-a}\beta}{2\nu'-\beta s^{1-a}}. \tag{29}$$

In the A limit, since $\psi(\beta,\beta')$ is exponentially damped for large β, and since $0<a$, we have approximately $\alpha'\to\kappa/2\nu'=\omega'$. We emphasize that this simplification is a consequence of the exponen-

tial damping of ψ – a reflection of the analyticity of (14). We thus obtain

$$\tilde{W} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{2\nu'} \int da\, \rho(a) \int_{s^a(2\nu'/s)}^{s^a(1-\omega')} d\beta\, s^{1-a} \times \psi(\beta, s^{1-a}\omega'). \quad (30)$$

Both of the arguments β and $\beta' = s^{1-a}\omega'$ in ψ in (30) are large and so for explicitness we take

$$\psi(\beta, \beta') \to e^{-H(\beta+\beta')/2}, \quad (31)$$

consistent with (24). Our analysis and results are insensitive to reasonable modifications of (31). Equation (30) then becomes

$$\tilde{W} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{(\kappa)^{1/2}} \int_{wN}^{ws/N} dy\, \rho\left(\frac{\ln y/w}{\ln s}\right) \times \exp\left[-\tfrac{1}{2}H(\kappa)^{1/2}\left(y + \frac{1}{y}\right)\right], \quad (32)$$

where $w \equiv (\kappa)^{1/2}/2\nu'$, and we have changed integration variables from a to $y = ws^a$. In the A limit, $w \to 0$ and $ws \to \infty$ so that (32) gives

$$\tilde{W} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{(\kappa)^{1/2}} K_{-1}(H(\kappa)^{1/2}) \tilde{\rho}\left(\frac{\ln\nu' - \frac{1}{2}\ln\kappa}{\ln s}\right), \quad (33)$$

where we have used the exponential damping to ignore $\ln y$ compared to $\ln s$ and where $K_{-1}(z)$ is the modified Hankel function. Notice that, contrary to its appearance, as a consequence of (25) and the fact that $\eta = \nu\nu'/s\kappa$ is fixed, (33) is symmetric under $\nu \leftrightarrow \nu'$. The final form of (33) is obtained from the asymptotic behavior of $K_{-1}(z)$:

$$\tilde{W} \underset{A}{\sim} (\text{const}) s^\alpha \kappa^{-3/4} e^{-H(\kappa)^{1/2}} \tilde{\rho}\left(\frac{\ln\nu' - \frac{1}{2}\ln\kappa}{\ln s}\right) \equiv \tilde{W}_A(\kappa, s, \nu, \nu'). \quad (34)$$

The next step is to incorporate the behavior (11) via the relation (17). Taking $\nu' \sim s^b$, (34) gives

$$\tilde{W}_A \underset{\substack{s/\kappa, \nu/\kappa, \nu'/\kappa \to \infty \\ \eta \text{ fixed}}}{\sim} (\text{const}) s^\alpha \kappa^{-3/4} e^{-H(\kappa)^{1/2}} \tilde{\rho}(b). \quad (35)$$

Since this must be independent of b (i.e., a function of only $\nu\nu'/s$), we learn that $\tilde{\rho}(a) = \text{const}$. This result is, of course, already clear from (34). Since $\tilde{\rho}$ is a function of ν' and s but not of $\nu\nu'/s$, it must be a constant function. Thus the final result is

$$\tilde{W}_A(\kappa, s, \nu, \nu') = A s^\alpha \kappa^{-3/4} e^{-H(\kappa)^{1/2}}, \quad (36)$$

for some constants A and H which have not been determined by our analysis. Note that (36) is independent of ν and ν'. From our derivation and assumptions, it is clear that (36) should be a good representation of W everywhere in the region $s, \nu, \nu' \gg \kappa$, with $\nu\nu'/s\kappa$ fixed, in particular, in either the P limit for large κ and fixed η or the A limit for large s/κ and fixed η.

Note, incidentally, that the form (34) explicitly vanishes in the fragmentation limits (s/κ, $\nu'/\kappa \to \infty$; η, ν/κ fixed or $\nu \leftrightarrow \nu'$) since $\tilde{\rho}(0) = \tilde{\rho}(1) = 0$. This shows that the decomposition (27) is correct.

We next consider the F limits. We write

$$\overline{W} = W^{(0)} + W^{(1)} \quad (37)$$

corresponding to the contributions from a near 0 and near 1, respectively. For $W^{(1)}$, we still have $\alpha' \to \omega'$ in the A limit [see Eq. (29)] and so approximately

$$W^{(1)} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{2\nu'} \int_{2\nu'}^{s(1-\omega')} d\beta\, \psi(\beta, \omega'), \quad (38)$$

where we have absorbed $\rho(1)$ into const. In (38) only β is large whereas $\beta' = \omega'$ is fixed. We therefore take

$$\psi(\beta, \omega') \to \psi(\omega') e^{-\kappa\beta/2}. \quad (39)$$

For ω' near 1, we expect the usual threshold behavior[10]

$$\psi(\omega') \sim (1 - \omega')^l, \quad \omega' \sim 1. \quad (40)$$

Equation (38) thus becomes

$$W^{(1)} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{\kappa} \varphi(\omega') e^{-\kappa\nu'}, \quad (41)$$

where $\varphi(\omega') \equiv \omega'\psi(\omega')$. Actually performing the a integration over $1 \geq a \geq 1 - \epsilon(s)$ changes this to

$$W^{(1)} \underset{A}{\sim} (\text{const}) \frac{s^\alpha}{\kappa} \varphi(\omega') e^{-\kappa\nu'/N}. \quad (42)$$

The behavior of $W^{(0)}$ is the same as (42) with $\nu \leftrightarrow \nu'$. Thus

$$\overline{W} \underset{A}{\sim} B s^\alpha \kappa^{-1}[\varphi(\omega') e^{-\kappa\nu'/N} + \varphi(\omega) e^{-\kappa\nu/N}] \equiv \overline{W}_A(\kappa, s, \nu, \nu'). \quad (43)$$

This expression automatically satisfies the scaling (12) in the limits $s/\kappa, \nu/\kappa \to \infty$, s/ν fixed, and $\nu \leftrightarrow \nu'$. Again the scaling is satisfied in the trivial sense that $\overline{W}_A$ is independent of ν (or ν') in the $\nu \to \infty$ ($\kappa\nu' \to \infty$) limit. The commutativity assumption is nevertheless not empty since it says that the form (43) is valid when $s, \nu \to \infty$ with ν/s, ν', and κ fixed first and then ν' and $\kappa \to \infty$ with ν'/κ fixed as well as when $\kappa \to \infty$ with $s/\kappa, \nu/\kappa, \nu'/\kappa$ fixed, and then $s/\kappa, \nu/\kappa \to \infty$ with ν'/κ fixed.

Since the minimum allowed value for ν or ν' is $\frac{1}{2}\kappa$, the form [see Eqs. (36) and (43)]

$$W_A = \tilde{W}_A + \overline{W}_A \quad (44)$$

should provide a good representation of W everywhere in the region $s \gg \kappa \gtrsim 2$. $\tilde{W}_A$ dominates for ν

and ν' both large (pionization region) whereas $\overline{W}_A$ dominates if only one is large. The very specific forms (36) and (43) [with (40)] illustrate the strength of the combined LC and multi-Regge assumptions.

At this point, we can inquire about the validity of our use of the leading LC singularity. Actually, the contribution of a nonleading singularity [e.g., $(x^2)^k \ln x^2$] will be more important than that of (2) if the associated spectral function $\psi_k(\beta,\beta')$ falls (exponentially) sufficiently slowly. This is obviously no problem since inclusion of this, or any similar term will not change the basic form of (44). We expect, however, on the basis of the general kinematical analysis and of the obtained Regge behavior [plus commutativity (17)], that the leading singularity will, in fact, dominate. One easily sees, moreover, that replacing $J(x)J(0)$ in (6) by $x^2J(x)J(0)$ gives a less leading contribution provided $\psi(\beta,\beta')$ falls slower than $\exp[-(\beta+\beta')^2]$.[11] It is interesting that the requirement of LC dominance can place such a strong restriction on ψ.

Let us conclude by briefly discussing some of the consequences of (44). We can compute various quantities of interest from the triple-differential cross section

$$\frac{d^3\sigma}{d\kappa d\nu d\nu'} = \frac{4\pi}{s^2}\, W(\kappa, s, \nu, \nu') \tag{45}$$

in terms of the few constants $A, B, H, K,\cdot$ and l. For example, integration over ν and ν' gives the cross section

$$\frac{d\sigma}{d\kappa} = \frac{d\tilde{\sigma}}{d\kappa} + \frac{d\overline{\sigma}}{d\kappa}\,. \tag{46}$$

The result is

$$\frac{d\tilde{\sigma}}{d\kappa} \sim s\kappa^{-3/4} e^{-H(\kappa)^{1/2}}, \tag{47}$$

$$\frac{d\overline{\sigma}}{d\kappa} \sim \kappa^{-l-1} e^{-K\kappa/N}\,, \tag{48}$$

where we have taken now $\alpha = 1$. The result for the total cross section

$$\sigma(s) = \tilde{\sigma}(s) + \overline{\sigma}(s) \tag{49}$$

is

$$\tilde{\sigma}(s) \sim s, \quad \overline{\sigma}(s) \sim (\text{const}) \tag{50}$$

for large s. We can similarly easily compute $d\sigma/dp_L$ (p_L = photon longitudinal momentum in lab), $d\sigma/d\kappa$, and $d\sigma/d\theta$ (θ = angle between photon and beam).

A similar analysis can obviously be carried out for the physically interesting case[5] in which $J(x)$ is replaced by the dimension three electromagnetic current $J_\mu(x)$. The expressions obtained for the various scalar amplitudes encountered do not differ essentially from the above results (36) and (43). Thus $d\sigma/d\kappa$ will be given essentially by (46)–(48). Since (48) dominates for small κ and (47) dominates for large κ, the experimentally observed shoulder is naturally explained by the interference of these two terms.[12] This and the other observed features of the results, including the data for $\sigma(s)$, $d\sigma/dp_L$, $d\sigma/d\kappa_\perp$, and $d\sigma/d\theta$, can all be accurately described by our expressions. A detailed theoretical development and comparison with experiment will be given elsewhere.[13]

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[7]This can be simply understood by noting that the nonleading terms in the asymptotic light-cone expansion can be written in a form analogous to the leading one but with c-number functions of milder singularities. The Regge analysis we perform in the following holds for the local operators $O_{\alpha_1\cdots\alpha_n}(0)$ of the nonleading ones as well. The uniform bound s^{α^n} ($\alpha = 1$) for such amplitudes excludes the occurrence of an expansion in the dimensionless quantity sx^2 which would make the nonleading LC singularities as important as the leading one for these processes.

[8]In particular, we assume there are no fixed poles. Fixed poles are not expected in such amplitudes.

[9]The argument, which we give for a function of one variable, is the following. If $f(\lambda,s) = f_0(\lambda/s) + s^{-1}f_1(\lambda/s) + \cdots$, with all derivatives $f_i^{(n)}(0)$ existing [so that the Fourier transforms $\hat{f}_i(\xi)$ are of fast decrease for $\xi \to \infty$], and if $\hat{f}(\alpha, s)$ has support $\alpha \in [0,1]$, then

$$s\hat{f}_0(\alpha a) \equiv \int d\lambda\, e^{-i\lambda\alpha} f_0(\lambda/s)$$
$$= \hat{f}(\alpha, a) - \hat{f}_1(\alpha s) - s^{-1}\hat{f}_2(\alpha s)\cdots,$$

so that

$$\int_0^1 d\alpha\, e^{i\alpha\lambda} s\hat{f}_0(\alpha s) = f(\lambda, s) + O(e^{-s})$$
$$= f_0(\lambda/s) + O(s^{-1}g(\lambda/s)).$$

There is no problem from $\alpha = 0$ since small α's will be seen to be excluded from our integrals.

[10]We expect from smooth threshold behavior (Ref. 1) that $l \gtrsim 3$.

[11]We might also comment, in this connection, on the recent paper by R. Jaffe [Phys. Letters 37B, 517 (1971)], who claims that the matrix element for massive μ-pair production is not LC dominated in the parton model. Although we are certainly not using the parton model, we would like to point out that, even though no LC singularity is present in the parton-model matrix element, $\langle J(x)\, J(0)\rangle_P$, the LC does dominate even here in the sense that substitution of $x^2\langle J(x)\, J(0)\rangle_P$ for $\langle J(x)\, J(0)\rangle_P$ gives a less leading contribution. Our approach does not, in fact, differ from that of the parton model in the question of LC dominance (Factors like $e^{-ix^2 s}$ would be necessary to ruin LC dominance. The parton model gives no such factor), but rather because the parton model does not exhibit the Regge behavior (19) at the five-point function level.

[12]The result is somewhat dependent on the specific form (31) chosen for ψ. Writing $\psi(\beta,\beta') = \Phi(\beta+\beta', (\beta\beta')^{1/2})$, the general statement is $d\bar{\sigma}/dx \sim \Phi((\kappa)^{1/2}, (\kappa)^{1/2})$ and $d\bar{\sigma}/d\kappa \sim \Phi(\kappa, (\kappa)^{1/2})$. Thus the requirement that the sum variable $\beta+\beta'$ be at least as important as the product variable $(\beta\beta')^{1/2}$ gives the result stated in the text. More generally the powers of κ in the exponentials can be left as free parameters and fit to the data.

[13]R. Brandt, A. Kaufman, and G. Preparata (unpublished).

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Scale and Conformal Transformations of Currents and Tensor-Meson Dominance

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We use the infinite-momentum limit and single-particle saturation to investigate consequences for baryon matrix elements of equal-time commutators of the generators of scale and conformal transformations with currents and their divergences. We show that the root-mean-square tensor mass radii are the same for all members of the baryon octet. We use Regge theory to show the validity of the procedure in this case. The result implies that some of the baryon gravitational form factors $F_1^B(q^2)$ or $F_2^B(q^2)$ must be subtracted. On demanding the subtractions to be SU(3)-symmetric we obtain $G_2(fNN)/G_1(fNN) = -1$, and find the f/d ratios to be the same for the two couplings. This is in agreement with the phenomenological analyses of Schlaile and of Strauss.

I. INTRODUCTION

In this paper we investigate consequences for baryon matrix elements of the behavior of currents and their divergences under scale and conformal transformations.[1] We make use of the infinite-momentum limit[2] (IML) and single-particle saturation of the commutation relations of scale and conformal generators Q_D and K_0 with currents and their divergences to show that the root-mean-square tensor mass radii are the same for all members of the nucleon octet. When combined with the usual tensor-meson-dominance (TMD) assumptions[3-5] these relations require some of the baryon gravitational form factors $F_1^B(q^2)$ and $F_2^B(q^2)$ to be subtracted. On demanding the subtractions to be SU(3)-symmetric we obtain $G_2(fNN)/G_1(fNN) = -1$, and find the f/d ratios in the two couplings to be the same.

In deriving our results we make use of the fact that the dimension of the time component of the currents J_μ^a, which are the vector V_μ^a or the axial-vector currents A_μ^a, is three. As is well known this follows from Gell-Mann's charge algebra if J_0^a has a dimension. Alternatively the same result holds if a state $|A\rangle$ exists such that $\langle A|J|A\rangle \neq 0$ and if under conformal transformations $[K_0(0), \partial_\mu J_\mu(0)] = 0$. The proof[6] makes use of the