Isham *et al.* (Ref. 10) refers to these three types as nonlocal, "just local," and local.

[9]G. V. Efimov, CERN Report No. 1087 (unpublished). This is an excellent review of the ideas and methods of NLFT, especially as applies to the earlier, Type-I theories.

[10]A review of more recent advances in NLFT is contained in C. J. Isham, A. Salam, and J. Strathdee, Phys. Rev. D 5, 2548 (1972). This and the previous two references will provide a guide to the literature.

[11]D. Fivel and P. K. Mitter, Phys. Rev. D 1, 3270 (1970).

[12]H. Lehmann and K. Pohlmeyer, Commun. Math. Phys. 20, 101 (1971).

[13]A. Salam and J. Strathdee, Phys. Rev. D 1, 3296 (1970).

[14]We shall hereafter refer to the present paper as I and to the following one as II.

[15]For notational simplicity, we often use only subscripts for space-time indices $\lambda = 0, 1, 2, 3$, although the Bjorken-Drell metric is being used ($g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$); also the Cabibbo-angle factors $\sin\theta$ and $\cos\theta$ are included in the currents $j_\lambda^{(q)}$.

[16]The importance of this condition has been stressed by Lehmann and Pohlmeyer, Ref. 12.

[17]For a discussion of weak-interaction cutoffs, see, for example, R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), Chap. 7.

[18]See, for example, W. Magnus, F. Oberhettinger, and R. P. Soni, *Formulas and Theorems for the Special Functions of Mathematical Physics* (Springer, New York, 1966).

[19]K. Sekine, Nuovo Cimento 11, 87 (1959); K. Hiida, Phys. Rev. 132, 1239 (1963); 134, B174 (1964).

[20]This procedure is outlined, for example, in J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), Chap. 8.

[21]The reader is reminded that Eq. (4.3) is a formal expression only. From it one is to construct the amplitude in the Euclidean region of momenta, perform all auxiliary integrations over Euclidean coordinates, and finally continue the amplitude to the appropriate physical momenta.

[22]The Kronecker $\delta_{\Sigma l_i, \Sigma m_i}$ expresses the conservation of charge of the φ particles.

[23]Our notation for the hypergeometric functions is that of Ref. 18, Chap. II.

[24]This definition of the degree of divergence is equivalent to the usual definition in momentum space. See, e.g., J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965); compare Eq. (4.9) of the present paper with Eq. (19.62) of this reference.

[25]S. Hori, Progr. Theoret. Phys. (Kyoto) 7, 578 (1952).

[26]We are especially indebted to Prof. M. Whippman for first suggesting that the *N*th-order superpropagator is the inverse of a determinant [Eq. (4.7)]. The method we follow here to derive the general formula (4.6) is similar to that used by G. V. Efimov, Zh. Eksp. Teor. Fiz. 44, 2107 (1963) [Sov. Phys.– JETP 17, 1417 (1963)], and by E. S. Fradkin, Nucl. Phys. 49, 624 (1963). This technique is also employed by Fivel and Mitter (Ref. 11) in a context very close to the one in which we are using it.

[27]See Ref. 18, p. 96.

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Theory of Higher-Order Weak Interactions and *CP*-Invariance Violation. II. The Neutral *K* System*

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Continuing our exposition of a nonpolynomial theory of higher-order weak interactions, we examine the neutral K system, with particular emphasis on the CP-invariance-violating amplitudes. These first appear in second order in the weak interactions. We use the free-quark model to estimate the short-distance singularity of products of hadronic currents, and we find that with appropriate choices of the minor coupling constants our theory is consistent with the known experimental results. In particular, we find $|\epsilon| \approx 10^{-3}$, and we give an argument leading to $\eta_{+-} \approx \eta_{00}$. The neutron dipole moment is a third-order weak effect in our theory and is estimated to be about 10^{-27} e cm. We calculate the production cross section for the superpropagating particles, and find it to be too small for the particles to have yet been observed.

I. INTRODUCTION

In the preceding paper[1] we have treated leptonic processes in higher-order weak interactions, using a particular nonpolynomial modification of the usual current × current interaction Lagrangian. In this paper, using the same Lagrangian as in I, we turn to two somewhat more speculative subjects: (i) the effect of our modification on hadronic weak processes (with special attention to CP noninvariance) and (ii) the production of the φ particles that are coupled nonpolynomially to the usual weak currents. We shall not repeat the derivation of our basic results here; rather, the reader is referred to Sec. II of paper I before continuing with the remainder of this paper.

II. THE K MASS MATRIX

The formula (2.9) of paper I for a second-order amplitude remains valid even when the external states or the interaction Lagrangians contain hadronic pieces. The main difference, of course, is that we can now no longer evaluate the matrix elements explicitly, as we did in paper I, so that even in principle we cannot calculate our second-order hadronic amplitudes exactly unless we solve the problem of the strong interactions first, a task that is considerably beyond the scope of the present work.

However, one of the lessons learned in I was that the strength of the singularity of the relevant matrix element as $x \to 0$ is of crucial importance in estimating the size of the amplitude. In order to make progress, therefore, we must find some method of calculating the degree of singularity of two hadronic operators (currents or Lagrangians) at short distances between the appropriate states.

We have chosen for this purpose to adopt the free-quark model for the hadronic weak currents. In our view, such a procedure amounts to deriving an upper bound for the degree of singularity of any given matrix element. In other words, currents built up of free, pointlike objects will exhibit a singularity structure at least as severe as that of the real currents; the effect of the strong interactions, we assume, will be possibly to smooth out some of the short-distance behavior, but never to make it worse. We remark in passing that the use of the free-quark currents to describe short-distance behavior is consistent with recent ideas about scaling in deep-inelastic scattering.[2]

To be specific, our discussion of hadron processes will focus on the neutral K system, with particular emphasis on CP-violating effects. We shall show, in fact, that with reasonable choices of the parameters in our theory, and reasonable arguments about the relative magnitudes of various amplitudes, it is possible to explain CP violation naturally as a second- (and higher-) order weak effect.[3]

Let us begin with a standard formula from the phenomenological description of CP violation[4]:

$$\epsilon \approx \frac{-T_{12}}{[(T_{22}-T_{11})^2-4T_{12}{}^2]^{1/2}} = \frac{-T_{12}}{im_{K^0}[\gamma_S-\gamma_L+2i(m_S-m_L)]}. \quad (2.1)$$

Here T_{ij} is the T-matrix element connecting the CP eigenstates K_i and K_j,

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) = CP|K_1\rangle,$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) = -CP|K_2\rangle, \quad (2.2)$$

and γ_S (γ_L) and m_S (m_L) are the lifetime and mass of the short- (long-) lived K meson, respectively. In terms of ϵ,

$$|K_{S,L}\rangle \cong |K_{1,2}\rangle + \epsilon|K_{2,1}\rangle. \quad (2.3)$$

Experimentally, of course, $|\epsilon| \approx 10^{-3}$.

In our theory, there is no contribution to T_{12} in first-order G, while in second order we have

$$T_{12} \equiv \langle K_1|\mathcal{T}|K_2\rangle = \tfrac{1}{2}i\int d^4x\, \langle K^0|T(\mathcal{L}_{10}(x)\mathcal{L}_{10}(0))|\bar{K}^0\rangle \times\left[\frac{1}{1-f_{10}{}^2\Delta^2(x)} - \frac{1}{1-f_{01}{}^2\Delta^2(x)}\right]. \quad (2.4)$$

As was discussed in paper I, we assume that unless f_{01} is purely real it has an infinitesimal imaginary part, so that the difference of superpropagators in (2.4) becomes proportional to a δ function. Following the philosophy outlined above, we now use the quark model to estimate the behavior of

$$M_{12}(x) \equiv \langle K^0|T(\mathcal{L}_{10}(x)\mathcal{L}_{10}(0))|\bar{K}^0\rangle \quad (2.5)$$

as $x \to 0$. Assuming that each hadronic weak current is of the form

$$j_\mu^{(a)}(x) = \tfrac{1}{2}\bar{q}(x)\gamma_\mu(1-\gamma_5)\lambda_a q(x), \quad (2.6)$$

we obtain the diagrams shown in Fig. 1. Diagrams 1(a) and 1(b) each have two quark propagators between the weak vertices, and the relevant contributions of the matrix element in (2.4) therefore behave as $1/x^6$ for small x. Diagram 1(c) has no such propagators, and its contribution is therefore nonsingular.

We may now draw the following conclusions:

(i) From (2.1), we see that T_{12} is of order $\epsilon m_{K^0}\gamma_S$.

(ii) From (2.4) and the quark-model result that $\mathcal{L}_{10}(x)\mathcal{L}_{10}(0)$ behaves like $1/x^6$ we deduce

$$T_{12} \approx \frac{G^2\sin^2\theta M^4}{r_{01}{}^2} + \text{terms of order } G^2\sin^2\theta M^6. \quad (2.7a)$$

Here θ is the Cabibbo angle, and M is a mass characteristic of the process under consideration; in this case, a reasonable choice would seem to be $M = m_{K^0}$. The first term in (2.7a) expresses the dependence of T_{12} on the cutoff length r_{01} [defined such that $|f_{01}\Delta(r_{01})|=1$] necessitated by the singularity in the integral as $r_{01}\to 0$. The other terms are only weakly dependent on r_{01}, and are therefore assumed to have the magnitude of a typical strangeness-changing second-order weak amplitude.

These assumptions, while they seem reasonable, lead us into difficulty. From (i) we have $T_{12}\approx 10^{-16}\times m_K^2$, while from (ii) we see

$$T_{12}\approx \frac{10^{-12}}{r_{01}^2}+10^{-12}m_K^2 . \tag{2.7b}$$

If we assume that the two terms in (2.7b) do not somehow miraculously cancel, then the best we can do is to choose r_{01} large enough so that the first term in (2.7b) is negligible. This still leaves us with T_{12} of order $10^{-12}m_K^2$, i.e. four orders of magnitude bigger than the experimental value given by (i).

There seem to be essentially only two ways to circumvent this dilemma:

(a) We can go back on our reasonability arguments that led to (2.7b) and claim that because of some unexpected property of strong-interaction

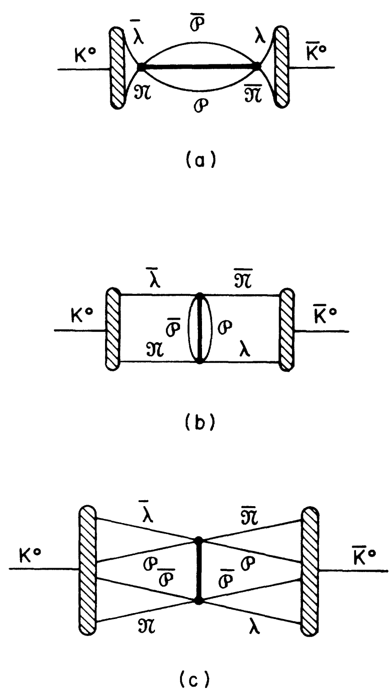

FIG. 1. The transition amplitude $K^0\to\bar{K}^0$ in the quark model, due to second-order nonleptonic processes. Shaded blobs indicate strong vertices. The heavy line is the superpropagator. The various species of quarks are indicated explicitly in each diagram.

dynamics the purely nonleptonic amplitudes, such as those appearing in (2.4), are suppressed; in the case at hand, this suppression is supposed to be roughly four orders of magnitude beyond what naive estimates would give.

(b) Looking at the superpropagator in (2.4), we see that the nonvanishing of T_{12} depends on $f_{01}^2 = f_{10}^{*2}$ having an imaginary part. (It is a general property of all CP-violating amplitudes that they vanish if the relevant minor coupling constant is *a priori* real.) It is therefore tempting to postulate that f_{01} is purely real; T_{12} as given by (2.4) then vanishes, and we must go to order G^3 to calculate ϵ. Without going into detail, we present in Fig. 2 a typical diagram that will contribute. For $|x_2-x_3|\to 0$, we see that this diagram is expected to have the cutoff dependence $T_{12}^{(3)}\approx (G^3 m_K^6 \sin^2\theta)\times(1/r_l^2)$, with r_l defined by $|f_{0l}f_{1l}\Delta^2(r_l)|=1$. Thus by choosing r_l about $\frac{1}{5}$ GeV^{-1} we have the desired result: $T_{12}^{(3)}\approx 10^{-16}m_K^2$. This estimate of the cutoff may vary by an order of magnitude or so, since our knowledge of third order is naturally even cruder than our knowledge of second.

At the level that we have been making these estimates, there is little to distinguish option (a) from option (b). One important piece of evidence, however, comes from another well-known formula,[4] this time for a CP-conserving amplitude:

$$i\,m_{K^0}[\gamma_S-2i\Delta m]\approx T_{22}-T_{11}$$
$$=iP\int d^4x\,\frac{\langle K^0|\,T(\mathfrak{L}_{10}(x)\,\mathfrak{L}_{10}(0))\,|\bar{K}^0\rangle}{1-(\mathrm{Re}f_{10}^2)\,\Delta^2(x)} . \tag{2.8}$$

Notice that in our formalism, the essential difference between a CP-conserving amplitude and a CP-violating one in second order is the appearance of the principal value prescription instead of the difference of two superpropagators (which becomes a δ function) that occurs in (2.4).

The left-hand side of (2.8) is of order $10^{-13}m_K^2$, which is typical for a second-order weak amplitude. Thus there is no evidence here for the supposed suppression of nonleptonic amplitudes in second order that we invoked in option (a).

III. THE DECAYS $K_{L,S}\to\pi\pi$

We turn now to the two-pion decay modes of the K_L and K_S mesons to see which, if any, of the lines of thinking begun in Sec. II will prove successful. Specifically, we shall construct the transition amplitudes needed for the evaluation of the

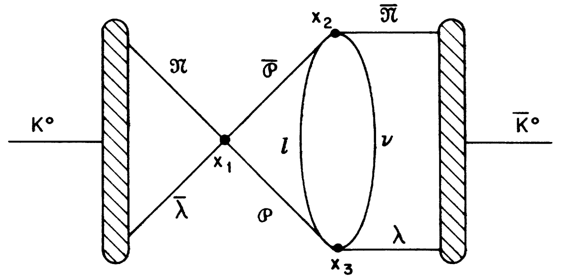

FIG. 2. A third-order diagram contributing to $K^0 \to \overline{K}^0$ in the quark model. Shaded blobs are strong vertices. The three weak vertices are labeled explicitly, and the lines connecting them are labeled according to the type of particle. The third-order superpropagator, while not drawn explicitly, should be thought of as connecting all the weak vertices in all possible ways.

quantities

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | \mathcal{T} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{T} | K_S \rangle}, \qquad \eta_{00} = \frac{\langle \pi^0\pi^0 | \mathcal{T} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{T} | K_S \rangle}. \tag{3.1}$$

Then we use the quark model to estimate the size of these amplitudes. Finally, we shall give an heuristic argument that the experimental result $\eta_{+-} \cong \eta_{00}$ is satisfied in our model.

Let $|\pi\pi\rangle$ denote any of the states $|\pi^+\pi^-\rangle$, $|\pi^0\pi^0\rangle$, $|2\pi, I=0\rangle$ or $|2\pi, I=2\rangle$. From Eq. (2.3),

$$\frac{\langle \pi\pi | \mathcal{T} | K_L \rangle}{\langle \pi\pi | \mathcal{T} | K_S \rangle} = \left(\epsilon + \frac{\langle \pi\pi | \mathcal{T} | K_2 \rangle}{\langle \pi\pi | \mathcal{T} | K_1 \rangle}\right)\left(1 + \epsilon \frac{\langle \pi\pi | \mathcal{T} | K_2 \rangle}{\langle \pi\pi | \mathcal{T} | K_1 \rangle}\right)^{-1}. \tag{3.2}$$

In our model, the lowest nonvanishing contribution to $\langle \pi\pi | \mathcal{T} | K_1 \rangle$ is first-order in G, while $\langle \pi\pi | \mathcal{T} | K_2 \rangle$ is at least second-order in G. Thus, since the currents have transformation properties consistent with CP conservation, we can approximate $\langle \pi\pi | \mathcal{T} | K_{1,2} \rangle$ by the following expressions:

$$\langle \pi\pi | \mathcal{T} | K_1 \rangle = \sqrt{2}\, \langle \pi\pi | \mathcal{L}_{01} | K^0 \rangle = G \sin\theta \cos\theta \langle \pi\pi | j_\lambda^{(1)\dagger}(0) j_\lambda^{(0)}(0) | K^0 \rangle, \tag{3.3}$$

and, with (sl) and (nl) denoting the double action of the semileptonic and nonleptonic interactions, respectively,

$$\langle \pi\pi | \mathcal{T} | K_2 \rangle \equiv \langle \pi\pi | \mathcal{T}^{(\mathrm{nl})} | K_2 \rangle + \langle \pi\pi | \mathcal{T}^{(\mathrm{sl})} | K_2 \rangle. \tag{3.4}$$

Here

$$\langle \pi\pi | \mathcal{T}^{(\mathrm{nl})} | K_2 \rangle = \frac{i}{\sqrt{2}} \sum_{a=0,1} \int d^4x \langle \pi\pi | T(\mathcal{L}_{01}(x)\mathcal{L}_{aa}(0)) | K^0 \rangle \{[1 - f_{01} f_{aa} \Delta^2(x)]^{-1} - [1 - f_{10} f_{aa} \Delta^2(x)]^{-1}\} \tag{3.5}$$

and

$$\langle \pi\pi | \mathcal{T}^{(\mathrm{sl})} | K_2 \rangle = \frac{i}{\sqrt{2}} \sum_{l=e,\mu} \int d^4x \langle \pi\pi | T(\mathcal{L}_{l1}(x)\mathcal{L}_{0l}(0)) | K^0 \rangle \{[1 - f_{l1} f_{0l} \Delta^2(x)]^{-1} - [1 - f_{l1} f_{l0} \Delta^2(x)]^{-1}\}. \tag{3.6}$$

Equation (3.5) tells us that $\langle \pi\pi | \mathcal{T}^{(\mathrm{nl})} | K_2 \rangle$ is negligible according to either of the possibilities (a) or (b) discussed in Sec. II. In case (a) we simply assume the suppression of nonleptonic amplitudes. Essentially this point of view was taken in an earlier paper[5]; however, this is difficult to justify in the quark model, where the diagrams are very similar to those representing $\langle \pi\pi | \mathcal{T}^{(\mathrm{sl})} | K_L \rangle$ (see Fig. 3). According to case (b), in which $f_{01} = f_{10}$, the nonleptonic contribution vanishes identically in $O(G^2)$. In view of our argument below that the result $\eta_{+-} \cong \eta_{00}$ can be understood in our model by retaining only the semileptonic part, Eq. (3.6), we shall therefore invoke the condition $f_{01} = f_{10}$ to remove $\langle \pi\pi | \mathcal{T}^{(\mathrm{nl})} | K_2 \rangle$ from further consideration.

Turning now to Eq. (3.6), extracting the leptonic factors and evaluating them as in paper I, we find for the $K_2 \to \pi\pi$ transition amplitude

$$\langle \pi\pi | \mathcal{T} | K_2 \rangle = 2^{3/2} G^2 \sin\theta \cos\theta\; i \sum_{l=e,\mu} \int d^4x \left(\frac{2x_\lambda x_\mu}{x^2} - g_{\lambda\mu}\right) \Delta_1(r; m_l)\, \Delta_1(r; 0) \langle \pi\pi | j_\lambda^{(1)\dagger}(x) j_\mu^{(0)}(0) | K^0 \rangle \times \{[1 - f_{l1} f_{0l} \Delta^2(r)]^{-1} - [1 - f_{l1} f_{l0} \Delta^2(r)]^{-1}\}. \tag{3.7}$$

Here we have paid attention to the fact that this integral is to be evaluated in the Euclidean region by expressing the propagators Δ and Δ_1 as functions of the Euclidean length $r = (-x^2)^{1/2}$ and by deleting any time-ordering instruction from the product of hadronic currents.

Since the difference of superpropagators in Eq. (3.7) is effectively a δ function requiring

$$r = r_l \cong |f_{l1} f_{0l} / 16\pi^4|^{1/4} \sim (10\,\mathrm{GeV})^{-1}, \tag{3.8}$$

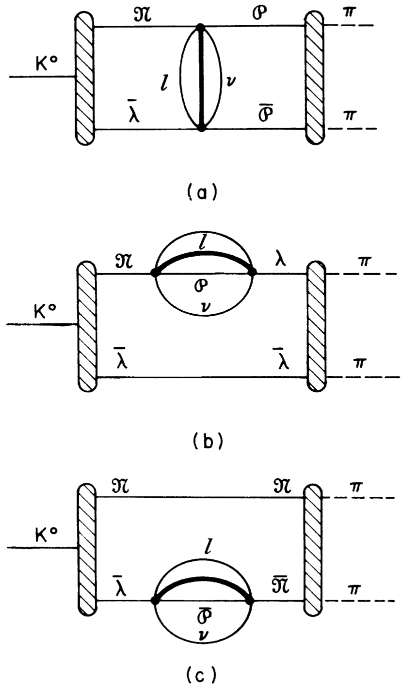

FIG. 3. Second-order semileptonic diagrams contributing to the process $K \rightarrow 2\pi$ in the quark model. Shaded blobs are strong vertices. The heavy line is the superpropagator. The lepton and quark lines are explicitly labeled.

the magnitude of $\langle\pi\pi|\mathcal{T}|K_2\rangle$ is largely governed by the short-distance behavior of the integrand in that equation. To determine this, we first recall that the leptonic factor behaves as r^{-6} for $r \rightarrow 0$. To estimate the small-r dependence of the hadronic factor $\langle\pi\pi|j_\lambda^{(1)}(x)j_\mu^{(0)}(0)|K^0\rangle$, we appeal to the quark-model diagrams in Fig. 3.

Apart from superpropagator corrections, diagram 3(a) exhibits the r^{-6} singularity due to lepton propagators, and no other. Diagrams 3(b) and 3(c) each have, in addition, a quark propagator, so that the total singularity for these contributions to the matrix element is r^{-9} (actually r^{-8} when symmetric integration is taken into account). However, we notice that the weak-interaction parts of Figs. 3(b) and 3(c) have exactly the same structure as the lepton self-energy diagram considered in Sec. III of paper I. It is therefore reasonable to interpret them as generalized self-energy diagrams for the quarks (analogous to the K^0 mass matrix discussed above), or, in other words, as renormalization diagrams for the hadronic weak currents. After renormalization has been properly carried out, we would expect the level of singularity to be reduced from r^{-8} to r^{-6}, as it was in the lepton case.

To summarize, based on the structure of diagrams in Fig. 3, we assume that once renormalization of various masses and coupling constants is carried out, the effect of the matrix element $\langle\pi\pi|T(\mathcal{L}_{l1}(x)\mathcal{L}_{0l}(0))|K^0\rangle$ is an r^{-6} singularity on the light cone. With the superpropagator $[1-f_{l1}f_{0l}\Delta^2]^{-1}$ no further removal of divergences is necessary, and we deduce that Eq. (3.7) takes the form

$$\langle\pi\pi|\mathcal{T}|K_2\rangle = 2\pi i \sum_{l=e,\mu} M_l \frac{G}{r_l^{\,2}} \operatorname{sgn} \operatorname{Im}(f_{l1}f_{0l}). \tag{3.9}$$

Here M_l is a constant which might depend on the lepton masses and on the charge state of the two pions, but only weakly (say, logarithmically) or not at all on the cutoff lengths r_l. Furthermore, M_l should have approximately the magnitude of the CP-conserving amplitude $\langle\pi\pi|\mathcal{T}|K_1\rangle$ in Eq. (3.3). It follows that

$$\left|\frac{\langle\pi\pi|\mathcal{T}|K_2\rangle}{\langle\pi\pi|\mathcal{T}|K_1\rangle}\right| \approx \begin{cases} 10^{-3} & \text{for } r_l^{\,-1} \sim 10\,\text{GeV} \\ 10^{-5} & \text{for } r_l^{\,-1} \sim 1\,\text{GeV}. \end{cases} \tag{3.10}$$

We now address ourselves to the question of whether our model is compatible with the result $\eta_{+-} \cong \eta_{00}$.[6] Following the notation in Ref. 4, we define amplitudes A_0 and A_2 by

$$\begin{aligned} \sqrt{2}\, e^{i\delta_0}\operatorname{Re}A_0 &= \langle\pi\pi; I=0|\mathcal{T}|K_1\rangle, \\ \sqrt{2}\, e^{i\delta_2}\operatorname{Re}A_2 &= \langle\pi\pi; I=2|\mathcal{T}|K_1\rangle, \\ i\sqrt{2}\, e^{i\delta_0}\operatorname{Im}A_0 &= \langle\pi\pi; I=0|\mathcal{T}|K_2\rangle, \\ i\sqrt{2}\, e^{i\delta_2}\operatorname{Im}A_2 &= \langle\pi\pi; I=2|\mathcal{T}|K_2\rangle. \end{aligned} \tag{3.11}$$

Here δ_0 and δ_2 are the strong-interaction $\pi\pi$ phase shifts in the $I=0$ and $I=2$ channels, respectively. We further introduce two parameters α and β:

$$\begin{aligned} \operatorname{Im}A_0 &= (\alpha+\beta)\operatorname{Re}A_0, \\ \operatorname{Im}A_2 &= \alpha\operatorname{Re}A_2. \end{aligned} \tag{3.12a}$$

If the CP-violating amplitudes $\operatorname{Im}A_0$ and $\operatorname{Im}A_2$ were governed by the $\Delta I = \frac{1}{2}$ rule to the same extent as the CP-conserving ones, then β would be zero. We have chosen to use α and β, rather than the more usual ϵ and ϵ', for later convenience.

We define one more parameter:

$$\omega = (\operatorname{Re}A_2/\operatorname{Re}A_0)\, e^{i(\delta_2-\delta_0)}. \tag{3.12b}$$

Using the branching ratio of $K_S \rightarrow \pi^+\pi^-$ to $K_S \rightarrow \pi^0\pi^0$ (Ref. 7) and the fact that[8] $|\delta_2-\delta_0| \cong 45°$, we obtain

$$|\omega| \cong 0.032. \tag{3.13}$$

From the usual phenomenological expressions[4] for η_{+-} and η_{00}, we may write

$$\frac{\eta_{00}}{\eta_{+-}} = 1 + \frac{i\beta}{\eta_{+-}}\left(\frac{1}{1-\sqrt{2}\,\omega} - \frac{1}{1+\omega/\sqrt{2}}\right).$$

This expression is exact. In view of the smallness of ω, it is nearly the same as

$$\frac{\eta_{00}}{\eta_{+-}} \cong 1+\frac{3i\beta\omega}{\sqrt{2}\,\eta_{+-}}\,. \tag{3.14}$$

Clearly, the value of η_{00}/η_{+-} predicted by our model rests on the calculation of β.

In order to estimate β, we begin by noting that from the definition of A_I, and using Eq. (3.7), we have (after continuation of the right-hand side to physical kaon and pion momenta)

$$e^{i\delta_I}\,\mathrm{Im}\,A_I = 2G^2\sin\theta\cos\theta\sum_{l=e,\mu}\int d^4x\left(\frac{2x^\lambda x^\mu}{x^2}-g^{\lambda\mu}\right)\Delta_1(r;m_l)\,\Delta_1(r;0)\langle\pi\pi;I\,|\,j_\lambda^{(1)\dagger}(x)\,j_\mu^{(0)}(0)\,|K^0\rangle\,F_-(f^2\Delta^2)\,, \tag{3.15}$$

where we have abbreviated

$$F_-(f^2\Delta^2) = 2\pi i\,\delta(1-f_{0l}f_{1l}\Delta^2(r;M))\,\mathrm{sgn}(\mathrm{Im}\,f_{0l}f_{1l})\,.$$

We have already used the free-quark model to deduce the singularity structure of the matrix element of current products in Eq. (3.15). Following Fritzsch and Gell-Mann[9] and Gross and Treiman,[10] who have discussed such products at short and lightlike distances, we shall further assume that the correct SU(3) $\otimes$SU(3) and tensor structure of $j_\lambda^{(1)\dagger}(x)\,j_\mu^{(0)}(y)$ in the limit $r^2=-(x-y)^2\to 0^+$ is obtained by using free-quark fields to construct these currents. Therefore we shall adopt the following simple, phenomenological form for this product:

$$j_\lambda^{(1)\dagger}(x)\,j_\mu^{(0)}(y) = -2i\,\frac{(x-y)^\sigma}{r}\,\Delta_1(r;m_P)\,j^\tau(x\,|\,y)\,\left[g_{\lambda\sigma}g_{\mu\tau}+g_{\lambda\tau}g_{\mu\sigma}-g_{\lambda\mu}g_{\sigma\tau}+i\epsilon_{\lambda\mu\sigma\tau}\right]+M_{\lambda\mu}(x\,|\,y)\,. \tag{3.16}$$

In the free-quark model, the bilocal operators $j_\tau(x\,|\,y)$ and $M_{\lambda\mu}(x\,|\,y)$ would be given in terms of quark fields by

$$j_\tau(x\,|\,y) = :\overline{\Lambda}(x)\,\gamma_\tau(1-\gamma_5)\,N(y):\,, \tag{3.17a}$$

$$\begin{aligned}M_{\lambda\mu}(x\,|\,y) &= :\overline{\Lambda}(x)\,\gamma_\lambda(1-\gamma_5)\,P(x)\,\overline{P}(y)\,\gamma_\mu(1-\gamma_5)\,N(y):\\ &= :j_\lambda^{(1)\dagger}(x)\,j_\mu^{(0)}(y):\,.\end{aligned} \tag{3.17b}$$

Based on these identifications, we assume the bilocal operators have the following properties:

(i) $j_\tau(x\,|\,y)$ has the same quantum numbers as the $(6-i7)$ vector–axial-vector current; in particular, it satisfies $|\Delta I|=\frac{1}{2}$.

(ii) $j_\tau(x\,|\,y)$ is expandable in terms of local operators about $x=y$ [the leading term ought to be $j_\tau^{(6-i7)}(\frac{1}{2}(x+y))$, but this is not crucial to our discussion]; hence, we shall write

$$\langle\pi\pi;I\,|\,j_\tau(x\,|\,y)\,|K^0\rangle = \langle\pi\pi;I\,|\,j_\tau^{(6-i7)}(\tfrac{1}{2}(x+y))\,|K^0\rangle + \sum_{n=1}^{\infty}\frac{(x-y)^{\alpha_1}\cdots(x-y)^{\alpha_n}}{n!}\langle\pi\pi;I\,|\,O_{\tau\alpha_1\cdots\alpha_n}(\tfrac{1}{2}(x+y))\,|K^0\rangle\,. \tag{3.18}$$

(iii) $M_{\lambda\mu}(x\,|\,y)$ is similarly expandable about $x=y$, and the leading term satisfies

$$\frac{G}{\sqrt{2}}\sin\theta\cos\theta\,g^{\lambda\mu}M_{\lambda\mu}(x|x) = \mathfrak{L}_{01}(x)\,. \tag{3.19}$$

As we shall see, the integral involving $M_{\lambda\mu}$ will contribute equally to $\mathrm{Im}A_0/\mathrm{Re}A_0$ and $\mathrm{Im}A_2/\mathrm{Re}A_2$ and will determine α. The j_τ term will contribute only to $\mathrm{Im}A_0$, and will provide the value of β.

To evaluate α, we make the by-now-standard assumption that the δ function in F_- fixes r at a small enough value so that we may make the small-distance approximation in the Δ_1 functions; furthermore, we need keep only the leading term, $M_{\lambda\mu}(\,|x|=r_l|0)\cong M_{\lambda\mu}(0\,|0)$. The integral can then be done explicitly, and we get, using (3.3) and (3.19),

$$\frac{\mathrm{Im}\,A_2}{\mathrm{Re}A_2} = \alpha \cong -(2^{3/2}\pi)^{-1}\sum_{l=e,\mu}\frac{G}{r_l^{\,2}}\,\mathrm{sgn}(\mathrm{Im}\,f_{0l}f_{1l})\,. \tag{3.20}$$

We turn now to the more troublesome term involving j_τ. We notice that by symmetric integration, only the odd terms in the series (3.18) survive. The $n=1$ term is logarithmically divergent, the others are finite. As we discussed above, this divergence, illustrated diagrammatically for the quark model in Figs. 3(b) and 3(c), corresponds to the need for renormalization. We shall assume, in order to obtain an order-of-magnitude estimate for β, that the effect of renormalization is to remove the $n=1$ term from the series (3.18). The terms in the series become rapidly smaller with n (since they involve higher powers of x where $|x|$ is small) so we shall consider only the $n=3$ term to estimate β. If there were a finite residue from the $n=1$ term after renormalization, we would expect it to be of the same order as the $n=3$ term, so in any case we expect the $n=3$ term to provide a fair indication of the magnitude of β.

The $n=3$ term is of the form

$$\frac{-16i}{3!}G^2\sin\theta\cos\theta\sum_{l=e,\mu}\int d^4x\,\frac{x^\mu x^\nu x^\lambda x^\rho}{r}[\Delta_1(r;0)]^3 \times\langle\pi\pi;I|O_{\mu\nu\lambda\rho}|K^0\rangle F_-(f^2\Delta^2), \quad (3.21)$$

where we have already made the small-distance approximation for Δ_1. The matrix element is independent of x; when we do the x integral, the factor $x_\mu x_\nu x_\lambda x_\rho$ becomes

$$\tfrac{1}{24}r^4(g_{\mu\nu}g_{\lambda\rho}+g_{\mu\lambda}g_{\nu\rho}+g_{\mu\rho}g_{\nu\lambda}).$$

Thus we must evaluate expressions such as

$$g^{\mu\nu}g^{\lambda\rho}\langle\pi\pi;I=0|O_{\mu\nu\lambda\rho}|K^0\rangle. \quad (3.22)$$

We know the phase of this matrix element is $ie^{i\delta_0}$. We let its magnitude be m^3, where m has the dimensions of mass. The calculation is then straightforward; putting it together with our earlier result (3.20) we obtain

$$\mathrm{Im}A_I\cong[(24\pi^3)^{-1}Gm^3\sin\theta\cos\theta\,\delta_{I,0}-(2^{3/2}\pi)^{-1}\mathrm{Re}A_I] \times\sum_{l=e,\mu}Gr_l^{-2}\,\mathrm{sgn}(\mathrm{Im}\,f_{0l}f_{1l}). \quad (3.23)$$

Thus β is given by

$$\beta=\frac{1}{24\pi^3}\frac{Gm^3\sin\theta\cos\theta}{\mathrm{Re}A_0}\sum_{l=e,\mu}Gr_l^{-2}\,\mathrm{sgn}(\mathrm{Im}\,f_{0l}f_{1l}). \quad (3.24)$$

The value of $|\mathrm{Re}A_0|$ is obtained from $K_S\to\pi\pi$ $(I=0)$ and is given by 0.346×10^{-6} GeV.[11] Furthermore, on the basis of our previous arguments, we know that $Gr_l^{-2}\cong10^{-3}$. Finally, using the value $|\eta_{+-}|\cong1.95\times10^{-3}$,[11] we can calculate the magnitude of the correction term

$$3i\beta\omega/\sqrt{2}\;\eta_{+-}=\eta_{00}/\eta_{+-}-1 \quad (3.25)$$

for various values of m. The results are displayed in Table I, in which we assume m to be a mass characteristic of the strong interactions. Now, even with the various assumptions we have made, the value of m is unknown. Furthermore, the value of β depends on m^3, and thus is quite sensitive to

TABLE I. Magnitude of the correction term (3.25), using Eq. (3.24) for various values of m.

	m (GeV) 0.5	1.0	5.0	10.0
β	1.27×10^{-6}	1.01×10^{-5}	1.27×10^{-3}	1.01×10^{-2}
$\left\lvert\frac{\eta_{00}}{\eta_{+-}}-1\right\rvert$	4.4×10^{-5}	3.5×10^{-4}	0.044	0.35

variations in m, as Table I demonstrates. Nevertheless, we see that the correction term, (3.25), is of order 10^{-4} for $m=0.5$ GeV, and reaches 0.35 when m is 10 GeV. Thus for "reasonable" values of m we can easily have $|\eta_{00}|\cong|\eta_{+-}|$, as is currently favored by experiment.[6,11]

This is, of course, not a derivation; we have presented the discussion for two reasons: first, to give the reader an idea of how far one can proceed before ignorance of the strong interactions begins to dominate the calculation, and second, to show that, without invoking any unusual or hitherto unexpected properties of the strong interactions, the result $\eta_{+-}\cong\eta_{00}$ is compatible with our model.

To summarize: We have assumed that the observed CP violation occurs via higher-order weak interactions. By examining the $K^0-\bar{K}^0$ mass matrix and the process $K^0\to2\pi$ under this assumption, we have found that our theory is indeed compatible with experiment, subject to the following conditions: (a) We choose $f_{0l}\equiv\frac{1}{2}(f_0+f_1^*)$ real, so that CP-violating effects occur only at leptonic or semileptonic vertices, in our case via the combination $f_{0l}f_{1l}$ of minor coupling constants; ϵ is then given by a third-order-weak matrix element. (b) The effective cutoff lengths r_{0l} and r_l are estimated to be approximately $\frac{1}{10}$ GeV^{-1} (to within an order of magnitude); this choice gives reasonable agreement with experiment, and furthermore predicts fairly sizeable CP violation in other processes, such as possibly μ decay and neutron β decay.

Note added in proof. One of us (K.L.) has calculated the second-order weak CP-violating amplitude for $K_L\to\mu^+\mu^-$, using the above choice of parameters. It is found to be large enough to interfere with the dominant absorptive amplitude (viz., that for $K_L\to\gamma\gamma\to\mu^+\mu^-$), thereby reducing the expected rate for $K_L\to\mu^+\mu^-$. Further, the theory is not in conflict with experimental upper bounds on the rates for $K_S\to\mu^+\mu^-$ or $K^+\to\pi^+l^+l^-$.

We close this section with a few remarks about the induced electric dipole moment of the neutron. In Nth order the appropriate matrix elements are of two types:

$$M_1^{(N)}=\langle n(p_2)|\,T[\mathcal{L}_{a_1b_1}(x_1)\,\mathcal{L}_{a_2b_2}(x_2)\cdots\mathcal{L}_{a_Nb_N}(x_N)\,j^\mu(0)]\,|n(p_1)\rangle\langle 0|\,T[\,:\exp(f_{a_1b_1}\varphi^\dagger\varphi(x_1)):\cdots:\exp(f_{a_Nb_N}\varphi^\dagger\varphi(x_N)):\,]|0\rangle$$

and

$$M_2^{(N)}=\langle n(p_2)|\,T[\mathcal{L}_{a_1b_1}(x_1)\cdots\mathcal{L}_{a_Nb_N}(x_N)]\,|n(p_1)\rangle\langle 0|\,T[\,:\exp(f_{a_1b_1}\varphi^\dagger\varphi(x_1)):\cdots:\exp(f_{a_Nb_N}\varphi^\dagger\varphi(x_N)):j^\mu(0)]\,|0\rangle\,.$$

Here $|n(p)\rangle$ denotes a neutron state of momentum p, and j^μ is the electromagnetic current. In M_1, the photon couples to the hadrons or leptons described by $\mathcal{L}_{ab}$, while in M_2 it couples to one of the charged φ particles.

We can dispose of M_2 very quickly by noting that $\varphi^\dagger\varphi(x)$, and hence $\exp\varphi^\dagger\varphi(x)$, is even under charge conjugation, while j^μ is odd. Therefore, the second factor in M_2 vanishes and $M_2=0$ for all N.

Presumably an electric dipole moment term will be generated by the appearance of CP violation in $M_1^{(N)}$. We now show that there is no CP violation for $N=1$ or 2.

For $N=1$, it is trivial, because the second factor in M_1 is just

$$\langle 0|\,T:\exp(f_{ab}\varphi^\dagger\varphi(x)):|0\rangle=1\,,$$

i.e., there is no superpropagator, and hence no CP violation.

For $N=2$, we must enumerate various cases. First we notice that, since there are no external leptons, if $\mathcal{L}_{a_1b_1}(x_1)$ has a lepton current,

$$\mathcal{L}_{a_1b_1}(x_1)=\mathcal{L}_{a_1l}(x_1)\,,$$

then there must be a compensating current in $\mathcal{L}(x_2)$,

$$\mathcal{L}_{a_2b_2}(x_2)=\mathcal{L}_{lb_2}(x_2)\,.$$

Then, since there is also no strangeness change between the initial and final states, if $a_1=0$, 1 or l' we must have $b_2=0$, 1 or l'. Thus the effective minor coupling constant $f_{a_1b_1}f_{a_2b_2}$ has $a_1=b_2$ and $a_2=b_1$, so it is of the form $|f_{ab}|^2$ and no CP violation can occur. If there is no lepton current, then, inasmuch as we have already assumed f_{01} is real, there can be no CP violation. Even if we drop that assumption, however, we can readily see that the only purely hadronic possibilities consistent with the quantum numbers of the external states are $f_{aa}f_{bb}$ ($a, b=0$ or 1) and $f_{01}f_{10}$; both of these are purely real, and there is therefore no CP violation.

Having exhausted all possibilities, we conclude that $M_1^{(2)}$ will produce no electric dipole term. In third order, we manifestly can have CP violation, e.g., from the product $\mathcal{L}_{l0}\mathcal{L}_{1l}\mathcal{L}_{01}$. Because of the lepton propagators, we estimate the magnitude of a term from this product to be

$$(GM_N^2)\left(\frac{G}{r_l^2}\right)d_1\,, \tag{3.26a}$$

where d_1 is a dipole moment that would be induced in first-order weak processes (say, by a theory where T violation occurs in j^μ). This latter is estimated to be[12]

$$d_1\approx 10^{-19}\ e\,\text{cm}\,,$$

so that, based on our theory, we would predict a dipole moment for the neutron of order

$$(10^{-5})\times(10^{-3})d_1\approx 10^{-27}\ e\,\text{cm}\,, \tag{3.26b}$$

which is orders of magnitude below present experimental limits.[13]

IV. PRODUCTION OF φ PARTICLES

We now turn to possibly an even more speculative subject than the preceding, namely, a calculation of the production cross section for the superpropagating φ particles. Here the speculation arises not so much in the details of the calculation itself, which is fairly straightforward, but rather because we must commit ourselves to a particular view of the true nature of the φ particles.

There are at least three viewpoints as to the meaning of the φ particles:

(i) The φ's are conventional particles which, once produced, should have the usual properties of scalar charged particles of mass M.

(ii) They represent some kind of average over the already observed particles, which can be exchanged in higher-order weak processes; in other words, in choosing a nonpolynomial factor and calculating to $O(G^2)$ we are really performing an effective sum over many processes which occur to all orders in G. The idea that such a selective summation procedure might produce a finite theory has been suggested in the past.[14]

(iii) The φ's are a manifestation of the short-distance nature of space-time, and do not represent any particles, averaged over or otherwise.

In what follows we shall assume that (i) is the correct choice, and calculate φ-particle production on that basis. It then remains to be understood why the φ particle, which otherwise behaves normally, has the distinction of being coupled nonpolynomially; we shall not attempt an explanation here, and shall cease further speculation at this point.

Proceeding with the calculation, we note that if the amplitude

$$A \to B \tag{4.1}$$

occurs in order G, then so does the amplitude

$$A \to B + N(\varphi^+\varphi^-). \tag{4.2}$$

In reaction (4.2), we let the momentum of state A be p_A, and that of state B be p_B. We denote the momenta of the N φ^+ particles by $k_1, \ldots, k_N$, and those of the N φ^- particles by $q_1, \ldots, q_N$. Furthermore, we let $\sum_{i=1}^N (k_i + q_i) = Q_N$. Then the relevant matrix element in first order is

$$\langle p_B, k_1, \ldots, k_N, q_1, \ldots, q_N | \mathcal{L}(0) | p_A\rangle = \sum_{a,b} \langle k_1, \ldots, k_N, q_1, \ldots, q_N | : \exp f_{ab}\, \varphi^\dagger \varphi(0) : |0\rangle \frac{G}{\sqrt{2}} \langle p_B | : j_\lambda^{(a)} j^{\lambda(b)\dagger} : | p_A\rangle$$

$$= \frac{G}{\sqrt{2}} \sum_{a,b} (f_{ab})^N \langle p_B | : j_\lambda^{(a)} j^{\lambda(b)\dagger} : | p_A\rangle . \tag{4.3}$$

We note the following features: First, due to the appearance of only $\varphi^\dagger\varphi$ in $\mathcal{L}$, the φ particles can be created only in pairs. Similarly, in an annihilation amplitude, the φ's must decay in pairs. A single φ particle would be stable. Second, in (4.3), note that the matrix element does not depend on the momenta k_i or q_i; it is simply proportional to that for reaction (4.1). This suggests the attractive possibility of using the basic process $A \to B$ to select out the various minor coupling constants f_{ab}. [For example, $\nu_e + e \to \nu_e + e + N(\varphi^+\varphi^-)$ measures f_{ee}, $n + p \to p + \Lambda + N(\varphi^+\varphi^-)$ measures f_{01}, etc.] Furthermore, by varying N, one can test whether the coefficient in (4.3) is really $(f_{ab})^N$, thereby checking whether the exponential we have used is correct, or whether perhaps a different nonpolynomial function is preferred.

However, this type of analysis depends on the N (pair) cross section $\sigma_{AB}^{(N)}$ for reaction (4.2) being reasonably large so that the indicated experiments can be done. We continue our calculation to see if this is so. From now on, we specialize to the reaction

$$p(p_1) + n(p_2) \to n(p_3) + p(p_4) + N\varphi^+(k_1, \ldots, k_N) + N\varphi^-(q_1, \ldots, q_N). \tag{4.2$'$}$$

We choose hadrons because (in the absence of colliding electron-neutrino beams) only with hadrons can one achieve fairly large center-of-mass energies, which are necessary in the first place simply to reach the production threshold for N $\varphi^+\varphi^-$ pairs, and in the second place because the cross sections described by (4.3) will in general grow significantly with increasing center-of-mass energy.

We make the following two approximations: (i) The reaction (4.2$'$) measures both f_{00} and f_{11}; however, we shall drop the f_{11} term for simplicity (it is expected to be smaller in any case by a factor of $\sin^2\theta$); (ii) we shall replace the basic hadronic amplitude in (4.3) by the equivalent high-energy form of a typical first-order leptonic amplitude, in order to have a definite form for our cross section.

The cross section is

$$\sigma_{np}^{(N)}(s) = \tfrac{1}{2}G^2 \frac{f_{00}{}^{2N}(2\pi)^4}{4E_1E_2|\vec{v}_1 - \vec{v}_2|} \int \frac{d^3p_3 d^3p_4}{(2\pi)^6 4E_3E_4} \left(\prod_{i=1}^N \frac{d^3k_i d^3q_i}{(2\pi)^6 4k_i^0 q_i^0}\right)$$

$$\times \delta^4(p_1 + p_2 - p_3 - p_4 - Q_N) 16M^4 \tfrac{1}{4}\sum_{\text{spins}} |\langle p_3, p_4 | j_\lambda^{(0)\dagger} j_\lambda^{(0)} | p_1, p_2\rangle|^2 . \tag{4.4}$$

Here M is the nucleon mass. According to the second approximation above, we make the replacement

$$16M^4 \tfrac{1}{4}\sum_{\text{spins}} |\langle p_3, p_4 | j_\lambda^{(0)\dagger} j_\lambda^{(0)} | p_1, p_2\rangle|^2 \approx 64(p_1\cdot p_2)(p_3\cdot p_4), \tag{4.5}$$

which is the result for $l + \nu \to l + \nu$.

The rest of the calculation is "just" phase space. In order to make some headway, we go to the laboratory frame and let the total momentum be $P \equiv p_1 + p_2$. Since the incident proton is taken to be extremely relativistic, we have, to a good approximation,[15]

$$p_1 \approx P,$$

$$p_3 \approx p_4 \approx k_i \approx q_i \approx \frac{1}{(2N+2)} P,$$

i.e., all the particles are produced forward, and all share approximately equally in the incident momentum. It follows that

$$p_3 + p_4 = \frac{1}{N+1} P \quad \text{and} \quad Q_N = \frac{N}{N+1} P .$$

Furthermore,

$$2p_1\cdot p_2 = s - 2M^2 \approx s$$

and

$$2p_3\cdot p_4 = \left(\frac{1}{N+1}\right)^2 P^2 - 2M^2$$

$$= \frac{1}{(N+1)^2}[s - 2(N+1)^2M^2] .$$

Also, the incident flux factor is given by

$$2E_1E_2|\vec{v}_1-\vec{v}_2|\approx 2p_1\cdot p_2\approx s\,.$$

Thus, in this approximation, the matrix element (4.5) is independent of the integration variables, and we may write

$$\sigma_{np}^{(N)}(s)\approx 8\pi G^2 f_{00}{}^{2N}\left[\frac{s-2(N+1)^2M^2}{(N+1)^2}\right]\Omega^{(N)}(s)\,, \quad (4.6)$$

where $\Omega^{(N)}(s)$ is the phase space for $2N+2$ scalar particles:

$$\Omega^{(N)}(s)=\frac{(2\pi)^3}{(2\pi)^{6N+6}}\int\frac{d^3p_3d^3p_4}{4E_3E_4}\prod_{i=1}^{N}\frac{d^3k_id^3q_i}{4k_i^0q_i^0}\times\delta^4(P-p_3-p_4-Q_N)\,. \quad (4.7)$$

When $2N+2\gg 1$ and $s\gg 4(N+1)^2M^2$, (4.7) can be approximated by[16]

$$\Omega^{(N)}(s)\approx\frac{s^{2N}}{(2\pi)^{4N+3}\Gamma(4N+4)(4N+4)^{1/2}} \quad (4.8)$$

so that, with the help of Stirling's approximation, the cross section becomes

$$\sigma_{np}^{(N)}(s)=\left[\frac{G^2s}{2\pi}\right]\left[\frac{f_{00}}{4\pi^2}\right]^{2N}\left[\frac{e^4}{128\pi}\,\frac{1}{(N+1)^6}\left(\frac{e^2s}{16(N+1)^2}\right)^{2N}\right]. \quad (4.9)$$

The first factor in (4.9) is the $pn\to pn$ elastic cross section at high energies, $\sigma_{pn}^{(0)}(s)$; the second factor explicitly demonstrates the dependence on the cutoff length

$$r_0\cong\left[\frac{f_{00}}{4\pi^2}\right]^{1/2},$$

and the third factor contains the rest of the dependence on s and N.

With the order-of-magnitude estimates of r_0 and the mass of the φ particle which we have made previously,

$$r_0\approx\tfrac{1}{10}\text{ GeV}^{-1}\,,\qquad M\approx 1\text{ GeV}\,, \quad (4.10)$$

the cross sections (4.9) turn out to be quite small, even at very high energies. For example, with the numerical values (4.10), and with $s=2\times10^4$ (GeV)^2 (corresponding to a lab energy of 10^4 GeV), we obtain $\sigma^{(1)}=0.16$, $\sigma^{(2)}=0.29$, $\sigma^{(3)}=0.17$, and $\sigma^{(4)}=0.042$, in units of 10^{-6} mb; $\sigma^{(N)}$ for $N>4$ continues to decrease.

Thus it is extremely unlikely that the φ particles will have any immediate experimental consequences. This is especially true since it is also not clear that they are electrically charged. They obviously carry a quantum number which is conserved, but nothing in the formalism requires this to be electric charge; if it is not, then the φ's will behave like massive neutrinos, and will hardly interact once they are produced.

In summary, we see that when we make the most "down to earth" choice and treat the φ's as conventional particles, they turn out to be very difficult to produce and perhaps even more difficult to detect, so that the question of their existence, even granting the validity of our theory, becomes a slippery one indeed.

V. SUMMARY AND CONCLUSIONS

In this and the preceding paper, we have begun an attempt to resolve two of the outstanding problems of weak-interaction physics, namely, the divergence difficulties in higher orders of perturbation theory and the observation of CP-violating effects in the neutral K-meson system. We have employed rather unconventional means to carry out this attempt, namely, the multiplication of the usual current$\times$current Lagrangian by a nonpolynomial function of the field of a hitherto undetected particle φ. However, if we insist on working in the context of a current$\times$current theory in order to retain the experimental successes of the lowest-order theory, then something like the "good" short-distance properties of nonpolynomial field theory is needed to overcome the divergences introduced by the current products. As we have seen, the particular choice of nonpolynomial modification made by us goes a long way toward solving these problems, since the divergences of the theory are now similar to those of quantum electrodynamics and since CP violation emerges in a very natural way.

The weak interactions of leptons in $O(G^2)$ were discussed in I. There we found that lepton-lepton scattering is finite, while the lepton self-energy is only logarithmically divergent. We showed that, in general, the finite part of any amplitude went inversely as the square of the naturally appearing cutoff length r_0. Further, any amplitude could be written as the sum of two terms, the first an entire function of the relevant invariant energies, and the second a dispersion integral over products of first-order weak amplitudes, as required by unitarity. Since the major thrust of our work so far has been to develop the tools needed to discuss renormalization of the weak interactions in all orders of perturbation in G, we have left undone many interesting calculations of second-order weak processes. These include especially situations which might involve time-reversal violation in leptonic processes. Apart from (possibly important) hadronic effects, these may be studied using the techniques of Secs. II and III of paper I.

As far as higher orders in G are concerned, we derived expressions for the time-ordered products, $\mathfrak{F}^{(N)}$, of any number $N\geq 2$ of nonpolynomial factors,

$:\exp[f\varphi^\dagger\varphi]:$. It was then a simple matter to show that the short-distance behavior of $\mathfrak{F}^{(N)}$ was exactly what was needed to make the naive degree of divergence independent of the order N. There remain the difficult problems of showing that the theory is unitary-analytic and renormalizable.

In this second paper, we turned our attention to the problem of CP-invariance violation in the neutral K-meson system. Here we were guided by the idea that CP violation is a higher-order weak effect, intimately connected with the short-distance behavior of the theory and with the structure of the superpropagator. Having realized that physical amplitudes have a definite analytic structure in the minor coupling constants $f_{ab}f_{cd}$, we identified CP-noninvariant terms with the discontinuity across the cut in the complex $f_{ab}f_{cd}$ plane. Although we are severely limited in our ability to make hard calculations of quantities involving hadronic currents, estimates based on the light-cone structure of quark currents led us to some definite relationships among f_0, f_1, f_e, and f_μ. Specifically, in order to obtain estimates for η_{+-}, η_{00}, and the K^0 mass matrix elements that were consistent among themselves and with experiment, it was necessary to choose f_{01} real and $f_{0l}f_{l1}$ complex, with $|f_{0l}f_{l1}/16\pi^4|^{-1/4} \lesssim 10$ GeV. This choice of the f's gives $K_2 \rightarrow \pi\pi$ as a second-order weak process, while $T_{12} \cong -\epsilon(\gamma_S + 2i\Delta m)m_{K^0}$ is a third-order weak quantity.

Finally, we calculated the production cross section for $\varphi^+\varphi^-$ pairs, and found results of (at most) 10^{-6} mb when incident energies are in the TeV range, thus making it unlikely that the φ-particles, if they exist, will be detected in the near future.

We have obviously left many problems unsolved. In addition to the T-violating leptonic processes mentioned above, there are many other second-order amplitudes which will attract new interest as high-energy weak-interaction experiments begin to be done. On a more fundamental level there is also the formidable problem of proving that our theory is renormalizable, and therefore worthy of serious consideration as a complete and consistent description of the weak interactions.

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[1]K. Lane and A. Chodos, preceding paper, Phys. Rev. D 6, 581 (1972) (hereafter referred to as I).

[2]See, for example, H. Fritzsch and M. Gell-Mann, in *Broken Scale Invariance and the Light Cone*, 1971 Coral Gables Conference on Fundamental Interactions at High Energy, edited by M. Dal Cin, G. J. Iverson, and A. Perlmutter (Gordon and Breach, New York, 1971), Vol. 2, p. 1.

[3]The idea that CP violation might arise from a difference in phase between first- and higher-order weak amplitudes was suggested by L. Wolfenstein, Phys. Letters 15, 196 (1965).

[4]The standard formalism of CP-invariance violation is presented in L. Wolfenstein, in *Theory and Phenomenology in Particle Physics*, edited by A. Zichichi (Academic, New York, 1969), Part A.

[5]K. Lane and A. Chodos, Phys. Rev. Letters 27, 966 (1971).

[6]The result $|\eta_{00}/\eta_{+-}| = 1.00 \pm 0.06$ has been obtained by the CERN-Aachen-Torino collaboration. We are unable to find a published account of this experiment; however, indirect references may be found in J. Cronin, lectures given at the Fourth Hawaii Topical Conference on Particle Physics, 1971 (unpublished); see also Ref. 3 of J. Nuyts, Phys. Rev. Letters 26, 1604 (1971).

[7]B. Aubert, in *Proceedings of the Topical Conference on Weak Interactions, CERN, 1969* (CERN, Geneva, 1969).

[8]J. R. Bensinger, A. R. Erwin, M. A. Thompson, and W. D. Walker, Phys. Letters 36B, 134 (1971), and references therein.

[9]H. Fritzsch and M. Gell-Mann, Ref. 2.

[10]D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 1059 (1971). These authors have examined the possible change in the light-cone structure of current commutators due to the addition of a vector gluon, and have concluded that the results of the free-quark model apparently persist in the presence of such an interaction.

[11]We have relied on Particle Data Group, Rev. Mod. Phys. 43, S1 (1971), for these experimental values.

[12]R. E. Marshak, Riazuddin, and C. P. Ryan, *Weak Interactions of Elementary Particles* (Wiley, New York, 1969), p. 659, Eq. (6.38). We use 10^{-19} instead of 10^{-20} since we prefer not to insert the factor $1/4\pi$ as they do.

[13]J. K. Baird, P. D. Miller, W. B. Dress, and N. F. Ramsey, Phys. Rev. 179, 1285 (1969).

[14]See, for example, T. Appelquist and C. E. Carlson, Phys. Rev. 187, 2119 (1969), and references therein.

[15]We thank J. Halpern for discussions on this point.

[16]V. A. Kolkunov, Zh. Eksp. Teor Fiz. 43, 1448 (1962) [Sov. Phys.–JETP 16, 1025 (1963)].