full three-body kernels instead of from the subsystem operators only. To find such terms in a reasonable way, however, is evidently very complicated.

[25]Here, we assume that the full potential V is given as the sum of the three two-particle potentials V_γ: $V = \sum_\gamma V_\gamma$. A slight generalization of the formalism allows us to incorporate also three-particle potentials.

[26]The normalization of $\vec{q}_\alpha$ is chosen such that $\vec{q}_\alpha^{\,2}$ represents the kinetic energy of the two colliding bodies in the over-all c.m. system.

[27]When replacing the right-hand side of (A13) by $R_{\alpha m}^{1/2} \lambda_{\alpha m}^{-1} \delta_{nm}$, with an arbitrary constant $R_{\alpha m}$, the residue of $(-\hat{\Delta}_\alpha)_{nm}$ at $z = \hat{E}_{\alpha r}$ becomes $\delta_{nr} R_{\alpha r} \delta_{rm}$. We then have to renormalize the transition amplitude $\tau_{\beta n, \alpha m}$ by a factor $R_{\beta n}^{1/2} R_{\alpha m}^{1/2}$, in order to get the conventional normalization.

[28]The proof is given in Appendix A of Ref. 10.

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Flux Quantization and Particle Physics

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Quantized flux has provided an interesting model for muons and for electrons: One closed flux loop of the form of a magnetic dipole field line is assumed to adopt alternative forms which are superposed with complex probability amplitudes to define the magnetic field of a source lepton. The spinning of that loop with an angular velocity equal to the *Zitterbewegung* frequency $2mc^2/\hbar$ implies an electric Coulomb field, (negative) positive, depending on (anti) parallelism of magnetic moment and spin. The model implies CP invariance. A quark may be represented by a quantized flux loop if interlinked with another loop in the case of a meson, with two other loops in the case of a baryon. Because of the link, their spinning is very different from that of a single loop (lepton). The concept of a single quark does not exist accordingly, and it is seen that a baryon with a symmetric spin-isospin function in the SU(2) × SU(3) quark representation might not violate the Pauli principle because the wave function representing the relative position of linked loops may be chosen antisymmetric. Weak interactions may be understood to occur when the flux loops involved in the interaction have to cross over themselves or over each other. Strangeness is readily interpreted in terms of the trefoil character of a λ quark: Strangeness-violating interactions imply crossing of flux lines and are thus weak and parity-nonconserving. $\Delta S = \Delta Q$ is favored in such interactions. Intrinsic symmetries may be interpreted in terms of topology of linked loops. Sections I and II give a short résumé of the 1971 paper.

I. INTRODUCTION

In recent years, several attempts have been made to move from an abstract description of quarks [successfully achieved in terms of the SU(3) and SU(6) symmetries] to a more specific model which might relate the internal and the external symmetries. In this connection the question has arisen whether magnetic monopoles can be considered as the physical counterparts to the formal definition of quarks. To extend this type of approach to a more conservative viewpoint, the obvious suggestion has been made that a quark may be considered as a closed quantized flux loop if interlinked with other flux loops. To verify the details of such a model, the known classifications of particles have been discussed in terms of the topological structure of linked quantized flux loops.

In our previous work we succeeded in formulating a charged-lepton theory (i.e., a muon or an electron) in terms of quantized flux. It was proposed that the lepton's magnetic field may be represented by the superposition of alternative forms which a quantized flux loop may adopt. These alternative loopforms should be superimposed with complex probability amplitudes in a manner similar to the superposition of alternative path histories in Feynman's space-time approach to quantum mechanics. With the appropriate choice of the spatial distribution of these complex amplitudes, the magnetic field of a muon-magneton or a Bohr-magneton source may be reconstructed. Furthermore, the electric Coulomb field has been shown to result from the spinning of the loop. In our previous work we have also made a quantitative proposal to explain the relation between the charge e of the electron and the Planck constant h.

To move from this heuristic to a more familiar and concise formulation of the theory, the probability-amplitude distributions of manifolds of loopforms are expressed in terms of a wave equation for these amplitudes (cf. the Appendix of the present paper). It has, however, been found that the preceding heuristic formulation of the theory and the analysis of its consistency[1] were essential in anticipating a more sophisticated formulation.

In the present paper we have proposed a topological configuration of the linked quantized flux loops to represent mesons and baryons. Mesons and baryons are represented by two and three interlinked quantized flux loops, respectively. We should note that the manifold of infinitely many loopforms of this type (defining a fibration of space) is needed for a full description of the particle. The superposition of this manifold, weighted appropriately with complex probability amplitudes, defines the internal structure of the particle.

Topologically these loops may be characterized by their winding numbers about the circular and about the straight axis (Sec. III, Figs. 1–5); they are assumed to be of the type of torus knots to avoid unnecessary singularities, and to permit independent spinning of the different quarks which make up a particle. The loop-antiloop (quark-antiquark) dichotomy is assumed to correspond to left-handedness–right-handedness which in the case of a neutrino or in the case of a λ quark implies the forms of left-handed-right-handed trefoils.

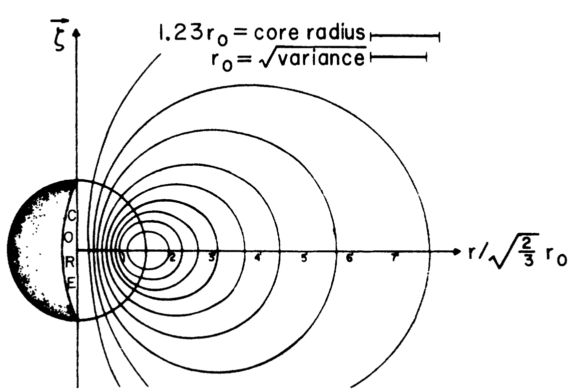

FIG. 1. Spinning-top model (spinning about a straight axis and a circular axis), referring to a muon or an electron. The figure shows alternative loopforms (magnetic flux lines) of an extended source corresponding to a spherically symmetric Gaussian distribution of polarization in $\vec{\zeta}$ direction. The variance of the distribution is denoted by r_0^2, the core radius is chosen so as to go through the circular axis of the magnetic field which is located at $1{,}23r_0$ for the Gaussian distribution. The total amount of flux is subdivided into 10 equal sheaves by the nine toroidal surfaces (which in the figure look like flux lines). This extended source model is supposed to visualize the quasinonlocal nature of a "single-particle" source, implied by the Pryce-Tani-Foldy-Wouthuysen transformation.

We assume that reactions which imply a loop crossing over itself, or over the loop with which it interacts, is slow (a weak process). Strangeness may therefore be interpreted in terms of left- or right-handedness of the trefoil (λ) knot. The reason for this is that two mirror-related trefoils may readily annihilate without flux-loop crossings.

Due to the topological constraints, the spinning of interlinked loops differs very much from that of free loops. Consequently the unlinkage of a quark (conditional to conservation laws) is not considered to lead to a "free quark" but to a lepton.

It was shown earlier[1] that the spinning of loops implies an electric potential by the very same assumption which defined flux in the first place. The signature of equivalent charge depends on parallelism (+) or antiparallelism (−) of magnetic moment and spin.

The quark loops are assumed to spin about both the circular and about the straight axes with the same angular velocity $2m_q c^2/\hbar$, in a left-handed spin for left-handed loops, and right-handed spin for right-handed loops so as to minimize electric field energy production. Thus the electric field will be seen to be proportional to the difference of the winding numbers which characterize a loop.

The ratio of absolute value of equivalent electric charge for a $\mathfrak{N}$ loop of winding numbers (2, 1), for a $\mathcal{P}$ loop of winding numbers (3, 1), and a λ loop of winding numbers (3, 2) are $2-1=1$ to $3-1=2$ to $3-2=1$; the signatures of the charges depend on the magnetic moment versus spin orientation, and will be determined by the following consideration.

Charge conservation (in reactions involving a replacement of a quark by another quark with accompanying change $\pm e$ or 0 of charge) implies that the different quark charges be ± 1 or 0 units of e apart. Only the assignment $-\frac{1}{3}e, +\frac{2}{3}e, -\frac{1}{3}e$ satisfies these conditions and implies the integer-charge spectra $0, \pm e, \pm 2e$ for $q\bar{q}$ and for qqq, but not for qq or $qq\bar{q}$ nor their antiparticles.

II. REVIEW OF A LEPTON'S ELECTROMAGNETIC FIELD IN TERMS OF QUANTIZED FLUX

Flux Quantization

Flux quantization arises from the possibility that the wave function of an electrically charged field particle, even though single-valued, may have a phase ϑ which is single-valued only modulo 2π. Lines in ordinary three-dimensional space around which the phase of a field particle changes by $\pm 2\pi$ define quantized flux. Indeed, if we set the gauge-invariant combinations of four-potential $A_k = (V, -\vec{A})$ with the derivative of the phase ϑ of any field particle's ψ function $|\psi| \exp i\vartheta$,

$$A_k - (\hbar c/e)\partial_k\vartheta = \mathfrak{A}_k = 0, \tag{1}$$

equal to zero, we simply state that a ψ field with a singularity of $\partial_k\vartheta$, characterized by a ϑ change of 2π, implies a singular potential A_k of quantized flux:

$$\begin{aligned} -2\pi &= \oint \nabla\vartheta \cdot d\vec{r} \\ &= (e/\hbar c) \oint \sum_1^3 A_k dr^k \\ &= -(e/\hbar c)\Phi_q . \end{aligned} \tag{2}$$

We want to understand a source particle in terms of quantized flux. In particular we assumed that a source lepton is to be understood as a single, closed, quantized flux loop which takes on the form of a closed magnetic field line of a magnetic dipole source.

Superposition of Loopforms

In a manner somewhat analogous to Feynman's space-time approach to quantum mechanics which constructs a quantum-mechanical path from a superposition of "alternative path histories" of classical paths, a superposition with complex probability amplitudes, we construct the magnetic dipole field of a source lepton from a complex amplitude superposition of the alternative loopforms which the quantized flux loop may adopt. (Such a superposition is not a superposition of different quantum states, but a superposition of alternative classical or semiclassical loopforms to construct a quantum state.)

A particular closed loop with its implication concerning the phase ϑ of the field particle's wave functions is to be considered as only one of a manifold of "loopforms" characterizing the source lepton (in terms of a corresponding manifold of field particle wave functions). The superposition of these loopforms is assumed to be made with probability amplitudes, continuous functionals of the loopforms, and result in a continuous magnetic and (as we shall see) electric field.

The flux loop is considered to be "attached" to the source, i.e., to the point at which the Maxwell-Lorentz equations have inhomogeneous terms. The probability amplitudes are to be chosen such as to result in a field satisfying the Maxwell-Lorentz equations. This leaves their phases still largely undetermined, a fact which was seen[1] to be altogether important for the possibility of constructing a consistent theory of leptons. The phases of the probability amplitudes have nothing to do with the above-mentioned phase ϑ of a field particle's wave function.

The Pertinent Fields

We have to distinguish three types of fields. First, the ψ functions of one of many field particles, its phase ϑ defining quantized flux (and as we shall see later, also an electric field). Second, the probability amplitudes of the loopform attached to one source lepton; they, *in toto*, represent the quantum-mechanical state of the source. Third, the electromagnetic field defined by the former fields; this field is a quantum-mechanical observable.

Heuristic Approach to Determine Probability Amplitudes

A central problem of the present and of the previous work[1] is how to make the appropriate choice for the probability amplitudes of the alternative loopforms. The usual way to do this is to find the appropriate (in this case "internal") wave equation for the system and use conventional techniques to carry through the analysis. In the Appendix to this paper a sketch of such an approach to this problem is given. This facilitates the transition from the ordinary techniques of the particle picture to the loop picture with all its topological refinements.

As it is inadvisable to speculate about possible wave equations and equations describing topological manifolds without previous study of a heuristic model, we took it as our task in the previous and present work to develop such a heuristic model which should determine the structure of the loop model and the physical meaning of the fields involved in it. This essentially qualitative model is made to fit the Maxwell-Lorentz equations, and the conservation laws and other data of particle physics. This heuristic step may help to find a precise quantitative formulation of the model, a formulation which might make use of fiber-space topology and differentiable manifolds. In the present paper we would like to check the consistency of that model and show how such a heuristic model makes it possible to understand some basic open issues in particle physics.

Magnetic Monopoles, Quantized Flux Loops, and Their Structure

Considerations similar to those relating to quantized flux had previously been made in order to introduce magnetic monopoles. With the introduction of magnetic charge, the physical basis for electromagnetic theory receives a drastic change, however: It may be an unnecessary complication of the well-defined discipline of quantum electrodynamics.

The introduction of closed quantized flux loops as a basic entity implies new mathematical techniques and concepts. Whereas magnetic monopoles may use the formalisms developed for point charges, the flux loops need, in order to describe their probability amplitude distributions, techniques which imply interesting topological concepts. But a model of particles in terms of flux loops does not imply a change from Maxwell-Lorentz theory as monopoles do. It seems therefore obvious that this more conservative loop model should be developed.

The basically new features which enter in the present theory are structural. In the previous work an essential issue was to characterize the manifolds of flux loopforms in terms of statistically independent bundles of loopforms of a lepton; it was essentially by geometrical means that these bundles were defined. In the present paper it is the topological characterization of linked loops of mesons and baryons which brings the interesting results. One may, with a certain amount of oversimplification, say that the present program is one of reducing problems of particle physics to topological issues of quantized magnetic flux loops.

Electromagnetic Field

Let us, for the moment, assume that this magnetic dipole is constructed such as to correspond to a moment $e\hbar/2mc$, and discuss later how the probability amplitudes will have to be chosen to properly relate this dipole moment to the quantized flux Φ_q. The effective (averaged) magnetic vector potential $\vec{A}_{\text{eff}}$ will be time-independent for a fixed lepton.

By the same definition, $\mathfrak{A}_k = 0$, which defined quantized flux through

$$\vec{A} = -(\hbar c/e)\nabla\vartheta, \tag{3a}$$

we find that moving quantized field lines define an electric potential

$$V = +(\hbar c/e)\partial\vartheta/\partial ct. \tag{3b}$$

We assume that every flux loopform performs a spinning motion with the *Zitterbewegung* angular velocity

$$\Omega = 2mc^2/\hbar \tag{4}$$

about its flux-orientation axis. The field $\vartheta(t, x, y, z)$ of the field lepton is carried along with that spinning motion (otherwise one would get an entanglement).

Any point which is "linked" with that loopform (Fig. 2 of Ref. 1), i.e., which is inside the perimeter of the loopform, experiences a $\partial\vartheta/\partial ct$ which amounts to a unidirectional rate of change of ϑ, a change $\pm 2\pi$ per spin period. An outside point Q, not linked with that loopform, experiences only periodical fluctuations of ϑ. We may thus calculate the expectation value of $V = (\hbar c/e)\partial\vartheta/\partial ct$ if we assume that the probability amplitude of the flux loopform corresponds to the total field of dipole moment $e\hbar/2mc$.

It may be shown that the manifold of loopforms of such a source lepton then gives rise to a Coulomb field, and this is not due to any additional assumptions but holds by virtue of the very same definition of the four-potential $A_k = (\hbar c/e)\partial_k\vartheta$ which defined quantized flux in the first place. Positive and negative charge of the source lepton corresponds to parallelism and antiparallelism, respectively, of magnetic moment and spin, again in agreement with well-known facts.

To illustrate this result in a greatly simplified manner we may evaluate, for a point P on the equator of the source lepton, the fraction F (of quantized flux $\Phi_q = 2\pi\hbar c/e$) of loopforms which are linked with P, as

$$F = (e^2/2mc^2)/r \tag{5}$$

because the effective magnetic dipole field implies

$$\int_r^\infty B_{\text{eff}}\, 2\pi r dr = (e\hbar/2mc)2\pi/r. \tag{6}$$

As the phase ϑ changes by $\pm 2\pi$ for each passage of a flux of the amount Φ_q, we have

$$\begin{aligned} V_{\text{eff}} &= (\hbar c/e)(\partial\vartheta/\partial ct)_{\text{eff}} \\ &= \pm(\hbar c/e)(1/c)(2\pi)F(\Omega/2\pi) \\ &= \pm(\hbar c/e)(1/c)(e^2/2mc^2 r)(2mc^2/\hbar) \\ &= \pm e/r. \end{aligned} \tag{7}$$

The result of a potential V_{eff} corresponding to an equivalent charge e, after we assumed the dipole moment to be $e\hbar/2mc$, does not surprise us; it is simply the reverse of Dirac's result of intrinsic moment $e\hbar/2mc$ which arises when a relativistic electron of charge e is considered.

A more detailed discussion[1] shows the isotropy of the electric field.

It is of great importance to note that the mass m of the lepton cancels out, rigourously, indeed; the effective electric charge of muon and of electron is thus identical.

The motion of an electric potential implicitly arising from spinning flux loops is one of the essential points of the theory. We derived this potential from the basic equation (1) rather than from an inappropriate application of the induction law to a situation of loops in spinning motion, and that with linear velocities beyond the velocity of light.

In the later sections of the present paper we shall make ample use of this simple relationship between magnetic moment and effective electric charge, and of the proportionality of electric charge to the number of "wings" of a loop (expressed in terms of its effective winding numbers).

The issue of the preceding work[1] was to account for the relationship of quantized flux $\Phi_q = 2\pi\hbar c/e$ and magnetic moment $e\hbar/2mc$ and also, to make the model consistent, to show how the electromagnetic energy accounts for mc^2 and the electromagnetic angular momentum for $\frac{1}{2}\hbar$.

Quasi-Nonlocality

The question now arises: How can we, with the simple loopform concepts, at least qualitatively satisfy these requirements; i.e., can we find a reasonable interpretation of charged leptons in terms of superposition of quantized flux loopforms, given these requirements.

The first obvious step in such a picture of the lepton in terms of semiclassical loopforms is the construction of the source lepton. If the source lepton (when we attach a manifold of semiclassical loopforms to it) were considered as a point source, not only would there be a true singularity of the magnetic field $\vec{B}_{\text{eff}}$ (unlike other points of the field at which only infinitesimal probability amplitudes contribute to each flux loopform), but also, for a given finite magnetic flux, the magnetic moment would be zero because almost all flux loopforms are of infinitesimally small size.

Though this circumstance seems to present a formidable problem, it has an obvious solution. It should be remembered that a source lepton, if used as a source to which semiclassical loopforms are attached, will have to be considered as a "single particle." Therefore, when we use a description of a source lepton in terms of a space-time distribution of loopforms, we necessarily have to attach those loopforms to the "mean position" of the lepton, the position of a single particle. The Pryce-Tani-Foldy-Wouthuysen[2] transformation of the Dirac electron from representation in ordinary position into a single-particle representation makes the particle's mean position an operator which is nonlocal of the extent $\hbar/mc$ in ordinary position space. For a stationary single particle this implies a nonlocality $\hbar/mc$ for the position of the particle as well as for the field lines emanating from it. As the underlying theory is truly local we might term this effect "quasi-nonlocality." We might then perhaps make the terribly crude hypothesis that we may take account of this quasi-nonlocality of the source by considering the source as an extended source of size $\hbar/mc$.

This then seems to imply a magnetic dipole moment of the order of $(\Phi_q/4\pi)(\hbar/mc)$, a moment 2 orders of magnitude too large compared with the Bohr or muon magneton, respectively. That expression for magnetic dipole moment would be valid if the total effective magnetic flux of the dipole field were equal to Φ_q. We assume, however, that such a statement were only correct if the complex probability amplitudes of the alternative loopforms were all in phase. Their actual phase difference is bringing about a superposition corresponding to a reduction factor in the calculation of effective flux from quantized flux, as well as in the calculation of electromagnetic energy and angular momentum.

We do not, in this paper, recapitulate the essential objective of the previous work, i.e., the answer to the aforementioned questions. This program also provided, as seen in Sec. X of Ref. 1, for an understanding of the electromagnetic interaction constant $e^2/\hbar c$, and was applied to the electron-muon problem.

Neutrino

The neutrino is proposed to be a loop of the form of a left-handed trefoil, the antineutrino a right-handed one. It is proposed to spin through space like a coasting three-blade propeller. (A surface $\vartheta = 0$, $\vartheta = 2\pi$ representing the "cut surface" of the multivalued pseudo gauge field ϑ may be chosen in the following way: Deform, i.e., dilate

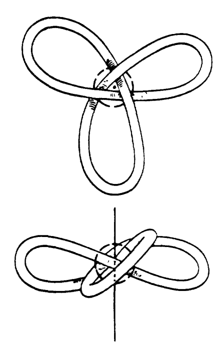

FIG. 2. A trefoil representing a neutrino loop which, like a coasting three-bladed propeller, moves in a helical spinning motion in the direction of the spin axis. In this and in subsequent figures, flux loops are drawn as double lines merely to better visualize the form of the loops. The loops are singular lines, the alternative forms of which define fibration of space. The question of orientation of the magnetic flux is still open; a neutrino might even be a superposition, not only of different loopforms, but also of both signatures of magnetic flux orientation. The difference between electron and muon neutrino is discussed in Sec. IV and in Appendix II of Ref. 1; the distinction is in regard to phase-related versus random-phased probability amplitudes superposition of the contributions of loopform bundles. A *single* loop of this form never represents anything else but a neutrino.

the central loop region which in the upper Fig. 2 projection shows a triangular form, into a form which has a circular projection; build this circle into a cylinder which is coaxial with the spin axis and which reaches up toward infinity. This cylindrical surface, together with three wing surfaces attached to the cylinder and extending out to the three loopwings, respectively, may represent this "cut surface." The helical coasting motion of that surface then implies no "sweeping," i.e., no $\partial\vartheta/\partial t$, and thus no electric field.)

This neutrino model is suggested by the possibility of explaining the helicity of the neutrino in terms of the seemingly general tendency of flux loops to spin in such a manner as to produce a minimal (in this case zero) electric field which is attained by the coasting rather than sweeping motion of the wings.

As the handedness of a neutrino does not change with any Lorentz transformation, whereas a neutrino of finite rest mass would change its helicity with a Lorentz transformation beyond its rest system, our model implies zero rest mass if helicity is interpreted in terms of loop handedness.

The knot character of the neutrino is also suggested because weak interactions involving neutrinos seem to imply the creation or destruction of a knotted loop: It is *assumed* that the crossing of a flux loop over itself or over a loop with which it is interacting, is a slow, i.e., weak process. The creation or destruction of a neutrino implies such processes. It is also to be remarked that two trefoils of opposite handedness, e.g., neutrino and antineutrino, have a means to annihilate each other or be pair-created without the crossing of loops. The neutrino-antineutrino dichotomy implies, apart from left-handedness–right-handedness of the trefoil loops, also opposite signature of the frequencies of their probability amplitudes.

The smallness of interaction of a neutrino with matter may be understood in terms of its zero electric field, and in terms of the weakness of a process involving the change of a neutrino trefoil into an ordinary loop.

As to the question of the "size" of such a loop one might consider that its lab energy indicates spinning frequency (we should note that the spinning frequency $2mc^2/\hbar$ referred to particles in their rest frame and may be interpreted as 2 lab energy/$\hbar$). As the essentially important ("first shell") loopforms may spin with linear velocities of the order of magnitude c, the radius (size) of a neutrino loop might be of the order of $\hbar c$/lab energy.

III. MESONS AND BARYONS, SPINNING-TOP MODEL

We shall outline two models of linked flux loops to represent hadrons. The basic assumptions are quite similar for both models. The first, the "spinning-top" model (Figs. 3–5), is a development of the model which we sketched in Appendix II of Ref. 1, now formulated in closer relationship to topology. The second, the "symmetric-axes"

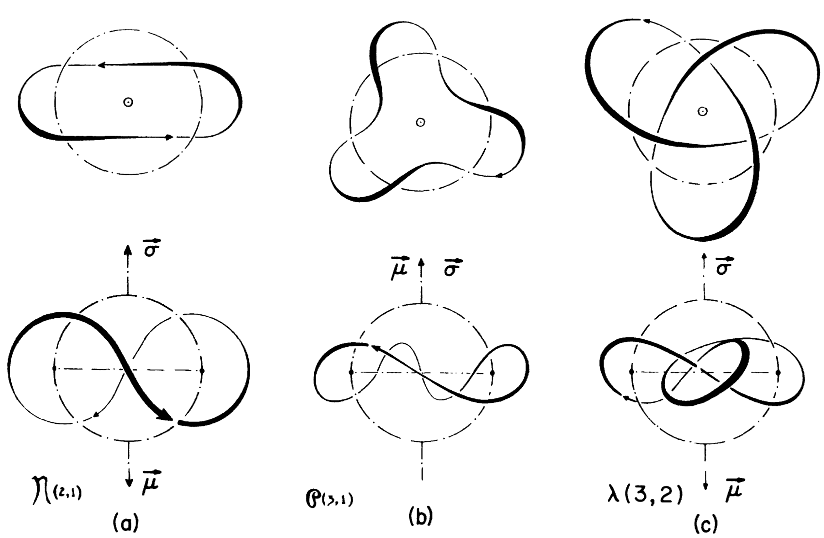

FIG. 3. Forms of quarks in the spinning-top model. These loops represent quarks only if interlinked with other loops as shown in Figs. 4 and 5. The difference of winding numbers about the two dash-dot-dash axes, i.e., $2-1=1(\mathfrak{N})$, $3-1=2(\mathcal{P})$, $3-2=1(\lambda)$, multiplied with the signature of spin with respect to magnetic moment, is proportional to the equivalent electric charge of the respective quarks. Quarks are assumed to be left-handed, antiquarks to be right-handed. Winding numbers have obviously a simple group-theoretical interpretation.

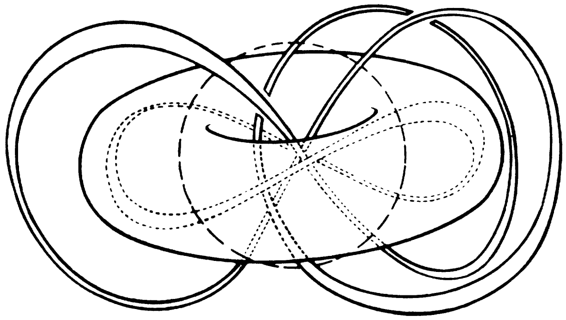

FIG. 4. Spinning-top model. λ and $\bar{\mathfrak{N}}$ quark interlinked, contributing to a meson. To illustrate the topological (knot-theoretical) relationships of the two loops, space is here subdivided by a toroidal surface [dashed lines in Fig. 4(a) which show a doughnut cut in half]. The λ is located entirely outside this doughnut shaped surface, the $\bar{\mathfrak{N}}$ entirely inside. This surface is dividing the fibrated space of λ loopforms from that of $\bar{\mathfrak{N}}$ loopforms; this toroidal interface may arbitrarily shrink or extend itself. Both loops pass through the spherical core region which is indicated by the dashed circle; the two loops may spin independently in a rolling-spinning motion about both the circular and the straight axes.

model (Figs. 6–11), is a generalization of the first model. We describe the basic ideas in terms of the spinning-top model which we consider the first choice to represent mesons and baryons. The symmetric-axes model is an alternative possibility; it also serves to illustrate the topological issues of flux loop linkage.

Linked Loops; Axes and Core

We assumed that a quark is a quantized flux loop if linked with another loop (to make a meson) or two other loops (to make a baryon). In terms of quantized flux loops, different quarks are defined by the form and orientation (direction of magnetic flux) of the linked loop, and by their spinning. A linked loop's mode of spinning is very different from that of a single loop. Accordingly the concept of an isolated single quark has no meaning in this theory. Loops, if able to dissociate themselves (in accord with conservation laws), behave as leptons.

We assumed that the linked loops are confined to regions between toroidal surfaces (cf. Figs. 4 and 5), which makes it possible for them to spin independently; this is an obvious requirement for loops representing quarks with spin.

For simplicity we assumed that such toroidal surfaces have the symmetry of a doughnut, i.e., that they have a circular axis and a straight central axis [which is perpendicular to the plane of the doughnut (Figs. 4 and 5)]. These toroidal surfaces are by no means fixed; they may shift altogether towards one or the other axis.

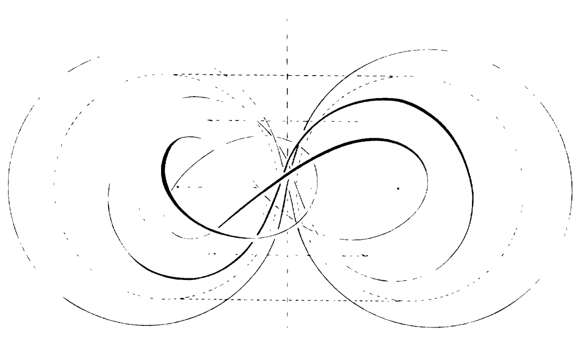

FIG. 5. Spinning-top model of a $\bar{\mathfrak{N}}\bar{\mathfrak{N}}\bar{\lambda}$ contribution to a baryon. The three loops define these fibrated space regions separated by the toroidal (dashed) interfaces. The subdivision of space permits independent spinning of the three quarks.

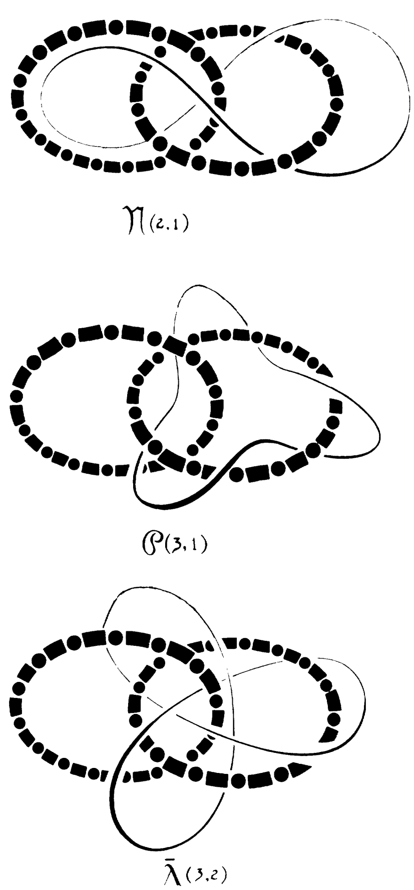

FIG. 6. Symmetric-axes model. Two interlinked axes are shown by thick dash-dot-dash lines. This is a generalization of the spinning-top model for which one of the axes is straightened out to reach ± infinity. The quarks $\mathfrak{N}(2,1)$, $\mathcal{P}(3,1)$, and $\bar{\lambda}(3,2)$ are shown in relation to one of the two axes.

In the model of the lepton we made the obvious assumption that the *Zitterbewegung* amplitude $\hbar/mc$, which corresponds to the linear extent of the quasi-nonlocality (which defines the radius of the "core" of the source lepton) is of the size of the radius of the circular axis (Fig. 1); this in turn is the extent of the inhomogeneous Maxwell-Lorentz region. In other words, what we called the "core equatorial ring" coincides with the circular axis. In the case of mesons and baryons, loops are linked in the manner illustrated in Figs. 4 and 5. In view of the discussion of (quasi-) nonlocality in Sec. V and Appendix I of Ref. 1, the region extending to the circular axis is to be considered again as the core, i.e., the "position" of the particle.

As every loop has, of course, to pass through this core (source) region, one may consider that as amounting to a linkage with the core. This linkage, as well as the linkage with the other loops of the same particle, may be simply expressed in terms of the above assumption and the following one:

To make an orderly linkage and motion possible it is assumed that the linked loops all share the same axes, and while spinning, do so with instantaneous spinning axes which are coaxial.

Some remarks about superposition of alternative loopforms may be useful here. The interlinked loops of a hadron are, if we refer to one of their "alternative forms," just two or three loops in space. The superposition of a continuous manifold of alternative (similar) forms which such a linked loop doublet or triplet may adopt, is formulated in terms of a superposition of products of three probability amplitude functions. The superposition defines a magnetic field in three-space. This may be considered to be a fibration of space, but a special one: It is made up of doublets or triplets of closed loopforms by definition because the superposition refers to alternative closed loopforms.

The superposition of the continuous manifold of loopforms, resulting in a fibration of space and formulated in terms of probability amplitude waves, implies a corresponding interpretation of coaxiality.

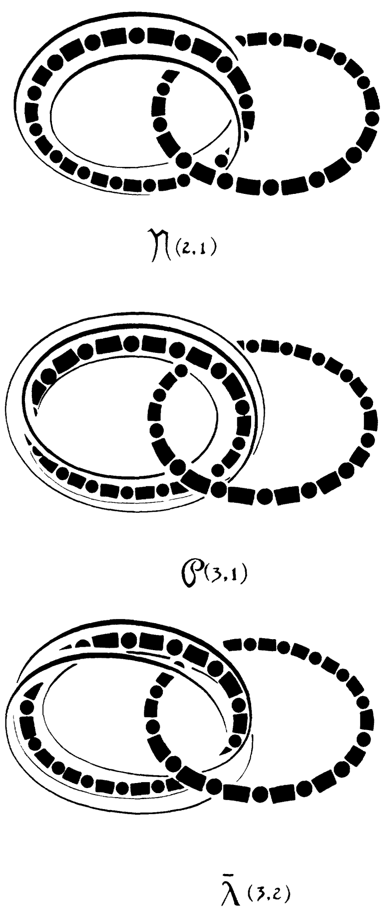

FIG. 7. The same quark loops as in Fig. 6 are shown here in relation to the other axes. This setting involves closer crowding of magnetic field lines and is thus expected to be less favored than the setting of Fig. 6, which shows the preferred setting of the loops.

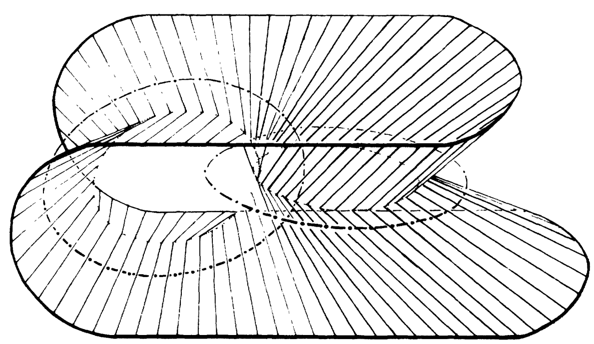

FIG. 8. The symmetric axes of the quarks shown in Fig. 6 are indicated as dash-dot-dash lines. The two flux loops are not shown here; they appear in Fig. 9. In order to permit the two meson loops to spin independently, these loops may be considered to be confined to the two domains separated by the shaded surface. To visualize this surface which reaches to infinity, it is in this figure, bounded by one long-winded line. The opening, connecting front to rear with the far right region, is free from shadings; we see an axis passing through that opening. The other opening, connecting upper to lower with the far left region, is hidden behind the shaded surface; the other axis passes through that opening. The surface is, as in the spinning-top model, not fixed at all. The spinning is again, as in the spinning-top model, a rolling, whirling motion about both axes, for each of the loops.

Form of Quark Loops; Winding Numbers

We shall assume that every quark loop is a torus knot, i.e., a closed loop which, if projected onto one of the doughnuts, does not intersect itself (Fig. 3). As a "loop" stands for the corresponding type of fibration of space, such an assumption permits us to avoid unwanted singularities of that fibration. (In our previous work[1] we did not yet adhere to this requirement.)

Every torus knot has two "winding numbers" which indicate how often it winds about the torus. We may denote by (1, 0) an electron or muon loop (Fig. 1) and by (2, 1) a loop which winds once about the hole of the doughnut, twice about the doughnut itself [Fig. 3(a)].

These winding numbers characterize the fibration. A fibration with rotational symmetry about a straight axis has also an axis which is circular; it may represent a topological singularity.

Handedness of Quark Loops

A loop (i.e., a fibration of space) has a handedness (right or left) which is defined by attaching two arrows to the (straight and circular) "axes," i.e., giving an orientation to the magnetic field at or near these axes. Handedness is, in the topologist's language, the "orientation" of the fibrated three-dimensional space.

A special case of handedness arises if the loop is a knotted one, i.e., a left-handed or right-handed trefoil [clover leaf knot, Fig. 3(c)]. We then assume that strangeness of a quark is represented by the topological character of the λ, $\bar{\lambda}$ loops. They are assumed to be of trefoil shape (λ and $\bar{\lambda}$ of opposite handedness) because an annihilation or pair creation of strange quarks must be fast if there is strangeness conservation: Indeed, two oppositely handed trefoils of opposite magnetic flux orientation may annihilate each other rapidly without crossing of flux loops. This is not so for two trefoils of equal handedness or for a trefoil and a simple nonstrange loop.

We assumed that the helicity of a neutrino, i.e., the handedness represented by the ψ function of the neutrino, corresponds to the handedness of the flux loop.

As the particle-antiparticle character of a neutrino (lepton) loop and of a λ quark loop is characterized by left-handedness and right-handedness, respectively, we might assume that this holds for all quarks.

As the segments of a line which form a loop seem to tend to spread out, "repelling each other" as magnetic field lines do to minimize magnetic field energy, the loops of type (2, 1), (3, 1), (3, 2) (which look like Figs. 6 if the left axes may be thought of as corresponding to the straight axes) seem to have preference over the loops (1, 2), (1, 3), (2, 3) (which look like Figs. 7 if the right axes may be thought of as corresponding to the straight axes) which more often go around the straight axes (around the holes of the doughnuts).

It is assumed that a flux loop crosses over itself, or over a loop with which it is interacting, in a weak process. As the unknotting of a knotted flux loop implies such flux loop crossing, we may look at a strangeness-nonconserving weak process as implying, in the simplest case, a transition from a trefoil to a simple loop. We therefore assumed the λ quark $S=-1$ to have the form of a (left-handed) trefoil and the $\bar{\lambda}$ ($S=+1$) the mirror form. Those two may annihilate each other or be

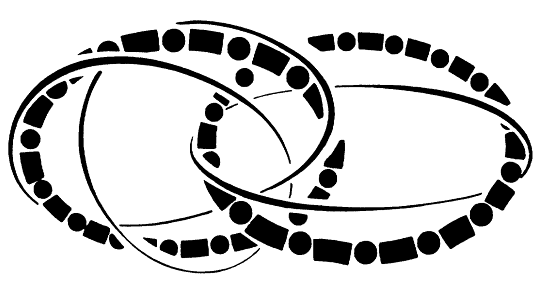

FIG. 9. A meson's loop-antiloop contribution in the symmetric-axes model (an $\mathfrak{N}\bar{\lambda}$ contribution). The interface between the fibrated space regions belonging to $\mathfrak{N}$ and $\bar{\lambda}$, respectively, is of the type shown in Fig. 8.

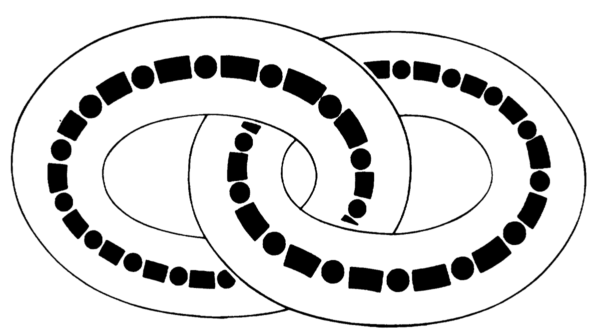

FIG. 10. Two toroidal surfaces separating space into three regions, for a baryon in the symmetric-axes model. The loops are not shown here, they are shown in the corresponding meson case of the spinning-top model Fig. 4(b). In the present case one loop is located altogether inside one doughnut (wound about its dash-dot-dash axis), another inside the other doughnut, and the third in between, i.e., the region which covers all outer space. Independent spinning is possible in this way.

pair-created without crossing of loops.

A strangeness-violating weak process then implies crossing of loops. It is interesting to note that such a process is, in terms of the topology of the loops, parity-nonconserving.

Frequencies of Probability Amplitudes

We mentioned handedness (and thus also strangeness) characterizing quark with respect to antiquark. For the muon we assumed that the particle-antiparticle character corresponds also to positive-negative frequencies of their probability amplitudes. Not only is this what might be expected in analogy with relativistic quantum mechanics; this frequency assignment also makes it plausible that particle-antiparticle annihilation and pair creation may occur while there is a loop conseration law, i.e., number of loops minus number of antiloops being conserved.

Spinning of the Loops

The spinning of the loopforms is assumed to occur with angular velocity $2m_q c^2/\hbar$ about both the straight axis and about the circular axis, in the latter case causing a rolling motion of the loopform about that circular axis.

The relationship of spinning motion to fibration of the flux loop field is assumed to be such that both have equal handedness. Such an assumption is equivalent to the assumption already made in regard to the neutrino, i.e., that the motion of the loopforms is such as to produce as little electric field energy as possible at the given spinning angular velocity. Such a kind of an assumption, similar to the assumption of forms of flux loops which correspond to a mininal magnetic field energy, is akin to a Maxwell-Lorentz field. One may shortly characterize this equalization of handedness of spinning and fibration as a tendency towards a coasting type of motion.

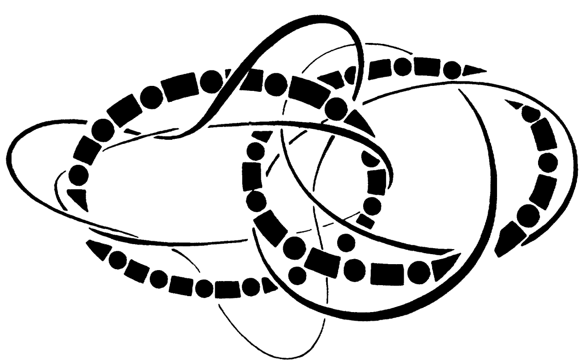

FIG. 11. Interlinkage of a baryon in the symmetric-axes model; an $\mathfrak{N}\mathfrak{N}\lambda$ contribution to a baryon (they are left-handed loops). The fibrated space corresponding to these quark loops has the interfaces represented by Fig. 10. In this symmetric-axes model of Figs. 9 and 11, orbital angular momentum might perhaps be accounted for by the rotational motion of the two axes about each other, spin being a matter of motion of the loop manifold about the two axes. The symmetric-axes model (Figs. 6–11) is shown to illustrate the topology of linkage; the spinning-top model (Figs. 3–5) has the advantage of simplicity.

Spinning and Equivalent Electric Charge

There is still the alternative of the resulting spin being parallel or antiparallel to the orientation of the resulting magnetic moment which, as we shall see, corresponds to positively charged quarks or negatively charged quarks, respectively (cf. Fig. 3).

It is most interesting to note that given, e.g., parallel orientation, the signature of the electric potential produced by the spinning depends on the difference of the winding numbers, i.e., is proportional to $+2-1=+1$ for a loop with winding numbers (2, 1). It is the difference because the coasting motion implies that the second winding number, i.e., 1 in case of (2, 1) [Fig. 3(a)] (with spinning parallel to flux orientation) counteracts the electric effect of the first winding number, i.e., 2 in this case of an $\mathfrak{N}$ quark (2, 1) in Fig. 3(a): The simultaneous spinning about both axes is exactly equivalent to a loop with "effective winding number" $2-1=1$ spinning only about the straight axis.

If we consider the absolute values of equivalent quark charges as proportional to the effective winding numbers, the equivalent electric charges of (2, 1), (3, 1), (3, 2) loops are proportional to 1 to 2 to 1, respectively. (In our former proposal[1] we counted the number of "wings" instead of the winding numbers because we did not yet pay attention to the spinning about both straight and circular axes).

The flux quantization proposal is based on gauge covariance of the definition of the fields. Charge conservation is thus implicit in the theory from the outset. Considering the integrity of electric charge of muon or of electron, all other reactions, directly or indirectly involving a muon or an electron, may only occur with integer changes of charge. As these reactions imply a quark-antiquark annihilation or production, [equivalent to a replacement of an (anti) quark by another (anti) quark] the difference of the equivalent electric charge of any two quarks should be integer, ±1, or 0, indeed.

Magnetic Moment

This charge assignment means that for $\mathfrak{N}$, $\mathcal{P}$, and λ quarks the spin and magnetic moments are parallel, antiparallel, and parallel, respectively. The opposite holds for the antiparticles.

A little careful consideration of the signatures of the field and loop quantities shows that with the loops of Figs. 3 and 6 the signature of the equivalent electric charge is indeed given by the parallelism or antiparallelism of spin and magnetic moment; in the case of the electron and muon that relationship was trivial.

It may also be noted that on the basis of the present assumptions, the proportionality between electric charge and magnetic moment is the general basic relation derived at the beginning of the previous paper.[1]

It is therefore no longer necessary to introduce an assumption (Appendix II of Ref. 1) relating the effective magnetic moment to the number of core traverses. This circumstance permits us now to define the forms of quark loops in terms of the topologically straightforward winding number assumptions which we formulated above and which we illustrated in Fig. 3, in perfect accord with the properties of the conventional quark model.

We may refer to further detailed discussions about baryons, mesons, neutrinos, electrons, and muons, given in the main body and Appendixes of Ref. 1.

IV. SOME PUZZLES OF THE QUARK MODEL

We noted above the difference of quark charges to be ±1 or 0 (due to charge conservation in interactions with leptons). We also noted that the ratio of the absolute values of the equivalent quark charges of $\mathfrak{N}$, $\mathcal{P}$, λ are 1 to 2 to 1, while their signatures depend on magnetic moment being parallel or antiparallel to spin, an open choice so far. One may ask what quark charge assignments and what quark combinations may satisfy these conditions and result in particle charges 0, $\pm e$, $\pm 2e$. Among the simple combinations it is only the $q\bar{q}$ and qqq, with charge assignments $-\frac{1}{3}e$, $+\frac{2}{3}e$, $-\frac{1}{3}e$ for q or for $\bar{q}$, which are compatible with those conditions.

We noted that the hypothesis of loop crossing being a slow process (when it comes to a loop trying to cross over itself or over the loop with which it may interact) leads to a topological interpretation of strangeness, to some understanding of weak interactions, and to an understanding of why strangeness-nonconserving weak interactions violate parity. We assumed that handedness of a quark loop itself, not only of the (probability amplitude) wave function referring to the loop, is related to parity.

Another interesting point is the following:

From the consideration of models of flux loops, their intrinsic handedness and their link with the axes, it becomes obvious that between a loop (3, 2) and a loop (3, 1), both of equal handedness, a transition is simple compared with a transition between (3, 2) and a (2, 1) loop. This fact may provide for an understanding of the ΔS-versus-ΔQ rule.

We discussed the muon-electron decay and noted that the presence of two types of neutrinos may be understood in terms of the muon-electron dichotomy. As a muon's probability amplitudes are assumed to be random phased, while the electron's are phase related, and because the transition muon to electron requires that the internal (and translational) k, ω distributions on both sides of the equation should match, a random-phased as well as a phase-related neutrino probability distribution must necessarily enter the picture.

In connection with loop crossing, the following issue is to be discussed. When a baryon's quark interacts with a meson's antiquark leading to an actual or virtual process involving annihilation and/or pair creation, the question arises whether the other (nonparticipating) quark loops are in the way, blocking any such $q\bar{q}$ interactions. It seems appropriate to assume that for an annihilation process the frequencies of the quark and of the antiquark with which it interacts (e.g., λ and $\bar{\lambda}$) are equal and of opposite signatures. In that case the wave function for the combination $q\bar{q}$ does not show beats, the absolute value of the wave function for $q\bar{q}$, in the course of time, does not frequently pass through zero (as other quark product wave functions do). Under such circumstances there is no cancellation of that $q\bar{q}$'s contribution to the magnetic field. Consequently for a possibility of fast annihilation the topological conditions will have to be met, i.e., the condition of opposite handedness of λ and of $\bar{\lambda}$, i.e., of strangeness conservation, so that annihilation may proceed without actual crossing of flux loops. For other quark products, the interference terms [written out in Eqs. (A1) and (A2) of Ref. 1] lead to many and frequent zero field values for the magnetic field. This circumstance should permit crossing of flux loops corresponding to such interfering pairs, and should permit the passage of the former $q\bar{q}$ quarks over regions in which other quarks are located. Accordingly annihilation of $q\bar{q}$ pairs becomes possible if the frequencies of the interacting quarks are equal, but then only between quark and antiquark of the same type; annihilation occurs under disregard of the presence of the other loops linked with q or with $\bar{q}$.

The consideration is also of relevance when it comes to the formation of the spatial part of quark wave functions. As they accordingly may exchange places in their distribution over the toroidal regions between the two axes, we may set up antisymmetric "spatial" wave functions as regards this distribution, or symmetric ones. We shall come back to this when discussing the applicability of the Pauli principle to quarks.

The loop picture has raised the following question: Clearly, a λ and $\bar{\lambda}$, being of opposite handedness, may readily annihilate when they approach each other. How may they coexist when attached to the same core of the meson? A simple model shows that in the latter case their opposite handedness prevents ready annihilation without flux lines crossing each other.

The question also arises about the absence of spin-$\frac{1}{2}$ baryons of the $\mathcal{N}\mathcal{N}\mathcal{N}$, of the $\mathcal{P}\mathcal{P}\mathcal{P}$, and of the $\lambda\lambda\lambda$ type. With total spin $=\frac{1}{2}$, there is in such a qqq always at least one pair of neighboring q's of opposite spin. As they are of equal charge, their magnetic field orientation is opposite. We may assume that those cannot coexist as nearest neighbors, attached to the same core, because they would repel each other.

The Question of Giant Quarks

We pointed out[1] that this spinning-top model brings up an interesting issue in regard to the higher-lying meson states (i.e., those above the pseudoscalar and the vector moments), as well as in regard to the higher baryon states. A spinning top has no orbital angular momentum. To introduce a $q\bar{q}$ orbiting about another $q\bar{q}$ introduces far too many unobservable states unless a plausible rule may be found to exclude them. It seems that the existence of the giant quarks might account for these higher lying states. A quark discussed as a spinning top indeed not only permits, but actually invites a spectrum of spins. A detailed discussion of this possibility has not been made.

Spin-Isospin Functions Without Violating the Pauli Principle

It should from the outset be remarked that it is an open question whether or not one may assign definite values of spin to quarks. As an individual isolated quark is a meaningless object in the present theory, it might also be questionable whether it may be given a definite spin.

The simple picture of quarks of spin $\frac{1}{2}$ and of magnetic moments proportional to their electric charges has, however, had such spectacular success with $SU(2)\times SU(3)$ that we shall stick to it for lower-lying meson and baryon states. The one drastic shortcoming was that the successful derivation of the ratio of magnetic moments of proton to neutron was in conflict with the Pauli principle. The successful symmetric spin-isospin functions for proton and neutron violate the Pauli principle because the quarks could not well be thought of as adopting an antisymmetrical orbital wave function as such a one is not expected to pertain to the lowest states as the nucleons are.

We like to consider now what new situation arises when the loop's distribution over the regions between the axes is considered. There is an innermost loop (i) (next to the doughnut's circular axis), a middle (m) and outermost loop (o) (closest to the straight axis). Were the loops confined to either (i) or (m) or (o), a simple product spin-wave function (as long as it corresponds to total spin $\frac{1}{2}$) would be appropriate for this loop triplet. If, however, there is a possibility of loops switching, i.e., permuting their locations (i), (m), and (o), we shall have to describe the situation again by the same spin-isospin function as in the case of nonlocalizable quark "particles." The same consideration holds in regard to quark loops spinning about linked axes, Fig. 11.

The proton's spin-isospin function, symmetric in the quarks, is

$$
\begin{aligned}
(18)^{-1/2}\{&2\mathcal{N}{\downarrow}\mathcal{P}{\uparrow}\mathcal{P}{\uparrow} - \mathcal{N}{\uparrow}\mathcal{P}{\downarrow}\mathcal{P}{\uparrow} - \mathcal{N}{\uparrow}\mathcal{P}{\uparrow}\mathcal{P}{\downarrow} \\
&+ 2\mathcal{P}{\uparrow}\mathcal{N}{\downarrow}\mathcal{P}{\uparrow} - \mathcal{P}{\uparrow}\mathcal{N}{\uparrow}\mathcal{P}{\downarrow} - \mathcal{P}{\downarrow}\mathcal{N}{\uparrow}\mathcal{P}{\uparrow} \\
&+ 2\mathcal{P}{\uparrow}\mathcal{P}{\uparrow}\mathcal{N}{\downarrow} - \mathcal{P}{\uparrow}\mathcal{P}{\downarrow}\mathcal{N}{\uparrow} - \mathcal{P}{\downarrow}\mathcal{P}{\uparrow}\mathcal{N}{\uparrow}\}\ .
\end{aligned}
$$

This function[3] (cf. Bég, Thirring, and Weisskopf[3]) thus applies not only to nonlocalized particle quarks, but also to localizable loops if these are occasionally interchanging their locations.

The spatial part of the three-quark wave functions could not be expected to be antisymmetric in the conventional model of quark particles. In the present model of quark loops there may, however, be no objection against antisymmetric spatial distribution (over the toroidal regions spanning from one axis to the other, of Fig. 5 or 11). There is accordingly no conflict with the Pauli principle and no need for introducing parastatistics.

The same arguments as for the proton hold for the neutron spin-isospin function, also, and the beautiful $SU(3)\times SU(2)$ result,

magnetic moment of neutron to that of proton $= -\frac{2}{3}$,

holds again, but now we might avoid a conflict with the Pauli principle.

We may finally remark that it might not be surprising if there would be some connection between the present quark proposal and the ones which consider dipole pairs of positive and negative magnetic monopoles as discussed by Barut.[4] It might be suggested that in this respect a meson might be represented by two dipoles, and a baryon by three dipoles. Magnetic monopoles, if they should exist at all, would not be expected to represent quarks.

V. SYMMETRIC-AXES MODEL

We want to discuss some modifications and generalizations of the spinning-top model simply to clarify a number of topological issues.

We may generalize the assumptions about the axes by bending the straight central axis so that we now consider two interlinked axes as the axes about which spinning occurs. (The straight form of one of the axes represents a special case of linkage of axes). Unless we state the contrary, we shall discuss the case of two equal size linked axes (Figs. 6 and 7). We may then show a few simple loops with winding numbers (2, 1), (3, 1), and (3, 2) in Fig. 6. These same loops also appear in the form of Fig. 7 (when their relation to the other axis is considered); they are topologically identical with the corresponding ones of Fig. 6. We shall later discuss why these loops may represent $\mathfrak{N}$, $\mathcal{P}$, and λ quarks, respectively.

The core maintains its significance as the region of inhomogeneous (i.e., source) terms of the Maxwell-Lorentz equations. It is now assumed to be bounded by the two axes.

These loops are either left-handed or right-handed, i.e., the fibrated space characterizing the magnetic field has a handedness. This handedness is again defined by attaching an arrow, i.e., an orientation to each of the two axes. Handedness characterizes a loop whether it is knotted (as a trefoil) or plain; for knotted loops it implies strangeness. One might suggest a quark to relate to a left-handed fibration, an antiquark to a right-handed fibration, in analogy to neutrino, antineutrino.

To generalize the assumption about spinning, we recall that one mode of spinning is a rolling, whirling motion about one axis; this motion, if smoothly continued throughout space, implies a tangential translation (circumferential) motion along the other axis, causing it to be displaced congruently upon itself. A second mode of spinning is possible with the role of the two axes interchanged.

We again assume that spinning occurs about both axes simultaneously. The spinning motion might then be left-handed or right-handed. We might again assume handedness of motion to coincide with handedness of fibration so as to minimize electric field energy.

In order to discuss the alternative of the forms of Fig. 3 or the forms of Fig. 6 as regards the axes, we first consider this question in regard to the muon or electron. Their field with winding numbers (1, 0) may be simply the dipole field of Fig. 1, only the spinning about that dipole axis matters.

We proceed to discuss the question of axes in the case of hadrons from now on. We first consider the relationship of a single quark loop to the two axes as illustrated in Figs. 6 and 7.

It is evident that *both* motions which a loop may perform with respect to the axes (the spinning-rolling motion about the two axes, be it Fig. 3 or be it Figs. 6 and 7) contribute to the generation of electric potential – both by the same argument which was made in sequel to Eq. (3b) and in Ref. 1. These contributions are expected to be proportional to the winding numbers.

The symmetric-axes model differs as follows from the spinning-top model: In the loop settings of the type of Fig. 7 the spinning about the left axis makes no electrical contribution in the limit of the loop converging toward the axis. On the other hand the loops of Fig. 3, loops setting close to the straight axis, contribute much to the electric field because these loops are big and sweep over large areas with each turn of the spinning motion. For those spinning top loops, wherever their setting, it is always the difference, 3 – 2 in the present example, which determines the electric potential.

With the above comments taken into consideration, it is, in the symmetric-axes model, effectively again the difference of the winding numbers which, as in the spinning-top model, is responsible for the equivalent electric charge.

Thus, as we already remarked, the two spinning motions occur simultaneously; we arrive again at the most interesting relationship between winding numbers and their equivalent charge. With this topological definition of quark loops and their properties, we achieve a definition of the ratios of the quark's electric charges, and also of their magnetic moments.

The forms of the antiquarks as previously mentioned are the mirror forms of the quarks. We might take as the mirror plane a plane perpendicular to the paper plane of Figs. 6 and 7, a plane which bisects both doughnuts, e.g., goes through the major axes of the elliptic projections of the interlinked axes.

The spinning motions may occur without the axes moving in space (except for the sliding motions along the axes, referred to above); there are furthermore the motions of the two linked axes in space. Ignoring deformations of those two axes and considering them for simplicity as equivalent in size and mutual relationship, these axes might perform a rigid body motion in space, characterized, apart from translations, by the rates of change of three Euler angles.

It might be possible that this latter motion may represent orbital angular momentum whereas the first two motions might correspond to spin. If that is a correct interpretation, there would no longer be need (as was suggested in Appendix II of Ref. 1) to assume giant quarks with higher spin to understand the higher-lying mesons, as well as the baryons. The spinning-top model on the other hand is preferable because of its simplicity, and may be more amenable to quantitative discussion.

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APPENDIX: WAVE EQUATION FOR LOOPFORMS OF QUANTIZED FLUX

The heuristic, geometrical picture, in terms of which we developed the present program, was designed to show the consistency of the idea of the quantized flux loop model. It permitted achieving approximate numerical results and it indicates to us how to develop a straightforward quantum-mechanical model.

It was evident from the beginning that a more axiomatic, rather than heuristic, formulation was eventually to be achieved. It would, however, have been too difficult to guess a correct analytical theory without first formulating the geometrical, heuristic model and showing its consistency. On this basis the appropriate choices and assumptions for an axiomatic analytic theory may then be made.

We start that effort by indicating what kinds of wave equations may appropriately chosen to describe the probability amplitudes of loopforms.

A general description of loopform probability amplitudes involves functionals, i.e., probability amplitudes for the various forms which a quantized flux loop may adopt. We assume that the probability amplitudes should reconstruct Maxwell-Lorentz electrodynamics, i.e., in the present case of muons or electrons, an ordinary point dipole field

$$\vec{B} = \mu[3r^{-5}(\hat{z}\cdot\vec{r})\vec{r} - r^{-3}\hat{z}]. \qquad (A1)$$

Expression (A1) is the resultant magnetic field; it is a superposition of contributions from sheaves of magnetic moment in the direction $\vec{\zeta}$, $|\vec{\zeta}| = 1$,

$$\vec{B}_{\vec{\zeta}} = \mu[1 + \cos(\vec{\zeta}, \hat{z})][3r^{-5}(\vec{\zeta}\cdot\vec{r})\vec{r} - r^{-3}\vec{\zeta}]. \qquad (A2)$$

For a given $\vec{\zeta}$, expression (A2) represents a 2-parametric manifold (aximuth α and size parameter σ of loopforms). Therefore (A1) can be considered as the resultant of a 4-parametric manifold of loopforms (flux orientation $\vec{\zeta}$, aximuth α, and size parameter σ; $\vec{\zeta}$, α are Euler angles), cf. Figs. 1 and 4 of Ref. 1. The size of a loopform was characterized by the size parameter σ which measures the "aphelion" distance from the source point of the loopform in question.

We notice that the shape of a loopform is the same for all sizes of loopforms, and evidently also for all orientation parameters $\vec{\zeta}$ and azimuth parameters α. This permits us to replace a functional description of the manifold of loopforms by a description in terms of probability amplitudes, functions of three angle parameters $\vec{\zeta}$, α and a size parameter σ.

In our previous paper we have discussed the motion of a lepton's loopforms and we have seen that the lepton can be appropriately described in terms of just one loop. Mesons and baryons imply two- and three-quark loops. When the motion of loopforms of quark loops is under consideration, we have to remember that the quark loops are assumed to spin about the main straight axis *and* about the circular axis (Figs. 3–5). This latter, rolling motion is equivalent to a spinning about the straight axis with a commensurable spinning frequency, the commensurability is determined by the ratio of the winding numbers. Thus, with caution, we may apply the spinning-top model even in the case of quark loops.

Returning to the simplest case of a muon or electron loop, we may start with the question: What may be the wave equation for the loopforms generating the magnetic dipole field of a *point* source, and thereafter discuss the issue of quasi-nonlocality of that source.

Considering the invariance of the loopforms with respect to $\vec{\zeta}$, α, i.e., with regard to the 3-parametric rotation group O_3, we follow Casimir's[5] spherical-top discussion. The homogeneous coordinates in terms of the unit vector $\vec{a}$ of the axis of rotation and of the angle ϕ of rotation are

$$\begin{aligned} \xi/\mathfrak{r} &= a_x \sin(\phi/2), \\ \eta/\mathfrak{r} &= a_y \sin(\phi/2), \\ \rho/\mathfrak{r} &= a_z \sin(\phi/2), \\ \chi/\mathfrak{r} &= \cos(\phi/2), \end{aligned} \tag{A3}$$

$$\mathfrak{r}^2 = \xi^2 + \eta^2 + \rho^2 + \chi^2. \tag{A4}$$

The angular part of the wave-equation operator for the symmetric top, analogous to the three-dimensional case for a mass point

$$r^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) - \left(\frac{\partial}{\partial r}\right) r^2 \left(\frac{\partial}{\partial r}\right) = \left(\frac{1}{\sin\theta}\right)\left(\frac{\partial}{\partial \theta}\right)\sin\theta\left(\frac{\partial}{\partial\theta}\right) + \left(\frac{1}{\sin^2\theta}\right)\left(\frac{\partial^2}{\partial\varphi^2}\right), \tag{A5}$$

is

$$\mathfrak{L}^2 = \mathfrak{r}^2\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \chi^2}\right) - \mathfrak{r}^{-1}\left(\frac{\partial}{\partial \mathfrak{r}}\right)\mathfrak{r}^3\left(\frac{\partial}{\partial \mathfrak{r}}\right) \tag{A6}$$

and has the eigenvalues

$$-4l(l+1). \tag{A7}$$

Whereas (A5) represents the Laplacian operating on the two-dimensional space of θ, φ, (A6) represents the Laplacian operating on the three-dimensional hypersurface spanned out by the three Euler angles, i.e., $\vec{\zeta}$, α; (A6) is the appropriate operator for the rigid spherical top.

Looking at the form (A6), (A4), and (A3) of the $\mathfrak{L}^2$ operator, we find that it permits the assignment of physical interpretation to a fourth parameter, i.e., $\mathfrak{r}$, along with the three Euler angles $\vec{\zeta}$, α. As the shape of the loopforms is not only the same for all values of $\vec{\zeta}$, α but also for all values of size σ, we may extend the 3-parametric loopform characterization (by $\vec{\zeta}$, α) to a 4-parametric characterization (by $\vec{\zeta}$, α, σ) and assume the interpretation

$$\mathfrak{r} \propto \sigma. \tag{A8}$$

We may therefore associate with every loopform a point in the four-dimensional space of the variables ξ, η, ρ, χ.

And we may assume that instead of the Laplacian operating on the 3-parametric manifold [Eq. (A6)], the Laplacian now operates on the 4-parametric manifold. The operators

$$\begin{aligned} &\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \chi^2}, \\ &\mathfrak{r}^2\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \chi^2}\right) \end{aligned} \tag{A9}$$

may now characterize this O(4)-invariant problem which characterizes the loopforms' invariance with respect to $\vec{\zeta}$, α, and σ or ξ, η, ρ, and χ. We may thus assume a wave equation

$$\left\{\mathfrak{r}^2\left(\frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2} + \frac{\partial^2}{\partial \rho^2} + \frac{\partial^2}{\partial \chi^2}\right) + \bar{\omega}^2 - C\right\}\psi = 0. \tag{A10}$$

The solution of such a wave equation corresponds to angular velocities Ω (of the loopforms) which are independent of $\mathfrak{r}$ because this equation is homogeneous of degree zero in $\mathfrak{r}$; ξ, η, ρ, and χ are proportional to $\mathfrak{r}$ [Eq. (A4)] This corresponds to our basic assumption of spinning of flux loops with *Zitterbewegung* angular velocity $2mc^2/\hbar$.

At this point it may be appropriate to comment on the proportionality (A8) between $\mathfrak{r}$ and σ. In the calculation of the electric field of the spinning flux loop, the mass of the electron or the muon cancels out rigorously. This implies the equality of the electric charge of these two leptons. The cancellation of the mass means that there is a scale invariance. Considering this we may write

$$\mathfrak{r} = \sigma/(\hbar/mc) \tag{A11}$$

as a parameter indicating the size of a loopform.

A Maxwell-Lorentz field $\vec{B}(\mathfrak{r}, \alpha)$ of the dipole form (A1) may be reconstructed by giving the probability amplitude $\psi(\xi, \eta, \rho, \chi)$ an $\mathfrak{r}$ dependence such that

$$B\mathfrak{r}d\mathfrak{r} \propto |\psi|^2 \mathfrak{r}^3 d\mathfrak{r}, \tag{A12}$$

i.e.,

$$|\psi| \propto \mathfrak{r}^{-5/2}. \tag{A13}$$

To discuss the solutions of a wave equation, we may be reminded of (A6) and (A7) which permit the solution of (A10) by separation of variables, leading to

$$\left[\mathfrak{r}^{-1}\left(\frac{d}{d\mathfrak{r}}\right)\mathfrak{r}^3\left(\frac{d}{d\mathfrak{r}}\right) - 4l(l+1) + \tilde{\omega}^2 - C\right]R(\mathfrak{r}) = 0. \tag{A14}$$

Since we want the ψ function to represent a dipole field, we have to take $R_{(\mathfrak{r})}$ to be proportional to $\mathfrak{r}^{-5/2}$,

$$R \propto \mathfrak{r}^{\kappa} = \mathfrak{r}^{-5/2}, \tag{A15}$$

$$-4l(l+1) + \tilde{\omega}^2 - C = -\kappa(\kappa+2)$$
$$= -\tfrac{5}{4} \tag{A16}$$

This represents the point dipole solution.

The solution (A15) has a singularity at $\mathfrak{r} = 0$, which corresponds to a point dipole source. Considering the Pryce-Tani-Foldy-Wouthuysen representation of a stationary single particle, the latter appears in ordinary position as smeared out. Replacing, accordingly, the point dipole source (a crude substitute for a transformation from mean position to position), we get rid of that singularity. We may effect this by introducing into the wave equation (A10) or (A14) a "potential" $U(\mathfrak{r})$ which is positive in the "core" region $0 \leq \mathfrak{r} \lesssim 1$ and goes to zero at the core surface $\mathfrak{r} \approx 1$ and is zero outside, $1 \lesssim \mathfrak{r} < \infty$, which ensures the $\mathfrak{r}^{-5/2}$ behavior of $R(\mathfrak{r})$ for large $\mathfrak{r}$.

Considering these, Eq. (A14) may be written as

$$\left[\mathfrak{r}^{-1}\left(\frac{\partial}{\partial\mathfrak{r}}\right)\mathfrak{r}^3\left(\frac{\partial}{\partial\mathfrak{r}}\right) - U(\mathfrak{r}) - 4l(l+1) + \tilde{\omega}^2 - C\right]R = 0. \tag{A17}$$

This equation might represent the wave equation for the loopform of an electron or a muon. The choice $C = \frac{9}{4}$ gives, by (A16) to achieve commensurability,

$$\tilde{\omega} = 2l + 1 = 1,\ 2,\ 3,\ \ldots . \tag{A18}$$

This relates to the spherical top. The eigenvalues of the symmetric top show a 2-parametric spectrum. Commensurabilities of $\tilde{\omega}$ permit phase-correlated motion of loopform amplitudes (angular group velocity) in the case of the electron, distinguished from random-phased muon amplitudes (angular phase velocity).

The wave equation is presumably to be written in a linearized form. The group-theoretical analysis of Eq. (A10) is particularly promising; Barut's analysis is expected to contain many of the relevant results.

We found in Sec. VIII B of Ref. 1 the important result that lepton-antilepton pairs, represented by generalized spherical harmonics, have the correct transformation properties under *CP* conjugation. These harmonics form bases of the irreducible representations of the continuous group O(3). Equation (A10) admits the O(4) group. The question arises whether the bundling of the continuous manifold of flux loopforms, Secs. VII, X, XI, and XIV of Ref. 1, into a discrete number of statistically independent bundles may be formulated in terms of the *discrete* subgroups of the aforementioned continuous groups. The counting in terms of the pentagondodecahedron/icosahedron, Fig. 8 of Ref. 1, has already pointed in that direction.

And the question arises about the role of irreducible representations of these discrete subgroups in the description of the electron, muon, and other particles. The representation of the continuous groups, in particular the generalized spherical harmonics, should, however, first of all be considered for a description of bundling of loopforms.

This bundling is an important issue because it was shown in Ref. 1 that the concept of superposition of complex probability amplitudes, with different phases for different bundles, may permit us to derive effective magnetic moments (= Bohr or muon magneton) and electric charge (= e), electromagnetic energy (=mc^2), and electromagnetic angular momentum (=$\hbar/2$), all from quantized flux Φ_q(=hc/e).

Note added. In our paper[1] we characterized the statistical independence (of the probability amplitudes) of the loopforms by assuming that a difference (in size, in orientation, in azimuth) greater than 1 rad makes them to be independent, whereas closely neighboring loopforms are correlated as regards their amplitudes. We thereby were led to group the loopforms into 207 bundles. This was done by simplified geometrical means, using graphical illustrations like a pentagondodecahedron whose corners are about 1 rad apart (or the faces of an icosahedron), and a flux tube picture. A more formal treatment is sketched in this Appendix.

To the qualitative discussion of electron versus muon we might note that the (angular) group and phase velocities of the terms bilinear in probability amplitudes are as 1 to 207. As the linear velocities of the spinning loops are of the order c to c and as the sizes (radii of the cores) stand in the ratio of 207 to 1, their electromagnetic energies are of the order of 1 to 207.

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