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High-Energy Total Cross Sections and Symmetry Relations

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Recent Serpukhov measurements of meson-baryon total cross sections allow comparisons with the Johnson-Treiman and Barger-Rubin relations. For 25–55-GeV/c incident meson momenta, the Johnson-Treiman relation, $\frac{1}{2}\Delta(Kp)=\Delta(\pi p)$, is well satisfied. The less restrictive Barger-Rubin relation holds for most of the Serpukhov points. Linear extrapolation of Serpukhov fits for $\Delta(Kp)$ and $\Delta(\pi p)$ to high momenta leads to the prediction of large violations of the Johnson-Treiman relations.

The recent measurements of π^+p, K^+p, and K^+d high-energy (20–65 GeV/c) total cross sections at Serpukhov[1] together with some earlier negative-beam measurements[2,3] at these energies allow us to check the Johnson-Treiman[4] and Barger-Rubin[5] relations. These relations among differences of meson-baryon total cross sections which involve only negative-charge-conjugation amplitudes are symmetry relations whose validity does not depend on detailed dynamical models. They were first proposed when only data up to 20 GeV/c was available, and it is of interest to see how good they are for the new high-energy data and what the results of the comparison imply.

In the present work we show that:

(a) The Johnson-Treiman relation, $\frac{1}{2}\Delta(Kp)=\Delta(\pi p)$, is satisfied for the Serpukhov data.

(b) The Barger-Rubin relation, $\Delta(Kp)=\Delta(\pi p)+\Delta(Kn)$, is satisfied for most, but not all of the Serpukhov points; its validity is somewhat ambiguous, primarily because of the large experimental uncertainties in $\Delta(Kn)$.

(c) An attempt to linearly extrapolate the Serpukhov $\Delta(Kp)$ and $\Delta(\pi p)$ fits to high momenta on a $\ln\Delta\sigma$ versus $\ln p_{\rm lab}$ plot leads to the prediction of large violations of the Johnson-Treiman relations. The assumption of the validity of the Barger-Rubin relation, together with the above-mentioned fits, leads to the prediction that $\Delta(Kn)=0$ at 600 GeV/c, namely, when $\Delta(Kp)=\Delta(\pi p)$, and that $\sigma_T(K^+n)>\sigma_T(K^-n)$ at still higher momenta.

The Johnson-Treiman relations[4]

$$\frac{1}{2}\Delta(Kp)=\Delta(\pi p)=\Delta(Kn) \tag{1}$$

were originally derived on the basis of SU(6) invariance of the forward scattering amplitudes and have since been obtained in many different ways. We define $\Delta(Kp)=\sigma_T(K^-p)-\sigma_T(K^+p)$, $\Delta(\pi p)=\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$, and $\Delta(Kn)=\sigma_T(K^-n)-\sigma_T(K^+n)$. Let us assume that the scattering amplitudes obey SU(3) invariance.[6] The Δ's are written in terms of the t-channel SU(3)-invariant amplitudes, $A(8_{FF})$, $A(8_{FD})$, and $A(10)$, in Eqs. (2)–(4). The octet subscripts indicate the symmetry (D) or antisymmetry (F) of the meson-meson and baryon-antibaryon states, respectively, reading from left to right:

$$\Delta(Kp)=2A(8_{FF})+A(10), \tag{2}$$

$$\Delta(\pi p)=A(8_{FD})+A(8_{FF})-A(10), \tag{3}$$

$$\Delta(Kn)=-A(8_{FD})+A(8_{FF})-A(10). \tag{4}$$

The Johnson-Treiman relations arise by letting the $A(10)$ and the $A(8_{FD})$ amplitudes $\equiv 0$. Letting $A(8_{FD})\equiv 0$ corresponds to saying that the symmetric coupling at the baryon vertex is $\equiv 0$, and that the only relevant coupling is the antisymmetric $A(8_{FF})$. Letting $A(10)\equiv 0$ means that there are no exotic $C=-$ amplitudes. Similarly, the much less restrictive Barger-Rubin relation

$$\Delta(Kp)=\Delta(\pi p)+\Delta(Kn) \tag{5}$$

is obtained simply by letting the $A(10)$ amplitude $\equiv 0$; $\Delta(\pi p)$ and $\Delta(Kn)$ individually contain both the $A(8_{FD})$ and $A(8_{FF})$ amplitudes but their sum is pure $A(8_{FF})$.

The $A(8_{FD})$ and $A(8_{FF})$ amplitudes and other t-channel SU(3)-invariant amplitudes are treated as the scattering analogs of Slater integrals, i.e., they are determined empirically from the data as functions of $p_{\rm lab}$. These amplitudes may contain contributions from Regge poles, cuts, etc., but for our purposes no detailed dynamical description of the amplitudes is necessary other than their assumed SU(3) invariance.

Our discussion is divided into three parts: (A) a comparison of the relations (1) and (5) with the data; (B) a comparison with low-energy analyses; (C) predictions which arise at higher energies from possible extrapolations of the presently available cross sections. In Table I we list the values of $\Delta(\pi p)$, $\frac{1}{2}\Delta(Kp)$, and $\Delta(Kn)$ obtained by combining

TABLE I. Data (see Refs. 1–3) used for checking Johnson-Treiman and Barger-Rubin relations. (a) $\Delta(\pi p)$; (b) $\frac{1}{2}\Delta(Kp)$; (c) $\Delta(Kn)$; (d) $\Delta(Kn)_{BR} = \Delta(Kp) - \Delta(\pi p)$. $\Delta(Kn)_{BR}$ is the value of $\Delta(Kn)$ predicted using the Barger-Rubin relation. Entries in brackets are obtained from Fig. 3 of Ref. 1.

p_{lab} (GeV/c)	(a) $\Delta(\pi p)$ (mb)	(b) $\frac{1}{2}\Delta(Kp)$ (mb)	(c) $\Delta(Kn)$ (mb)	(d) $\Delta(Kn)_{BR}$ (mb)
25	1.36± 0.13	1.56± 0.06	1.9± 0.6	1.77± 0.18
30	1.46± 0.14	[1.45± 0.08]	2.4± 0.5	1.44± 0.21
35	[1.30± 0.14]	[1.30± 0.08]	1.8± 0.5	1.30± 0.21
40	1.28± 0.13	1.22± 0.08	1.2± 0.6	1.16± 0.20
45	1.23± 0.14	1.28± 0.07	1.9± 0.6	1.34± 0.20
50	1.19± 0.14	1.02± 0.07	1.9± 0.6	0.84± 0.20
55	1.15± 0.14	1.03± 0.10	2.0± 1.0	0.91± 0.25

data from the three reported Serpukhov experiments. In our analysis, as in all others that deal with differences of total cross sections, we are plagued by the difficulties of working with data gotten by taking differences between two large but comparable quantities. In addition to the statistical errors listed, there are individual systematic errors for the positive- and negative-beam experiments of (0.4–0.5)%, so the Δ's obtained by combining data from the separate experiments may be more uncertain than is evident. Hopefully, this situation will be somewhat rectified when the results of new negative-beam experiments on liquid deuterium and liquid hydrogen analogous to the positive-beam ones are available.[7] The most uncertain numbers of all are the K^-n total cross sections $\sigma_T(K^-n)$ and the corresponding values of $\Delta(Kn)$ based on their use. The $\sigma_T(K^-n)$ were extracted from the $\sigma_T(K^-d)$ obtained in the first Serpukhov experiment[2] which used a gas target and are not as precisely specified as are the π^-p and K^-p measurements which were repeated using a liquid-hydrogen target.[3]

(A1) Johnson-Treiman relations. A point by point comparison of $\frac{1}{2}\Delta(Kp)$ and $\Delta(\pi p)$ [Fig. 1(a)] shows that they are equal over the range of the data. The equality may also be obtained from the fits to $\Delta(Kp)$ and $\Delta(\pi p)$ published in Ref. 1, and listed here: $\Delta(Kp) = (19.2 \pm 1.3)p^{-0.56 \pm 0.02}$ mb; $\Delta(\pi p) = (3.88 \pm 0.35)p^{-0.31 \pm 0.04}$ mb. These values lead to the ratio

$$\Delta(Kp)/2\Delta(\pi p) = (2.47 \pm 0.39)p^{-0.25 \pm 0.06}. \qquad (6)$$

This ratio decreases from 1.13 ± 0.26 at 25 GeV/c to 0.90 ± 0.21 at 55 GeV/c. Thus the value of $\Delta(Kp)/2\Delta(\pi p)$ *is consistent with unity* for the range of momentum considered, i.e., 25–55 GeV/c. The implications of extrapolating these fits to higher momenta and their relation to lower-energy data will be discussed below.

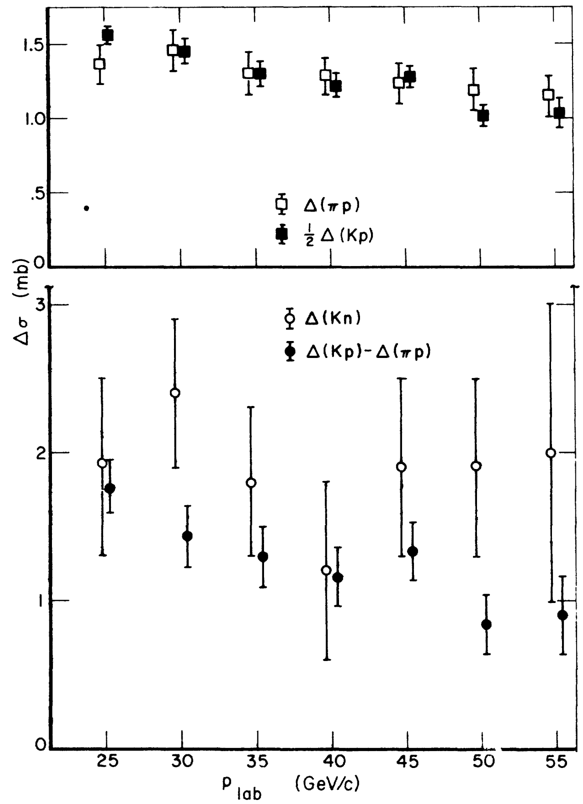

FIG. 1. (a) Data for check of Johnson-Treiman relation, $\frac{1}{2}\Delta(Kp) = \Delta(\pi p)$; (b) data for check of Barger-Rubin relation, $\Delta(Kn) = \Delta(Kp) - \Delta(\pi p)$. In both (a) and (b), points at the same p_{lab} are separated for display purposes.

The $\Delta(Kn)$ points lie consistently higher, ~50%, than either $\Delta(\pi p)$ or $\frac{1}{2}\Delta(Kp)$. The average value of $\Delta(Kn)/\Delta(\pi p) = 1.5 \pm 0.6$ but because of the large $\Delta(Kn)$ uncertainty, $\Delta(Kn)$ might even be said to agree with $\Delta(\pi p)$ or $\frac{1}{2}\Delta(Kp)$ within error. We can turn the procedure around, assume that the Johnson-Treiman relations hold, use $\sigma_T(K^+n)$ and $\Delta(\pi p)$ [or $\Delta(Kp)$] as input and predict the values of $\sigma_T(K^-n)$ to be obtained in the new Serpukhov experiment. This prediction is listed in column (b) of Table II.

TABLE II. K^-n total cross sections. (a) $\sigma_T(K^-n)_{exp}$; (b) $\sigma_T(K^-n)_{predicted} = \frac{1}{2}\Delta(Kp) + \sigma_T(K^+n)$.

p_{lab} (GeV/c)	(a) $\sigma_T(K^-n)_{exp}$ (mb)	(b) $\sigma_T(K^-n)_{predicted}$ (mb)
25	19.7± 0.6	19.3± 0.2
30	20.1± 0.5	19.1± 0.2
35	19.9± 0.5	19.4± 0.2
40	19.4± 0.5	19.4± 0.2
45	20.2± 0.5	19.6± 0.3
50	19.9± 0.6	19.0± 0.2
55	20.4± 1.0	19.5± 0.3

On the whole, the values of $\sigma_T(K^-n)$ predicted this way are about 0.6 mb lower than those obtained in the gas target experiment[2] [listed in column (a) of Table II]. We might expect the new data to produce this result because the $\sigma_T(K^-p)$ which were remeasured using the liquid-hydrogen targets, averaged about 0.5 mb lower than those originally obtained from the gas target.

(A2) Barger-Rubin relations. The comparison of the data with Eq. (5) is displayed in Fig. 1(b) by plotting $\Delta(Kp)-\Delta(\pi p)$ and $\Delta(Kn)$ as functions of $p_{\rm lab}$. [The comparison is made in this form because both $\Delta(Kp)$ and $\Delta(\pi p)$ are better determined than $\Delta(Kn)$.[8]] The two quantities overlap within the errors except for the points at 30 and 50 GeV/c; the values of $\Delta(Kp)-\Delta(\pi p)$ are always less than those of the observed $\Delta(Kn)$. The Barger-Rubin relation may be used to predict $\Delta(Kn)$, with $\Delta(\pi p)$ and $\Delta(Kp)$ as input. As may be seen from Table I, the values obtained in this way are rather similar to those of $\Delta(\pi p)$ or of $\frac{1}{2}\Delta(Kp)$; this is due to the near equality of $\Delta(\pi p)$ and $\frac{1}{2}\Delta(Kp)$.

(B) Comparison with low-energy analyses. Inasmuch as comparisons with the Johnson-Treiman and Barger-Rubin relations have been carried out for the lower-energy (6–20 GeV/c) data in the past,[5,9] we should see how the results of our analysis relate to earlier ones. $\Delta(Kp)/2\Delta(\pi p)$ varies from 1.52 at 6 GeV/c to 1.3 at 18 GeV/c with an uncertainty of about 25%. Although the errors are large, the ratio decreases slowly with incident $p_{\rm lab}$ and is somewhat larger than unity; its behavior is well represented by Eq. (6). Over this same momentum range, with even greater uncertainty, $2\Delta(Kn)/\Delta(Kp)\simeq 1.3$ and $\Delta(Kn)/\Delta(\pi p)\simeq 1.7$. As pointed out in Ref. 5, $\Delta(Kp)\simeq\Delta(\pi p)+\Delta(Kn)$ over the range of the data, indicating that the Barger-Rubin relation is satisfied.

We observe, therefore, that the Johnson-Treiman relation $\frac{1}{2}\Delta(Kp)=\Delta(\pi p)$ is not satisfied at lower energies, but *is* satisfied at Serpukhov energies, albeit with a possible gradual decrease from 1.1 to 0.9. If the fits obtained in Ref. 1 are adopted then the crossover for $\Delta(\pi p)$ and $\frac{1}{2}\Delta(Kp)$ has occurred at 37.5 GeV/c, and the fact that the Johnson-Treiman relation holds in the region 25–55 GeV/c may be regarded as a fluke. This may even be true, for the fit of $\Delta(\pi p)=(3.88)p^{-0.31}$ mb, contrary to general belief, is not a new and startling deviation from an expected power behavior of $p^{-0.5}$. In 1967, Foley *et al.*[10] had already obtained a fit to $\Delta(\pi p)$ of $3.85p^{-0.306}$ mb. The high-energy $\Delta(Kp)$ fit is in remarkable agreement with a fit that we have carried out for the very low-energy (2.35 GeV/c–3.30 GeV/c) K^+p and K^-p data of Abrams *et al.*[11] Consequently, we have no reason to dispute the use of the Serpukhov fits. The Barger-Rubin relation, on the other hand, holds rather well over the low-energy range, but some deviation may be indicated at the higher-energy points as considered above. One measure of the deviation is the value of $\Delta(\pi p)+\Delta(Kn)-\Delta(Kp)$. It changes slowly from ~ -0.3 mb at 6 GeV/c to ~ 1.1 mb at 55 GeV/c. Unfortunately, the errors are comparable in size to the quantity itself. If the new measurements for $\sigma_T(K^-n)$ come down ~ 0.6 mb, then the Barger-Rubin relation may be regarded as quite good at the higher energies.

A departure from the Barger-Rubin relation could possibly be due to the neglect of the exotic $A(10)$ amplitude in Eqs. (2)–(4). We know that there *are* exotic amplitudes, even though no exotic mesons have been found. Such amplitudes may be due to two particle exchange, for example, and might even be expected to grow with momentum. The magnitude of this cut amplitude is proportional to the deviation from the Barger-Rubin relation; solving Eqs. (2)–(4) for the $A(10)$ amplitude we find that

$$A(10)=\tfrac{1}{3}[\Delta(Kp)-\Delta(\pi p)-\Delta(Kn)]. \tag{7}$$

The determination of whether or not such an amplitude plays a significant role must await more precise measurements for the Δ's than are presently available.

(C) Extrapolation to very high energy. It is tempting to try to extend our present knowledge about total cross-section differences to ranges of high incident meson momenta by making simple extrapolations of present data behavior. A simple exercise of this sort is the extension of the Serpukhov fits for $\Delta(Kp)$ and $\Delta(\pi p)$ (Ref. 1) as straight lines on a plot of $\ln\Delta\sigma$ versus $\ln p_{\rm lab}$ to NAL energies and beyond. Of course, we have no *a priori* reason to assume that such straight-line extrapolations are correct, especially in view of current work on the importance of cuts.

One result of these extrapolations is that they predict that the Johnson-Treiman relation would be violated; at 300 GeV/c, $\frac{1}{2}\Delta(Kp)$ would be 0.39 mb, whereas $\Delta(\pi p)$ would be 0.66 mb. Nevertheless, the Barger-Rubin relation might still be satisfied, and in the form

$$\begin{aligned}\Delta(Kn)&=\Delta(Kp)-\Delta(\pi p)\\ &=19.2p^{-0.56}\text{ mb}-3.88p^{-0.31}\text{ mb}\end{aligned} \tag{8}$$

would predict a value $\Delta(Kn)=0.12$ mb at 300 GeV/c. Such a value for $\Delta(Kn)$ leads to a gross violation of the other Johnson-Treiman relations, i.e., $\frac{1}{2}\Delta(Kp)=\Delta(Kn)$, and $\Delta(\pi p)=\Delta(Kn)$. Relation (8) yields the further prediction that $\Delta(Kn)=0$ at 600 GeV/c and that at higher energies $\Delta(Kn)$ becomes slightly negative; i.e., there is a crossover and $\sigma_T(K^+n)$

$> \sigma_T(K^- n)$.

We wish to thank Professor S. B. Treiman for very useful suggestions and discussions, Professor T. Kycia for sending us a prepublication copy of Ref. 1, and Professor J. V. Allaby for interesting discussions. We wish to acknowledge the hospitality of the Aspen Center for Physics where part of this work was carried out.

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[7] J. V. Allaby (private communication).

[8] V. Barger and R. J. N. Phillips (unpublished) have compared the Barger-Rubin relation to experiment, plotting both $\Delta(\pi p)$ and $\Delta(Kp) - \Delta(Kn)$ against $p_{\text{lab}}^{-1/2}$ for the range 20–55 GeV/c.

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[10] K. J. Foley *et al.*, Phys. Rev. Letters 19, 193 (1967). See footnote 15.

[11] R. J. Abrams *et al.*, Phys. Rev. D 1, 1917 (1970).

PHYSICAL REVIEW D VOLUME 6, NUMBER 1 1 JULY 1972

Errata

Two-Photon Processes in Colliding-Beam Experiments, C. E. Carlson and Wu-Ki Tung [Phys. Rev. D 4, 2873 (1971)]. The phase-space volume element given in terms of the BW variables, Eq. (A5), and used in Eq. (16) and Eq. (28) is incorrect. The correct expression in the general case turns out to be rather complicated. This renders the use of the BW variables as actual integration variables in the general case somewhat impractical. For specific physically interesting regions of phase space where the kinematics simplifies, one can either use the lab variables directly or, if the BW variables are preferred, evaluate the Jacobian (with the simplified transformation formulas) and compute the phase-space volume element accordingly.

Goldhaber Distributions for Four-Pion Decays of Isosinglet Bosons, G. V. Weller and A. C. Dotson [Phys. Rev. D 1, 2169 (1970)]. The graphs and correlation coefficients are not accurate because they were based on values of $d^2R/dm_{12}^2dm_{34}^2$ instead of $d^2R/dm_{12}dm_{34}$. However, calculations performed by Mr. Wilson C. Lu suggest that the primary result (significant form dependence only for type-2 scalars) would not be altered by the correcting of this error. Also, the word "text" in the first line of the Abstract should be replaced by "test," the "Sec. V" mentioned near the end of the Introduction should be replaced by "Sec. IV," and the first line of the final paragraph in the Appendix should read "Now in all cases, θf and . . . "