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Scaling in Multi-Triplet Quark Models*

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Asymptotic bounds for the ratio F_2^{en}/F_2^{ep}, found in the three-triplet quark model, are not obeyed by the SLAC data. In the model, this observation implies the breaking of scaling behavior in the transition from present energies to extremely high ones. Such possible breaking in multi-triplet quark models is discussed.

The light-cone commutation relations of currents abstracted from the free-quark model[1] and (equivalently) the parton-quark model[2] have been very useful in analyzing the present – and predicting the future – data of deep-inelastic lepton-nucleon scattering.

Some of the quantum numbers, in particular the charges, of the fundamental fields (or constituents) may be, in principle, determined in these inclusive processes.[3] However, it has been recently emphasized[4] that this is not a very useful way of distinguishing between quark models with different quark charges. In a typical multi-triplet integral-charge-quark model one needs an appreciable production of "charmed" particles[5] in order to distinguish this model from others. Therefore, these particles may as well be directly observed in exclusive experiments.

The inclusive experiments to be performed at the National Accelerator Laboratory (NAL) have been proposed for the study of the scaling phenomenon at very high energies. Therefore, it may still be quite useful to anticipate the different ways in which scaling may show up in the various quark models.

As an illustration, we consider, in this note, the deep-inelastic electron-nucleon scattering in the three-triplet model.[6] We first determine a certain quantitative feature of this process at extremely high energies. We find that our result is not satisfied by the findings of the SLAC experiment. Conclusions and an interpretation are then proposed in terms of a possible breaking of scale invariance within the framework of this model.

In the three-triplet model[6] one assumes the existence of nine fundamental fields which form a $(3,3)$ representation of the group $SU(3)\times SU(3)'$, where the first group in the product is the familiar SU(3), and SU(3)′ is a new symmetry group. The nine quarks all have baryon number $\frac{1}{3}$ and integral charges. The charge operator is defined by the generators of the group as

$$Q=I_3+\tfrac{1}{2}Y+Y' . \tag{1}$$

The lowest-lying states are assumed to be SU(3)′ singlets.

The deep-inelastic electron-nucleon cross section is expressed in terms of the single-nucleon matrix element of the commutator of the electromagnetic current at a lightlike separation. In the three-triplet model this current is given by the following combination of components of SU(3) $\times$ SU(3)′ representations:

$$J_\mu^{\text{e.m.}}=J_\mu^{(3,0)}+\frac{1}{\sqrt{3}}\,J_\mu^{(8,0)}+\frac{2}{\sqrt{3}}\,J_\mu^{(0,8)} . \tag{2}$$

0, 3, and 8 denote the SU(3) [or SU(3)′] scalar, third, and eighth components of an octet, respectively.

Using the free-quark-model commutation relations for the leading singularity of the current commutators near the light cone, one obtains, in a standard manner,[1] an expression for the deep-inelastic e-p structure function:

$$x^{-1}F_2^{ep}(x)=\frac{4}{3}\left(\frac{2}{3}\right)^{1/2}A^0(x)+\frac{1}{3\sqrt{3}}\,A^8(x)+\frac{1}{3}\,A^3(x), \tag{3}$$

where $x=-q^2/2M\nu$ is the scaling variable, and $A^l(x)$ are defined by the Fourier transforms of single proton matrix elements of the bilocal densities[1] on the light cone ($y^2=0$):

$$\langle p|J_\mu^{(l,0)}(y,0)-J_\mu^{(l,0)}(0,y)|p\rangle =2(\tfrac{2}{3})^{1/2}p_\mu\int e^{-ix(p\cdot y)}A^l(x)dx+y_\mu\cdots . \tag{4}$$

In deriving Eq. (3) one assumes that the proton is an SU(3)′ singlet. A similar expression is obtained for the neutron structure function:

$$x^{-1}F_2^{en}(x)=\frac{4}{3}\left(\frac{2}{3}\right)^{1/2}A^0(x)+\frac{1}{3\sqrt{3}}A^8(x)-\frac{1}{3}A^3(x). \tag{5}$$

The combinations of $A^l(x)$, which correspond to the 3×3 basis matrices E_i $[(E_i)_{jk}=\delta_{ij}\delta_{ik}]$ in the U(3) space, are, by definition, positive definite. Hence[7]

$$(\tfrac{2}{3})^{1/2}A^0 + (\tfrac{1}{3})^{1/2}A^8 \pm A^3 \geq 0,$$
$$(\tfrac{2}{3})^{1/2}A^0 - 2\,(\tfrac{1}{3})^{1/2}A^8 \geq 0. \tag{6}$$

From Eqs. (3), (5), and (6) one obtains the following bounds for the ratio $F_2^{en}(x)/F_2^{ep}(x)$ in the three-triplet model:

$$\tfrac{1}{2} \leq F_2^{en}(x)/F_2^{ep}(x) \leq 2. \tag{7}$$

The same bounds can be easily obtained in the parton model using the integral charge values of the nine fundamental constituents. Since the proton is an SU(3)′ singlet, the momentum distribution functions of the constituents do not depend on their SU(3)′ index. The singlet character of the nucleon state plays an important role in the determination of these bounds. Omitting such correlations among the wave functions of the fundamental constituents, the ratio turns out to be unbounded.[3]

Experimentally the neutron-to-proton ratio drops from ~1 near $x=0$, to around $\frac{1}{4}$ when x approaches unity.[8] This cannot be considered yet as evidence against the three-triplet model, since, as long as there is no appreciable production of "charmed" states the results of the electroproduction experiments are expected to coincide with the predictions of the fractionally charged quark model.[4] The latter model allows for values as low as $\frac{1}{4}$ for the neutron-to-proton ratio.[3,7]

It should be noticed that the high values of x, in which the available experimental results disagree with Eq. (7), correspond to rather low values of the final hadron invariant mass ($M_x < 3$ GeV). Since $M_x{}^2$ falls off linearly with x for fixed energy loss ν,

$$M_x{}^2 = M^2 + 2M\nu(1-x), \tag{8}$$

one needs rather high-energy leptons in order to confront the three-triplet model with a severe test. That is where the NAL experiments may be of much help.

Equation (7) is expected to hold, in the three-triplet model, once experimental energies appreciably exceed the threshold for excitations of "charmed" states. [A violation of this relation at high energies does not necessarily rule out the three-triplet model. It may arise from the existence of a component, which is not an SU(3)′ singlet, in the nucleon wave function.] The lower bound in this equation is violated in the presently observed scaling domain. Since scaling requires the ratio F_2^{en}/F_2^{ep} to be independent of the energy variable ν separately, we conclude that in the framework of our model *scale invariance should be broken* in the transition from presently available energies to extremely high ones.

One need not be surprised by the possibility of breaking of scale invariance in multi-triplet models. On the contrary, the existence of an, up to now unobserved, energy scale, M_c, which determines the excitation energy of the first "charmed" state in the spectrum of hadrons, requires that "ultimate" scaling should only hold when experiments are finally performed above this energy region. The presently observed scaling phenomenon is expected to be retained as long as the final-state hadron mass does not exceed M_c. Only the first two terms in Eq. (2) contribute then to the electroproduction cross section, and predictions of the light-cone (or parton) approach with third integral charge quarks may hold.

Breaking of scaling is anticipated while M_x obtains values in the M_c region; only when produced hadron masses are sufficiently higher than M_c should "ultimate" scaling prevail. With 500-GeV leptons at NAL ($M_x \leq 30$ GeV) one may reach the asymptotic scaling region if, for example, M_c is a few GeV.

The parton model (unlike the light-cone approach in its most general framework) predicts a scaling behavior in the process of massive lepton pair production in hadronic collisions[9]:

$$p + p \to \mu^+ + \mu^- + \text{anything}.$$

The cross section for this process may be explicitly calculated[10] (for low values of the appropriate scaling variable) in terms of the electroproduction data and the charges associated with partons. Assuming that a diffractive behavior characterizes the small-x electroproduction data, one finds

$$\frac{d\sigma}{dQ^2}(\tau < x_P{}^2) = \frac{1}{\sum_i \lambda_i{}^2}\,\frac{F(\tau)}{(Q^2)^2}\,, \tag{9}$$

where $\tau = Q^2/s$ is the scaling variable, Q^2 is the invariant mass squared of the μ pair, and s is the total energy squared of the colliding protons in their center of mass frame. x_P is the highest value of x for which the Pomeranchuk contribution still seems to dominate the electroproduction structure functions, and λ_i are the charges of the different types of partons.

The scaling function, $F(\tau)$, is explicitly given in terms of the electroproduction data[10]:

$$F(\tau) = \tfrac{4}{3}\pi\alpha^2 F_2^{ep}(0)\left(\int_{x_P}^1 \frac{dx}{x}\,F_2^{ep}(x) + \tfrac{1}{2}F_2^{ep}(0)\ln(x_P{}^2/\tau)\right). \tag{10}$$

Within the framework of the three-triplet parton model one may expect the observed scaling to hold as long as no "charmed" hadrons are produced in the final state. This phenomenon is expected to be violated in an intermediate energy region (in which hadrons with invariant mass $M_x \gtrsim M_c$ are produced), and ultimately reappear at extremely high energies. In the lower-energy scaling domain only the average charge of each parton SU(3)′ triplet (in the singlet state) is measured, while in the "ultimate" scaling region one measures the instantaneous charges. These two domains are characterized by $\sum\lambda_i^2=\frac{2}{3},4$, respectively, and have two different sets of structure functions. Relating the lepton-nucleon inclusive cross-section data to those of μ pair production through Eqs. (9) and (10) should be done at the same energy region.

To conclude, the presently observed scaling phenomenon may be "ultimate" in the fractionally charged quark model, while in certain multi-triplet models it is certainly just a temporary effect. When energies in the 500-GeV region become available one may be able to distinguish between these two possibilities.

The possibility of the breakdown of scaling has already been discussed.[11] The existence of additional significant masses in the theory due to the cluster structure of partons is proposed in that work. In our approach the new energy scale arises due to the additional hidden symmetry.

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