versed. The original source from which this is taken also contains this error. All functions used in this work are as defined in this book.

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Energy Spectrum of the Hydrogen Atom in a Strong Magnetic Field*

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We obtain the energy spectrum for the 14 lowest states of the hydrogen atom in a strong magnetic field B. Emphasis is placed on values of $B \approx 10^6$–10^8 G, characteristic of those found in magnetic white dwarfs. Our results for the ionization energy of the ground state are significantly better than those of Cohen, Lodenquai, and Ruderman for fields $\lesssim 7\times10^{11}$ G.

The discovery of strong magnetic fields $\approx 10^6$–10^7 G in some white dwarfs,[1] and the possible existence of superstrong magnetic fields $\sim 10^{12}$ G in pulsars, has stimulated interest in the behavior of atoms in intense magnetic fields. Mueller, Rau, and Spruch[2] considered the *total* ground-state binding energies of atoms and positive ions, whereas Cohen, Lodenquai, and Ruderman (CLR)[3] obtained the binding energy of the *last* electron for atoms and ions. These papers were largely motivated by the discovery of pulsars and thus concentrated on *superstrong* magnetic fields ($B \gtrsim 3\times10^{10}$ G). By contrast, Rajagopal, Chanmugam, O'Connell, and Surmelian,[4] motivated by the discovery of magnetic white dwarfs,[1] initiated a program to study the behavior of atoms in fields of *any* strength, with emphasis on B values from about 10^4 to 10^{12} G. That paper[4] concentrated on the ionization energies of hydrogen in magnetic white dwarfs, and the essence of the calculation was the use of a trial wave function which was hydrogenlike, in contrast to the oscillatorlike trial wave function used by CLR. By the use of merely a four-parameter trial function,[4] the values of the ionization energy obtained were significantly better than those of CLR for fields $\lesssim 3\times10^{10}$ G. A general procedure for carrying out a multiparameter calculation was also outlined. It is our purpose here to present the results of such a calculation, not only for the ground state, but also for the next 13 lowest-energy eigenstates.

The Hamiltonian for the hydrogen atom (we neglect spin) in a magnetic field B oriented along the z axis is given by

$$H=\frac{p^2}{2m}-\frac{e^2}{r}+\omega_L L_z+\tfrac{1}{2}m\omega_L{}^2r^2\sin^2\theta, \qquad (1)$$

where $\omega_L = eB/2mc$. Since it is invariant under rotations about the z axis and under inversion, the eigenstates can be labeled by the eigenvalues of L_z and the parity. Thus, a general form of the trial solution may be written

$$\psi_m^{\pm}(\vec{r}) \equiv \psi_t = \sum_{il}\left(a_i^{(t)}r^{l}+b_i^{(t)}r^{l+1}\right)\exp(-\beta_i^{(t)}r)Y_{lm}(\theta,\phi), \qquad (2)$$

where $a_i^{(t)}$, $b_i^{(t)}$, and $\beta_i^{(t)}$ are parameters.

The sum on l in (2) over all even integers leads to the state with even parity (+), and the sum over odd l, to the odd-parity (−) state. Here m is the eigenvalue of L_z. This choice is consistent with the condition that the solution associated with Y_{lm} in the hydrogen atom must have the behavior r^l near $r=0$. It has the advantage that we obtain an explicit evaluation of the matrix elements entirely in terms of the well-known Γ functions. For superstrong B fields we used a partial-wave expansion with values of l up to $20+|m|$ included in summation (2). This was found to be necessary in order to obtain convergence of the expansion of $\psi_m^{\pm}$. Up to 9 Slater-type orbitals were employed in the description of the radial function for the ground state, and 12 for the excited states. Figure 1 gives the ionization energy E_I (in eV) of the ground state of hydrogen as a function of the magnetic field B (in G). We compare our results (curve a) with the results of CLR[3] (curve b) and with those using perturbation[5] theory (curve c). For $B \lesssim 7\times10^{11}$ G, our value for the ground-state energy level is lower than that of CLR.[3] Previously, our result[4] was superior only for $B \lesssim 3\times10^{10}$ G. Thus, we have significantly extended the range in which the Slater-

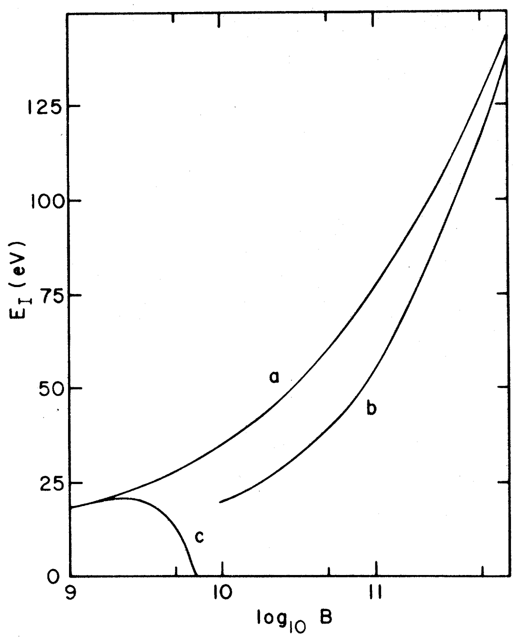

FIG. 1. The ionization energy of the ground state of hydrogen as a function of the magnetic field B calculated using our variational wave function (curve a). Shown for comparison are the results of CLR (curve b) and those obtained using perturbation theory (curve c).

type variational results are better than those of CLR.[3]

In Fig. 2, we present the energy spectrum for the 13 lowest states above the ground state for B values from 10^6–10^8 G. The labeling of the curves corresponds to the usual labels for the hydrogenic energy levels in the absence of a magnetic field.

We have compared these results with the perturbation results[5] and find that the latter are inadequate at B values of about 10^8 G and 3×10^7 G

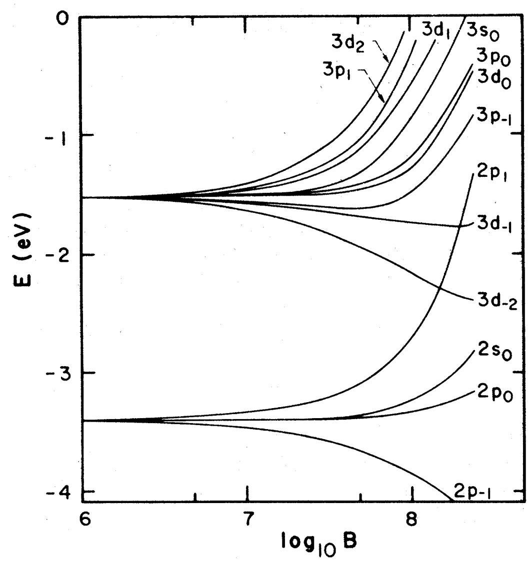

FIG. 2. The energy spectrum of hydrogen in a magnetic field for the 13 lowest states above the ground state.

for $n=2$ and 3, respectively. This is in conformity with the well-known result[5] that, for a particular B value, the perturbation results are less reliable for the higher states. Presented elsewhere are results for oscillator strengths and transition probabilities.[6] Elsewhere, we will consider the implications relating to (a) the deduction of magnetic field values in white dwarfs from the quadratic Zeeman effect,[7] (b) the effect of a magnetic field on the opacities of the atmospheres of magnetic white dwarfs, and (c) solid state problems involving excitons.

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