Model of the Decay $K_2 \to \pi^0 \gamma\gamma$ and a Unitarity Constraint on the Decay Rate

L. M. Sehgal
III. Physikalisches Institut, Technische Hochschule, Aachen, Germany
(Received 7 February 1972)

A model is presented for the decay $K_2 \to \pi^0\gamma\gamma$, in which the amplitude is determined in terms of the amplitude for $K_2 \to \pi^+\pi^-\pi^0$. The decay rate is found to be 13 $\sec^{-1}$. Using unitarity, a model-independent lower bound is obtained on the spectrum and decay rate of this reaction in the kinematical region $s > M^2 - 7\mu^2$, where s is the invariant mass squared of the photon pair and M and μ are the kaon and pion masses.

I. INTRODUCTION AND KINEMATICS

The radiative nonleptonic decays of the K meson hold interest for the possible insight that they might offer into the dynamics of weak nonleptonic processes.[1] One such decay is $K_2 \to \pi^0\gamma\gamma$. In the limit of CP invariance (which we assume), this process involves only the parity-conserving part of the nonleptonic Hamiltonian. Therefore, the general form of its matrix element is similar to that for a parity-conserving process like $\pi\pi \to \gamma\gamma$, and is given by

$$\begin{aligned} M = {} & F(\epsilon\cdot\epsilon' k\cdot k' - \epsilon\cdot k'\epsilon'\cdot k) \\ & + G(\epsilon\cdot\epsilon' k\cdot Q k'\cdot Q + k\cdot k'\epsilon\cdot Q\epsilon'\cdot Q \\ & - \epsilon\cdot Q\epsilon'\cdot k k'\cdot Q - \epsilon\cdot k' k\cdot Q\epsilon'\cdot Q), \end{aligned} \tag{1}$$

where the symbols are defined in Fig. 1. The expressions multiplying F and G are the two possible Lorentz- and gauge-invariant amplitudes that can be constructed from the available four vectors.[2] F and G are, in general, functions of the two independent invariants s and t, where we define

$$\begin{aligned} & s = (Q-p)^2 = (k+k')^2, \\ & t = (Q-k)^2, \\ & t' = (Q-k')^2, \\ & s + t + t' = M^2 + \mu^2, \end{aligned} \tag{2}$$

M and μ being the K-meson and pion masses, respectively. The physical region of the invariants is shown in the Dalitz plot of Fig. 2 and is bounded by the curves

$$\begin{aligned} & s = 0, \\ & (t-t')^2 = (s - M^2 - \mu^2)^2 - 4M^2\mu^2. \end{aligned} \tag{3}$$

Note, in particular, that s, the invariant mass squared of the photon pair, lies in the interval

$$0 < s < (M-\mu)^2. \tag{4}$$

One model that has been proposed for $K_2 \to \pi^0\gamma\gamma$ is based on the assumption that the process is dominated by the η pole: $K_2 \to \eta \to \pi^0\gamma\gamma$.[3] In such a model the amplitude becomes proportional to the amplitude for $\eta \to \pi^0\gamma\gamma$. The latter has been studied in a vector-meson-dominance model[4] which yields a structure of the form (1) with both F and G dependent on s and t. There exists, unfortunately, no reliable estimate of the K_2-η vertex, nor is it clear that other intermediate states (such as π^0 and η') are unimportant. In any event, it seems worthwhile to consider alternative models that can be

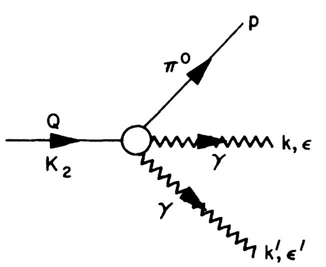

FIG. 1. Kinematic notation for $K_2 \to \pi^0\gamma\gamma$.

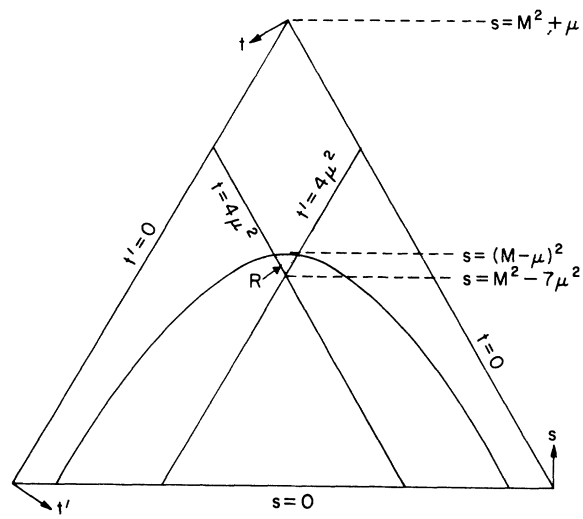

FIG. 2. Dalitz plot for $K_2 \to \pi^0\gamma\gamma$. The physical region is bounded by the curve and the line $s=0$. The shaded area (R) is the region in which unitarity constraints are obtained on the Dalitz-plot density of this reaction.

calculated unambiguously and which can be made plausible from the point of view of current algebra. A further question of importance that deserves investigation is to what extent one can make a *model-independent* statement about this process on the basis of general arguments like unitarity.

We present here a model in which the amplitude of $K_2 \to \pi^0\gamma\gamma$ is completely determined in terms of the known amplitude for $K_2 \to \pi^+\pi^-\pi^0$. A feature of the model is that it produces an amplitude containing only the first of the two gauge-invariant forms given in (1). Furthermore, the function F that emerges depends only on s and not on t, implying that the Dalitz-plot density varies only in the vertical direction, and not in the horizontal direction. The amplitude (which is obtained in closed form) is shown to possess a plausible soft-pion limit. Finally, we extract from the calculation that part which is a consequence only of unitarity, and derive a model-independent lower bound on the decay rate of $K_2 \to \pi^0\gamma\gamma$ into a specific region of the 2γ invariant mass.

II. THE MODEL

The model is illustrated in Fig. 3. We assume a point coupling for the $K_2 \to \pi^+\pi^-\pi^0$ vertex and minimal electromagnetic coupling of the pions to the photons. The assumption of a point-coupling for $K_2 \to \pi^+\pi^-\pi^0$ is justified to the extent that the Dalitz plot for this reaction is fairly uniform, there being a small variation along the vertical axis (that is, with the energy of the π^0), but no discernible variation in the horizontal direction. (We attempt later to take some account of the nonuniformity.) The graphs (a) and (b) in Fig. 3 are divergent but this divergence is exactly cancelled by the divergence of the "seagull" graph (c). We observe in particular that for $s > 4\mu^2$, the amplitude has an absorptive part which can be obtained by setting the intermediate pions on the mass shell. Defining the invariant amplitude by

$$\langle\pi^0(p)\gamma(k,\epsilon)\gamma(k',\epsilon')|M|K_2(Q)\rangle = A(s,t)F_{\mu\nu}F'^{\mu\nu} = 2A(s,t)(\epsilon\cdot\epsilon' k\cdot k' - \epsilon\cdot k'\epsilon'\cdot k)\,, \tag{5}$$

we have

$$\mathrm{Im}\langle\pi^0\gamma\gamma|M|K_2\rangle = \tfrac{1}{2}\sum \langle\pi^+(p_+)\pi^-(p_-)|M|\gamma(k,\epsilon)\gamma(k',\epsilon')\rangle^* \left(\frac{1}{4p_{+0}p_{-0}}\right)$$
$$\times\langle\pi^+(p_+)\pi^-(p_-)|M|K_2(Q)\pi^0(-p)\rangle(2\pi)^4\delta^4(Q-p_+-p_-), \tag{6}$$

where $\sum$ stands for an integration over the phase space of the intermediate $\pi^+\pi^-$ state. Carrying out this integration, we obtain

$$\mathrm{Im}\langle\pi^0\gamma\gamma|M|K_2\rangle = \left(\frac{ge^2}{4\pi}\right)\frac{2\mu^2}{s}\ln\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\,\theta(s-4\mu^2)(\epsilon\cdot\epsilon' k\cdot k' - \epsilon\cdot k'\epsilon'\cdot k)\,, \tag{7}$$

where g is the coupling constant for $K_2 \to \pi^+\pi^-\pi^0$,

$$\langle\pi^+\pi^-\pi^0|M|K_2\rangle = g\,, \tag{8}$$

and is numerically equal to 0.71×10^{-6}.[5] Equation (7) implies that

$$\mathrm{Im}A(s) = \left(\frac{ge^2}{4\pi}\right)\frac{\mu^2}{s^2}\ln\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\,\theta(s-4\mu^2). \tag{9}$$

(Observe that the function A is independent of t. This is a direct consequence of the fact that the Dalitz plot of $K_2 \to \pi^+\pi^-\pi^0$ has no horizontal variation.) To obtain the real part of the amplitude corresponding to

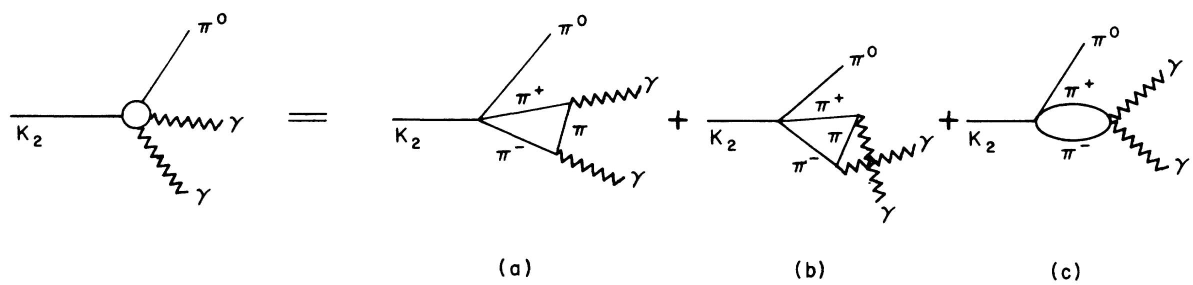

FIG. 3. Model for $K_2 \to \pi^0\gamma\gamma$.

the Feynman diagrams of Fig. 3, we make use of an unsubtracted dispersion relation for $A(s)$,[6] that is

$$\mathrm{Re}A(s) = \frac{1}{\pi}\ \mathrm{P}\int_{4\mu^2}^{\infty} \frac{\mathrm{Im}A(s')ds'}{s'-s}\ . \tag{10}$$

The result is[7]

$$\mathrm{Re}A(s) = \frac{1}{4\pi}\left(\frac{ge^2}{4\pi}\right)\frac{1}{s}\left\{\left[\frac{2\mu^2}{s}\left(\pi^2 - \ln^2\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\right) - 2\right]\theta(s-4\mu^2)\right.$$
$$\left. + \left[\frac{8\mu^2}{s}\left(\tan^{-1}\frac{1}{(4\mu^2/s-1)^{1/2}}\right)^2 - 2\right]\theta(4\mu^2 - s)\right\}. \tag{11}$$

Note in particular that

$$\mathrm{Re}A(s=4\mu^2) = (\tfrac{1}{2}\pi^2 - 2)\left(\frac{1}{4\pi}\frac{ge^2}{4\pi}\frac{1}{4\mu^2}\right)\ ,$$
$$\mathrm{Re}A(s=0) = \frac{2}{3}\left(\frac{1}{4\pi}\frac{ge^2}{4\pi}\frac{1}{4\mu^2}\right)\ . \tag{12}$$

A plot of the real and imaginary parts of $A(s)$ is shown in Fig. 4.

The decay rate of $K_2 \to \pi^0\gamma\gamma$ (in the rest frame of the K meson) is

$$\Gamma = \frac{1}{2}\ \frac{1}{2M}\int \frac{d^3p}{(2p_0)(2\pi)^3}\ \frac{d^3k}{(2k_0)(2\pi)^3}\ \frac{d^3k'}{(2k_0')(2\pi)^3}\ (2\pi)^4\delta^4(Q-p-k-k')\sum_{\mathrm{pol}}|M|^2 \tag{13}$$

$$= \frac{1}{2^{12}\pi^3M^3}\left(\frac{ge^2}{4\pi}\right)^2\int_0^{(M-\mu)^2} ds\ [(M+\mu)^2 - s]^{1/2}[(M-\mu)^2 - s]^{1/2}(I^2+R^2), \tag{14}$$

where I and R are related to $\mathrm{Im}A$ and $\mathrm{Re}A$ by

$$I(s) = 4s\,\mathrm{Im}A(s)\Big/\left(\frac{ge^2}{4\pi}\right)\ ,$$
$$R(s) = 4s\,\mathrm{Re}A(s)\Big/\left(\frac{ge^2}{4\pi}\right)\ . \tag{15}$$

The differential decay rate $d\Gamma/ds$ as a function of s, the 2γ invariant mass squared, is exhibited in Fig. 5, where the contributions of the absorptive and dispersive parts of the amplitude are shown separately. A notable feature of the spectrum is that $d\Gamma/ds$ is small for low values of s, but rises sharply near $s = 4\mu^2$. The integrated rates are

$$\Gamma_{\mathrm{abs}} = 7.5\ \mathrm{sec}^{-1},$$
$$\Gamma_{\mathrm{disp}} = 5.6\ \mathrm{sec}^{-1}, \tag{16}$$
$$\Gamma_{\mathrm{total}} = \Gamma_{\mathrm{abs}} + \Gamma_{\mathrm{disp}} = 13.1\ \mathrm{sec}^{-1}.$$

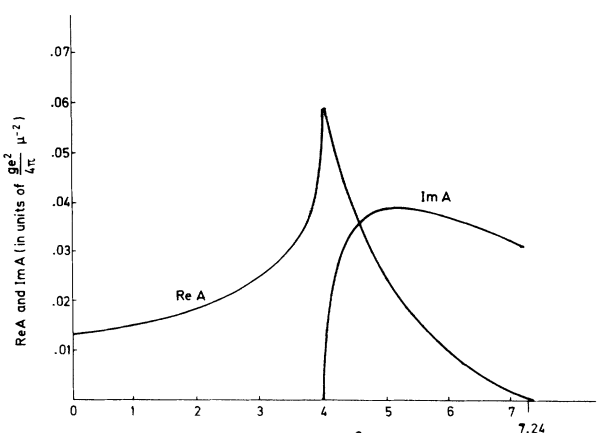

FIG. 4. A plot of the real and imaginary parts of $A(s)$.

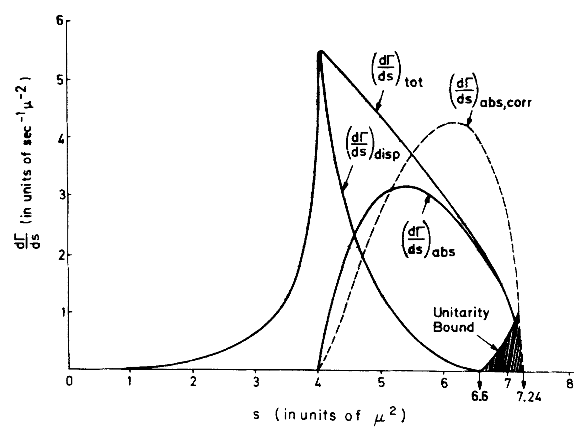

FIG. 5. A plot of $d\Gamma/ds$, the 2γ invariant mass distribution of $K_2 \to \pi^0\gamma\gamma$. The full lines show the absorptive, dispersive, and total spectrum predicted by the model of Fig. 3. The dashed curve is the absorptive contribution, corrected for the variation of the τ_0 decay amplitude. The shaded area under the dashed curve is the lower bound obtained from unitarity.

It will be interesting to see whether or not experiments find any significant enhancement over such a low value.

Let us now examine the soft-pion limit $p\to 0$ of the model. This limit is equivalent to taking $s\to M^2$, and from Eqs. (9) and (11), we find

$$(\mathrm{Im}A)_{p\to 0}=\frac{\tilde{g}e^2}{4\pi}\frac{\mu^2}{M^4}\ln\frac{\sqrt{M^2}+(M^2-4\mu^2)^{1/2}}{\sqrt{M^2}-(M^2-4\mu^2)^{1/2}},$$
$$(\mathrm{Re}A)_{p\to 0}=\frac{1}{4\pi}\frac{\tilde{g}e^2}{4\pi}\frac{1}{M^2}\left[\frac{2\mu^2}{M^2}\left(\pi^2-\ln^2\frac{\sqrt{M^2}+(M^2-4\mu^2)^{1/2}}{\sqrt{M^2}-(M^2-4\mu^2)^{1/2}}\right)-2\right], \tag{17}$$

where $\tilde{g}$ is the limit of the $K_2\to\pi^+\pi^-\pi^0$ amplitude as $p\to 0$, that is,

$$\tilde{g}=\lim_{p\to 0}\langle\pi^+\pi^-\pi^0(p)|M|K_2\rangle. \tag{18}$$

(Notice that the invariant amplitude for $K_2\to\pi^0\gamma\gamma$ does *not* vanish when the pion is made soft.) To make the result (17) meaningful, we recall a model of $K_1\to 2\gamma$ which assumes the reaction to proceed via a $\pi^+\pi^-$ intermediate state (Fig. 6).[8] Defining the invariant amplitude for $K_1\to 2\gamma$ as

$$\langle\gamma(k,\epsilon)\gamma(k',\epsilon')|M|K_1\rangle=BF_{\mu\nu}F'^{\mu\nu}=2B(\epsilon\cdot\epsilon'k\cdot k'-\epsilon\cdot k'\epsilon'\cdot k), \tag{19}$$

the real and imaginary parts of B may be calculated in exactly the same way as for $K_2\to\pi^0\gamma\gamma$. If h is the coupling constant for $K_1\to\pi^+\pi^-$,

$$\langle\pi^+\pi^-|M|K_1\rangle=h, \tag{20}$$

the result is

$$\mathrm{Im}B=\frac{he^2}{4\pi}\frac{\mu^2}{M^4}\ln\frac{\sqrt{M^2}+(M^2-4\mu^2)^{1/2}}{\sqrt{M^2}-(M^2-4\mu^2)^{1/2}},$$
$$\mathrm{Re}B=\frac{1}{4\pi}\frac{he^2}{4\pi}\frac{1}{M^2}\left[\frac{2\mu^2}{M^2}\left(\pi^2-\ln^2\frac{\sqrt{M^2}+(M^2-4\mu^2)^{1/2}}{\sqrt{M^2}-(M^2-4\mu^2)^{1/2}}\right)-2\right]. \tag{21}$$

Comparing (17) and (19), we see that

$$\frac{\lim_{p\to 0}\langle\pi^0(p)\gamma(k,\epsilon)\gamma(k',\epsilon')|M|K_2\rangle}{\langle\gamma(k,\epsilon)\gamma(k',\epsilon')|M|K_1\rangle}=\frac{\tilde{g}}{h}=\frac{\lim_{p\to 0}\langle\pi^+\pi^-\pi^0(p)|M|K_2\rangle}{\langle\pi^+\pi^-|M|K_1\rangle}. \tag{22}$$

But this is precisely the relation between $K_2\to\pi^0\gamma\gamma$ and $K_1\to\gamma\gamma$ predicted on general grounds using current-algebra and soft-pion techniques.[9] We conclude that our model of $K_2\to\pi^0\gamma\gamma$, when coupled with a reasonable model of $K_1\to\gamma\gamma$, satisfies the constraint of current algebra.

Finally, we consider the corrections to the above model arising from the fact that the Dalitz-plot density for $K_2\to\pi^+\pi^-\pi^0$ does show a variation with π^0 energy. Empirically, it is found that the density can be parametrized as

$$|\langle\pi^+\pi^-\pi^0|M|K_2\rangle|^2=|gD(s)|^2=|g|^2\left[1+\alpha\frac{s-s_0}{\mu^2}+\beta\left(\frac{s-s_0}{\mu^2}\right)^2\right], \tag{23}$$

where s is the invariant mass of the $\pi^+\pi^-$ pair, related to the π^0 energy E by $s=M^2+\mu^2-2ME$, and s_0 is the value of s at the center of the Dalitz plot. From the data of Albrow *et al.*,[10] we infer that $A\approx 7.66\,\mu^2/M^2$ and $B\approx -1.58\mu^4/M^4$. Since the absorptive part of the $K_2\to\pi^0\gamma\gamma$ amplitude in our model involves only the physical (mass-shell) amplitude for $K_2\to\pi^+\pi^-\pi^0$, we obtain the correction to the absorptive part of $d\Gamma/ds$ by simply multiplying the previous result by the factor

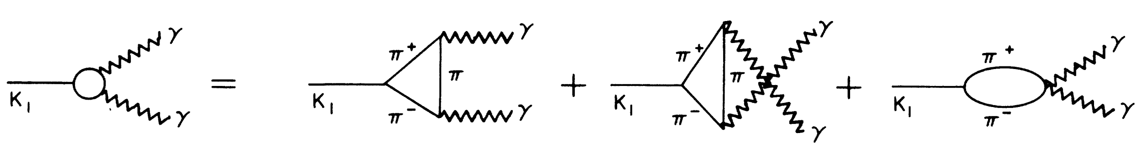

FIG. 6. Model for $K_1\to\gamma\gamma$, related to the model of $K_2\to\pi^0\gamma\gamma$ in Fig. 3.

$$\left[1+\alpha\frac{s-s_0}{\mu^2}+\beta\left(\frac{s-s_0}{\mu^2}\right)^2\right].$$

The corrected distribution $(d\Gamma/ds)_{\text{abs corr}}$ is

$$\left(\frac{d\Gamma}{ds}\right)_{\text{abs corr}}=\frac{1}{2^{12}\pi^3M^3}\left(\frac{ge^2}{4\pi}\right)^2[(M+\mu)^2-s]^{1/2}[(M-\mu)^2-s]^{1/2}\left(\frac{4\mu^2}{s}\right)^2\left(\ln\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\right)^2|D(s)|^2\,, \tag{24}$$

and is shown in Fig. 5. In particular, the integrated absorptive contribution is now

$$\Gamma_{\text{abs corr}}=9.3\ \text{sec}^{-1}. \tag{25}$$

The correction to $\text{Re}A(s)$, and consequently to the dispersive part of $d\Gamma/ds$, is more problematic. In order to agree with the empirical distribution (23), an expansion of the amplitude $\langle\pi^+\pi^-\pi^0|M|K_2\rangle$ in powers of $(s-s_0)$ must go up to at least the quadratic term. However, any such expansion is valid only in the limited region $4\mu^2<s<(M-\mu)^2$, and the behavior of the amplitude for large s remains essentially unknown. Indeed, if a quadratic form is assumed for all s, the high-s behavior of $\text{Im}A(s)$ is affected in such a way that an unsubtracted dispersion relation (UDR) can no longer be written. A UDR is still possible, however, if the matrix element $\langle\pi^+\pi^-\pi^0|M|K_2\rangle$ is approximated by an expression linear in s. We mention here the results obtained if we adopt the parametrization

$$\langle\pi^+\pi^-\pi^0|M|K_2\rangle=g(a+bs) \tag{26}$$

and assume it valid for all s. [The parameters a and b are chosen to agree with (23) as far as the linear term.] The dispersion relation for $\text{Re}A(s)$ gives

$$\text{Re}A(s)=\frac{1}{4\pi}\left(\frac{ge^2}{4\pi}\right)\frac{1}{s}\left[\left(\frac{2\mu^2}{s}\,a+2b\right)\left(\pi^2-\ln^2\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\right)-2a\right]\theta(s-4\mu^2)$$
$$+\left[\left(\frac{8\mu^2}{s}\,a+8b\right)\left(\tan^{-1}\frac{1}{(4\mu^2/s-1)^{1/2}}\right)^2-2a\right]\theta(4\mu^2-s). \tag{27}$$

The principal effects produced on the dispersive part of $d\Gamma/ds$ are: (i) There is a general enhancement of the distribution, especially for $s>4\mu^2$. (ii) The sharp peak at $s=4\mu^2$ broadens. (iii) The integrated dispersive rate is larger by a factor of about 6 compared to the prediction (16) of the simple model. Because of the simplisitic form chosen in (26), however, these corrections to the dispersive part cannot be regarded too seriously, and we have chosen not to exhibit them.

III. A LOWER BOUND FROM UNITARITY

It is clear that a knowledge of the absorptive part of the $K_2\to\pi^0\gamma\gamma$ amplitude in any part of the physical region would enable us to give a lower limit on the decay rate into the corresponding portion of the Dalitz plot. In general, the absorptive part at any point (that is, for any value of s and t in the physical region) will receive contributions from all real intermediate states that can appear in any of the three channels s, t, and t'. To order GE^2, the only relevant intermediate state is the two-pion state, which can appear in the s channel for $s>4\mu^2$ and in the t (t') channel for t (t')$>4\mu^2$. The absorptive part in the s channel is amenable to calculation, as it requires only a knowledge of the $K_2\to3\pi$ and $2\pi\to2\gamma$ amplitudes. On the other hand, the absorptive parts in the t and t' channels are difficult to estimate, since they involve the amplitudes $K_2\to\pi\pi\gamma$ and $\pi\gamma\to\pi\pi$. To circumvent this difficulty, we restrict our attention to that part of the physical region for which $s>4\mu^2$ and simultaneously t, $t'<4\mu^2$. This region is indicated by the shaded area in Fig. 2, and represents a portion of the Dalitz plot in which the absorptive part of the amplitude arises solely from the presence of a real 2π state in the s channel. [Note that the lines $t=4\mu^2$ and $t'=4\mu^2$ in Fig. 2 intersect at $s=M^2-7\mu^2$ and cut the upper boundary of the physical region at $s=\frac{3}{4}(M^2-4\mu^2)$.]

In the restricted region defined above (which we will call R), we may obtain the imaginary part of the $K_2\to\pi^0\gamma\gamma$ amplitude by means of an s-channel unitarity relation, in which the intermediate states $\pi^+\pi^-$ and $\pi^0\pi^0$ are included. In principle, therefore, knowledge of the amplitudes $K_2\to\pi^+\pi^-\pi^0$, $K_2\to\pi^0\pi^0\pi^0$, $\pi^+\pi^-\to\gamma\gamma$, and $\pi^0\pi^0\to\gamma\gamma$ determines $\text{Im}A(s)$ in a model-independent way. For the present purpose, we assume that the $\pi^0\pi^0\to\gamma\gamma$ amplitude may be neglected in relation to $\pi^+\pi^-\to\gamma\gamma$, and that the latter is adequately represented by the Born approximation. This leads to the result

$$\text{Im}A(s)=\frac{ge^2}{4\pi}\,\frac{\mu^2}{s^2}\left(\ln\frac{\sqrt{s}+(s-4\mu^2)^{1/2}}{\sqrt{s}-(s-4\mu^2)^{1/2}}\right)D(s)\quad(\text{valid in }R), \tag{28}$$

where $D(s)$ is defined in Eq. (23), and $|D(s)|^2$ is known empirically. On the basis of Eq. (27), we are thus able to obtain a lower bound on the Dalitz-plot density in the region R, given by

$$\frac{d^2\Gamma}{ds dx} \geq \frac{1}{X(s)}\left(\frac{d\Gamma}{ds}\right)_{\text{abs corr}}, \tag{29}$$

where $x=(1/\sqrt{3})(t'-t)$ is the horizontal coordinate of the Dalitz plot, $X(s)$ is the total horizontal width of the physical region at a given value of s,

$$X(s)=(2/\sqrt{3})[(s-\mu^2-M^2)^2-4\mu^2M^2]^{1/2}, \tag{30}$$

and $(d\Gamma/ds)_{\text{abs corr}}$ is given by Eq. (25) and plotted in Fig. 5.

If the Dalitz-plot density of $K_2\to\pi^0\gamma\gamma$ is not measured, but only the 2γ invariant mass distribution $d\Gamma/ds$, the above analysis provides a lower bound on $d\Gamma/ds$ in the region $s>M^2-7\mu^2$. We obtain

$$\frac{d\Gamma}{ds} \geq \left(\frac{d\Gamma}{ds}\right)_{\text{abs corr}} W(s), \quad s>M^2-7\mu^2 \tag{31}$$

where $W(s)$ is a "reduction factor" expressing the fact that the shaded area of Fig. 2 is only a fraction of the allowed physical region. Explicitly,

$$W(s)=\begin{cases} \dfrac{s-(M^2-7\mu^2)}{[(s-M^2-\mu^2)^2-4M^2\mu^2]^{1/2}}, & M^2-7\mu^2<s<\frac{3}{4}(M^2-4\mu^2) \\ 1, & \frac{3}{4}(M^2-4\mu^2)<s<(M-\mu)^2 \end{cases} \tag{32}$$

The "unitarity bound" on $d\Gamma/ds$ is shown in Fig. 5 by the shaded area. As is obvious, the bound is useful only for the very highest values of s. Integrating the unitarity bound, we find

$$\Gamma(K_2\to\pi^0\gamma\gamma;\, s>M^2-7\mu^2)>0.28\ \text{sec}^{-1}. \tag{33}$$

IV. A COMMENT ON THE DECAY $K^+\to\pi^+\gamma\gamma$

Unlike the decay $K_2\to\pi^0\gamma\gamma$, the reaction $K^+\to\pi^+\gamma\gamma$ can proceed both by the parity-conserving and parity-violating parts of the weak interaction. However, insofar as the total decay rate or the 2γ invariant mass distribution is concerned, there is no interference between the parity-conserving amplitude and the parity-violating one. Thus a model similar to that discussed in Sec. II can be used for estimating the parity-conserving contribution. The shape of the $d\Gamma/ds$ distribution is the same as in Fig. 5, and the integrated rate is

$$\Gamma(K^+\to\pi^+\gamma\gamma,\ \text{parity conserving})=76\ \text{sec}^{-1}, \tag{34}$$

(where the $K^+\to\pi^+\pi^+\pi^-$ coupling constant has been taken to be 1.72×10^{-6},[5] and the small variation of the τ^+ Dalitz plot has been neglected). By the reasoning of Sec. III, a lower bound based on unitarity can also be obtained for this reaction and the results are closely similar, the only difference arising from the difference in the $K^+\to\pi^+\pi^+\pi^-$ and $K_2\to\pi^0\pi^+\pi^-$ coupling constants, and the slight difference between the π^+ and π^0 masses.

ACKNOWLEDGMENTS

I wish to thank J. Cleymans, D. Rein, and H. Reithler for conversations on the subject.

---

[1]For a review, see, e.g., D. G. Sutherland, in *K* Decay: Proceedings of the Daresbury Study Weekend, 1971, edited by A. Donnachie and D. G. Sutherland (unpublished).

[2]B. R. Desai, Phys. Rev. 124, 1248 (1961).

[3]S. Oneda, Phys. Rev. 158, 1541 (1967); G. Fäldt, B. Petersson, and H. Pilkuhn, Nucl. Phys. B3, 234 (1967).

[4]G. Oppo and S. Oneda, Phys. Rev. 160, 1397 (1967).

[5]C. Itzykson, M. Jacob, and G. Mahoux, Suppl. Nuovo Cimento 5, 978 (1967).

[6]The use of a dispersion relation avoids certain ambiguities that arise when one attempts to calculate the Feynman diagrams of Fig. 3 by the usual perturbation-theoretic methods. See K. Nishijima, *Fields and Parti-*

cles (Benjamin, New York, 1969), p. 350.

[7]To evaluate the principal-value integral, we follow Nishijima, Ref. 6, p. 356, correcting a certain error. See also B. R. Martin, E. de Rafael, and J. Smith, Phys. Rev. D 2, 179 (1970).

[8]Martin, de Rafael, and Smith, Ref. 7.

[9]A. della Selva, A. D. Rujula, and M. Mateev, Phys. Letters 24B, 468 (1967); R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley, New York, 1968), p. 674.

[10]M. G. Albrow *et al.*, Phys. Letters 33B, 516 (1970).