fore, the insertion of the operator u will require no additional subtractions.

[22]These are the only two operators for a given n which are charge conjugation even and may have nonvanishing, spin-averaged matrix elements between two identical pseudoscalar or fermion states.

[23]For simplicity we use the same notation to represent analogous quantities in the pseudoscalar and vector theories. It should always be clear from the context to which theory a given symbol refers.

[24]S.-J. Chang and P. Fishbane, Phys. Rev. D 2, 1084 (1970). See also M. Kugler and S. Nussinov, Nucl. Phys. B28, 97 (1971); R. Gatto and P. Menotti, Nuovo Cimento 2A, 881 (1971).

[25]P. Fishbane and J. Sullivan, Phys. Rev. D 4, 2516 (1971).

[26]A. Mason (unpublished).

[27]The requirement (56a) is valid only if we set equal to zero the direct, renormalized photon coupling constant f, introduced in Eqs. (A19) and (A20).

[28]If the right-hand side of Eq. (65a) is expanded in powers of g^2, we find a term behaving as $1/g^2$. This term is independent of x^2 and is therefore annihilated by the derivatives appearing in Eq. (2).

[29]For a derivation of the Callan-Symanzik equations in quantum electrodynamics that follows similar lines, see A. Sirlin, Phys. Rev. D 5, 2132 (1972).

[30]Throughout our discussion of the vector theory we work in the Feynman gauge using $\delta_{\mu\nu}\Delta(k^2, m, \mu)$ for the photon propagator.

[31]A somewhat different asymptotic behavior is implied if g_∞ is a multiple root or an essential singularity of β. For a discussion of these various possibilities see S. Adler, IAS report (unpublished).

[32]G. F. Dell'Antonio (unpublished).

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Eikonal Cancellations in a Solvable Model*

H. M. Fried

Physics Department, Brown University, Providence, Rhode Island and Division de Physique Thêorique, † Institut de Physique Nuclêaire, Tour 16-1er êtage, 11 quai St-Bernard, 75230 Paris Cedex 05, France

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Two versions of a high-energy field-theoretic eikonal amplitude are examined in a special limit of large internal mass, where infinite sums over all n-point, connected, eikonal graphs become calculable. Both examples exhibit cancellations which tend to reduce the energy dependence of σ_T below the Froissart bound.

I. INTRODUCTION

Recent very-high-energy experiments displaying a constant pp total cross section[1] have acted as a spur to the estimation of σ_T and related multiplicity distributions. In particular, one would like to understand how the eikonal "tower graph" calculations of Cheng and Wu,[2] and the strong-coupling eikonal Regge calculation of Chang and Yan,[3] which generate $\sigma_T \sim \ln^2 s$, might be improved; and it has been suggested[4,5] that neglected crossed-channel multiparticle (connected) amplitudes can provide sufficient cancellations to remove the unobserved energy dependence. The purpose of this note is to describe a special version of a field-theoretic model previously discussed in an approximate way[5]; and to exhibit in an exact way two distinct forms of such cancellation in the special limit of large vector-meson mass (while the mass of emitted scalar "pions" remains finite). The first computation displays deviations from the form of a previous result of Aviv, Sugar, and Blankenbecler,[4] which arise from the inclusion of the next, more complicated set of fundamental graphs employed in the construction of the eikonal. The result of the second calculation, exact in its model context, sums over all contributing, nontrivial graphs, and produces an eikonal function independent of incident particle energy. While this does agree with the experimental $\sigma_T \sim$const, the main value of these computations lies in the construction of explicit examples which exhibit eikonal cancellations.

The starting point of the analysis is the specification of an interaction Lagrangian, coupling nucleon, neutral vector meson (NVM), and scalar pion fields,

$$\mathcal{L}' = ig\bar{\psi}\sum_\mu \gamma_\mu W_\mu \psi + \tfrac{1}{2}\lambda\Pi \sum_\mu W_\mu{}^2 . \tag{1}$$

A formal construction of the eikonal amplitude in which arbitrary numbers of NVM's are exchanged between a pair of scattering protons ($p_1+p_2 \to p_1'+p_2'$), while all possible pions are exchanged between the virtual NVM's, has been given[6] and employed[5] elsewhere; it is

$$e^{i\chi}=\exp\left(-\tfrac{1}{2}i\int\frac{\delta}{\delta\Pi}D_c\frac{\delta}{\delta\Pi}\right)\exp\left(ig^2\int\mathfrak{F}_{\mathrm{I}}^{\mu}\overline{\Delta}_c[\Pi]\mathfrak{F}_{\mathrm{II}}^{\mu}\right)\Big|_{\Pi=0}, \quad (2)$$

where

$$\overline{\Delta}_c[\Pi]=\Delta_c[1+\lambda\Pi\Delta_c]^{-1},$$

$$\mathfrak{F}_{\mathrm{I,II}}^{\mu}(w)=p_{1,2}^{\mu}\int_{-\infty}^{+\infty}d\xi\,\delta(w-z_{1,2}+\xi p_{1,2}),$$

D_c and $\delta_{\mu\nu}\Delta_c$ denote pion and (Feynman gauge) NVM propagators, $z_{1,2}$ are configuration-space coordinates of the (assumed distinguishable) nucleons, and $\Pi(u)$ represents a fictitious, c-number pion field. The two-dimensional impact parameter $\vec{b}$ of the small-angle c.m. scattering amplitude

$$T(s,t)=\frac{is}{2m^2}\int d^2b\,e^{i\vec{q}\cdot\vec{b}}\,(1-e^{i\chi(s;b)}) \quad (3)$$

is here given by $\vec{b}=\vec{z}_1-\vec{z}_2$, with $s=-(p_1+p_2)^2$, $t=-(p_1-p_1')^2=-\vec{q}^{\,2}$.

All Feynman graphs for this elastic process may be built up by simple expansion of the exponential factors of (2) in appropriate powers of the coupling constants g and λ. Typical graphs of order $g^6\lambda^6$ and $g^8\lambda^4$ are pictured in Fig. 1 and Fig. 2. The reader is referred elsewhere[5,6] for derivation of (2) and (3), which need not be repeated here. In the next section, the ASB model will be defined, in the context of the interaction (1), and then generalized. The detailed form of this generalization will suggest in Sec. III, a method of carrying through the functional differentiation operations of (2) without approximation.

II. GENERALIZATION OF THE ASB MODEL

That approximation which employs but a single pion line emanating from any virtual NVM line, for elastic nucleon scattering and multiple pion production processes, permits a simple computation of the eikonal function. The fundamental Feynman graph allowed is pictured in Fig. 3, with typical application to elastic and inelastic processes pictured in Figs. 4. The eikonal of this model

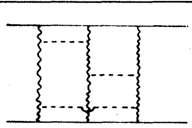

FIG. 1. A typical ladder graph of order $g^6\lambda^6$, with vertical NVM lines and horizontal pion lines.

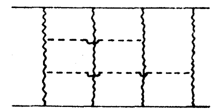

FIG. 2. A typical Feynman graph of order $g^8\lambda^4$.

may easily be obtained directly from (1), for the approximation simply entails the replacement of $\overline{\Delta}_c[\Pi]$ by its first-order expansion in Π,

$$\overline{\Delta}_c(x,y|\Pi)\simeq\Delta_c(x-y)-\lambda\int d^4u\,\Delta_c(x-u)\Pi(u)\Delta_c(u-y), \quad (4)$$

and the operations of (2) may be performed in an elementary manner, since $ig^2\int\mathfrak{F}_{\mathrm{I}}\cdot\overline{\Delta}_c\cdot\mathfrak{F}_{\mathrm{II}}$ is then but a linear functional of $\Pi(u)$. One obtains

$$i\chi=i\chi_0+i\chi_{\mathrm{ASB}},$$

with

$$i\chi_0=-\frac{ig^2}{2\pi}K_0(\mu b),$$

$$i\chi_{\mathrm{ASB}}=\tfrac{1}{2}i\int f(u)D_c(u-v)f(v)\,d^4u\,d^4v, \quad (5)$$

where

$$f(w)=-g^2\lambda\int\mathfrak{F}_{\mathrm{I}}^{\mu}(u)\Delta_c(u-w)\times\int\Delta_c(w-v)\mathfrak{F}_{\mathrm{II}}^{\mu}(v)\,d^4u\,d^4v$$

$$=\frac{\lambda}{2}\left(\frac{g}{\pi}\right)^2\delta((z_2-w)_{(-)})\delta((z_1-w)_{(+)})\times K_0(m|\vec{z}_1-\vec{w}|)K_0(m|\vec{z}_2-\vec{w}|), \quad (6)$$

and $a_{(\pm)}\equiv a_3\pm a_0$, $\vec{a}=(a_1,a_2)=\vec{a}_T$. Because the $s\to\infty$, $t/s\to 0$ limits have already been taken, in the derivation of (2), the $D_c(u-v)$ of (5) will depend on its transverse configuration-space coordinates only, and hence displays a Fourier transform logarithmically divergent in longitudinal and energy variables,

$$\int dk_{(+)}\int dk_{(-)}\,(k_{(+)}k_{(-)}+\vec{k}_T^{\,2}+\mu^2-i\epsilon)^{-1}\to i\pi\ln\left(\frac{s}{s_0}\right)\equiv i\pi Y, \quad (7)$$

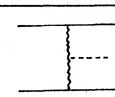

FIG. 3. The fundamental graph of the ASB model.

(a) (b)

FIG. 4. (a) A typical Feynman graph for elastic scattering in the ASB model. (b) A typical pion production graph of the ASB model.

so that

$$D_c(u-v) \to (2\pi)^{-2}\tfrac{1}{2}(i\pi Y)\delta^{(2)}(\vec{u}-\vec{v}) .$$

In writing (7), it has been noted that all transverse momenta of (5) are limited, and therefore s_0 may be considered a constant $\gtrsim \mu^2$. Evaluation of (5) is straightforward, and yields ($\Lambda \equiv \lambda^2/4\pi$, $G \equiv g^2/4\pi$)

$$i\chi_{\text{ASB}} = -\frac{YG^2\Lambda}{(2\pi)^2}\int d^2u \int d^2v\,\delta(\vec{b}-\vec{u}-\vec{v})K_0{}^2(mu)K_0{}^2(mv) , \tag{8}$$

leading to an energy-dependent σ_T. While the particular form of this dependence is considerably weaker[7] than that of the Froissart bound, what is of interest here is the possible nature, and energy dependence, of the corrections to (8).

One set of calculable corrections may be written down immediately, with numerical values dependent upon the resolution of a linear integral equation. They follow from the inclusion of all graphs constructed by the emission of a pair of pions from any virtual NVM line, as in Fig. 5, in addition to those of Fig. 3. In the present model, one simply expands $\overline{\Delta}_c[\Pi]$ retaining up to quadratic dependence on Π,

$$ig^2\int \mathfrak{F}_{\text{I}}\cdot\overline{\Delta}_c[\Pi]\cdot\mathfrak{F}_{\text{II}} \to i\chi_0 + i\int f\Pi + \tfrac{1}{2}i\int \Pi B \Pi , \tag{9}$$

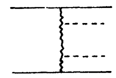

FIG. 5. The additional Feynman graph used to generalize the ASB model.

where

$$B(u,v) = g^2\lambda^2\int \mathfrak{F}_{\text{I}}^{\mu}(u')\Delta_c(u'-u)\Delta_c(u-v)\Delta_c(v-v')$$
$$\times \mathfrak{F}_{\text{II}}^{\mu}(v')d^4u'd^4v' + (u \leftrightarrow v) . \tag{10}$$

Again, the functional operations of (2) may be performed,[8] and one obtains

$$i\chi = i\chi_0 + i\chi'_{\text{ASB}} + i\chi_D ,$$

where

$$i\chi'_{\text{ASB}} = \tfrac{1}{2}i\int f D_c(1-BD_c)^{-1}f \tag{11}$$

and

$$i\chi_D = -\tfrac{1}{2}\,\text{Tr}\ln(1-BD_c) . \tag{12}$$

Clearly, $i\chi'_{\text{ASB}}$ represents the sum of all connected eikonal graphs containing a pair of NVM lines with one-virtual-pion exchange ("f lines"), and arbitrary numbers of NVM lines with two-pion-virtual exchanges ("B lines"). Graphs with more than one pair of f lines are disconnected, and cannot appear in $i\chi$. Similarly, $i\chi_D$ represents the sum of all connected graphs involving B lines only. That term of (12) involving a single B line represents a radiative correction to $i\chi_0$, and shall be omitted.

Both (11) and (12) are given in terms of quadratures over the unknown function $\overline{D}_c(x,y;\xi)$, which is the solution of the integral equation

$$\overline{D}_c(x,y;\xi) = D_c(x-y) + \xi\int D_c(x-u)B(u,v)\overline{D}_c(v,y;\xi)d^4u\,d^4v , \tag{13}$$

namely,

$$i\chi'_{\text{ASB}} = \tfrac{1}{2}i\int f(u)\overline{D}_c(u,v;1)f(v)d^4u\,d^4v , \tag{14}$$

and

$$i\chi_D = \tfrac{1}{2}\int_0^1 \xi d\xi\int B(u,v)D_c(v-u')B(u',v')\overline{D}_c(v',u;\xi)\,d^4u\cdots d^4v' . \tag{15}$$

Explicit symmetry of $B(u,v)$ under the exchange of its two variables is required, in order to generate properly both ladder and crossed graphs [corresponding to repeated use of Fig. 5, or the iteration of (13)],

$$B(u,v) = -\left(\frac{\lambda g}{\pi}\right)^2\Delta_c(u-v)\{\delta([z_1-u]_{(+)})\delta([z_2-v]_{(-)})K_0(m|\vec{u}-\vec{z}_1|)K_0(m|\vec{v}-\vec{z}_2|) + (u \leftrightarrow v)\} . \tag{16}$$

With (16), one is then instructed to compute (13), order by order if necessary, and insert the results into (14) and (15).

A vast simplification of this analysis is obtained upon taking the limit of very large NVM mass *between* pion emissions on any B line, $\Delta_c(u-v) \to \delta^{(4)}(u-v)/m^2$, and

$$B(u,v)\to -2\left(\frac{\lambda g}{\pi m}\right)^2 \delta(u-v)\delta([z_1-u]_{(+)})\delta([z_2-u]_{(-)})K_0(m|\vec{u}-\vec{z}_1|)K_0(m|\vec{u}-\vec{z}_2|)\ . \tag{17}$$

This approximation, which replaces Fig. 5 by Fig. 6, requires some elucidation, because of the singular functions $\delta^{(2)}(\vec{0})$ which subsequently appear. Before the limit leading to (17) is taken, the eikonal limits ($s\to\infty$, $t/s\to 0$) generate the logarithmically divergent s dependence of (7), with all transverse-momentum integrals finite. Upon making the replacement (17), and *continuing the identification (7)*, certain pion lines will be permitted to absorb arbitrarily large amounts of transverse momenta, and corresponding integrals shall diverge quadratically. Physically, this may be viewed as the property of arbitrarily large NVM masses to possess arbitrarily large transverse momenta, which may be transferred to the pions. Hence, any $\delta^{(2)}(\vec{0})=(2\pi)^{-2}\int d^2k$ may be rewritten as a pion transverse-momentum cutoff parameter, κ^2, and understood to be proportional to, or limited by m^2; one expects the approximation to be physically sensible if κ^2 enters only in the ratio κ^2/m^2, which can then be considered a finite parameter. In the present calculation, $m^{-2}\delta^{(2)}(\vec{0})$ appears only as a multiplicative factor in $i\chi_D$, and not at all in $i\chi'_{\text{ASB}}$.

With the replacement (17), it is a straightforward matter to carry through iteration of (13), and the corresponding evaluation of (14) and (15). One finds

$$i\chi'_{\text{ASB}}=-\frac{YG^2\Lambda}{(2\pi)^2}\int d^2u\int d^2v\,\delta(\vec{b}-\vec{u}-\vec{v})K_0{}^2(mu)K_0{}^2(mv)\left(1+i\frac{Y\Lambda G}{\pi m^2}K_0(mu)K_0(mv)\right)^{-1} \tag{18}$$

and

$$i\chi_D=-\delta^{(2)}(\vec{0})\int d^2u\int d^2v\,\delta(\vec{b}-\vec{u}-\vec{v})\left[\ln\left(1+i\frac{Y\Lambda G}{\pi m^2}K_0(mu)K_0(mv)\right)-i\frac{Y\Lambda G}{\pi m^2}K_0(mu)K_0(mv)\right]\ , \tag{19}$$

using the notation of (8). In obtaining both (18) and (19), it has been supposed that

$$\frac{Y\Lambda G}{\pi m^2}K_0(mu)K_0(mv)<1,$$

in order to sum the series corresponding to each iteration, after which this condition may be relaxed. For arbitrary, real values of Y, G, Λ, and m^2, both integrals are finite, complex numbers. Limiting values of very large Y may be taken under the integrals, and one obtains

$$i\chi'_{\text{ASB}}\sim i\frac{m^2G}{4\pi}\int d^2u\int d^2v\,\delta(\vec{b}-\vec{u}-\vec{v})\times K_0(mu)K_0(mv) \tag{20}$$

and

$$i\chi_D\sim i\frac{Y\Lambda G}{\pi}\left(\frac{\kappa^2}{m^2}\right)\int d^2u\int d^2v\,\delta(\vec{b}-\vec{u}-\vec{v})\times K_0(mu)K_0(mv)\ . \tag{21}$$

It is interesting to note that, in this limit, $i\chi'_{\text{ASB}}$ becomes independent of Λ and Y, and that while the ASB eikonal has been rendered finite, its original linear Y dependence has been, in effect, transferred to the B line, double pion emissions, $i\chi_D$. One may speculate that inclusion of triple pion emissions from any NVM line will tend to damp away the contribution of (21), etc.; but the immediate point to be emphasized is how, in this simple passage from (18) to (20), the higher n-point connected, crossed-channel amplitudes constructed in this model of double pion emission qualitatively change the ASB result. The assumption that κ/m does not grow with s is crucial if (21) is to have an energy dependence no stronger than linear in Y.

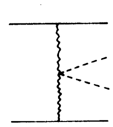

FIG. 6. A pictorial representation of Fig. 5 in the limit of large NVM mass between pion emissions.

III. AN EXACT SOLUTION

The simplifications found in the preceding section suggest that a model in which the emission of arbitrary numbers of pions in the manner of Fig. 7(a) is replaced by pion emission in the form of Fig. 7(b) will be fully solvable in the sense that all operations of (2) may be performed. This is indeed the case.

It is convenient to first obtain a suitable representation for the source dependence of $\overline{\Delta}_c[\Pi]$. To

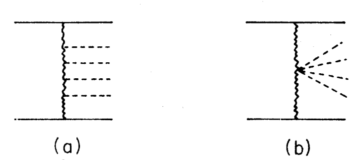

FIG. 7. (a) A typical, fundamental graph of the exact theory. (b) The pictorial representation of Fig. 7(a) in the limit of large NVM mass between pion emissions.

this end, the integral equation

$$\bar{\Delta}_c[\Pi] = \Delta_c - \lambda \int \Delta_c \Pi \bar{\Delta}_c[\Pi]$$

may be iterated once and rewritten in the form

$$\bar{\Delta}_c = \Delta_c - \lambda \int \Delta_c H \Delta_c \ ,$$

where H satisfies the relation

$$H(x,y) = \Pi(x)\delta(x-y) - \lambda \Pi(x)\Delta_c(x-y)\Pi(y) + \lambda^2 \Pi(x) \int \Delta_c(x-u) H(u,v) \Delta_c(v-y)\Pi(y) \ . \tag{22}$$

Equation (22) is exact. The transition from Fig. 7(a) to Fig. 7(b) is obtained by replacing, in (22), every Δ_c by $m^{-2}\delta$ of appropriate argument,

$$H(x,y) \to \Pi(x)\delta(x-y) - \frac{\lambda}{m^2}\Pi^2(x)\delta(x-y) + \frac{\lambda^2}{m^4}\Pi(x)H(x,y)\Pi(y) \ . \tag{23}$$

The solution to (23) is immediate:

$$H(x,y) = \Pi(x)\delta(x-y)\left(1 + \frac{\lambda}{m^2}\Pi(x)\right)^{-1} , \tag{24}$$

with a corresponding solution for $\bar{\Delta}_c$,

$$\bar{\Delta}_c(x,y\,|\,\Pi) = \Delta_c(x-y) + m^2 \int \Delta_c(x-u)\Delta_c(u-y)d^4u \times \int_0^\infty d\xi\, e^{-i\xi} \frac{\partial}{\partial \xi} \exp\left(-i\xi \frac{\lambda}{m^2}\Pi(u)\right) \ . \tag{25}$$

A parametrization of (24) convenient for subsequent manipulations has been introduced in (25). These forms recall those[9] appearing in discussions of nonpolynomial Lagrangians, and, indeed, the present computation may be viewed as that rare application of the quantum fluctuations of such Lagrangian theories which could conceivably have some relation to experimental physics.

With (25) one may perform all the functional operations of (2) exactly:

$$e^{i\chi} = e^{i\chi_0} \sum_{n=0}^{\infty} \frac{1}{n!} \exp\left(-\tfrac{1}{2} i \int \frac{\delta}{\delta \Pi} D_c \frac{\delta}{\delta \Pi}\right) \times \left[i g^2 m^2 (p_1 \cdot p_2) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} da\, db \int d^4u\, \Delta_c(z_1 - a p_1 - u)\Delta_c(u + b p_2 - z_2) \int_0^\infty d\xi\, e^{-i\xi} \frac{\partial}{\partial \xi} \exp\left(-i\xi \frac{\lambda}{m^2}\Pi(u)\right)\right]^n \Bigg|_{\Pi=0} . \tag{26}$$

The $n=0$ contribution to this sum is unity, while the $n=1$ term (together with any other self-linkages) corresponds to a radiative correction to an NVM line, and is dropped. Retaining only pion linkages between different NVM lines, for $n=2$ one finds the amount

$$\frac{i^2}{2!}\left(\frac{gm}{2\pi}\right)^4 \int d^2u_1 \int d^2u_2 K_0^2(m u_1) K_0^2(m|\vec{u}_2 - \vec{b}|) \int_0^\infty d\xi_1 e^{-i\xi_1} \int_0^\infty d\xi_2 e^{-i\xi_2} \frac{1}{\kappa^2}\delta(\vec{u}_1 - \vec{u}_2) \frac{\partial}{\partial \xi_1}\frac{\partial}{\partial \xi_2} \exp(-\xi_1\xi_2 \Lambda Y \kappa^2/2m^4) \ . \tag{27}$$

To obtain (27), one proceeds from the result of the functional operations,

$$\exp[i\xi_1\xi_2(\lambda/m^2)^2 D_c(u_1 - u_2)] ,$$

and observes that the eikonal limits permit this D_c to depend only upon its transverse variable, $\vec{u}_1 - \vec{u}_2$. The replacement of $D_c(\vec{u}_1 - \vec{u}_2)$ by

$$(2\pi)^{-2}(i\tfrac{1}{2}\pi Y)\delta^{(2)}(\vec{u}_1 - \vec{u}_2),$$

carrying the same physical interpretation $[\delta^{(2)}(\vec{0}) \leftrightarrow \kappa^2]$ as in Sec. II, together with the association

$$\frac{\partial}{\partial \xi_1}\frac{\partial}{\partial \xi_2}\exp[-\alpha\xi_1\xi_2\delta(\vec{u}_1 - \vec{u}_2)] \to \frac{1}{\kappa^2}\delta(\vec{u}_1 - \vec{u}_2)\frac{\partial}{\partial \xi_1}\frac{\partial}{\partial \xi_2}\exp(-\alpha\xi_1\xi_2\kappa^2) \ , \tag{28}$$

then leads to (27), or to

$$\frac{i^2}{2!}\frac{G^2}{\pi^2}\left(\frac{m^2}{\kappa^2}\right) m^2 \int d^2u K_0^2(mu) K_0^2(m|\vec{u} - \vec{b}|) I_2\left(\frac{\Lambda}{m^2} Y \frac{\kappa^2}{m^2}\right) , \tag{29}$$

where

$$I_n(\alpha) \equiv \int_0^\infty d\xi_1 \cdots \int_0^\infty d\xi_n e^{-i(\xi_1 + \cdots + \xi_n)} \frac{\partial}{\partial \xi_1} \cdots \frac{\partial}{\partial \xi_n} \exp\left(-\tfrac{1}{2}\alpha \sum_{i<j=1}^{n} \xi_i \xi_j\right) . \tag{30}$$

All the Y dependence of this special limiting form of interaction between a pair of extremely massive NVM's (and of finite ratio κ^2/m^2) is contained in the $\alpha = (\Lambda/m^2)(\kappa^2/m^2)Y$ variable of (30).

One may anticipate the form of $I_2(\alpha)$ in the limit $Y \to \infty$, by a comparison with conventional derivations of field-theoretic Regge models. There,[3] one isolates nested ladder graphs of the source of leading-logarithmic contributions, and finds in order $g^2\lambda^{2m}$ a contribution proportional to $(1/m!)Y^m$, with the $1/m!$ arising because of ordered emission of pion rapidities along either NVM line. Summing over all $m (\geq 1)$ pions produces the schematic form $\sim \exp(\lambda^2 Y) - 1 \sim (s/s_0)^{\lambda^2}$. In the present computation, one again has Y^m leading-logarithmic dependence; but because the pions are emitted from a single configuration-space point along the NVM line, the relative ordering of rapidities is absent, and hence the $1/m!$ factor is missing. One then expects the schematic form

$$\sim \sum_{m=1}^{\infty} (\lambda^2 Y)^m = \lambda^2 Y (1 - \lambda^2 Y)^{-1} ,$$

and the limit $Y \to \infty$ will produce a result independent of s. Thus the sum over those graphs which generate the usual Regge amplitude is already independent of energy, by itself producing the simplest sort of non-shrinking diffraction scattering. This may be viewed as an example of (vertical) cancellation, distinct from from the (horizontal) examples of the previous section.

It is a simple matter to evaluate $I_n(\alpha)$ in the limit $\alpha \to \infty$, a procedure simplified by the repeated use of integration by parts,

$$\int_0^\infty d\xi e^{-i\xi} \frac{\partial}{\partial \xi} \psi(\xi) = i \int_0^\infty d\xi e^{-i\xi} [\psi(\xi) - \psi(0)] ,$$

for the appropriate $\psi(\xi)$ of this problem. For large α, it is only the $\psi(0)$ dependence in all but one variable which yields the leading contributions [first corrections are $O(\alpha^{-1} \ln\alpha)$]; there are $n-1$ such factors, and one obtains

$$I_n(\alpha)|_{\alpha \to \infty} \sim (n-1)(-)^{n-1} , \tag{31}$$

a result independent of Λ and Y (and κ^2/m^2) as $Y \to \infty$. The quantity $I_n(\alpha)$ appears in the nth term of (26),

$$\frac{i^n}{n!}\left(\frac{mg}{2\pi}\right)^{2n} \int d^2u K_0{}^n(mu) K_0{}^n(m|\vec{u} - \vec{b}|) \left(\frac{1}{\kappa^2}\right)^{n-1} I_n(\alpha) , \tag{32}$$

using the previous notation and repeated use of the technique of (28). With (31), as $Y \to \infty$ each term is independent of Y, and may be summed to yield

$$e^{i\chi} = e^{i\chi_0} \left\{ 1 - \rho \frac{\partial}{\partial \rho} \frac{1}{\rho} m^2 \int d^2u \left[\exp\left(-i \frac{G}{\pi} \rho K_0(mu) K_0(m|\vec{u} - \vec{b}|)\right) - 1 \right] \right\} \Bigg|_{\rho = m^2/\kappa^2} \tag{33}$$

as the complete, energy-independent expression for the eikonal. The "cancellations" of (33) are only those due to the exponential rearrangement of energy-independent terms, with (vertical) cancellation of all energy dependence occurring at the earlier stage of (31). From this point of view, the complete model is not as interesting as the examples of Sec. II; however, assuming the validity of the interchange of $\sum_{n=0}^\infty$ and $Y \to \infty$ limits, within the context of the model it is an exact result.

IV. SUMMARY

The special, limiting model described above is certainly too simple to stand a close comparison with the most recent pp experiments (e.g., shrinkage exists), although the feature of constant σ_T is exhibited. The eikonal of this field-theoretic model becomes calculable when the limit of large, internal NVM mass is taken, and one finds strong, explicit cancellations from the Regge or Regge-eikonal picture. These cancellations are not as strong as the approximate ones of Ref. 5, but act in the same direction, reducing σ_T from the Froissart bound.

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†Laboratoire associé au Centre National de la Recherche Scientifique.

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[7]Writing

$$i\chi_{\mathrm{ASB}} = -(YG^2\Lambda/2\pi)F(b),$$

$F(b)$ may be represented as

$$(2\pi)^{-1}\int d^2k\, e^{i\vec{k}\cdot\vec{b}}\left[\int d^2u\, e^{-i\vec{k}\cdot\vec{u}}K_0{}^2(mu)\right]^2 .$$

The Fourier transform of $K_0{}^2$ may be calculated exactly (Bateman Project), and hence

$$F(b)=\int_0^\infty dk\,[k\Omega(k)]^{-1}J_0(kb)\ln^2[(\Omega+k)/(\Omega-k)]\,,$$

where $\Omega(k)\equiv(k^2+4m^2)^{1/2}$. For large b, only small k is important, and $\ln^2[\;]$ may be replaced by its value for small k, $(k/m)^2$, so that $F(b)\sim m^{-2}K_0(2mb)$. This corresponds to a $b_{\max}\sim \ln Y$, and hence $\sigma_T\sim\ln^2(\ln s)$. It must be emphasized that ASB discuss different unitary models [viz., their Eq. (58)], and hence obtain different estimates of σ_T.

[8]See, for example, the Appendix of the paper by C. Sommerfield, Ann. Phys. (N.Y.) 26, 1 (1964); or Ref. 6, Chap. 3.

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Inelastic e^+e^- Annihilation in Perturbation Theory

Paul M. Fishbane*

Physics Department, University of Virginia, Charlottesville, Virginia 22901

and

J. D. Sullivan†

National Accelerator Laboratory, Batavia, Illinois 60510

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The structure functions for the annihilation process $e^+ + e^- \to \bar{p} + X$ are calculated in the neutral-vector-gluon model in the Bjorken limit. Bjorken scaling is broken by the presence of $\ln q^2$ factors in a way which is closely related to the situation in inelastic scattering. All calculations are carried out in a leading-logarithm approximation. In particular there is a multiplicity $n\sim\ln^2 q^2$ and a close interplay between the damping of the elastic form factor and the excitation of inelastic channels. The annihilation structure functions are shown to be related to their inelastic scattering counterparts by analytic continuation and by a physical-region reciprocal relation. The reciprocal relation is observed to have a number of interesting consequences if it applies, in some approximate sense, to pions, protons, etc. In addition to the leading-logarithm calculations contained in this paper the discussions given here of discontinuities of the virtual Compton amplitude and the longitudinal-impact-parameter representation are of general interest and applicability.

I. INTRODUCTION

This paper is one in a series of papers[1-3] in which we study the neutral-vector-gluon model [massive quantum electrodynamics (QED)] in the Bjorken scaling[4] limit. The major topic of this paper is the annihilation channel[5-7] $e^- + e^+ \to \bar{p} + X$, and the relation of the annihilation structure functions to their counterparts in inelastic scattering, $e^- + p \to e^- + X$[8]. These relations may transcend the particular field theory studied here.

Here, as in all renormalizable[9,10] (in contrast to