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General Chiral-Symmetry Breaking and σ Terms*

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We systematically investigate the properties of general chiral-symmetry breaking in a σ model. We obtain not only the conventional results of nonlinear pion models, but also the effects of the symmetry breaking upon σ terms. We show that the σ terms depend upon the fraction of the pion mass coming from the chirally symmetric part of the model, as well as upon the representation of the symmetry-breaking part of the model.

I. INTRODUCTION

A study of the linear σ model is unavoidable if one wishes to study the effects of the σ particle in pion processes. Although the nonlinear pion Lagrangian model is an easier tool to use in the analysis of multiple-pion processes,[1] it provides no insight into the effects of a finite-mass σ particle upon these processes.

We shall generalize the Gell-Mann–Lévy σ model[2] in order to treat systematically the symmetry-breaking term $\mathcal{L}_m$ that transforms according to the representation $(\frac{1}{2}m, \frac{1}{2}m)$; i.e., like a traceless symmetric tensor of rank m. We shall note that the symmetry-breaking term is an elementary solution of the ordinary hypergeometric equation and is a homogeneous binomial. Since $\tilde{\sigma}^2+\Phi^2$ is chirally invariant, we can multiply any symmetry-breaking term by an arbitrary power of $\tilde{\sigma}^2+\Phi^2$ without affecting the chiral transformation properties of the term. We shall see that the physics is similarly unaffected by the multiplication and is solely a function of m.

The terms of zero order in μ_π^2/μ_σ^2 in the scattering amplitudes for $\pi\pi\to\pi\pi$ and $\pi N\to\pi N$ reproduce the results previously obtained from the nonlinear chiral model. The size of the contribution of the so-called "σ term" (the term proportional to μ_π^2/μ_σ^2) to the scattering amplitudes will be seen to be arbitrary unless further constraints are imposed on the model. For example, if one imposes the condition that the sole contribution to the pion-mass term $-\frac{1}{2}\mu_\pi^2\Phi^2$ comes from the symmetry-breaking portion of the Lagrangian (unlike the contribution in the original Gell-Mann–Lévy model), one finds a specific m-independent value for the σ term.

II. PROPERTIES OF SYMMETRY BREAKING

We shall first write down a general homogeneous binomial P_N of Φ^2 and $\tilde{\sigma}^2$, then perform the transformation $\tilde{\sigma}=\sigma+f$,[3] and then demand that the linear relations that exist among the coefficients of $\sigma\Phi^2$, Φ^2, σ^2, and σ and among the coefficients of $(\Phi^2)^2$, Φ^2, σ^2, and σ be independent of M when we multiply P_N by $(\Phi^2+\tilde{\sigma}^2)^M$. These coefficients are all that are needed to calculate the $\pi N\to\pi N$, $\pi\pi\to\pi\pi$, and $\pi N\to 2\pi N$ amplitudes.

Let

$$P_N=\sum_{j=0,1,\ldots} A_{N-j}\,\tilde{\sigma}^{2(N-j)}(\Phi^2)^j\,. \tag{1}$$

Then

$$P_N(\Phi^2+\tilde\sigma^2)^M = A_N\{2(N+M)\sigma+(N+M)[2(N+M)-1]\sigma^2\}+(A_{N-1}+MA_N)[\Phi^2+2(N+M-1)\sigma\Phi^2] + [A_{N-2}+MA_{N-1}+\tfrac{1}{2}M(M-1)A_N](\Phi^2)^2+\cdots\,, \tag{2}$$

where for convenience we are choosing $f=1$.

We can now isolate the coefficients, $c(i)$, of the vertices, i:

$$\begin{aligned} c(\sigma) &= 2(N+M)A_N\,,\\ c(\sigma^2) &= (N+M)[2(N+M)-1]A_N\,,\\ c(\Phi^2) &= A_{N-1}+MA_N\,,\\ c(\sigma\Phi^2) &= 2(N+M-1)(A_{N-1}+MA_N)\,,\\ c(\Phi^4) &= A_{N-2}+MA_{N-1}+\tfrac{1}{2}M(M-1)A_N\,. \end{aligned} \tag{3}$$

The linear relationship among the $\sigma\Phi^2$, σ, σ^2, and Φ^2 coefficients is then

$$c(\sigma\Phi^2)=k_1c(\sigma)+k_2c(\sigma^2)+k_3c(\Phi^2)\,. \tag{4}$$

The k_i's are to be independent of M. Isolating the terms quadratic in M, linear in M, and independent of M leads to three equations that may be solved for k_1, k_2, and k_3. If we now let f equal its actual value, then

$$k_1=\frac{(1-2N)A_N+2A_{N-1}}{2A_Nf^2}\,,\qquad k_2=\frac{1}{f}\,,\quad k_3=-\frac{2}{f}\,. \tag{5}$$

We may similarly relate the $(\Phi^2)^2$, σ, σ^2, and Φ^2 coefficients:

$$c(\Phi^4)=h_1c(\sigma)+h_2c(\sigma^2)+h_3c(\Phi^2)\,. \tag{6}$$

Requiring that the h_i's are independent of M, we obtain

$$\begin{aligned} h_1 &= \frac{-(N+\frac{1}{4})A_N+A_{N-1}-h_3A_N}{2A_Nf^3}\,,\\ h_2 &= \frac{1}{4f^2}\,,\\ h_3 &= \frac{-\frac{1}{2}N(1+N)A_N+NA_{N-1}-A_{N-2}}{(NA_N-A_{N-1})f^2}\,. \end{aligned} \tag{7}$$

The constraint that the k_i's and the h_i's are independent of M is sufficient to guarantee that the physics is unaffected when $\mathcal{L}_m$ is multiplied by an arbitrary function of the chiral-invariant $\tilde\sigma^2+\Phi^2$.

The result that the values of k_2 and h_2 are independent of the choice of binomial is crucial for the extraction of the nonlinear pion Lagrangian amplitudes from the linear σ model amplitudes.

III. STRUCTURE OF SYMMETRY BREAKING

We shall now derive the general structure of $\mathcal{L}_m(\tilde\sigma,\Phi^2)$. Remember that in the σ model we have the following commutation relations of the fields with the axial charge:

$$[Q_i^5,\Phi_j]=i\delta_{ij}\tilde\sigma\,,\quad [Q_i^5,\tilde\sigma]=-i\Phi_i\,. \tag{8}$$

We define $\mathcal{L}_m$ by the condition that it transform according to the representation $(\frac{1}{2}m,\frac{1}{2}m)$ (Ref. 4); i.e.,

$$\sum_{i=1}^{3}[Q_i^5,[Q_i^5,\mathcal{L}_m]]=m(m+2)\mathcal{L}_m\,. \tag{9}$$

We set

$$\nu^2-1=m(m+2)\,,\quad \nu=+(\nu^2)^{1/2}\,, \tag{10}$$

and rewrite Eq. (9) in the form

$$\sum_{i=1}^{3}[Q_i^5,[Q_i^5,\mathcal{L}_\nu]]=(\nu^2-1)\mathcal{L}_\nu\,. \tag{11}$$

To solve Eq. (11) we shall switch from the variables $\tilde\sigma$ and Φ^2 to the variables y and z defined by

$$y=\tilde\sigma^2+\Phi^2\,,\quad z=\frac{\Phi^2}{\tilde\sigma^2+\Phi^2}\,. \tag{12}$$

Using Eq. (8), we find that

$$[Q_i^5,y]=0\,,\quad [Q_i^5,z]=\frac{2i\Phi_i\tilde\sigma}{y}\,. \tag{13}$$

If we write Eq. (11) in terms of the variables y and z, we obtain an ordinary hypergeometric equation,

$$z(1-z)\frac{\partial^2\mathcal{L}_\nu}{\partial z^2}+\tfrac{1}{2}(3-4z)\frac{\partial\mathcal{L}_\nu}{\partial z}+\tfrac{1}{4}(\nu^2-1)\mathcal{L}_\nu=0\,. \tag{14}$$

Equation (14) has two well-known solutions.[5] The one of physical interest is the one regular at $z=0$:

$$\mathcal{L}_\nu(z)={}_2F_1(\tfrac{1}{2}(1+\nu),\tfrac{1}{2}(1-\nu);\tfrac{3}{2};z) = \frac{\sin\nu\theta}{\nu\sin\theta}\,, \tag{15}$$

where

$$z=\sin^2\theta = \frac{\Phi^2}{\tilde\sigma^2+\Phi^2}\,.$$

Since the result of multiplying $\mathcal{L}_\nu(z)$ [given by Eq. (15)] by an arbitrary function of $\tilde\sigma^2+\Phi^2$ still satisfies Eq. (11), we choose to multiply $\mathcal{L}_\nu(z)$ by $(\tilde\sigma^2+\Phi^2)^{(\nu-1)/2}$, thereby obtaining a homogeneous binomial $P_{(\nu-1)/2}$ of degree $\frac{1}{2}(\nu-1)$ in $\tilde\sigma^2$ and Φ^2. We shall write down the first five binomials that transform according to the representations $(\frac{1}{2}m,\frac{1}{2}m)$; $m=0,1,2,3,4$:

$$m=0:\quad P_0=1\,,$$
$$m=1:\quad P_{1/2}=\tilde\sigma\,,$$
$$m=2:\quad P_1=\tilde\sigma^2-\tfrac{1}{3}\Phi^2\,, \tag{16}$$
$$m=3:\quad P_{3/2}=\tilde\sigma^3-\tilde\sigma\Phi^2\,,$$
$$m=4:\quad P_2=\tilde\sigma^4-2\tilde\sigma^2\Phi^2+\tfrac{1}{5}(\Phi^2)^2\,.$$

For arbitrary ν we have

$$P_{(\nu-1)/2}=\sum_{j=0,1,\ldots}A_{(\nu-1)/2-j}(\tilde\sigma^2)^{(\nu-1)/2-j}(\Phi^2)^j\,, \tag{17}$$

where

$$A_{(\nu-1)/2-j}=\exp(i\pi j)\frac{\Gamma(\nu)}{\Gamma(2j+2)\Gamma(\nu-2j)}$$
$$=\frac{\exp(i\pi j)}{2j(2j+1)}B^{-1}(2j,\nu-2j) \tag{18}$$

and $B^{-1}(2j,\nu-2j)$ is the inverse of the beta function defined by

$$B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}\;. \tag{19}$$

In particular, we have

$$A_{(\nu-1)/2}=1\,,$$
$$A_{(\nu-3)/2}=-\frac{(\nu-1)(\nu-2)}{3!}\,, \tag{20}$$
$$A_{(\nu-5)/2}=\frac{(\nu-1)(\nu-2)(\nu-3)(\nu-4)}{5!}\;.$$

We can now explicitly write down as functions of ν^2 the coefficients of the linear relations for the $\sigma\Phi^2$ and $(\Phi^2)^2$ vertices discussed above. Plugging Eq. (20) into Eqs. (5) and (7) and noting that N is to be everywhere replaced by $\frac{1}{2}(\nu-1)$, we obtain

$$k_1(\nu)=\frac{4-\nu^2}{6f^2}\;,\quad k_2(\nu)=\frac{1}{f}\,,$$
$$k_3(\nu)=-\frac{2}{f} \tag{21a}$$

and

$$h_1(\nu)=\frac{7}{20f}k_1(\nu)\,,\quad h_2(\nu)=\frac{1}{4f^2}\,,$$
$$h_3(\nu)=-\frac{1}{f^2}\left(\frac{\nu^2+11}{20}\right). \tag{21b}$$

Equations (21a) and (21b) contain information that depends solely upon the transformation properties of the nonderivative portion of the π-σ Lagrangian.

IV. MASS TERMS AND THE LAGRANGIAN

We can now construct our pion and σ mass terms as well as the $\sigma\Phi^2$ and $(\Phi^2)^2$ interactions for arbitrary ν^2.

Let us assume that the nonderivative part of the Lagrangian has two components: One transforms as a scalar ($\nu^2=1$) under commutation with the axial charge and the other transforms according to Eq. (11) with an arbitrary value of ν^2, which we shall call the transformation parameter.

From Eqs. (4), (6), and (21), we may write

$$c_T^\nu(\sigma\Phi^2)=k_1(1)c_1(\sigma)+k_1(\nu)c_\nu(\sigma)+k_2(1)c_1(\sigma^2)+k_2(\nu)c_\nu(\sigma^2)+k_3(1)c_1(\Phi^2)+k_3(\nu)c_\nu(\Phi^2)$$
$$=\frac{1}{2f^2}c_1(\sigma)+\frac{4-\nu^2}{6f^2}c_\nu(\sigma)+\frac{1}{f}[c_1(\sigma^2)+c_\nu(\sigma^2)]-\frac{2}{f}[c_1(\Phi^2)+c_\nu(\Phi^2)]\,, \tag{22a}$$

and similarly

$$c_T^\nu(\Phi^4)=\frac{7}{40f^3}c_1(\sigma)+\frac{7}{120f^3}(4-\nu^2)c_\nu(\sigma)+\frac{1}{4f^2}[c_1(\sigma^2)+c_\nu(\sigma^2)]-\frac{3}{5f^2}c_1(\Phi^2)-\frac{\nu^2+11}{20f^2}c_\nu(\Phi^2)\,, \tag{22b}$$

where c_ν refers to the contribution from the portion of the Lagrangian with transformation parameter ν^2. The total coefficients, c_T^ν, of the $\sigma\Phi^2$ and $(\Phi^2)^2$ vertices are given by Eqs. (22a) and (22b).

We shall impose three constraints upon Eq. (22):

$$c_T^\nu(\sigma)=c_1(\sigma)+c_\nu(\sigma)$$
$$=0\,, \tag{23a}$$

$$c_T^\nu(\Phi^2)=c_1(\Phi^2)+c_\nu(\Phi^2)$$
$$=-\tfrac{1}{2}\mu_\pi^{\,2}\,, \tag{23b}$$

and

$$c_T^\nu(\sigma^2)=c_1(\sigma^2)+c_\nu(\sigma^2)$$
$$=-\tfrac{1}{2}\mu_\sigma^{\,2}\,. \tag{23c}$$

Equation (23a) guarantees that σ mesons do not couple to the vacuum. Equations (23b) and (23c) give the pions and σ mesons their masses.

We then find that

$$c_T^\nu(\sigma\Phi^2)=\frac{\nu^2-1}{6f^2}c_1(\sigma)-\frac{\mu_\sigma^{\,2}}{2f}+\frac{\mu_\pi^{\,2}}{f}\,, \tag{24a}$$

$$c_T^\nu(\Phi^4) = \frac{7}{120 f^3}(\nu^2 - 1)c_1(\sigma) - \frac{\mu_\sigma^2}{8f^2} + \frac{\nu^2 - 1}{20 f^2}\, c_1(\Phi^2) + \frac{\nu^2 + 11}{40}\,\frac{\mu_\pi^2}{f^2}\,. \tag{24b}$$

Since we demand that in the limit $\mu_\pi^2 \to 0$ the total Lagrangian becomes chirally symmetric, we require that

$$\lim_{\mu_\pi^2 \to 0} c_\nu(\Phi^2) = 0\,.$$

Using Eq. (23b), we set

$$c_1(\Phi^2) = -\tfrac{1}{2}\alpha\mu_\pi^2\,, \tag{25}$$

where α must not have any singularities at $\mu_\pi^2 = 0$.[6] The constant of proportionality α is the fraction of pion mass contributed by the chirally symmetric nonderivative portion of the Lagrangian. We shall assume that α and μ_σ^2 are free and independent parameters of our model.

One may easily verify by use of Eqs. (3) and (20) that for any chirally invariant function of $\tilde\sigma$ and Φ^2, we get

$$c_1(\sigma) = 2f c_1(\Phi^2)\,. \tag{26}$$

Equations (23a), (25), and (26) imply that the pion mass must vanish in any Lagrangian model in which the total nonderivative portion of the Lagrangian is chirally invariant.

We use Eqs. (24) through (26) to produce the final results for the coupling coefficients of the $\sigma\Phi^2$ and $(\Phi^2)^2$ vertices:

$$c_T^\nu(\sigma\Phi^2) = \frac{[(1-\nu^2)\alpha + 6]\mu_\pi^2 - 3\mu_\sigma^2}{6f}\,, \tag{27a}$$

$$c_T^\nu(\Phi^4) = \left(\frac{(1-\nu^2)\alpha}{12} + \frac{\nu^2+11}{40}\right)\frac{\mu_\pi^2}{f^2} - \frac{\mu_\sigma^2}{8f^2}\,. \tag{27b}$$

Therefore, our total nucleon, pion, and σ Lagrangian $\mathcal{L}_T$ is given by

$$\mathcal{L}_T = \bar\Psi[i\gamma\cdot\partial - M_n - G(\sigma - i\vec\tau\cdot\vec\Phi\gamma_5)]\Psi + \tfrac{1}{2}[(\partial_\mu\Phi)^2 - \mu_\pi^2\Phi^2] + \tfrac{1}{2}[(\partial_\mu\sigma)^2 - \mu_\sigma^2\sigma^2] + c_T^\nu(\sigma\Phi^2)\sigma\Phi^2 + c_T^\nu(\Phi^4)(\Phi^2)^2\,, \tag{28}$$

where we have kept only those terms necessary for the calculation of the amplitudes for $\pi N \to \pi N$, $\pi\pi \to \pi\pi$, and $\pi N \to 2\pi N$.

V. DISCUSSION

Calculating π-π scattering in the tree approximation and keeping terms to order μ_π^2/μ_σ^2, we find

$$\mathfrak{M}_{abcd}(s,t,u) = \left[\frac{1}{f^2}\left[\tfrac{1}{5}(9-\nu^2)\mu_\pi^2 - s\right] - \left(\frac{s - \frac{1}{3}[(1-\nu^2)\alpha + 6]\mu_\pi^2}{f\mu_\sigma}\right)^2\right]\delta_{ab}\delta_{cd} + \text{crossing-symmetric permutations}\,. \tag{29}$$

One may similarly calculate the σ-exchange contribution to πN scattering. This scalar exchange contributes only to $A^{(+)}$:

$$A^{(+)} = -\frac{G}{f}\left[1 + \frac{t}{\mu_\sigma^2} - \tfrac{1}{3}[(1-\nu^2)\alpha + 6]\left(\frac{\mu_\pi}{\mu_\sigma}\right)^2 + O\left(\frac{\mu_\pi^4}{\mu_\sigma^4}\right)\right] \tag{30}$$

The isospin-even πN scattering length is therefore[3]

$$a^{(+)}(\pi N) = -\frac{G^2}{12\pi(M_n + \mu_\pi)}[(1-\nu^2)\alpha + 6]\frac{\mu_\pi^2}{\mu_\sigma^2}\,. \tag{31}$$

Using Eq. (29), one may relate ν^2 to ξ, the parameter[1] that measures the departure from the Weinberg and Gell-Mann–Lévy form of chiral symmetry breaking:

$$\xi = \tfrac{2}{5}(4 - \nu^2)\,. \tag{32}$$

For the Weinberg[7] and Gell-Mann–Lévy[2] models, $\nu = 2$ and thus $\xi = 0$. For the Schwinger form of symmetry breaking,[8] we have shown in an earlier work that $\xi = 1$.[1] This second value of ξ corresponds to a noninteger and irrational value of ν: $\nu = (\frac{3}{2})^{1/2}$. Note that for real ν, Eq. (32) implies $\xi \leq \frac{8}{5}$.

By analyzing pion production using a nonlinear pion Lagrangian, we have shown that $-1 \lesssim \xi \lesssim 0$.[9] If one requires the chiral symmetry-breaking portion of the Lagrangian to have integer values of ν, then the restriction upon the effective value of ξ suggests that there is a symmetry-breaking $\nu = 3$ component in addition to the customary $\nu = 2$ component. The nonlinear Lagrangian model which has only a $\nu = 3$ symmetry-breaking component is

$$\tfrac{1}{2}(\partial_\mu\Phi)^2 + \tfrac{1}{2}[\partial_\mu\tilde\sigma(\Phi^2)]^2 - \tfrac{1}{2}\mu_\pi^2\Phi^2\,,$$

where $\tilde\sigma = (f_\pi^2 - \Phi^2)^{1/2}$.[10]

One should note from Eqs. (29) and (30) that the form of the symmetry breaking will affect the σ terms except when $\alpha = 0$, i.e., when the pion mass comes entirely from the chiral-symmetry-breaking portion of the Lagrangian. In the Gell-Mann–

Lévy σ model, $\alpha = 1$.

Finally we should mention that the effects of symmetry breaking upon higher-order terms such as $\sigma^3, \sigma^2\Phi^2, \ldots$ have not yet been investigated. These higher-order terms contribute to $\pi N \to 3\pi N$ and other processes of high multiplicity.

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[5]*Higher Transcendental Functions* (Bateman Manuscript Project), edited by A. Erdélyi (McGraw-Hill, New York, 1953), Vol. I, p. 101.

[6]An example where α has a singularity at $\mu_\pi^2 = 0$ is the Lagrangian whose nonderivative part is

$$\frac{\mu_\sigma^2 - 3\mu_\pi^2}{4}(\tilde{\sigma}^2 + \Phi^2) + \frac{\mu_\pi^2 - \mu_\sigma^2}{4f}\tilde{\sigma}(\tilde{\sigma}^2 + \Phi^2) + \left(\frac{3\mu_\pi^2 + \mu_\sigma^2}{4}\right) f\tilde{\sigma}.$$

In this model

$$\alpha = \tfrac{1}{2}(3 - \mu_\sigma^2/\mu_\pi^2),$$

and therefore

$$\lim_{\mu_\pi^2 \to 0} c_2(\Phi^2) = -\tfrac{1}{4}\mu_\sigma^2 \neq 0.$$

In the limit $\mu_\sigma \to \infty$ this model cannot yield the conventional nonlinear $\nu = 2$ amplitudes.

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[10]Weinberg's solution for $\nu = 3$ (see Ref. 4) differs from ours by a canonical transformation of the pion field.

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$\pi\pi$ Scattering and the πN σ Term*

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The methods of an earlier work are modified so that the unitarity constraints on the $\pi\pi$ amplitude are better satisfied. The modification permits us to examine the extent to which $\pi\pi$ scattering affects the determination of the σ term in πN scattering.

Some renewal of interest has developed in the venerable problem of calculating $\pi\pi$ scattering from the general principles of analyticity, unitarity, and crossing symmetry. The impetus has come from current algebra. What the local-operator methods mean to an S-matrix approach is twofold. First, the Ward identities obtained from the current commutation relations provide equations relating matrix elements which are analytic in the invariant variables. As such they offer a vehicle for invoking unitarity. For a low-energy treatment this represents a distinct advantage over the use of partial-wave dispersion relations because only local analyticity needs to be employed. Secondly, the low-energy theorems[1] of current algebra are incorporated and effectively normalize the results of the analytic approach.

Schnitzer[2] has proposed methods for such a scheme, and an analysis of what can be predicted has been carried out.[3] The purpose of the present investigation is to show how a slight modification of what was done in Ref. 3 leads to considerable improvement on the extent to which unitarity is satisfied. This is achieved by making a minor alteration in the parametrization. Of course a more general treatment of the $\pi\pi$ problem admits other