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Study of the Presence Probability Distribution in Autointerference States

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An argument by one of us is first reproduced, demonstrating that the bare negation of the consequences imposed by the completeness hypothesis, concerning the particular formal fact that a quantum-mechanical free wave packet can spread out indefinitely, leads necessarily to predictions on the presence probability distribution in interference states that diverge from the corresponding quantum-mechanical predictions. It follows that, contrary to what seemed to be almost generally accepted, it is erroneous to admit systematically that the completeness problems are essentially unconnected to observation. This result is similar to one previously established by Bell and seems to support a suggestion by Ballentine, that any hidden-variable theory might necessarily lead to predictive divergences from quantum mechanics, if internal "pathologies" are not admitted. The above-mentioned argument entails a first minimal definition of a nonorthodox significance of a state vector. In the continuation of this work this definition is tentatively developed in a constructive way. This development is realized by a modification of de Broglie's model of a microsystem. This modification, imposed by an examination of de Broglie's double-solution theory in the light of our conclusion demonstrated before, is in fact rather profound; in particular, it forces the abandonment of the central idea of two solutions with distinct physical meanings, of a unique evolution law. However, the theory called into consideration by this modification may be capable of leading to objective proofs of the specific descriptive power of the essence of de Broglie's conception on the structure of microsystems. Indeed, inside the general nonorthodox framework tentatively outlined, a detailed definition of the presence probability distribution is elaborated that coincides with the quantum-mechanical one outside interference states, but diverges from it in interference states, in an experimentally testable way. This is a verifiable constructive embodiment of the conclusion only critically obtained at the beginning of this work. A final verdict on the value of the specific basic conception of the outlined theory and of the new perspectives it brings forth can be obtained only by a further systematic theoretical elaboration and by the corresponding experimental tests.

INTRODUCTION

The old and persistent controversy on the completeness of the quantum-mechanical formalism is surreptitiously entering a new phase. The slow but continuous evolution that kept modifying the accepted subject of this debate now finally touches the very nature assigned to the problem. Wigner's demonstration[1] closed the period of formulation of "paradoxes" and of "answers" to these by clearly establishing the price ineluctably required for a coherent integration of the completeness hypothesis (duality of the evolution laws[1] or a solipsistic subjectivity of description[2]), whereas various constructive attempts tend to demonstrate the possibility also of a coherent development of the incompleteness hypothesis, in more "natural" epistemological conditions. Meanwhile, certain results obtained recently impose the conclusion that the completeness problem, in certain particular forms at least, if fully analyzed, can be stated in terms of testable divergences from the predictions of quantum mechanics (Q.M.), inside the specific realm of validity of this theory (atomic dimensions and Newtonian energies). The essentially "metaphysical," "interpretative" nature of the questions about the physical significance of the quantum-mechanical formalism has then been but a long-lasting semblance.

Indeed, in 1964, Bell demonstrated[3] that if hidden variables of an arbitrary form are supposed to be associable with spin states, then it follows that a certain specified spin experiment necessarily yields results that do not coincide in general with the corresponding quantum-mechanical predictions if nonlocality is excluded. In spite of the certainly fallacious[4,5] character of von Neumann's well-known general demonstration, this result reestablishes after all von Neumann's conclusion, for the particular case of hidden parameters for spin states (such parameters are predictively incoherent with Q.M.). Moreover, Bell's result can also be envisaged, as Ballentine suggests,[6] as a symptom of the possible, more general fact that a theory of microsystems that introduces any sort of descriptive elements undefined in Q.M. necessarily leads to specific predictions, if internal "pathologies" are rejected (von Neumann's conclusion integrally).

Likewise, one of us showed[7] that within the de Broglie–Bohm interpretation of Q.M., if the *a priori* acceptance of the general validity of the quantum-mechanical distribution postulates is ignored, it is possible to establish deductively the conclusion that a certain experimental procedure applied to an interference state must lead to the registration of a non-quantum-mechanical "guidance" distribution of values for the momentum. This conclusion stands in obvious contradiction with the observational hiddenness asserted in the de Broglie–Bohm interpretation for the distributions of "guidance" values of the momentum, as well as with the acceptance, in this theory, of the general validity of the quantum-mechanical distribution postulates (in the realm of atomic dimensions and nonrelativistic energies). But in the present context the significant feature of the above-mentioned result consists in its possible analogy with Bell's result for spin variables: The definition of certain non-quantum-mechanical parameters, here values of the momentum, is found to lead to testable divergencies with Q.M.

The present work is a progressive development, first critical and then constructive, of the line of thought sketched above.

The first section of this work is purely critical. The conclusion reached in it is the following: The negation (by postulation of appropriate, minimally specified, non-quantum-mechanical parameters) of the consequences imposed by the completeness hypothesis concerning the particular fact that free wave packets can spread out indefinitely, necessarily leads, concerning the presence probability distribution in interference states, to predictions which are different from the corresponding quantum-mechanical predictions.

An experimental procedure capable of deciding between the two possibilities defined is indicated in the second section.

Whereas the first and second sections of this work are purely analytical and take the quantum-mechanical formalism as the object of analysis, the subsequent sections are of a rather opposite nature: On the basis of the established conclusion, they are tentatively aimed, outside Q.M., towards a constructive solution for the completeness problem in terms that are not essentially statistical and with an explicit *a priori* allowance for predictive specificities with respect to Q.M. (Such an allowance is the natural constructive prolongation of the possibility of observational impact of the completeness problem.) This aim is far from being fulfilled here. Yet we believe that a certain amount of progress has been achieved in the stated direction:

The nonorthodox theory first conceived by de Broglie,[8] then rediscovered by Bohm,[9] and since systematically developed by de Broglie and his school[10-18] under the name of double-solution (D.S.) theory, seems to us to be the best structured among all the causal descriptions of microsystems proposed up to now. For reasons of coherence with its basic conception of physical reality and of a satisfactory description of it, the D.S. theory should yield an interpretation of the possibility of unlimited spreading of a free wave packet that is acceptable to those who reject the completeness hypothesis. Yet it certainly fails to do this in a coherent way. Indeed, though it negates globally the completeness hypothesis, the D.S. theory admits nevertheless *a priori* the *general* validity of the quantum-mechanical distribution postulates. Now, as mentioned, we demonstrated that the quantum-mechanical postulate on the presence probability distribution stands in contradiction with the negation of the specific implications of the completeness postulate concerning the particular spreading-wave problem. It seemed then efficient to us, in order to orient the constructive effort we had assigned to ourselves, to start with a detailed research of the obstacles that obstruct in D.S. theory an exhaustive and coherent solution of the spreading-wave problem. We identified these obstacles, and thereby also the modifications required to remove them. These modifications are in fact very profound. Besides a clear dropping of the *a priori* exigency of a rigorous predictive identity with Q.M., they consist also of the attribution of a more radically statistical significance to a state vector. This entails the dissolution of a central idea in de Broglie's theory, the idea of a double solution of a unique evolution law (to which the title alludes). Thereby, however, there comes naturally into consideration a new theory which may prove capable of liberating finally the descriptive power of de Broglie's conception of microsystems, masked and immobilized until now by the ramification of artificial restrictions issued from the *a priori* condition of a rigorous predictive identity with Q.M.

In the final, purely constructive section of this work, we propose a first tentative outline of this new theory of microsystems. Inside the general conceptual frame outlined we have been able to elaborate a detailed nonorthodox definition of the presence probability distribution. This definition coincides with the quantum-mechanical one outside interference states, but diverges from it in interference states in a testable way. (Thus the conclusion of the first part obtains a constructive embodiment.) The precise and verifiable character of the particular result stated above makes possible a first evaluation of the specific basic conception of the outlined theory and of the various new perspectives it creates tentatively. Of course, only further systematic theoretical elaboration and experimental tests could lead to a final conclusion.

I. THE QUANTUM-MECHANICAL DEFINITION OF THE PRESENCE PROBABILITY DISTRIBUTION IN AUTOINTERFERENCE STATES, THE SPREADING OF WAVE PACKETS, AND THE OBJECTIVITY OF THE COMPLETENESS PROBLEM

A. Introductory

Let us consider one single microsystem that undergoes an evolution during which an autointerference evolves. The descriptive element by which quantum mechanics (Q.M.) represents such an evolution is the concept of a normalized wave packet. Let us examine the quantum-mechanical definition of the distribution of the presence probability, obtained by use of this descriptive element.

B. The Example of a Step Potential

1. *The Quantum-Mechanical Definition of the Presence Probability*

We consider first the classical one-dimensional case of one microsystem, sent from left to right towards a step potential. Q.M. represents the evolution of this microsystem by means of a spatially limited wave packet ψ_1, at first incident on the wall, which is then partly reflected back in a packet ψ_2, and partly transmitted. At any moment $t_i \in \Delta t$ contained inside the duration Δt of the process of reflection, there exists a non-null domain $\Delta r(t_i)$ such that at a point $r \in \Delta r(t_i)$, the state vector has the additive form

$$|\psi(\vec{r}, t_i)\rangle = |\psi_1(\vec{r}, t_i) + \psi_2(\vec{r}, t_i)\rangle ,$$

and the presence probability of the microsystem S is

$$\Pi(\vec{r}, t_i) = |\psi_1(\vec{r}, t_i) + \psi_2(\vec{r}, t_i)|^2 ,$$

whereas at a point $r \in \Delta r(t_i)$, the presence probability of S is either $\Pi(\vec{r}, t_i) = |\psi_1(\vec{r}, t_i)|^2$ if it is ψ_1 that is non-null at $\vec{r}$ at the moment t_i, or $\Pi(\vec{r}, t_i) = |\psi_2(\vec{r}, t_i)|^2$ if ψ_2 is non-null at $\vec{r}$ at the moment t_i. At a moment $t \in \Delta t$, the state vector is everywhere either $|\psi_1\rangle$ or $|\psi_2\rangle$ and the presence probability of S is $\Pi(\vec{r},t) = |\psi_1(\vec{r},t)|^2$ or $\Pi(\vec{r},t) = |\psi_2(\vec{r},t)|^2$ depending on whether t occurs before or after the process of reflection.

2. *The Spreading of Wave Packets and the Completeness Problem: Demonstration of the Incompatibility of the Quantum-Mechanical Formalism, with a Particular Negation of the Completeness Hypothesis*

Thus Q.M. postulates, at each moment $t_i \in \Delta t$, the interference form $\Pi(\vec{r}, t_i) = |\psi_1(\vec{r}, t_i) + \psi_2(\vec{r}, t_i)|^2$ of the presence probability inside the whole zone of superposition $\Delta r(t_i)$ of $\psi_1(\vec{r}, t_i)$ and $\psi_2(\vec{r}, t_i)$. We shall now reproduce a demonstration by one of us[19] of the incompatibility of this quantum-mechanical postulate with the negation of the consequences which the completeness hypothesis implies concerning the spreading of a wave packet: As soon as one drops the implications of the completeness hypothesis concerning the significance of the formal fact that a free wave packet spreads out, the interference form of $\Pi(\vec{r},t_i)$ can no longer be coherently postulated inside the *whole* zone of superposition of $\psi_1(\vec{r},t_i)$ and $\psi_2(\vec{r},t_i)$ (the superposition zone and interference zone must be distinguished). Since the real dimension of the interference zone is recordable, this will permit transposition of the completeness problem into an experimentally solvable alternative.

The quantum-mechanical formalism implies the following consequence concerning the dimensions of the interference zone: The initial packet $|\psi_1\rangle$ will always contain more than one monochromatic component, so that it will spread out. Its dimension at the moment at which it reaches the step – hence the maximal dimension of the successive zones $\Delta r(t_i)$ (t_i varies inside Δt) where the incident packet ψ_1 and the reflected packet ψ_2 superpose – will then be larger, the bigger the distance is between the source and the step (with every other condition fixed). Since this distance can be increased indefinitely, it follows within Q.M. that the maximal spatial extension of the ψ_1 and ψ_2 superposition zones $\Delta r(t_i)$, and hence of the zones where the presence probability distribution has the interference form, can also be increased indefinitely.

Let us now confront successively this consequence of the quantum-mechanical formalism, first with the acceptance and then with the negation of the completeness hypothesis.

(a) The implications of the completeness hypothesis are the following:

(1) As is well known, according to the completeness hypothesis – specific to any "orthodox" interpretation of the quantum-mechanical formalism – a state vector $|\psi\rangle$ describes maximally every individual microsystem to which it is associated. This hypothesis can be incorporated into a rich variety of shades of orthodoxy (d'Espagnat gives a detailed and subtle characterization of these[20,21]), differing from one another by the metaphysical postulates adopted concerning the existence and the nature of physical reality, and by the characteristics subjectively required for a "satisfactory" description of this reality. But as soon as the bare existence of some reality is accepted outside the moments of observation (it is hardly conceivable that a phys-

icist will not do this, no matter how algorithmical a significance he assigns to a state vector), whatever be the nature and properties attributed to this reality, the completeness hypothesis entails that the possibility of an indefinite spreading of a wave packet is an expression of the fact that the reality corresponding to one single microsystem is somehow able to spread out indefinitely. (A criterion for the "singleness" of a microsystem can be found in the formal circumstance that Q.M. excludes, at any given moment, more than one localized observable effect on registering devices.)

(2) If, in particular, an interference evolution causes the spreading wave packet of one microsystem to divide into parts that superpose, the completeness hypothesis entails that the possibility of an indefinite increase of the superposition zone of these parts is an expression of the fact that the domain which one single microsystem is somehow able to occupy entirely by an interaction with itself can be arbitrarily increased.

(3) The following sequence of assertions has then to be accepted concerning the process that leads to an observable interference pattern: It is a fact that an interference pattern can be obtained only by use of a whole statistical ensemble of microsystems S of the same type, subjected to practically identical macroscopic conditions. But these microsystems can constitute a very low-intensity beam, so that they shall certainly act separately. Now, according to (2), at any moment $t_i \in \Delta t$ of its own evolution, every one, separately, of the microsystems S that contribute to the interference pattern interacts with itself at each point of the whole $\Delta r(t_i)$ zone in which the parts $\psi_1(t_i)$ and $\psi_2(t_i)$ of its state-vector $|\psi(t_i)\rangle$ superpose at t_i. This autointeraction somehow works on the probability that a localized observable effect be registered at a given point of $\Delta r(t_i)$, if what is called a "position-recording" takes place, thereby causing the mentioned probability to take on the interference form. It is a fact that this spatially extended self-action on the probability for registering a localized effect manifests itself observably only at one point in $\Delta r(t_i)$, for one single microsystem S contributing to the interference pattern so that its spatial structure remains unapparent after one single registration. (As is known, in order to obtain a syntactically coherent description of microphenomena, this last fact has to be described either by help of the "reduction" concept – correlated to the acceptance of two distinct evolution laws for a microsystem[1] – or by an indefinite subjective splitting of universes[2]; no other coherent solution has been proposed so far. But the particular conceptual structure in which an orthodox physicist chooses to integrate this fact is irrelevant to our present argument; the fact alone has to be taken into account, as appears just below.) However, even though the above-mentioned modifying self-action of the probability of localization results in an observable *manifestation* at one single point of $\Delta r(t_i)$ for one single microsystem S, and therefore becomes perceptible only for a statistical ensemble of replicas of S, the completeness hypothesis implies ineluctably that this self-action *itself* is realized for every individual microsystem S throughout the whole superposition domain $\Delta r(t_i)$ of $\psi_1(t_i)$ and $\psi_2(t_i)$, no matter how much $\Delta r(t_i)$ has been increased by increasing the distance between the source of $|\psi_1\rangle$ and the reflecting step.

If this implication of the completeness hypothesis is accepted, then there effectively follows the quantum-mechanical prediction of the interference form $\Pi(\vec{r}, t_i) = |\psi_1(\vec{r}, t_i) + \psi_2(\vec{r}, t_i)|^2$ of the presence probability distribution throughout the whole, indefinitely increasable zone $\Delta r(t_i)$ – but only if this implication is accepted. Indeed,

(b) let us now negate tentatively the particular implication of the completeness hypothesis brought into evidence above. We accept instead the following noncompleteness (n.c.) hypothesis related to the significance of a spreading wave packet.

[H(n.c.)]: The reality corresponding to one single microsystem, whatever be its nature and properties, or any part of this reality, cannot cover an indefinitely spreading spatial domain.

(Obviously this negation of a particular implication of the completeness hypothesis also touches the completeness hypothesis as a whole.)

To begin with, we shall transpose this *in abstracto* negation into more specified affirmative terms:

The state vector associated to one free microsystem can spread out indefinitely. It follows from [H(n.c.)] that the domain covered by the state vector and that covered by the reality corresponding to one single microsystem are not in general coextensive. More specifically, it follows from [H(n.c.)] that

$\{H_1\}$: The domain $\mathfrak{D}_s(t)$ covered at a moment t by the physical reality corresponding to one single microsystem S is in general distinct from the domain $\mathfrak{D}_\psi(t)$ covered at t by the state vector $|\psi\rangle$ associated with S, being less extended than $\mathfrak{D}_\psi(t)$:

$$\mathfrak{D}_s(t) \subseteq \mathfrak{D}_\psi(t) .$$

It also follows from [H(n.c.)] that

$\{H_2\}$: $\mathfrak{D}_s(t)$ is not subjected to the unlimited Schrödinger expansion.

But the dimensions of $\mathfrak{D}_s(t)$ are not defined inside Q.M. Consequently, they constitute a particular type of "hidden" (to the formalism of Q.M.) parameters (corresponding, *not* to a *dynamical* quantity, but to hypothetical *geometrical* individual properties of a microsystem). The bare negation H(n.c.) of the specific implications which the completeness hypothesis imposes concerning the particular problem of the significance of the spreading of free wave packets introduces these parameters *necessarily*. The assumption of their minimally characterized existence, as it is expressed by the concept $\mathfrak{D}_s(t) \neq \mathfrak{D}_\psi(t)$ contained in H_1 and H_2, is then just the transposition in affirmative terms that we wanted to obtain for our tentative negation of the implications of the completeness hypothesis concerning the particular spreading-wave problem.

Let us now make explicit the consequences of the affirmative logical equivalent $(H_1 + H_2)$ of our tentative negation H(n.c.). They are the following:

(1) Let us first emphasize again the fact that whatever significance one attributes to the spreading of a wave packet (an individual one, or not), any verification of the quantum-mechanical postulate on the presence probability distribution requires a whole statistical ensemble of microsystems S of the same type, submitted to identical macroscopic conditions. Now, the association of H_1 with the experimental fact that the probability for registering at t the presence of an S is in general non-null at *every* point of the domain $\mathfrak{D}_{\psi_s}(t)$, which is submitted to the Schrödinger spreading, suppresses the possibility of assigning to this spreading a purely individual significance: It entails that the Schrödinger spreading of $\mathfrak{D}_{\psi_1}(t)$ is a formal expression of the fact that, whereas the physical reality corresponding to one given S among all those used for verifying the presence probability definition is located, at a time t after its emission, in a limited region $\mathfrak{D}_s(t)$ inside $\mathfrak{D}_{\psi_1}(t)$ (H_1), the reality corresponding to another one of the considered S is located, at a time t after *its* emission, in another (in general) limited region $\mathfrak{D}_s(t)$ inside the $\mathfrak{D}_{\psi_1}(t)$ of its own $|\psi_1\rangle$, which is superposable in shape to the $\mathfrak{D}_{\psi_1}(t)$ of the preceding experiment. Only the maximal distance possible at t between such two distinct locations of two distinct $\mathfrak{D}_s(t)$ represented simultaneously in one single $\mathfrak{D}_{\psi_1}(t)$, with respect to a fictive common origin for the different t, can now be conceived to increase indefinitely with time, but no longer the domain $\mathfrak{D}_s(t)$ covered by one single microsystem S (H_2). (It can be mentioned that this possibility of an indefinite increase of the maximal distance defined above can be conceived as a consequence of some difference in the initial states of the two corresponding microsystems S of the same type and submitted to identical macroscopic conditions. Also, the view described enables one to deal conceptually with the localized character of a position registration, without being forced to introduce either the notion of "reduction," or indefinitely splitting universes. But neither of these two remarks is part of our present argument.)

The statistical significance of the spreading of a free wave packet entailed by $(H_1 + H_2)$ implies that the domain $\mathfrak{D}_\psi(t)$ covered by a state vector $|\psi\rangle$ at an arbitrary moment t of its evolution is the union (with the meaning the theory of ensembles ascribes to this term) of all the domains $\mathfrak{D}_s(t)$ covered at t by all the possible locations of a microsystem S describable by $|\psi\rangle$. Thus we can write

$$\{(H_1 + H_2)'\}: \ \mathfrak{D}_\psi(t) = \bigcup \mathfrak{D}_s(t)\,.$$

Thus, in rigorous consequence of the mere negation $(H_1 + H_2)$ of the completeness hypothesis, $\{(H_1 + H_2)'\}$ defines certain purely geometrical features of a nonorthodox significance of a state vector. But we emphasize that $(H_1 + H_2)$ yields *no indication whatsoever concerning the physical nature of the reality corresponding to a microsystem S which is located inside a spatial domain* $\mathfrak{D}_s(t)$. Only a further constructive development of the bare negation $(H_1 + H_2)$ of the completeness hypothesis can introduce by independent postulates assumptions concerning the physical reality located in a $\mathfrak{D}_s(t)$.

Let us now continue our argument.

(2) If in particular an interference evolution causes the wave packet to divide into parts ψ_1 and ψ_2 that superpose during a certain period Δt, then the consequence $\{(H_1 + H_2)'\}$ of $(H_1 + H_2)$ implies that the possibility of an unlimited increasing of the superposition zone of these parts is due to the fact that the individual domains $\mathfrak{D}_s(t_i)$ $(t_i \in \Delta t)$, the simultaneous representation of which is covered at $t_i \in \Delta t$ by $\mathfrak{D}_{\psi_1 + \psi_2}(t_i)$, *do not arrive at the reflecting step at the same time (nor in the same region of the step, in a two- or three-dimensional treatment).* But *whatever be the moment (or the region) at which one given* $\mathfrak{D}_s$ *reaches the wall, when its turn comes to be reflected, the reality contained in this one* $\mathfrak{D}_s$ *superposes on itself over a spatial domain that is adjacent to the step and that obviously cannot exceed in extension this* $\mathfrak{D}_s$ *itself.* Now, *according to* H_2 *a* $\mathfrak{D}_s$ *cannot spread out indefinitely*. Consequently

(3) the following sequence of assertions has to be admitted concerning the process that leads to an observable interference pattern:

The superposition with itself of the reality corresponding to one microsystem S causes some specific internal evolution of S that influences the result of the interaction of S with an apparatus that is able to record what is named the "position of S." This influence is perceivable on the statistical level of observation by the fact that around a point where it exists, the distribution of impacts registered by a position recorder acquires the well-known interference form. Since the individual processes that are (by summation of their successively recorded punctual manifestations) the source of this specific statistical distribution of the impacts cover each time only a domain which cannot exceed one $\mathcal{D}_s$ and which is adjacent to the reflecting step, the whole interference distribution of impacts obviously cannot cover *more* than a domain maximized by $\mathcal{D}_s$ (and adjacent to the reflecting step). Since the $\mathcal{D}_s$ do not spread out as the superposition zone $\Delta r(t_i)$ of the parts $\psi_1(t_i)$ and $\psi_2(t_i)$ of the state vector $|\psi(t_i)\rangle$, it follows that *when the distance from source to wall is increased, contrary to the quantum-mechanical prediction, the domain in which the interference form of the presence probability distribution is realized does not spread out like the superposition zone $\Delta r(t_i)$ of ψ_1 and ψ_2*. This is the consequence of H(n.c.) that we wanted to demonstrate. It cannot be avoided, we believe, unless some *ad hoc* repostulation of a Schrödinger spreading is made for $\mathcal{D}_s(t)$, by a more or less explicitly circular reasoning.

C. The Young-Interference Example

In practice, the interference by reflection on a step potential is rather inappropriate for an experimental examination. Interferences of progressive waves propagating in the same sense (along different directions) are much easier to study. The preceding demonstration can be immediately transposed to such examples. Let us consider for instance a Young interference (see Fig. 1).

In this case the state vector keeps constantly the additive form $|\psi\rangle = |\psi_1 + \psi_2\rangle$ in the whole right-hand half-space, since the ψ_1 and ψ_2 packets propagate simultaneously for one microsystem. At a given moment, in any point of the superposition domain of ψ_1 and ψ_2, Q.M. predicts the interference form of the presence probability, $\Pi(\vec{r}) = |\psi_1(\vec{r}) + \psi_2(\vec{r})|^2$, whereas in a point outside of this superposition domain that is covered only successively by the terms ψ_1 and ψ_2 the presence probability predicted by Q.M. is $\Pi(\vec{r}) = |\psi_1|^2 + |\psi_2|^2$.

The argument developed for the step-potential example concerning the compared dimensions of the superposition zone of ψ_1 and ψ_2 and of the zone where the interference form of $\Pi(r)$ is realized can be entirely transposed to the Young-interference example.

D. Interference of Plane-Wave Packets

Finally, analogous considerations can be developed also for an interference of practically plane-wave packets, like that represented in Fig. 2.

E. Conclusion

Thus the quantum-mechanical postulate for the presence probability distribution contains, built into it, consequences of the completeness hypothesis. As soon as the *general* validity of the quantum-mechanical formalism is accepted, no choice is left concerning the completeness hypothesis; this hypothesis is already accepted also, implicitly. Its negation, then, leads to non-quantum-mechanical predictions. *Thus the completeness problem is an essentially objective one;* it is not a metaphysical, subjective, purely interpretive problem that one is free to refuse to consider, or for which a solution can be freely chosen independently of the acceptance of the quantum-mechanical formalism.

II. AN EXPERIMENTAL PROCEDURE CAPABLE OF DETERMINING THE DEFINED ALTERNATIVE

This very short section is reserved for a more specific description of the crucial experiment already discussed throughout the preceding section. This experiment is studied elsewhere in all technical details.[22,23]

A very low-intensity source $\mathcal{S}$ sends, one by one,

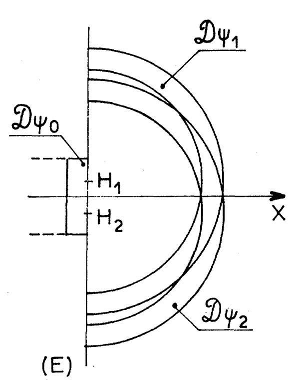


FIG. 1. Young interference.

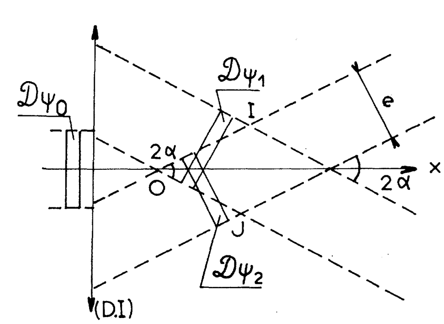


FIG. 2. Interference of two practically plane-wave packets, obtained by use of a dividing interferometer.

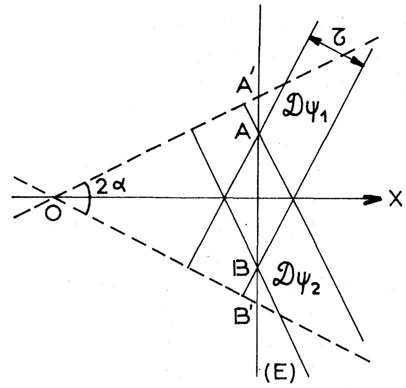


FIG. 3. The domain of superposition of the two packets ψ_1 and ψ_2 is limited, on the screen E, by the points A and B, if $AB < A'B'$.

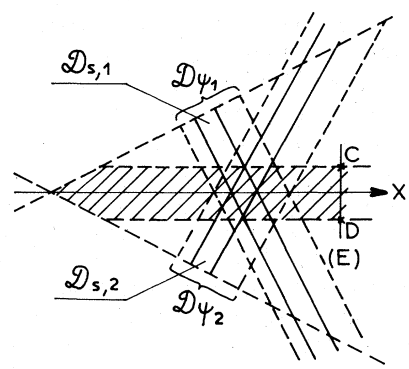


FIG. 4. The domain of superposition of the two parts $\mathfrak{D}_{s,1}$ and $\mathfrak{D}_{s,2}$ of the domain $\mathfrak{D}_s$.

electrons representable by a sufficiently monokinetic wave packet $|\psi_0\rangle$ towards an interferometer with divisor of the front of the wave packet D.I. (Fig. 2). Such an interferometer was first described by Faget.[24] The two packets ψ_1 and ψ_2, obtained from $|\psi_0\rangle$, superpose in a domain the intersection of which with the sensitive screen E is limited by the points A, B (Fig. 3).

If τ represents the depth of the packets ψ_1 and ψ_2, then the relation $AB = \tau/\sin\alpha$ is obviously fulfilled. Now, according to the Schrödinger law of evolution, τ is an increasing function of the distance $\mathcal{S}$ – D.I. (in free propagation the length of the front of the wave packet $|\psi_0\rangle$ is equally an increasing function of $\mathcal{S}$ – D.I., but the interferometric device contains diaphragms that recreate for ψ_1 and ψ_2 a value of the length of the front of the wave independent of $\mathcal{S}$ – D.I., whereas they do not affect the depth τ inherited by ψ_1 and ψ_2 from $|\psi_0\rangle$). Thus, if $\mathcal{S}$ – D.I. is increased, τ increases for $|\psi_0\rangle$ as well as for ψ_1 and ψ_2, and, according to Q.M., the length AB of the zone on the screen where an interference distribution of the position is recorded should also increase correspondingly (a calculation has been made[23]). But according to the statistical significance of $|\psi_0\rangle$, ψ_1, and ψ_2 entailed by H(n.c.) the superposition domain of ψ_1 and ψ_2 has to be distinguished from the domain of superposition of the parts $\mathfrak{D}_{s,1}$ and $\mathfrak{D}_{s,2}$ of the domain $\mathfrak{D}_s$ covered by the physical reality corresponding to one electron (Fig. 4), and the zone on the screen where a periodic distribution of the position is recorded does *not* change with the distance $\mathcal{S}$ – D.I. [at least, it does not change as the intersection of E with the superposition zone of ψ_1 and ψ_2 (H_2)]. The small values of α (see Fig. 3) required for the observability of the interferences imply difficulties that can be overcome.[22]

We add two remarks:

(a) The mere fact that the depth τ of the wave packet associated with a free electron is often mentioned as a measurable characteristic of an electron (the value estimated by some authors has an order of magnitude of 1 μ) stands in contradiction with the unlimited spreading entailed for τ by the Schrödinger evolution. The τ characteristic of an electron seems to be much more probably a dimension of the individual non-quantum-mechanical domain $\mathfrak{D}_s(t)$ necessarily introduced by H(n.c.), rather than a dimension of the quantum-mechanical domain $\mathfrak{D}_\psi(t)$.

(b) Let us consider the following question:

At a moment $t_i \in \Delta t$, in the example of a step potential, how large exactly is the superposition zone $\Delta r(t_i)$ of ψ_1 and ψ_2 where, according to Q.M., the expression $\Pi(\vec{r},t_i) = |\psi_1(\vec{r},t_i) + \psi_2(\vec{r},t_i)|^2$ of the presence probability of S is valid?

In principle Q.M. yields a perfectly definite answer to this question: The dimensions of $\Delta r(t_i)$ are calculable from the initial geometrical and dynamical structure of the incident packet ψ_1 – determined by the macroscopic and microscopic structure and state of the source of ψ_1 at the moment of emission – and from the distance from the source of ψ_1 to the reflecting step; if all these data are known, the time-dependent Schrödinger equation describes ψ_1 at the moment at which it reaches the wall, as well as the progressive transformation of ψ_1 into ψ_2 by the reflection process, and, in particular, it determines for any $t_i \in \Delta t$ the dimension of the zone $\Delta r(t_i)$ where it has to be written $\Pi(\vec{r},t_i) = |\psi_1(\vec{r},t_i) + \psi_2(\vec{r},t_i)|^2$. The difficulty is that in practice these necessary data are never known rigorously, so that the zone $\Delta r(t_i)$ can be experimentally identified, but not predicted by calculation. Therefore, on the theoretical level, as soon as the state vector $|\psi\rangle$ is written as a sum normalized to unity, $|\psi\rangle = |\sum_j \psi_j\rangle$ of solutions of the time-evolution equation, the problem arises of finding criteria for deciding at any given moment on the spatial extension of the interference form of the presence probability distribution. Feynman and Hibbs,[25] for instance, consider that if *in principle* one could have determined in which of the states ψ_j the system was, then the amplitudes of the ψ_j do not interfere and the probabilities add: $\Pi(r) = \sum_j |\psi_j(r)|^2$. Otherwise, the amplitudes add and $\pi(r) = |\psi(r)|^2$ so that there is interference. Schulman[26] gives a more formalized criterion.

But any such *a posteriori* criterion, as long as it is used in an absolute, noncomparative way, is obviously fundamentally incapable of bringing into evidence a distinction between an abstract superposition of domains $\mathfrak{D}_\psi(t)$ and a physical superposition of domains $\mathfrak{D}_s(t)$, if such a distinction really exists. To bring into evidence such a distinction, *at least two different experiments are necessary, corresponding to different distances from source to screen.*

III. EXAMINATION OF THE DEFINITION OF THE PRESENCE PROBABILITY DISTRIBUTION IN INTERFERENCE STATES IN THE DE BROGLIE-BOHM INTERPRETATION OF QUANTUM MECHANICS

A. Introductory

The conclusion established in Sec. I is void of any hints concerning the nature and properties of the individual physical reality located in one domain $\mathfrak{D}_s(t)$ assigned to one single microsystem S in consequence of H(n.c.). For a further constructive development of this conclusion, such hints may be very fruitful. It seems to us that the model for a microsystem introduced by de Broglie in his thesis[27] is particularly suggestive in this respect. But, as we pointed out in the Introduction, our preceding conclusion implies that this model has been integrated into a theoretical structure where the completeness problem certainly does not obtain an integral and coherent solution. Therefore, before trying a nonorthodox constructive development of our conclusion, we shall first analyze de Broglie's theory, with the explicit aim of identifying the features, in his model for a microsystem, that hinder a coherent and integral constructive solution for the completeness problem.

B. History and General Framework

In 1927, immediately after his thesis, de Broglie outlined a deterministic description of microphenomena,[8] but in a form still too schematic to be convincing. The development of the orthodox description continued therefore its inertial course despite the well-known "paradoxes" and the various interpretative difficulties formulated by Einstein, Schrödinger and many others (Solvay Conference, 1927). In 1952, Bohm[9] expressed in a more detailed fashion an essentially equivalent view, and obtained a certain attention. Since 1956, de Broglie[10-14] and his school[15-18] developed systematically this view under the name of double-solution (D.S.) theory. This theory, we believe, already represents a very remarkable conceptual construct – the best structured in fact among all the nonorthodox descriptions proposed up to now, and particularly rich in suggestive views. It possesses the conceptual capacity – though not yet realized in all directions – to dissolve most of the features which, in the orthodox description, appear as unsatisfactory to a constantly increasing number of physicists. [An image of this growing dissatisfaction concerning the orthodox description of microphenomena can be acquired by reading the following works: Bohm,[9] Bunge,[28,29] Destouches,[30,31] Renninger,[32] Wakita,[33] Lande,[34-36] Ludwig,[37] Margenau,[38,39] D'Espagnat,[20,21] Bell,[5] Bub,[40] Bohm and Bub,[41] Scott,[42] Lamb,[43] Robinson,[44] Tutsch,[45] Ballentine,[6] Pearle,[46] Popper,[47] Mugur-Schächter,[4] Evrard,[22] and many others.]

Let us recall concisely the essential contents of D.S.:

We consider first only the Schrödinger time-evolution equation for a zero-spin microsystem with nonrelativistic energy.

1. *The Double-Solution Principle*

For every regular solution $\psi = a_\psi e^{i\varphi_\psi/\hbar}$ of this equation, the *double-solution principle* asserts the existence of a second solution $u = a_u e^{i\varphi_u/\hbar}$, having the same phase

$$\varphi_u \equiv \varphi_\psi \tag{1}$$

and an amplitude a_u containing a point singularity (in general mobile) arbitrarily located inside u. This second solution is considered to describe a physically existing corpuscular field, the mobile point singularity of this field representing the corpuscular part (not "aspect") of the microsystem, whereas its regular part represents the wavelike part (not "aspect") of the microsystem. Each one of these two parts of the microsystem exists permanently and is the source of the corresponding "aspects" or "manifestations" of the microsystem, wavelike or corpuscular, of which it is spoken in the orthodox quantum theory, and to which the principle of complementarity refers.

2. *The Guidance Relation*

The double-solution principle [in particular, the identity (1)] implies the following expression for the velocity of the singularity representing the corpuscular part ($\mathfrak{C}$) of the corpuscular wave u:

$$\vec{v}_{\mathfrak{C}} = -c^2 \frac{\vec{\nabla}\varphi_\psi + (\epsilon/c)\vec{A}}{\partial\varphi_\psi/\partial t - \epsilon V} \tag{2}$$

(where $\vec{A}$ and V are the exterior vector and scalar potentials, ϵ is the electronic charge, and c is the velocity of light).

If $V=0$ and $\vec{A}=0$, this implies for the momentum of the corpuscular part

$$\vec{p}_{\mathfrak{C}} = -(\vec{\nabla}\varphi_\psi)_{M_{\mathfrak{C}}}, \tag{2'}$$

where $M_{\mathfrak{C}}$ indicates the point at which is located the singularity representing the corpuscular part. The values $\vec{p}_{\mathfrak{C}}$ do not belong in general to the quantum-mechanical spectrum of the momentum. In an interference state (in particular, in a stationary interference state) these values $\vec{p}_{\mathfrak{C}}$ of the momentum are distributed otherwise than the quantum-mechanical eigenvalues of $\vec{p}$, and are considered in D.S. as hidden, unobservable, though really existing before a $\vec{p}$ measurement.

3. *The Quantum Potential*

The following equality is also demonstrated:

$$-\frac{\hbar^2}{2m}\left(\frac{\Delta a_\psi}{a_\psi}\right)_{M_{\mathfrak{C}}} = -\frac{\hbar^2}{2m}\left(\frac{\Delta a_u}{a_u}\right)_{M_{\mathfrak{C}}} = Q, \tag{3}$$

where the expressions in parentheses are calculated at the point $M_{\mathfrak{C}}$. Q is shown to act on the corpuscular part like an additional potential, the de Broglie–Bohm "quantum potential," that can be nonzero even if $V=0$ and $\vec{A}=0$ (this happens if obstacles produce interference or diffraction); the "quantum forces" $(-\vec{\nabla}Q)_{M_{\mathfrak{C}}}$ derived from Q change, when they are non-null, the momentum $\vec{p}_{\mathfrak{C}}$ of the corpuscular part of the microsystem.

4. *The Eigenvalues Postulate*

It is also shown that if "fluctuations of a subquantum medium"[48] are assumed, it can be coherently accepted that $\Pi(\vec{r}) = |\psi|^2$ represents the distribution of the presence probability of the corpuscular part of a microsystem described in Q.M. by ψ. More generally, the quantum-mechanical eigenvalues-distribution postulate $\Pi(q_i) = |\langle\varphi_{q_i}|\psi\rangle|^2$ is accepted: Any measurement process of a quantity q is assumed to produce the quantum-mechanical distribution of eigenvalues q_i of q, alone observable, out of a non-quantum-mechanical distribution of values of q, existing before the measurement and considered as hidden.

All this can be generalized for relativistic microsystems of zero or nonzero spin (Klein-Gordon, Dirac theory), and can be completed by thermodynamical considerations on the interaction of a microsystem with the hypothetical subquantum medium.

C. More About the ψ-u Relation

To incorporate organically the corpuscular part of a microsystem and its dynamics into the microsystem's wave, it is necessary to conceive that, rigorously, u is solution of a nonlinear equation of evolution. It is admitted in D.S. that this equation is, moreover, very well approximated by the linear quantum-mechanical equation for ψ, everywhere in the domain covered in common by u and ψ, except at the location $M_{\mathfrak{C}}$ of the singularity, where the nonlinear terms are important.

Finally, and this is the important point in the present discussion, "to be able to explain the success of the usual calculus of interference and diffraction phenomena, and also the success of the usual calculus of eigenfunctions of the stationary-state energies in quantized systems" (Ref. 11, pp. 54–55; our translation), de Broglie and Vigier have been led to accept a further hypothesis concerning the structure of the function $u = a_u e^{i\varphi_u/\hbar}$. u is representable as a sum, $u = u_0 + v$, of two solutions of the linear quantum-mechanical equation, where u_0 is a singular solution having an important contribution only in the immediate proximity of the singularity, whereas v is a regular solution of the same equation that must, "in general at least, coincide, except for a constant factor, with the form usually admitted for the wave ψ in the considered problem" (Ref. 11, p. 55; our translation). Therefore it is admitted that $\psi = Cv$, C being a constant that ensures normalizability of ψ. On this basis, de Broglie represents the subjective evolution of the predictive contents of ψ as a function of the information on the microsystem acquired by the observer by changes of the value of the constant C of proportionality between the objective v and the informational ψ: If ψ, during a "second-type measurement" (Ref. 11, pp. 102–106; our translation), separates in several packets ψ_i, u separates correspondingly in several distinct parts u_i, and since u_0 is unique, it exists in only one of these parts $u_k = u_0 + v_k$, the other ones containing only a v part, $u_i = v_i$ $(i \neq k)$. Before knowing in which u part u_0 is, the observer has to write $\psi_i = Cv_i$ for all the i, where C ensures normalization of the whole ψ. But, as soon as he has learned, by a registration, that u_0 is in the u part u_k, he has to rewrite $\psi_k = C_k v_k$, $\psi_{i\neq k} = 0.v_i$, where C_k now ensures normalization of ψ_k alone. *This representation of a measurement eliminates the objective reduction, leaving only an informational reduction.* Thus ψ appears as "a construct of the mind, having a subjective character, and the unique role of which is to permit calculation of certain probabilities, but it ought to be built by the physicist from the relation $\psi = Cv$, insofar as his informations on the v function are exact. It is because the ψ function is thus built out of v, which is an objective reality, that it permits an exact estimation of the probabilities despite its subjective character" (Ref. 11, p. 59; our translation).

The assumption of coextensivity of ψ and v soon raised objections. It has been remarked in particular that the quantum-mechanical wave ψ often spreads out over infinite domains, whereas it is difficult to conceive that the objective v wave of one microsystem spreads correspondingly. de Broglie has therefore (Ref. 10, pp. 247–263) been led to "weaken" the coextensivity assumption. For instance, in Q.M. the isotropic emission of a point source of monochromatic corpuscles is represented by a spherical ψ wave, $\psi = (A/r)e^{-ikr}e^{iEt/\hbar}$. de Broglie accepts that "the diverging $\psi = (A/r)e^{ikr}$ would no longer be *in this case* a wave associated with each corpuscle, but simply a statistical representation of the global emission of the source, spherically isotropic" (our translation and our italics). But a one-to-one extensional relation ψ-u has never been clearly dropped in D.S.

D. Critique of the ψ-u Relation Defined in D.S.

As its designation shows, the double-solution principle asserts a one-to-one (extensional) relation between the quantum-mechanical wave function ψ associated with a microsystem, and the real corpuscular wave u attributed to this microsystem in D.S. Let us confront this relation with the definability in D.S. of a satisfactory distribution of the presence probability in interference states.

1. D. S. Description of the Step-Potential Example

To make a choice, we shall examine the D.S. description of the autointerference realized by reflection on a step potential of microsystem S, sent one by one.

To the quantum-mechanical incident wave packet ψ corresponds, according to the double-solution principle, an incident objective wave u_1, at any moment coextensive to ψ_1. A permanent localized region of high amplitude of vibration, constituting the corpuscular part of S, is included in u_1 as long as this region has not yet left u_1 by transmission or by reflection. To the quantum-mechanical reflected wave packet ψ_2 corresponds analogously an objective, reflected part u_2, of u_1, coextensive to ψ_2 at any moment, and including the corpuscular part of S, starting from the moment at which this corpuscular part has reached the step and has been reflected. (If, however, it has not been integrated into the transmitted part, u_3, of u_1, corresponding to the transmitted wave packet ψ_3; in the special case in which the step has a practically infinite height, the transmitted u_3 and ψ_3 vanish, and u_2 certainly contains the corpuscular part of S, from the moment when u_1 has been entirely reflected.) The phases and the amplitudes of ψ_1 and u_1 are related, respectively, according to the relations (1) and (3), and this is equally asserted concerning the phases and amplitudes of ψ_2 and u_2. In the superposition domain of ψ_1 and ψ_2, which coincides with the superposition domain of u_1 and u_2, in consequence of the assumption of coextension of u_1 with ψ_1 and of u_2 with ψ_2, the phase φ_ψ of $|\psi\rangle = |\psi_1 + \psi_2\rangle$ identical to the phase φ_u of $u = u_1 + u_2$ takes on an interference form. This interference form of $\varphi_u \equiv \varphi_\psi$ transmits an interference value to the momentum $\vec{p}$ of the corpuscular part of S, by the guidance relation (2′) (if the corpuscular part of S is contained in $u = u_1 + u_2$ at the considered moment). Thereby, the dynamics of a corpuscular part contained in the superposition domain is specifically modified, and it is this modification that leads, on the statistical level of observation of the positions, to the interference form of the position distribution.

2. D. S. Prediction for the Positions Distribution

In D.S. the quantum-mechanical eigenvalues postulate is entirely maintained. For the special case of the quantity position, this postulate states that the distribution of the presence probability is $\Pi(\vec{r}) = |\psi(\vec{r})|^2$. Hence the D.S. prediction is identical to the quantum-mechanical prediction: In D.S., exactly as in Q.M., the quantum-mechanical wave function ψ is considered to be a descriptive element *sufficient* for yielding *alone* the definition of the presence probability distribution. The concept of an objective wave u of one microsystem S, though admitted and mathematically defined inside D.S., makes no specific contribution to the definition of the presence probability distribution. It is true that the proportionality relation $\psi = Cv$ makes it possible to express $\Pi(\vec{r})$ also as a function of the continuous amplitude v of u, by writing $\Pi(\vec{r}) = |\psi(\vec{r})|^2 = C^2 |v(\vec{r})|^2$. But this does not introduce a change in the measurable aspects of the distribution.

3. The Relation of D. S. to the Argument of Sec. I

It can be easily found that, rather surprisingly, the argument contained in Sec. I remains in itself rigorously unchanged in D.S. However, the contents of this argument acquire a more explicit character in D.S., in consequence of the definition stated here for u. Indeed, what was called, throughout the argument developed in Sec. I "the objective reality of one microsystem – whatever it be" becomes in D.S. the mathematically defined concept of an objective u wave of one microsystem, including a permanent localized and mobile region of high amplitude of vibration, representing the corpuscular part of one microsystem. Now, this explicit definition of "the objective reality of one microsystem" brings forth the following important sequence of remarks:

(1) The definition of a permanent localized and mobile element in the u wave of one microsystem S, that does not exist in the corresponding ψ wave of S, permits, as has been shown in c of the present section, the elimination of the necessity of accepting an objective reduction in order to explain the unique and localized result of each measurement operation, despite the non-null extension of ψ. Now, the reduction problem has always been considered by the unorthodox physicists as a typical manifestation of the nonvalidity of the completeness postulate. The introduction of an additional descriptive element for one microsystem, its corpuscular part, constituted in D.S. a negation of the completeness postulate. The dissolution of the reduction problem appears in D.S. as a consequence of this partial negation of the completeness postulate.

(2) But the reduction problem is not the unique quantum-mechanical conclusion considered as a symptom of the incompleteness of Q.M. The possibility of an unlimited spreading of certain quantum-mechanical ψ waves that describe, according to the completeness hypothesis, one single microsystem S was also always considered by the unorthodox physicists as a typical manifestation of the incompleteness of Q.M. (de Broglie himself expresses the same attitude concerning the spherical spreading ψ wave that represents in Q.M. a particle emitted by a point source). Now, the definition of a localized element in the representation of one microsystem – though it resolves the reduction problem – does *not* eliminate also the orthodox conclusion of the possibility of an unlimited spreading of one microsystem (more specifically, in terms of its description by a u wave, of the wave part of one microsystem): The reduction problem and the problem of the possibility of an unlimited spreading of a part of the objective reality of one microsystem are not identical; *they are independent problems*.

As long as a one-to-one extensional relation ψ-u is asserted, and as long as ψ and its corresponding u are admitted to be governed by the same equation (the double-solution principle does assert such a relation and such a uniqueness of the equation of motion), every indefinite spreading that takes place for a ψ wave according to the quantum-mechanical laws is transferred to a u wave also. Thereby the problem of the possibility of an unlimited spreading of a part of the objective reality of one single microsystem – considered by most unorthodox physicists as a manifestation of incompleteness of the adopted description – is transferred from Q.M. into D.S. It follows that *the consequences of the completeness hypothesis are not integrally suppressed in D.S.* This is rather paradoxical, concerning a theory created in order to permit the substitution into Q.M. of a complete description of microphenomena.

(3) The specific, direct negation H(n.c.) of the possibility of an unlimited spreading of the objective reality of one microsystem leads necessarily to a more profoundly statistical view on the significance of a ψ wave than that implied by the definition alone of a corpuscular part of a microsystem, located somewhere inside the domain of ψ: Namely, it leads to the assertion (H_1+H_2) of a statistical relation between the domain covered by ψ and the domain covered by the objective wave of one microsystem. Hence, in order to suppress entirely the incompleteness problems, it is necessary to substitute for the double-solution principle a more totally statistical relation ψ-u, assuming that one quantum-mechanical ψ wave corresponds to a whole (virtual) ensemble of u waves, each of which covers a spatial domain in general much smaller than that covered by ψ, and to derive the fundamental consequences of this assumption. Of course, the possibility of a one-to-one extensional relation ψ-u has to be left open for nonspreading bound states of small dimensions (like the electron states in atoms, or the particle states in nuclei). But this can be conceived as a limiting case of a statistical extensional relation ψ-u, whereas such a statistical relation cannot be described as a limiting case of a one-to-one extensional relation ψ-u.

(4) If now the argument developed in Sec. I is taken into consideration, it follows from it that a statistical reconsideration of the double-solution principle is incompatible with the assumption of a general validity of the quantum-mechanical postulate $\Pi(\vec{r})=|\psi(\vec{r})|^2$ on the distribution of the position probability. In the particular case of autointerferences produced by reflection on a step potential, for instance, another distribution law, which is not deducible from ψ alone but depends also on the dimension of one u wave, will be obtained.

(5) Thus the important conclusion is reached that a clear choice has to be made in D.S. between the following two alternatives:

(i) acceptance of the possibility of an unlimited spreading of the objective u wave of one microsystem (of a one-to-one extensional relation ψ-u), *and* conservation of the quantum-mechanical probability distribution, *or*

(ii) negation of the possibility of an unlimited spreading of one objective u wave (substitution of a statistical extensional relation ψ-u to the one-to-one extensional relation ψ-u), *and* acceptance of the nonvalidity, in interference states, of the quantum-mechanical postulate for the distribution of the position probability.

The first of these possibilities does not eliminate entirely the incompleteness problems. The second possibility raises the problem of a redefinition of the presence probability distribution that shall be coherent with the admitted statistical relation ψ-u.

(6) This last point is simply a remark: Professor de Broglie, who does not accept the possibility of an unlimited spreading of one microsystem, does in fact envisage – among other solutions – a statistical relation ψ-u, every time he is directly faced with a particular spreading state (cf. Ref. 10, pp. 238–260). But nowhere does he define systematically such a relation, nor does he accomplish a full development of its implications (that should include a dissolution of the very principle of a double solution of a unique equation of evolution). In particular, he seems not at all aware of the fact that a statistical extensional relation ψ-u is not generally compatible with the quantum-mechanical definition of the presence probability distribution and that consequently a clear choice has to be made in this respect.

E. Conclusion

In D.S. the consequences of the completeness hypothesis appear to be only *partially* eliminated; the definition of a permanent localized element in the representation of one microsystem entails the possibility of a solution for the reduction problems, but it does not entail an elimination also of certain other quantum-mechanical conclusions, which are equally considered by the unorthodox physicists as unacceptable consequences of the incompleteness of Q.M., and which are specifically rooted in the possibility of an unlimited spreading of a ψ wave packet. In order to eliminate these conclusions it is necessary also to assign to a quantum-mechanical ψ wave packet a meaning more radically statistical than that attributed to it by D.S.

Indeed, the ψ-u relation asserted in D.S. by the double-solution postulate attributes already to a ψ wave packet a statistical meaning, but exclusively by the fact that the corresponding u function – conceived as coextensive to ψ and as governed by the same differential equation as ψ – possesses a singularity arbitrarily located inside the common domain of u and ψ, so that one single u-wave part of the u function corresponds to one ψ wave packet, but this u-wave part is associated in fact to an ensemble of u functions, representing an ensemble of possible microsystems, differing by the location of their corpuscular part. Now, as long as a u wave is conceived as coextensive to the corresponding ψ wave packet, and governed by the same evolution equation as ψ, any unlimited outspreading of a ψ wave packet, as well as all the quantum-mechanical consequences of this outspreading, are transmitted to the corresponding u wave, and thereby persist in D.S. To suppress this transmission, the domain of a ψ wave packet has to be considered as distinct (in general) from that of the corresponding u wave, namely it has to be considered as the union of a whole statistical ensemble (virtual) of smaller domains of *possible* u waves of the unique microsystem represented by ψ. This makes necessary the adjunction of a new statistics to that issued exclusively from the arbitrary location of the singular part of a u function inside the domain covered by the wave part of u. Only such an enriched statistical meaning of a ψ wave packet contains the power of entirely eliminating the implications of the completeness hypothesis. But at the same time, it leads to predictive divergences with Q.M. concerning the interference states.

Moreover, by the fully statistical relation ψ-u outlined, the u-function description could acquire a degree of independence with respect to the ψ-wave-packet description that could permit an understanding of the dependence of the ψ wave packet of a microsystem of given type on the characteristics of the source and of the emission process of this microsystem, and could also permit acceptance of an unlimited spreading of such a packet. This spreading, in the new context, would be compatible with the concept of a nonspreading objective singular u wave characterizing fully, by its properties, one single microsystem. A ψ wave packet and a singular u wave that possess this degree of independence with respect to one another ought to be governed respectively by two *distinct* – though closely related – equations, the first one linear, the second one nonlinear. Thus the *double*-solution principle of *one* equation of motion dissolves entirely, and has to be replaced by the definition of a new relation ψ-u. These views may lead perhaps to a prediction concerning the dimensions of the interference zone of the presence probability distribution that possesses an operational definiteness superior to that of the corresponding quantum-mechanical prediction (while being different from it). Criteria such as those proposed by Feynman and Hibbs[25] and by Schulman[26] (cf. Sec. II) may then cease to be necessary.

IV. DEFINITION OF AN UNORTHODOX DISTRIBUTION OF THE PRESENCE PROBABILITY

A. Introductory

A critical analysis of the quantum theory of measurement processes led us to a progressive outline of a possible unorthodox theory of microphenomena[4,7,22,23,49–51] based on D.S. theory, but possessing specific conceptual features and, in consequence of these, leading – contrarily to D.S. theory – to verifiable experimental predictions nonexistent in Q.M.

We shall begin with a summary of the primitive outline of this theory.

We shall then graft upon this primitive view an embodiment, coherent with it, of the radically statistical conception of the significance of a quantum-mechanical state vector of one microsystem, already characterized *in abstracto* in the course of the critical considerations from the preceding sections. Within the theoretical structure thus obtained, it will be shown that a detailed definition of interference states can be carried out, leading to a redefinition of the presence probability distribution, that coincides in ordinary states with the quantum-mechanical definition, but diverges from it in interference states. Moreover, we shall indicate other specific consequences of the above-mentioned, constructively elaborated, statistical view on the significance of a wave function.

B. Primitive Outline: The Two Types of Measurement Theory

(1) To begin with, a doubt has been established concerning the validity, in interference states, of the quantum-mechanical probability distribution of the momentum; a systematic analysis of the quantum theory of measurement processes leads to the conclusion that for interference states the quantum-mechanical probability distribution of the momentum, $\Pi^{\psi}(\vec{p}_i) = |C^{\psi}_{\vec{p}_i}|^2 = |\langle\psi|\varphi(\vec{p}_i)\rangle|^2$, is devoid of any existing experimental basis, and that, moreover, several strong conceptual reasons support the opinion that this quantum-mechanical probability distribution is probably false in such states (cf. Refs. 4 and especially 49). The doubt thus established leads outside Q. M. It can be developed along two distinct paths: On the one hand, it can be continued by a purely experimental investigation, aimed towards a specific confirmation or infirmation of the validity in interference states of the quantum-mechanical distribution postulate for the momentum. On the other hand, this doubt can be followed by a theoretical investigation elaborated outside Q.M. Hence, whereas an experimental investigation has been proposed,[4,7,49,52] new theoretical steps have also been taken tentatively, starting from the de Broglie–Bohm interpretation of Q.M., namely:

(2) It has been shown first[7] that the acceptance of the validity of the quantum-mechanical probability distribution $\Pi^{\psi}(\vec{p}) = |C^{\psi}_{\vec{p}_i}|^2$ for the momentum is logically inconsistent with the rest of D.S. theory. Indeed, in D.S. theory, the nonquantum distribution of values of $\vec{p}$ defined by the guidance relation is considered to be hidden; that is to say, any experimental procedure that leads to registrations interpretable in terms of a distribution of values of $\vec{p}$ – in other words, any measurement process of $\vec{p}$ – is admitted to transform these non-quantum-mechanical values into a distribution of quantum-mechanical eigenvalues of $\vec{p}$, different in general from the distribution of the guidance values of $\vec{p}$ in ψ, before the measurement. Now, notwithstanding this assumption, if the *a priori* acceptance of the quantum-mechanical distribution postulate is ignored, it is possible to derive deductively from the rest of the D.S. formalism the conclusion that certain experimental procedures, applied to an interference state, lead to the registration of a nonquantum distribution of guidance values possessed by $\vec{p}$ in this state. These experimental procedures are therefore named "noneigenvalues measurements," to distinguish them from the "eigenvalues measurements" that lead to the quantum-mechanical distribution. This conclusion is obviously in contradiction with both the acceptance of the general validity of the quantum-mechanical postulate and the assumption of hiddenness of the distribution of guidance values of $\vec{p}$.

(3) On the other hand, it has been remarked[7] that the assumption of the general validity of the quantum-mechanical distribution postulate is not organically integrated into the logical structure of D.S. theory; if this assumption is dropped, the rest of the D.S. formalism is left essentially unaltered.

(4) On the basis of all the preceding conclusions, the *a priori* acceptance, by postulate, of the quantum-mechanical eigenvalues distribution law is tentatively dropped, whereas the rest of the D.S. formalism is tentatively conserved. Inside the new logical structure that results in this way, the contradiction brought into evidence at point (2) is obviously eliminated.

Moreover, it can now be shown deductively inside this structure liberated from the *a priori* acceptance, by postulate, of the quantum-mechanical eigenvalues distributions, that the particular category of the eigenvalues measurements (which are considered in Q.M. as the only possible measurements) effectively leads, according to the specific D.S. formalism, to the registration of eigenvalues;

the quantum-mechanical eigenvalues postulate is thereby demonstrated for eigenvalue measurements.[50] Thus the ensemble of all the possible measurements appears as consisting of two distinct categories: the noneigenvalues measurements, connected *deductively* with specific experimental previsions inexistent in Q.M., and the eigenvalues measurements, for which the theory leads deductively to conclusions identical to those admitted in Q.M. by postulate. Therefore the theory obtained starting out from the D.S. formalism, by dropping the quantum-mechanical eigenvalues distribution postulate, was originally called "the theory of two types of measurements."[7,50] The further outline proposed below expresses a much more profound view on microphenomena, brought into evidence by developments subsequent to those summarized above.

C. Further Outline: Radically Statistical Redefinition of the ψ-u Relation and Characterization of Interference States

We shall now build upon the two-types-of-measurement theory, a radically statistical redefinition of the ψ-u relation for one microsystem, permitting a detailed characterization of interference states. (The importance of such a redefinition has been perceived first by Evrard, who was struck by the unacceptable implications of a one-to-one relation in the step-potential example.[22]) This redefinition is based on the conclusions of Secs. I and II. We shall then study the consequences of this redefinition.

1. Spatial Relation ψ-u

We define first the purely spatial aspects of the radically statistical relation ψ-u we propose on the basis of the conclusions obtained in the preceding sections.

Let $\psi = a_\psi e^{i\varphi_\psi/\hbar}$ be a solution of the Schrödinger equation of a given problem, concerning one microsystem S. This solution will then have the form of a ψ wave packet. Let $\mathfrak{D}_\psi$ be the domain of ψ. Since ψ is a packet, $\mathfrak{D}_\psi$ is a function of time, $\mathfrak{D}_\psi(t)$.

Let us now conceive a virtual statistical set $\{u(j,k)\}$ of de Broglie-type singular functions $u(j,k) = a_{u(j,k)}\, e^{i\varphi_{u(j)}/\hbar}$ representing a virtual set $\{S(j,k)\}$ of identical replicas of the microsystem S, only one of which is physically existent in one experiment. Let $\mathfrak{D}(j)$ be the domain of $u(j,k)$. We admit that in general $\mathfrak{D}_\psi \gg \mathfrak{D}(j)$, but it is also possible that $\mathfrak{D}_\psi \approx \mathfrak{D}(j)$. Let M_{jk} designate the quasipoint region located somewhere in $\mathfrak{D}(j)$, where the amplitude $a_{u(j,k)}$ of $u(j,k)$ has a singularity representing the corpuscular part of the replica $S(j,k)$ of S. Thus each $\mathfrak{D}(j)$ corresponds in fact to a virtual set $\{u(j,k)\}$ of singular wave functions $u(j,k)$ having all the same domain at any moment but including differently located singularities labeled by the index k.

Every $u(j,k)$ is conceived as a possible solution of a still unknown nonlinear evolution equation that defines in particular $\mathfrak{D}(j)$ and the location of M_{jk}, and prevents spreading of $\mathfrak{D}(j)$.

We admit that the domain $\mathfrak{D}_\psi$ of ψ is the union (this term is used here with the meaning it has in the theory of ensembles) of all the domains $\mathfrak{D}(j)$ of the virtual set of functions $\{u(j,k)\}$ describing the virtual set $\{S(j,k)\}$ of replicas of S.

It can then be formally written

$$\mathfrak{D}_\psi(t) = \bigcup_j \mathfrak{D}(j,t)\,. \tag{4}$$

[In (4) the $\mathfrak{D}(j,t)$ are assigned a physical content that was absent in the $\mathfrak{D}_s(t)$ from $\{(\mathrm{H}_1+\mathrm{H}_2)'\}$ in Sec. I.]

If the considered state is a propagating state, then, even if the initial ψ packet is chosen such that $\mathfrak{D}_\psi(t_0)$ shall coincide at an initial moment t_0 with one domain $\mathfrak{D}(j)$ in which all the waves $\{u(j,k)\}$ corresponding to the virtual ensemble $\{S(j,k)\}$ of microsystems superpose at t_0, as time passes, $\mathfrak{D}_\psi(t)$ spreads out indefinitely, since ψ is governed by the linear Schrödinger equation. Consequently $\mathfrak{D}_\psi(t)$ certainly ceases to coincide with one $\mathfrak{D}(j)$ at $t > t_0$. We conceive this as an effect of a certain dispersion at t_0 ineluctably existing in the initial conditions concerning the singularities of the $u(j,k)$ waves of the virtual ensemble $\{S(j,k)\}$ of microsystems S.

The spreading of $\mathfrak{D}_\psi$ is a physical fact (it corresponds to the spreading of the registrable presence probability) that has to be described inside a complete theory of microsystems. But, moreover, this fact is conceived here as a statistical-level fact, not as an individual-level one. By the radically statistical significance we assign to it, a ψ wave packet is perfectly appropriate for describing this statistical-level fact. The distinct time evolutions of the elements $\mathfrak{D}_\psi(t)$ and $\mathfrak{D}(j,t)$ related in (4) illustrate strikingly that a quantum-mechanical wave packet, even if one tries to constrain it to describe one microsystem at the maximal degree possible inside Q.M. by imposing initially upon it the dimensions of one $\mathfrak{D}(j)$, still remains an essentially statistical descriptive device (contrary to what Bohm and Bub believe; cf. Ref. 41), since the k statistics cannot be eliminated even at an initial moment, whereas the j statistics, if initially suppressed, are irrepressibly resurrected as time passes, in consequence of the initial k statistics.

For these reasons we believe that the linear character of the evolution equation of ψ corresponds to an essential and useful descriptive capacity of ψ, and has to be *conserved*; it must not be suppressed by submitting ψ to a nonlinear equation, as is envisaged in D.S. theory, and as Wigner,[1] Ludwig,[37] and other authors also suggested. Other descriptive elements, the $u(j,k)$ waves, describing individual-level aspects of microphenomena, have to be governed by a nonlinear equation.

Let us now consider the case of a bound state. If the microsystem is bounded inside a region of the order of one $\mathfrak{D}(j)$ of a microsystem S, both $\mathfrak{D}_\psi$ and $\mathfrak{D}(j)$ are immobile, and (4) takes on the particular form

$$\mathfrak{D}_\psi = \mathfrak{D}(j)\,. \tag{4$'$}$$

If the well is of dimensions superior to one $\mathfrak{D}(j)$ of an S microsystem, then we assume that the transitory process constituting the genesis of the bounded state (that ought to be describable in a complete theory) is such that it transforms the continuous structure of the ψ packet of the system, in a superposition of monochromatic ψ waves possessing frequencies that form a discontinuous series of harmonics. This superposition fills up the well, so that $\mathfrak{D}_\psi$ coincides with the well and is immobile. But the $u(j,k)$ of the virtual ensemble $\{u(j,k)\}$ (the structural relation of which with the ψ_i waves will be defined later) have domains $\mathfrak{D}(j) < \mathfrak{D}_\psi$ that are mobile inside $\mathfrak{D}_\psi$. If then we denote by $\mathfrak{R}(j)$ the region swept out by $\mathfrak{D}(j,t)$ it can be formally written

$$\mathfrak{D}_\psi = \bigcup_j \mathfrak{R}(j,t)\,. \tag{4$''$}$$

Since each $\mathfrak{D}(j)$ corresponds to a statistical ensemble $\{u(j,k)\}$, according to (4) and (4$''$) *the domain of the quantum-mechanical wave ψ contains in fact in general a double-level statistics, with respect to both indices j and k.* If in particular the domain $\mathfrak{D}_\psi$ is of the order of a domain $\mathfrak{D}(j)$ – as can happen for systems bounded in nuclear or even atomic structures – then the statistics on j vanish and alone the statistics on k are left.

Thereby the D.S. statistics on the location of the corpuscular part of a microsystem inside the objective wave that includes it (exclusively "k statistics") are supplemented by a superadded statistics concerning the location of $\mathfrak{D}(j)$ inside $\mathfrak{D}_\psi$ (a "j statistics"). The statistical view on the significance of a quantum-mechanical wave function is thus carried up to its limit and the orthodox completeness postulate for Q.M. is exhaustively negated.

2. *Spatial Characterization of Interference States*

From the radically statistical point of view on the significance of a ψ wave adopted here, the orthodox characterization of interference states, by exclusive use of ψ waves, appears as insufficient. Let us construct a more detailed characterization, connecting the concept of interference state to that of an objectively existing u wave of a microsystem. We shall achieve this characterization by a sequence of definitions. We begin by considering only the spatial aspects of an interference state.

Space point, ψ point, $u^r(j,k)$ point. At any given moment t all the $u(j,k)$ are virtual except a single one that is real. Let us denote it by $u^r(j,k)$, and its domain by $\mathfrak{D}^r(j,t)$.

We distinguish in what follows between a *space point $M(x,y,z)$* of the physical space, a *ψ point $M_\psi(x_\psi,y_\psi,z_\psi)$* of the domain $\mathfrak{D}_\psi$ of the ψ wave of the studied microsystem, and a *$u^r(j,k)$ point $M_{u^r(j,k)}(x_j, y_j, z_j)$* conceived as a material constitutive point of the body of $u^r(j,k)$. Any $u^r(j,k)$ point obviously is a ψ point and a space point, and every ψ point is a space point.

Individual interference point, ψ superposition point, one ψ point. Individual interference domain, ψ superposition domain. Individual interference state, ψ superposition state. Whether a given space point is or is not also a ψ point, or also a ψ point and a $u^r(j,k)$-point, is a function of time. From this point of view the following situations can be distinguished:

A given space point $M(x,y,z)$ can be, at a given moment t, in one of the following three situations, with respect to the domain $\mathfrak{D}_\psi(t)$ of all the ψ-points M_ψ:

(α) It may not be covered by any ψ point M_ψ [$x \neq x_\psi$, $y \neq y_\psi$, $z \neq z_\psi$, for all $M_\psi \in \mathfrak{D}_\psi(t)$].

(β) It may be covered by one single ψ point $M_\psi \in \mathfrak{D}_\psi(t)$ [$x = x_\psi$, $y = y_\psi$, $z = z_\psi$, for one $M_\psi \in \mathfrak{D}_\psi(t)$].

(γ) It may be covered simultaneously by $2, 3, \ldots, q$ different ψ points.

The significance of this assertion is the following: It may happen that the Schrödinger equation of the considered problem requires at the moment t a ψ solution that consists in a superposition of n different ψ_i wave packets, $|\psi(t)\rangle = \sum_{i=1}^n |\psi_i(t)\rangle$ (in the step-potential example, at a moment t belonging to the time interval in which the reflection process of the incident wave ψ_1 takes place, the complete solution of the Schrödinger equation of the problem is, at certain points at the left of the wall, the superposition $|\psi(t)\rangle = |\psi_1(t) + \psi_2(t)\rangle$ of a still incident part of the initially entirely incident packet ψ_1, with an already reflected part of ψ_1. In such a case $2, 3, \ldots, q$ distinct wave packets can be all defined, at t, in one given space point $M(x,y,z)$.

Let us now consider the $u^r(j,k)$ points. A given space point $M(x,y,z)$ can be, at a given moment t, in one of the following three situations with respect to these points:

(α') It may not be covered by any $u^r(j,k)$ point $M[u^r(j,k)]$ $[x\neq x_j,\ y\neq y_j,\ z\neq z_j$, for all $M[u^r(j,k)] \in \mathfrak{D}^r(j,t)]$.

(β') It may be covered by one single $u^r(j,k)$ point $M[u^r(j,k)]$ of $\mathfrak{D}^r(j,t)$ $[x=x_j,\ y=y_j,\ z=z_j$, for one $M[u^r(j,k)]\in\mathfrak{D}^r(j,t)]$.

(γ') It may be covered simultaneously by $2, 3, \ldots, s$ different $u^r(j,k)$ points $M[u^r(j,k)]$ $[x=x_1=x_2=\cdots=x_s,\ y=y_1=\cdots=y_s,\ z=z_1=\cdots=z_s,\ j=1,2,\ldots,s$, for s points $M[u^r(j,k)]\in\mathfrak{D}^r(j,t)]$.

The significance of this last assertion is the following:

According to the ψ-u relation we defined, a ψ wave corresponds to a virtual collective $\{u(j,k)\}$ containing one single real wave $u^r(j,k)$. If the ψ solution of the problem has at t the form of a superposition $|\psi(t)\rangle = |\sum_i \psi_i(t)\rangle$, then a given $\psi_i(t)$ either contains no $u^r(j,k)$ at all [though *a priori* it has a non-null probability to contain $u^r(j,k)$ at the moment t], or it contains integrally $u^r(j,k)$ [as, for instance, in the wall example, the reflected packet ψ_2, if the wall is infinitely high, and if, moreover, t is posterior to the time interval during which the reflection process of $u^r(j,k)$ takes place], or, finally, it contains a part of $u^r(j,k)$ [as, in the step-potential example, the incident wave ψ_1 and the reflected wave ψ_2, at a moment during the reflection process of $u^r(j,k)$; or as each one of the Young packets at the right of the holes; or as each one of the packets represented in Fig. 2, at the right of the interferometer. In the last two examples, each one of the waves ψ_1 and ψ_2, the superposition of which constitutes $|\psi\rangle$, corresponds to a virtual collective of parts of a $u(j,k)$ wave].

Now, if the last case is realized, some of these parts of $u^r(j,k)$ contained in different ψ_i (or all of them) can, in particular, superpose at a given moment, in a given space point.

Let us now formulate some further definitions:

We say that the space point $M(x,y,z)$ is, at the moment t,

an individual-interference point of degree s, if (γ') is realized (the point A in Fig. 5), and we shall denote it $M[u_s^r(j,k)]$ (usually $s=2$);

a *ψ superposition point of degree q,* if (γ) is realized (the points A, B, C in Fig. 5), and we shall denote it $M(\sum_{i=1}^q \psi_i)$ (usually, $q=2$);

a *one-ψ point,* if (β) is realized (the points D, E in Fig. 5), and we shall denote it $M(\psi)$.

We call

individual interference domain of degree s, in ψ at t, the ensemble of all the individual interference points at the moment t, and we denote it by $\mathfrak{D}^\psi[u_s^r(j,k),t]$ (usually $s=2$);

individual interference domain of degree s, in ψ, the union of the $\mathfrak{D}^\psi[u_s^r(j,k),t]$ corresponding to all the moments of the evolution of ψ, and we denote it $\mathfrak{D}^\psi[u_s^r(j,k)] = \bigcup_t \mathfrak{D}^\psi[u_s^r(j,k),t]$ (the shaded domains in Fig. 5);

ψ-superposition domain of degree q, in ψ, at t, the ensemble of the ψ-superposition points at t, and we denote it by $\mathfrak{D}_t^\psi(\sum_{i=1}^q \psi_i)$;

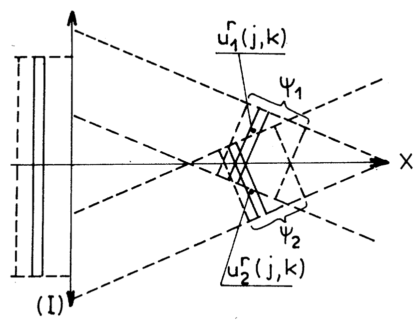
ψ-superposition domain of degree q in ψ, the union of the $\mathfrak{D}_t^\psi(\sum_{i=1}^q\psi_i)$ corresponding to all the moments of the evolution of ψ, and we denote it $\mathfrak{D}^\psi(\sum_{i=1}^q\psi_i) = \bigcup_t \mathfrak{D}_t^\psi(\sum_{i=1}^q\psi_i)$ (the shaded and dotted domains in Fig. 5).

We call

individual-interference state of degree s, a state for which $\mathfrak{D}^\psi[u_s^r(j,k)]$ is not void;

ψ-superposition state of degree q a state for which $\mathfrak{D}^\psi(\sum_{i=1}^q\psi_i)$ is not void.

If we confront the above definitions with the ψ-u relation we have defined, it follows that $(\beta)\not\rightarrow(\beta')$ whereas $(\beta')\rightarrow(\beta)$ [a ψ point is not necessarily a $u^r(j,k)$ point, whereas a $u^r(j,k)$ point is always a ψ point]; this expresses in other terms the inclusion $\mathfrak{D}(j,t)\subset\mathfrak{D}_\psi(t)$ essential to our views. Another,



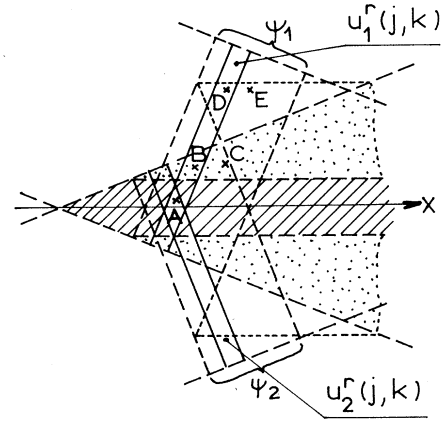


FIG. 5. Interference of two practically plane-wave packets: ψ and $u^r(j,k)$ superposition domains. (a) General representation. (b) Details: shaded areas, $\mathfrak{D}^\psi \times [u_s^r(j,k)]$; shaded and dotted areas, $\mathfrak{D}^\psi(\sum_{t=1,2}\psi_i)$.

less trivial implication is that $(\gamma') \to (\gamma)$ whereas $(\gamma) \not\to (\gamma')$ (any individual-interference point at t is a superposition point, but a superposition point is *not* necessarily an individual-interference point). This expresses the fact that *in general*

$$\mathfrak{D}^{\psi}[u_s^r(j,k)] \subset \mathfrak{D}^{\psi}\left(\sum_{i=1}^{q} \psi_i\right)$$

in consequence of the unlimited spreading of freely propagating ψ_i packets, corresponding to $\mathfrak{D}(j)$ that do *not* spread out indefinitely. Only in the particular case in which (4′) is realized,

$$\mathfrak{D}^{\psi}[u_s^r(j,k)] = \mathfrak{D}^{\psi}\left(\sum_{i=1}^{q} \psi_i\right) = \mathfrak{D}_{\psi} = \mathfrak{D}(j)$$

[stationary interference of $u^r(j,k)$]. But it is also possible, as another limiting case, that whereas $\mathfrak{D}^{\psi}(\sum_{i=1}^{q}\psi_i)$ is finite, $\mathfrak{D}^{\psi}[u_s^r(j,k)]$ may be void: This happens, for instance, if the ψ superposition to which $\mathfrak{D}^{\psi}(\sum_{i=1}^{q}\psi_i)$ corresponds is not "coherent."

3. *Structural Relation ψ-u and Dynamical Characterization of Interference States*

So far we have taken into consideration exclusively the spatial aspects of the relation ψ-u. Let us now state very briefly what structural relations we assume for the moment. A more complete account of these relations is given in another study.[51]

In D.S. it is admitted that the ψ wave packet of one microsystem corresponds "in general" to one objective singular wave u, ψ and u being governed by the same equation of evolution, the rigorous unknown form of which is nonlinear, but which is well approximated outside the singular region of u, for nonrelativistic energies, by the linear Schrödinger equation. The phases of ψ and u are asserted to be always identical, and their amplitudes are considered to be "in general" proportional everywhere, except around the location of the singularity of u.

In our conception, the ψ wave packet of one microsystem is *rigorously* governed by the linear Schrödinger equation (for nonrelativistic energies), and corresponds to a whole virtual collective of objective singular waves $\{u(j,k)\}$, each of which is governed by an unknown nonlinear equation, different from the Schrödinger equation of ψ. We conceive the amplitude of the ψ wave packet as representing aspects of the described reality, essentially different from the aspects described by the amplitude of a $u(j,k)$. The amplitude of ψ is directly related, in our view, only to the statistical distribution of the locations of the singularities of the $u(j,k)$ from $\{u(j,k)\}$, but it has *no direct relation with the amplitude of the wave part of one $u(j,k)$*, whereas the amplitude of an $u(j,k)$ describes the structure and shape of a possible objective wave of the studied microsystem. Therefore we drop any coextensivity and proportionality relation between the amplitude of ψ and that of the $u(j,k)$, even at the level $\mathfrak{D}(j)$, and we are unable yet to state some other relation between these two amplitudes; in particular we can state no equivalent of (3) from Sec. III.

This influences the assumptions that we can coherently make concerning the relations implying the momentum:

Let us first consider a ψ point that is not an individual interference point at the considered moment t. Such a point can be covered at t only by a one-piece $u^r(j,k)$ or part of $u^r(j,k)$. Therefore, the amplitude and the phase of an $u^r(j,k)$ possible at t in such a point are independent of one another. Hence, even though we are unable for the moment, starting from ψ and from experimental data, to make assumptions concerning the amplitude of $u^r(j,k)$, it may be possible to find a basis for stating tentative assumptions concerning the phase of $u^r(j,k)$. Indeed, if the ψ wave packet of the studied microsystem has the form of an approximately plane wave almost everywhere in $\mathfrak{D}_{\psi}$ (if it is approximately an eigenwave of the momentum), then experience shows that the measured momentum $\vec{p}$ is yielded by $-\vec{\nabla}\varphi_{\psi} = \vec{p}$. Now, in our view, an individual objective event, like the registration of one value of $\vec{p}$, must be directly related to $u^r(j,k)$, hence this registration has to be regarded as produced by the corpuscular part $\mathfrak{C}$ of $u^r(j,k)$, whereas $\vec{\nabla}\varphi_{\psi}$, inasmuch as it is derived from the abstract statistical ψ, cannot possess directly an individual objective significance: $\vec{\nabla}\varphi_{\psi}$ and $\vec{p}$ in de Broglie's relation $-\vec{\nabla}\varphi_{\psi} = \vec{p}$ must be, in the limiting case of an approximate plane-wave packet, the two ends of a chain containing in the middle some element of $u^r(j,k)$.

Now, we established[51] that, for an arbitrary $\psi = \sum_{i=1}^{q}\psi_i$, the general $\vec{p}$-ψ relations that are coherent with the significance we assign to a state-vector (the two ends of the above-mentioned chain) are the following:

$$\langle \vec{p}_{\mathfrak{C}}[M(j,k),t]\rangle_{\text{av}} = -[\vec{\nabla}\varphi_{\psi}(t)]_{M(\sum_{i=1}^{q}\psi_i)\equiv M(j,k)}\,, \qquad (5)$$

$$\langle \vec{p}_1[M(j,k),t]\rangle_{\text{av}} = -[\nabla\varphi_{\psi_1}(t)]_{M(\sum_{i=1}^{q}\psi_i)\equiv M(j,k)}\,. \qquad (6)$$

In (5), $\langle \vec{p}_{\mathfrak{C}}[M(j,k),t]\rangle_{\text{av}}$ is the mean value of the momenta of all the corpuscular parts of all the different objective waves $u(j,k)$ (at most one of which is real) that can be located at $M(j,k)$ at the moment t (the other notations have obvious significance). In (6), $\langle \vec{p}_1[M(j,k),t]\rangle_{\text{av}}$ is the mean value of the moments of all the corpuscular parts of the different objective waves $u(j,k)$ (at most one of which is real) constituting the virtual set corresponding to *one*, ψ_1, of the ψ_i from $\psi=\sum_{i=1}^{q}\psi_i$, that can be located at $M(j,k)$ at the moment t.

If $q=1$, (5) and (6) identify and reduce to a statistical generalization of the D.S. guidance relations. But if $q\neq 1$, in a ψ-superposition point, the existence of (6), besides (5), indicates an important specificity with respect to D.S. theory. The relations (5) and (6) characterize dynamically, in our view, a noninterference point.

As to the $\vec{p}$-$u(j,k)$ relation, the middle of the $\vec{p}$-u-ψ chain, only the individual theory implied by the statistical significance that we assign to a state vectors will be able to yield it (maybe it will be found that $\vec{p}_{\mathfrak{C}}=\vec{\nabla}\varphi_{u^r(j,k)}$, as de Broglie's intuition asserts).

Let us now consider a point that is an individual-interference point at a given moment t. In this circumstance the relations (5) and (6) are no longer valid, according to our results.[51] All that we can coherently assume in an individual-interference point is the relation, unconnected to ψ,

$$\vec{p}=F[u_s^r(j,k)],$$

where $F[u_s^r(j,k)]$ is an unknown function of $u_s^r(j,k)$, itself unknown as long as the nonlinear equation determining $u^r(j,k)$ is still unidentified. *A priori* this relation differs in general from the D.S. guidance relation in an interference point. It expresses however the conception that the momentum of the corpuscular part of $u^r(j,k)$ is influenced by all the parts of $u^r(j,k)$ that superpose in the point $M(j,k)$ where the corpuscular part of $u^r(j,k)$ is located, creating a "quantum field." This quantum field, however, is fundamentally different from that one defined by de Broglie and by Bohm on the basis of the amplitude of ψ, which, in our view, is not connected in a known way to $u^r(j,k)$. This characterizes dynamically, in our view, an individual-interference point.

4. *The Presence Probability Distribution in Interference States*

What definition of the presence probability distribution in an interference state is coherent with the conception just outlined?

If we possessed a precise knowledge of the initial conditions of $u^r(j,k)$, as well as of the form of the assumed nonlinear equation of one $u^r(j,k)$, the characteristics and the evolution of the objective wave $u^r(j,k)$ of one microsystem could be predicted for any moment with certainty (our fundamental conception is deterministic). But we do not yet possess such a knowledge. Consequently our predictions concerning the evolution of one microsystem will still be probabilistic, as are the quantum-mechanical ones, but they will also bear the imprint of our deterministic underlying view, that introduces the idea of one $u^r(j,k)$ associated to any quantum-mechanical ψ wave packet. It can therefore be said in advance that these predictions will have a mixed, hybrid character, implying at the same time statistical and individual features.

Let us begin again by considering the step-potential example. The arguments developed throughout the preceding sections describe with much detail, though qualitatively, the basis on which our definition has now to be established:

Concerning an individual-interference point $M[u_s^r(j,k)]\in\mathfrak{D}^{\psi}[u_s^r(j,k)]$ according to our conception we can assert that $u^r(j,k)$ is divided in such a point into two parts that superpose, one belonging to ψ_1, the other one to ψ_2, so that $u^r(j,k)$ in its integrality belongs there simultaneously to both ψ_1 and ψ_2, that is to say, to the sum $|\psi_1+\psi_2\rangle$.

The physical modifications in the structure of $u^r(j,k)$ that lead to a specific "interference form" of the presence probability for the corpuscular part $\mathfrak{C}$ of $u^r(j,k)$ can take place only in an individual-interference point where two or more parts of $u^r(j,k)$ superpose (where a quantum field acts).

Though the specific form taken on by the presence probability, at a moment t, of the corpuscular part $\mathfrak{C}$ of $u^r(j,k)$ in an individual-interference point is, physically, an effect of the superposition with itself of the objective wave $u^r(j,k)$, experiment demonstrates so far that formally, this specific form of the presence probability of $\mathfrak{C}$ is describable in a correct way by the expression $\Pi(M,t) = |\sum_{i=1,2}\psi_i|^2$, built up exclusively from the quantum-mechanical wave-packet functions ψ_1 and ψ_2 to which $u^r(j,k)$ belongs simultaneously at t, in the considered interference point.

Analogous remarks hold for an individual-interference point of a Young interference or of the interference state illustrated by Fig. 5.

These reasons lead, within our conception, to the postulation, at a moment t in an individual-interference point $M[u_s^r(j,k)]\in\mathfrak{D}_t^{\psi}[u_s^r(j,k)]$, of the following form of the presence probability of $\mathfrak{C}$:

$$\Pi^{\psi}(M,t)=\left|\sum_{i=1}^{s}\psi_i(M,t)\right|^2=|\psi(M,t)|^2,$$
$$M\equiv M[u_s^r(j,k)]\in\mathfrak{D}_t^{\psi}[u_s^r(j,k)]. \tag{7}$$

This definition is formally identical to the definition asserted in Q.M. for any ψ-superposition point, but it differs from this definition by the domain of validity we attribute to it, since in general

$$\mathfrak{D}_t^{\psi}[u_s^r(j,k)] \subset \mathfrak{D}_t^{\psi}\left(\sum_{i=1}^{s} \psi_i\right) \quad .$$

Concerning a ψ-superposition point of the step-potential example, which is not an individual-interference point, we can assert, according to our conception:

$u^r(j,k)$ belongs entirely – contrary to the quantum-mechanical assertion – to only one of the wave-packets ψ_1, ψ_2 but we cannot tell to which of them.

If we know that it belongs entirely to ψ_1, experiment shows that the presence probability of $\mathfrak{C}$, though it concerns a part of $u^r(j,k)$, is formally describable by aid of the expression $|\psi_1(M,t)|^2$ ($M = M(\sum_{i=1,2}\psi_i)$, $M \notin \mathfrak{D}_t^{\psi}[u_s^r(j,k)]$), built up from the quantum-mechanical ψ_1. An analogous assertion holds for ψ_2. But all the knowledge available, in fact, is that $u^r(j,k)$ belongs either to ψ_1 or to ψ_2.

Similar remarks can be made concerning the points $M(\sum_{i=1,2}\psi_i) \notin \mathfrak{D}_t^{\psi}[u_s^r(j,k)]$ of a Young interference or of the interference represented in Fig. 5.

Finally it can be concluded that the available knowledge, according to our conception, concerning a ψ-superposition point of degree q that is not an individual-interference point, has to be expressed, to agree with the current rules of the probability calculus, by the expression

$$\Pi^{\psi}(M,t) = \sum_{i=1}^{q} |\psi_i(M,t)|^2 , \qquad (8)$$

$$M \equiv M\left(\sum_{i=1}^{q} \psi_i\right) \begin{cases} \in \mathfrak{D}_t^{\psi}\left(\sum_{i=1}^{q} \psi_i\right) \\ \notin \mathfrak{D}_t^{\psi}[u_s^r(j,k)] \end{cases} .$$

This definition is different from the quantum-mechanical one that asserts the relation (7) for *any* ψ-superposition point. If in particular $q=1$, Eq. (8) reduces to the definition of $\Pi^{\psi}(M,t)$ in a one-ψ point $M(\psi)$ that is identical to the quantum-mechanical one.

It has to be noted that the normalization condition compatible with (7) and (8), $\int \Pi^{\psi}(M,t)dM = 1$, has to be realized, even though its expression, in terms of ψ wave packets, is different from the quantum-mechanical one, in consequence of the form (8) of $\Pi^{\psi}(M,t)$ at the points

$$M\left(\sum_{i=1}^{q} \psi_i\right) \begin{cases} \in \mathfrak{D}_t^{\psi}\left(\sum_{i=1}^{q} \psi_i\right) \\ \notin \mathfrak{D}_t^{\psi}[u_s^r(j,k)] \end{cases}$$

for $q \neq 1$.

In the most general case, a given space point can be at certain moments an individual interference point (of a degree function of the moment) and at other moments only a ψ-superposition point (of a degree function of the moment, in particular, of degree 1) [point A, Fig. 5(b)]. Therefore, if an integration with respect to time is performed on $\Pi^{\psi}(M,t)$ in a fixed space point, one has to add in general an integrated term of type (7), corresponding to the moments that can contribute to the progressive constitution of an interference pattern on a sensitive screen, and a term of type (8), corresponding to the moments that can superpose a uniform distribution, to the interference distribution (7). An explicit calculation of these two terms requires the knowledge of the particular features of the considered state [analytic form, propagation velocity $u^r(j,k)$, etc.]. The term of type (8) is usually not taken into account on the theoretical level, and is introduced afterwards, as a deviation from the theoretical conditions, to express the existence of a "noise." Of course, a given space point can also be populated, in particular, exclusively by individual-interference points (points on the axis, Fig. 5) or exclusively by ψ-superposition points that are not individual-interference points. In the first case, the time-integrated distribution has a "noiseless" interference structure; in the second case it has a uniform structure.

The definitions (7) and (8) integrate Feynman's criterion into our virtual statistical view on the significance of quantum-mechanical wave packet, thus connecting this criterion to an objectively existing $u^r(j,k)$ wave.

On the final calculational level, the presence probability $\Pi^{\psi}(M,t)$ is still defined exclusively in terms of the quantum-mechanical abstract wave packet ψ of the microsystem, determined by the Schrödinger equation of the problem, and thereby *the statistical descriptive resources specifically possessed by the quantum-mechanical ψ waves are integrally conserved*. But these resources are utilized in a way different, in interference states, from the quantum-mechanical one. Moreover, by the intermediary of the concepts of a ψ-superposition point and of an individual-interference point, $\Pi^{\psi}(M,t)$ is explicitly connected to the objective wave $u^r(j,k)$ and is thereby rooted into an underlying, unorthodox conception, where a virtually statistical meaning is attributed to a ψ wave packet and where the nonlinear equation of one $u^r(j,k)$ acts, determining in principle the respective non-quantum-mechanical domains of the definitions (7) and (8) expressed in terms of the quantum-mechanical ψ wave packet.

Of course, in the present stage of development of the theory, since the nonlinear equation of one $u(j, k)$ is not yet identified, the domains of the probability definitions postulated above cannot be calculated. But what can be done is to confront these definitions directly with the experimental position distributions obtained in states containing an individual interference domain (an experiment is outlined in Sec. II). Such a confrontation can yield experimentally the domains of the two distributions postulated, and thus it can verify qualitatively our theory, and at the same time – if this theory proves true – it can offer significant data for the identification of the nonlinear equation of one $u(j,k)$. Moreover, the theory outlined above could throw light on certain experiments already performed, where correlations have been very accurately registered (cf. Ref. 53).

A final remark: Every position of the corpuscular part $\mathcal{C}$ of the $u^r(j, k)$ successively implied in the N measurement processes that lead to the registration of a position distribution (7) or (8) is conceived in our view to exist before the registration takes place; they are not assumed, as in orthodox Q.M., to be created by the measurement, via a reduction of the ψ wave packet of one microsystem. The unique corpuscular part $\mathcal{C}$ of the unique $u^r(j, k)$ of the virtual collective $\{u(j, k)\}$ represented by the ψ wave packet of one microsystem simply imprints some mark upon a position recorder where it happens to be located at the moment at which the recording takes place: The values making up the distributions (7) and (8) are permanently defined observables. The experimental procedure proposed in Sec. II can be reconsidered in the more detailed terms available in consequence of the definitions (7) and (8).

5. *Remarks on the Momentum Probability Distribution in Individual Interference States*

We add now a few important remarks on the momentum probability distribution, in order to complete the picture, though these remarks are in fact exterior to the specific object of this article. The basis of these remarks may be found elsewhere.[51]

The expressions (5) and (6) concern in our conception averages calculated over values of $\vec{p}_{\mathcal{C}}$ possibly possessed before measurement by the corpuscular part $\mathcal{C}$ of the single objective wave $u^r(j, k)$ from the virtual collective $\{u(j, k)\}$ described by ψ, if $\mathcal{C}$ happens to be located, at the considered moment, at the point concerned by the definition of $\vec{p}_{\mathcal{C}}$. Moreover, we assume that a measurement of the momentum at any moment t does not create a new value of $\vec{p}_{\mathcal{C}}$, previously nonexistent, nor does it imply a reduction of the wave packet in the $\vec{p}$ space, but that it simply registers a preexisting value, the only real one contributing to the averages expressed in (5) and (6). These expressions do not concern "hidden" values of $\vec{p}_{\mathcal{C}}$, as in D.S. theory, nor values "created by the measurement" as in Q.M., but permanently defined momentum observables. Moreover, *the value of such a permanently defined momentum observable is implied in a mean value that is a function of the location of the measurement interaction.* Relating this dependence on the location – surprising from the point of view of Q.M. – to the definitions (7) and (8) of the presence probability, one obtains a distribution of the momentum observables which is a function of the position distribution. It can be shown that this distribution of permanently defined momentum observables can be coherently assumed to coincide, in noninterference states, with the quantum-mechanical distribution of momentum eigenvalues, but that it diverges from the quantum-mechanical distribution in interference states. The difficult problem of the possibility of joint probabilities for the position and the momentum seems to admit a solution, if these probabilities are defined in systematic agreement with the statistical significance we assign to ψ.[51]

This implies specific experimental consequences, permitting a test of the outlined theory, and also the collection of data significant for the study of the $u^r(j,k)$ waves and of the nonlinear equation by which these waves are assumed to be governed.

Analogous conclusions can be obtained, we hope, concerning the other dynamical quantities.

The observable and permanently (independently of the realization of a measurement) defined character possessed, according to our view, by the values defined for any dynamical quantity of a microsystem, are a fundamental characteristic of the theory outlined above.

6. *Remark on the Spatial Structure of ψ and on Its Time Evolution*

The ψ wave packet associated with a microsystem is strongly dependent on the structure of the source and on the process of emission (a spherical emission by a quasipoint source yields a ψ radically different from that describing an emission by an extended source in one definite direction, even if the microsystems emitted in the two cases are the same). Moreover, a ψ wave packet that propagates freely spreads out indefinitely, and this spreading does correspond to a physical fact: The emission of a quasipoint source creates effectively, after a certain time, a non-null probability of presence in a whole spherical layer centered on the source, and this layer does effectively spread out indefinitely. This is a fact, and it has to be described, so that the capacity of an unlimited spreading of ψ is a necessary feature, to be conserved.

But on the other hand, the objective $u^r(j;k)$ wave of one microsystem certainly has another type of dependence on the source and on the process of emission, and also another type of time evolution since *it cannot be conceived to spread out without limitation,* accordingly to our view.

Therefore a one-to-one extensional relation ψ-u, as in D.S. theory, as well as the idea that ψ and u are governed by the same equation of motion, constitute a ψ-u connection too restrictive to permit a free representation of these differences between ψ and u that have to be *a priori* admitted.

But the statistical relation ψ-u we propose could remove this obstacle, because it introduces a much more radical separation between a ψ wave and its corresponding u waves. If a ψ wave is conceived as describing global aspects of the evolution of a whole ensemble (virtual) of $u(j,k)$ waves, then the dependence of the form of the domain it covers on the geometry and nature of the source, as well as on the process of emission, become clearly understandable, and its capacity to spread out indefinitely becomes at the same time a natural and a necessary characteristic, adequately describable by a convenient superposition of solutions of a linear equation of motion, whereas the objectively existing $u^r(j,k)$ wave of one microsystem has then to be conceived as governed by another equation of motion, nonlinear, that determines the dimensions, shape, structure, as well as the dynamical properties of the studied microsystem, as sufficiently stable characteristics of this single microsystem (and of the fields that act on it), and moreover that forbid an unlimited spreading of the substance of this single microsystem. Obviously, the two distinct equations governing ψ and $u^r(j,k)$, as well as their solutions, have to be closely related.

7. *Conclusion*

We developed in this section a radically statistical conception concerning the significance of a quantum-mechanical ψ wave packet associated with one microsystem. This conception permitted a detailed characterization of the interference states, which led to a redefinition of the presence probability in such states, divergent from the quantum-mechanical one in an experimentally verifiable way. The same view leads, moreover, to the definition of position-dependent values of the momentum, which are shown elsewhere to be distributed as the quantum-mechanical eigenvalues of the momentum in noninterference states, but to have a non-quantum-mechanical distribution in interference states.

Contrary to the quantum-mechanical eigenvalues conceived in usual orthodox interpretations (an exception is found in Ref. 2) as being created by reduction during a measurement process, and contrary to the de Broglie-Bohm "hidden" values, all the values defined in our conception are permanently defined observables.

CONCLUSION

A systematic research of the implications of the completeness hypothesis, concerning specifically the possible interpretation of the unlimited spreading of free wave packets, introduces a first, minimally defined (in purely geometrical terms) unorthodox definition of a state vector. If de Broglie's model of a microsystem is modified so as to become coherent with this first definition of a state vector, this definition, in turn, can be completed on the basis of the modified model by physical assumptions that permit the development of a new conception of microsystems issuing from de Broglie's D.S. theory, but essentially different from D.S. theory. This conception permits a very detailed characterization of the structure of an interference state. Via this characterization it leads in principle to the construction of a new definition of the position probability distribution, diverging from the quantum-mechanical one in interference states, and experimentally recordable. At the same time a position-dependent definition of the momentum values is given that involves a new, position-dependent definition of the momentum probability distribution not established in this work.

This definition deviates also from the quantum-mechanical one, in interference states. Thus *the interference states appear as the Achilles' heel of Q.M.* This may seem a paradoxical result, concerning a theory that confers to the wavelike aspects of microphenomena a clear domination over the corpuscular aspects. (However, it certainly becomes less surprising as soon as the fact is taken into account that the only one-microsystem states that permit microinteraction are the auto-interference states.) Finally, the results obtained involve the dissolution of the ψ-reduction problem, as well as stimulating consequences, to be developed elsewhere, concerning the degree of statisticity of a given description of microsystems, and concerning the profound significance of the Schrödinger equation.

Despite the tentative character of the performed investigation, the results obtained may be interesting, we hope, in at least two respects:

In the first place, they yield an explicit formulation of the potentialities contained in the statistical hypothesis we made. This permits a clearer appreciation of the value of this hypothesis.

In the second place, they constitute a theoretical basis for further experimental research, concerning at the same time the completeness problem and certain characteristics of individual microsystems.

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