Proposed 3⁻ Recurrence of the 1⁻ Nonet*

R. H. Graham and T. S. Yoon
Department of Physics, University of Toronto, Toronto, Canada
(Received 24 January 1972)

A $J^P = 3^-$ nonet consisting of the states $\rho(1670)$, $\omega(1680)$, $K^*(1760)$, and $\phi(1820)$ is proposed. These mesons are taken to be the Regge recurrences of the 1^- nonet, and their various decays into 0^- and 1^- mesons are studied. A consistent scheme emerges, in which the 3^- nonet and its couplings to the lower-lying mesons may be described in terms of SU(3) symmetry and the quark model. Of particular interest is the result that the isosinglet members of the 3^- nonet appear to mix "ideally."

In the past few years the possibility that Regge trajectories rise linearly to infinity has been seriously entertained. There has been some supporting evidence for this view, the most notable perhaps being the $\rho(1670)$ or g meson with $J^P = 3^-$ which is assumed to be the recurrence of the ρ meson. It is well known that the linear Regge trajectory determined by the ρ and g mesons has an intercept at $s = 0$ approximately equal to the value ($\approx\frac{1}{2}$) which is obtained from the analysis of πN charge-exchange scattering experiments. In this note we want to present evidence for the existence of a nonet of $J^P = 3^-$ resonances which are the linear recurrences of the vector-meson nonet. These are

$\rho(1670)$: $I^G = 1^+$, $Y = 0$,

$\omega(1680)$: $I^G = 0^-$, $Y = 0$ (see Ref. 1),

$K^*(1760)$: $I = \frac{1}{2}$, $Y = \pm 1$,

$\phi(1820)$: $I^G = 0^-$, $Y = 0$.

With these assignments all 3^- trajectories (ρ, ω, K^*, and ϕ) on the Chew-Frautschi plots are closely parallel to one another.

While the assignment of these resonances to the 3^- nonet is tentative, we find that a consistent picture emerges in which the couplings of these resonances to lower-lying mesons are governed by SU(3) symmetry and the quark model.

We begin with a brief summary of the experimental status of these resonances.

$\rho(1670)$ or g Meson

The spin-parity of the g meson is well established to be 3^-.[2-6] However, the mass and the width are subject to wild fluctuations from one experiment to another. Relying heavily on some recent experimental work,[4,5,7] we adopted the following values for our study: $M = 1670$ MeV, $\Gamma = 150 \pm 50$ MeV.

Recent studies indicate that the main decay modes of the g meson are $\pi\pi$, $K\bar{K}$, $\bar{K}^*K$, and 4π. Spin-parity analysis has been possible only on the $\pi\pi$ channel. There is some controversy as to the identity of the 4π enhancement which appears at a mass slightly higher than the 2π g meson. The 4π decay is further shown to be mainly to $\rho\rho$ and $\omega\pi$, but not to $A_2\pi$.[7] We have based our analysis on a tentative assumption that the 4π decay mode exclusive of the $\omega\pi$ mode is due to $\rho\rho$. Experimentally, it is difficult to separate the $\rho 2\pi$ (2π not in ρ) mode from the $\rho\rho$ mode. The branching ratios used for the present analysis are given in Table I. 4π and $\omega\pi$ branching fractions are from Ref. 7 and the $K\bar{K}$ and the $K^*\bar{K}$ branching fractions are from Ref. 4.

$K^*(1760)$

The observation of a $I = \frac{1}{2}$, $J^P = 3^-$ resonance decaying to $(K\pi)^0$ and $(K\pi\pi)^0$, final states in K^+d reactions at 9 GeV/c was first reported by Carmony *et al.*[8] The presence of a resonant $K\pi$ amplitude at about this mass with $J^P = 3^-$ was also reported by Firestone *et al.*[9] However, strong interference with the background processes prevented them from making reliable determination of the mass and the width. We have used for our study the mass and the width given by Carmony *et al.*, $M = 1759 \pm 10$ MeV and $\Gamma = 60 \pm 20$ MeV, respectively. The two branching fractions reported by Carmony *et al.* are listed in Table I.

$\omega(1680)$

$\omega(1680)$ was discovered by Armenise *et al.*[10] and was later confirmed by Kenyon *et al.*[11] and by Matthews *et al.*[12] Its only established decay mode is $\rho\pi$. The mass and the width used for our analysis are obtained from the weighted averages of the two latest experiments.[11,12] These are

$M = 1675 \pm 14$ MeV,

TABLE I. Total widths and branching fractions used for the fit are tabulated. The third column represents the results of a 5-parameter fit with $R=0.2$ F. The last column represents a 3-parameter fit with f and f' fixed at -1.4 and 4, respectively ($R=0.2$ F also).

Name	Input values	Fitted values 5-parameter fit	3-parameter fit
Γ_g	150 ± 50 MeV	150 MeV	198 MeV
$\Gamma_{\omega(1680)}$	144 ± 19 MeV	144 MeV	134 MeV
$\Gamma_{K^*(1760)}$	60 ± 20 MeV	84 MeV	84 MeV
$\dfrac{g \to 2\pi}{g \to (\rho\rho + \omega\pi)}$	0.30 ± 0.10	0.30	0.26
$\dfrac{g \to \omega\pi}{g \to (\rho\rho + \omega\pi)}$	0.22 ± 0.04	0.22	0.24
$\dfrac{g \to K\bar{K}}{g \to 2\pi}$	0.08 ± 0.03	0.13	0.13
$\dfrac{g \to K^*\bar{K} + \bar{K}^*K}{g \to 2\pi}$	0.10 ± 0.03	0.08	0.08
$\dfrac{K_N(1760) \to K\pi}{K_N(1760) \to K\rho + K^*\pi}$	0.40 ± 0.10	0.48	0.48
$\dfrac{K_N(1760) \to K^*\pi}{K_N(1760) \to K\rho + K^*\pi}$	0.40 ± 0.15	0.61	0.61

$\Gamma = 144 \pm 19$ MeV.

The spin-parity of the resonance is not well determined. Kenyon *et al.* favor 1^+ and 2^- over 1^-, 2^+, 3^- assignments. Matthews *et al.* favor natural spin-parity assignments 1^-, 2^+, and 3^- over unnatural assignments 1^+, 2^-, 3^+. Both experiments rule out 0^-. The possibility of this being a recurrence of $\omega(784)$ was first speculated by Matthews *et al.*

$\phi(1820)$

One possible candidate for the recurrence of $\phi(1020)$ is the (K_1^0MM, MM$>K^0\pi^0$) enhancement seen by French *et al.* in $\bar{p}p$ annihilations at 3.0, 3.6, and 4.0 GeV/c.[13] The mass and the width are $M=1820 \pm 12$ MeV, $\Gamma = 50 \pm 23$ MeV, respectively. Since the nature of this decay is not well established we have not made use of this data. Rather, we explored the consequence of the existence of $\phi(1820)$ at this mass and studied various branching modes.

In order to test the above assignments we have studied the three types of two-body decays of the 3^- nonet into vector (1^-) and pseudoscalar (0^-) mesons: $3^- \to 0^- + 0^-$, $3^- \to 0^- + 1^-$, and $3^- \to 1^- + 1^-$. Consistent with the absence of πA_2 decay mode for the g meson we have excluded the $3^- \to 2^+ + 0^-$ channel in this analysis. We make the assumption that the couplings of the 3^- mesons to the 1^- and 0^- mesons are given by exact SU(3).[14] The partial widths of the decays then have the general form

$$\Gamma = g^2 C^2 \Phi ,$$

where g is an over-all coupling constant, C is determined from SU(3) Clebsch-Gordan coefficients, and Φ is a phase-space and barrier-penetration factor. SU(3)-symmetry-breaking effects are taken into account by making use of the physical masses in Φ and through the mixing of $\omega(784)$ with $\phi(1020)$ and $\omega(1680)$ with $\phi(1820)$.

The relevant expressions for C are given in Table II. They depend on the usual mixing angles θ_1 and θ_3 of the 1^- and 3^- nonets and, in the case of the $3^- \to 1^- + 0^-$ decays, on the parameters f and f' as well. f measures the relative coupling of a 1^- singlet to 3^- and 0^- octet states and f' that of a 3^- singlet to 1^- and 0^- octets. Consistency with our assumption that the 3^- nonet is the recurrence of the 1^- nonet requires that $\theta_3 = \theta_1 = \theta$. The adopted assignment of the 3^- nonet indeed leads to an octet-singlet mixing angle of $\theta_3 = 28° \pm 10°$. Since available data do not allow an independent determination of this mixing angle we have assumed that θ takes its "ideal" value[15] given by $\tan\theta = 1/\sqrt{2}$, that is, $\theta = 35°$.

We have chosen for the phase space factor Φ a modified version of a form suggested by Quigg and von Hippel.[16] Namely, we take

$$\Phi = f(M^2)\rho^{-1}|h_l^{(1)}(\rho)|^{-2}, \quad \rho = qR$$

TABLE II. Relevant 3^- decay modes are tabulated. The subscripts 1 and 3 refer to the 1^- and 3^- nonets, respectively. We have adopted the shorthand notation S_1, C_1, S_3, and C_3 for $\sin\theta_1$, $\cos\theta_1$, $\sin\theta_3$, and $\cos\theta_3$, respectively. Column 2 gives the SU(3) Clebsch-Gordan coefficients. Column 3 gives the partial widths calculated from the 5-parameter fit with $R = 0.2$ F. Column 4 gives the partial widths obtained from the 3-parameter fit with f and f' fixed at -1.4 and 4, respectively ($R = 0.2$ F also).

	Decay mode	SU(3) coefficients	Partial widths (MeV) 5-parameter fit	3-parameter fit
$3^- \to 0^- + 0^-$	$\rho_3 \to 2\pi$	$\frac{1}{3}\sqrt{3}$	33	39
	$\rho_3 \to K\bar{K}$	$\frac{1}{6}\sqrt{6}$	4.2	5.0
	$\omega_3 \to K\bar{K}$	$-(1/\sqrt{2})S_3$	4.3	5.1
	$K_3^* \to K\pi$	$\frac{1}{2}$	19	22
	$K_3^* \to K\eta$	$\frac{1}{2}$	8	9.5
	$\phi_3 \to K\bar{K}$	$(1/\sqrt{2})C_3$	17	20
$3^- \to 1^- + 0^-$	$\rho_3 \to \omega\pi$	$-S_1 + C_1 f$	24	37
	$\rho_3 \to \rho\eta$	1	2.1	2.4
	$\rho_3 \to K^*\bar{K}, \bar{K}^*K$	$\frac{1}{2}\sqrt{6}, \frac{1}{2}\sqrt{6}$	2.7	3.3
	$\rho_3 \to \phi\pi$	$C_1 + S_1 f$	0	0
	$\omega_3 \to \rho\pi$	$\sqrt{3}\, S_3 + \frac{1}{4}\sqrt{6}\, C_3 f'$	133	123
	$\omega_3 \to K^*\bar{K}, \bar{K}^*K$	$\mp\frac{1}{2}\sqrt{2}\, S_3 \pm \frac{1}{2}C_3 f'$	4.7	3.5
	$\omega_3 \to \omega\eta$	$-S_3S_1 + \frac{1}{4}\sqrt{2}\, C_3S_1f' - S_3C_1f$	1.9	2.2
	$\omega_3 \to \phi\eta$	$S_3C_1 - \frac{1}{4}\sqrt{2}\, C_3C_1f' - S_3S_1f$	0	0
	$K_3^* \to \rho K$	$\frac{3}{2}$	16	18
	$K_3^* \to K^*\pi$	$\frac{3}{2}$	24	28
	$K_3^* \to \phi K$	$-\frac{1}{2}C_1 + S_1 f$	0.7	1.1
	$K_3^* \to \omega K$	$+\frac{1}{2}S_1 + C_1 f$	2.7	5.4
	$K_3^* \to K^*\eta$	$\frac{1}{2}$	0.4	0.5
	$\phi_3 \to \rho\pi$	$-\sqrt{3}\, C_3 + \frac{1}{4}\sqrt{6}\, S_3 f'$	2.2	0
	$\phi_3 \to K^*\bar{K}, \bar{K}^*K$	$\pm\frac{1}{2}\sqrt{2}\, C_3 \pm \frac{1}{2}S_3 f'$	38	35
	$\phi_3 \to \phi\eta$	$-C_3C_1 - \frac{1}{4}\sqrt{2}\, S_3C_1f' + S_1C_3f$	3.4	3.9
	$\phi_3 \to \omega\eta$	$C_3S_1 + \frac{1}{4}\sqrt{2}\, S_1S_3f' + C_3C_1f$	0.6	0
$3^- \to 1^- + 1^-$	$\rho_3 \to \rho\rho$	$\frac{1}{3}\sqrt{3}$	85	113
	$K_3^* \to K^*\rho$	$\frac{1}{2}$	41	55
	$K_3^* \to K^*\omega$	$\frac{1}{2}S_1$	11	15
	$\phi_3 \to K^*\bar{K}^*$	$\frac{1}{2}\sqrt{2}\, C_3$	12	16

where $h_l^{(1)}$ is a Hankel function of the first kind, l is the orbital angular momentum of the final decay state, M is the mass of the 3^- meson, q is the c.m. momentum of a final meson, and R is the strong-interaction radius. We take l to be (3, 3, 1) and $f(M^2)$ takes[17] the form $(M^{-2}, 1, M^{-2})$ for the $(0^-0^-, 0^-1^-, 1^-1^-)$ channels, respectively. The Hankel function in the phase-space expressions results from an attempt to take into account centrifugal-barrier effects associated with a finite strong-interaction radius.

Three observed total widths and six branching fractions listed in Table I are compared with the model by means of a least-squares fitting method.

The unknown parameters are the three over-all coupling constants for the three decay channels ($3^- \to 0^-0^-$, 1^-0^-, and 1^-1^-) and f and f' for the 1^-0^- decay. To compare a total width with the corresponding experimental value all partial widths for allowed decay modes were calculated and added whether such modes were observed or not.

The effect of R, the strong-interaction radius, on the fit was studied over all possible values of R. Figure 1 gives the variation of χ^2 as a function of R. Although the meager data available so far favor $R \to 0$, it is clear that any fit with $R < 0.4$ **F** is acceptable. It should be pointed out that the higher χ^2 at larger R is mainly due to the R dependence of the branching fraction, $[\rho(1670) \to K\bar{K}]/[\rho(1670) \to \pi\pi]$. In the following, we choose to discuss the results of a fit with the strong interaction radius fixed at 0.2 **F**. The χ^2 value of 6.8 with 4 degrees of freedom corresponds to a confidence level of 30%.[18]

The fitted total widths and the branching fractions are listed in Table I to be compared with the input values. It is clear that the model fits the data remarkably well. The following points are noteworthy:

(1) In the quark model, the decay $g(1670) \to \phi\pi$ is forbidden if the mixing angle is such that $\phi(1020)$ is a pure strange quark-antiquark system. Similar arguments hold for the decay of $\phi(1820) \to \rho\pi$. The model indeed predicts small partial widths for both of these decay modes (Table II). The SU(3) factors for the respective processes are identically zero if (see Table II)

$$f = -\cot\theta = -1.4,$$

$$f' = 2\sqrt{2}\,\cot\theta_3 = 4.$$

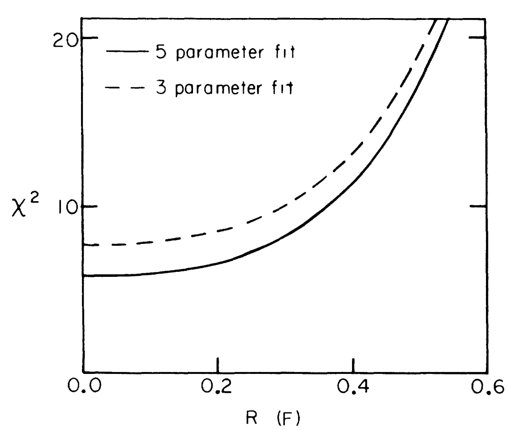

FIG. 1. Plot of χ^2 vs R, the strong-interaction radius. The solid line represents a 5-parameter fit with 4 degrees of freedom. The dashed line is a 3-parameter fit (6 degrees of freedom) with f and f' fixed at -1.4 and 4, respectively (see text).

The values obtained from the least-squares fitting are

$$f = -1.2 \pm 0.38,$$

$$f' = 4.8 \pm 1.5.$$

These are remarkably close to the "ideal" values, which lends support for the quark model. A fit with f and f' fixed at their "ideal" values, -1.4 and 4, respectively, is shown in the last columns of Tables I and II; the corresponding χ^2 dependence on "R" is shown as a dashed line in Fig. 1.

(2) The computed total width for $\phi(1820)$ is 72.5 MeV and the most important decay mode is to $K^*(890)\bar{K} + \bar{K}^*(890)K$. Both of these features are consistent with the enhancement observed by French *et al.*[13]

(3) Corresponding to the $\rho\rho$ decay of the g meson, strong $K^*\rho$ and $K^*\omega$ decays are predicted for the $K^*(1760)$. Observation of such modes would render strong support for the 3^- nonet scheme proposed here.

(4) Consistent with the idea that the 3^- nonet is a Regge recurrence of the 1^- nonet, all mass formulas applying to 1^- nonets are well satisfied for the 3^- nonet. For example, the Schwinger mass formula[19]

$$(\phi^2 - \omega_8{}^2)(\omega^2 - \omega_8{}^2) = -\tfrac{8}{9}(K^{*2} - \rho^2)^2,$$

when applied to the 3^- nonet, yields -0.04 ± 0.03 GeV^4 for the left-hand side and -0.08 ± 0.03 GeV^4 for the right-hand side.

(5) The couplings of the g meson to $\pi\pi$ and $\pi\omega$ have been related to the corresponding couplings of ρ to $\pi\pi$ and $\pi\omega$.[20] This relation, which is derived on the basis of bootstrap requirements placed on finite-energy sum rules involving linearly rising trajectories, is given by[21]

$$\frac{g_{\rho_3\pi\pi}\, g_{\rho_3\omega\pi}}{g_{\rho\pi\pi}\, g_{\rho\omega\pi}} = 0.87 \times 10^{-1}\ \text{GeV}^{-4}.$$

This may be tested, up to a sign, from our fit. Converting this into a relation among the respective widths and taking[22] $g_{\rho\omega\pi}{}^2/4\pi$ to be 23 GeV^{-2}, we obtain

$$\frac{\Gamma(\rho_3 \to 2\pi)\Gamma(\rho_3 \to \omega\pi)}{m_\rho \Gamma(\rho \to 2\pi)} = (2.0 \pm 0.4) \times 10^{-2}.$$

Using a ρ mass and width of 760 MeV and 130 MeV, respectively, and Table II, the left-hand side is found to be 0.8×10^{-2} for the 5-parameter fit while the fit with the "ideal" f and f' yields 1.5×10^{-2}.

In conclusion, we feel that the evidence presented here for the existence of a 3^- nonet is more than circumstantial. Although the spin and parity of some of the proposed states have not been determined, the assignments are consistent with their being the recurrence of the 1^- nonet along linear Regge trajectories. The description of the 3^- nonet and its decays into lower-lying mesons lends itself to a simple description in terms of the quark model.[23] In this scheme the 3^- nonet would be composed of excited quark-antiquark states and the two isosinglets would mix "ideally."

---

*Supported in part by the National Research Council of Canada.

[1]This has been usually referred to as $\phi(1680)$. In this note we adopt more informative notation based on the assumption of its being a recurrence of $\omega(784)$.

[2]D. J. Crennel, U. Karshon, K. W. Lai, J. M. Scarr, and I. O. Skillicorn, Phys. Letters 28B, 136 (1968).

[3]T. F. Johnston, J. D. Prentice, N. R. Steenberg, T. S. Yoon, A. F. Garfinkel, R. Morse, B. Y. Oh, and W. D. Walker, Phys. Rev. Letters 20, 1414 (1968).

[4]J. Bartsch, G. Kraus, N. Tasnos, C. Grote, H. Kotzan, A. Meyer, E. Ryseck, A. Angelopoulous, G. Ascoli, K. Barnham, V. T. Cocconi, P. F. Dalpiaz, R. Ely, J. D. Hansen, D. R. O. Morrison, J. Schreiber, and H. Tofte, Nucl. Phys. B22, 109 (1970).

[5]J. A. J. Matthews, J. D. Prentice, T. S. Yoon, J. T. Carroll, M. W. Firebaugh, and W. D. Walker, Nucl. Phys. B33, 1 (1971).

[6]G. Grayer, B. Hyams, C. Jones, P. Schlein, W. Blum, H. Dietl, W. Koch, E. Lorenz, G. Lutjens, W. Manner, J. Meissburger, W. Ochs, U. Stierlin, and P. Weilhammer, Phys. Letters 35B, 610 (1971).

[7]T. S. Yoon, T. F. Johnston, J. A. J. Matthews, M. Meiners, J. D. Prentice, P. N. Robillard, N. R. Steenberg, J. T. Carroll, M. W. Firebaugh, A. F. Garfinkel, R. Morse, B. Y. Oh, and W. D. Walker, in *Particles and Fields–1971,* proceedings of the 1971 Rochester Meeting of the Division of Particles and Fields of the American Physical Society, edited by A. C. Melissinos and P. F. Slattery (A.I.P., New York, 1971).

[8]D. D. Carmony, D. Cords, H. W. Clopp, A. F. Garfinkel, R. F. Holland, F. J. Loeffler, H. B. Mathis, L. K. Rangan, J. Erwin, R. L. Lander, D. E. Pellet, P. M. Yager, F. T. Meiere, and W. L. Yen, Phys. Rev. Letters 27, 1160 (1971).

[9]A. Firestone, G. Goldhaber, D. Lissauer, and G. H. Trilling, Phys. Letters 36B, 513 (1971).

[10]N. Armenise, B. Ghidini, V. Picciarelli, A. Romano, A. Forino, R. Gessaroli, L. Lendinara, G. Quareni, A. Quareni-Vignudelli, A. Cartacci, M. Dagliana, G. di Caporiacco, G. Parrini, M. Barrier, J. Laberrique-Frolow, and J. Quinguard, Phys. Letters 26B, 336 (1969).

[11]I. R. Kenyon, J. B. Kinson, J. M. Scarr, I. O. Skillicorn, H. O. Cohn, R. D. McCulloch, W. H. Bugg, G. T. Condo, and M. M. Nussbaum, Phys. Rev. Letters 23, 146 (1969).

[12]J. A. J. Matthews, J. D. Prentice, T. S. Yoon, J. T. Carroll, M. W. Firebaugh, and W. D. Walker, Lett. Nuovo Cimento 1, 361 (1971).

[13]B. R. French, J. B. Kinson, R. Rigopoulos, V. Simak, F. McDonald, G. Petmezas, and L. Riddiford, Lett. Nuovo Cimento 52A, 442 (1967).

[14]Our approach is in the same spirit as earlier work on the 1^- and 2^+ nonets. See, e.g., S. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963); S. Glashow and R. H. Socolow, Phys. Rev. Letters 15, 325 (1965); M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, Nuovo Cimento 45A, 169 (1966).

[15]S. Okubo, Phys. Letters 5, 165 (1963); J. J. Sakurai, Phys. Rev. 132, 434 (1963).

[16]C. Quigg and F. Von Hippel, in *Experimental Meson Spectroscopy,* edited by C. Baltay and A. H. Rosenfeld (Columbia Univ. Press, New York, 1970), p. 477; also F. Von Hippel and C. Quigg, ANL report, 1971 (unpublished).

[17]We have chosen the mass dependence $f(M^2)$ such that, in the limit $R \to 0$, Φ takes the forms obtained from a description of the decays by Lagrangians with minimal derivative coupling.

[18]We have considered other forms of $f(M^2)$, namely 1 and $1/M^2$ for all three channels. These yielded somewhat higher χ^2 values than our choice of $f(M^2)$. We have also considered the barrier factor employed by Glashow and Rosenfeld (Ref. 14). Although the χ^2 values converge to our result as $R \to 0$, the fit is much poorer at finite R; e.g., at $R = 0.2$ F, we obtain χ^2 of 26 for the 3-parameter fit.

[19]J. Schwinger, Phys. Rev. 135, B816 (1964).

[20]M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. 176, 1095 (1968).

[21]The couplings are defined in Ref. 20.

[22]This estimate has been based on the Gell-Mann–Sharp–Wagner model: M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters 8, 261 (1962).

[23]For a discussion of meson nonets, and their possible recurrences, based on the quark model see, e.g., R. H. Dalitz, in *Meson Spectroscopy,* edited by C. Baltay and A. H. Rosenfeld (Benjamin, New York, 1968), p. 497.