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Value of the ξ Parameter in K_{l3} Decay from the Pion-Gauge Condition*

A. O. Barut and Z. Z. Aydin†
Institute for Theoretical Physics, University of Colorado, Boulder, Colorado 80302
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It is shown with the help of the pion-gauge condition that the divergence of the most general boson current in K_{l3} decay vanishes in the limit of zero four-momentum of the pion. This in turn implies $\xi=-1$ at the (unphysical) point $t=m_K^2$. Inasmuch as ξ does not seem, experimentally, to vary much with t, the result is significant in the physical region and agrees with experiment.

Recently, Fischbach *et al.*[1,2] reported reasonably good values for some of the K_{l3} decay parameters by assuming a zero in the so-called scalar form factor $f_0(t)=(m_K^2-m_\pi^2)f_+(t)+tf_-(t)$ at the threshold $t=t_0\equiv(m_K+m_\pi)^2$, which they obtained using part of the Kemmer currents for bosons. It was then correctly pointed out[3] that the vanishing of $f_0(t)$ at $t=t_0$ cannot be purely kinematical, as claimed.

We show in this note that the most general boson current in Kemmer form coupled to leptons is conserved, not at $t=t_0$ as previously claimed,[1,2] but at the (unphysical) point $p_\pi^\mu\to 0$. This is achieved by applying the so-called "pion-gauge condition" to the K_{l3} decay. According to this condition the invariant amplitude vanishes at the (unphysical) point $p_\pi^\mu\to 0$.[4-8]

If we impose this condition, we find a zero in $f_0(t)$ at $t=t_0'\equiv m_K^2$, i.e., the zero-pion-mass limit of $t_0=(m_K+m_\pi)^2$ of Ref. 1. This condition gives us, as we shall see, a value $\xi(m_K^2)=f_-/f_+=-1$, instead of $\xi((m_K+m_\pi)^2)=-(m_K-m_\pi)/(m_K+m_\pi)$ of Ref. 1 (which by the way goes over to our value for $m_\pi^2=0$). Inasmuch as ξ does not seem to depend much experimentally on t (see Fig. 1), our result is also significant in the physical region.

We now show the details of the argument. There are three vector currents in the Kemmer formalism:

$$J_1^\lambda\equiv i\bar{u}_\pi\beta^\lambda u_K=\tfrac{1}{2}(p_K^\lambda/m_K+p_\pi^\lambda/m_\pi)\,, \tag{1}$$

$$J_2^\lambda\equiv q^\lambda\bar{u}_\pi u_K/(m_K+m_\pi)=q^\lambda(t_0-t)/4m_K m_\pi(m_K+m_\pi)\,, \tag{2}$$

$$J_3^\lambda\equiv i\bar{u}_\pi\beta^\lambda C u_K=\tfrac{1}{2}(p_K^\lambda/m_K-p_\pi^\lambda/m_\pi)\,, \tag{3}$$

where $q=p_K-p_\pi$, $t=q^2$, and C is a 5×5 matrix given explicitly in Ref. 9. Fischbach *et al.*[1] have chosen only the first two terms. However, the last term can be written as

$$J_3^\lambda=-\frac{m_K-m_\pi}{m_K+m_\pi}J_1^\lambda+\frac{4m_K m_\pi}{t_0-t}J_2^\lambda\,. \tag{4}$$

Now if one starts with the combination

$$V^\lambda\equiv g_1(t)J_1^\lambda+g_2(t)J_2^\lambda+g_3(t)J_3^\lambda\,, \tag{5}$$

which can be rearranged as

$$V^\lambda=\left(g_1(t)-g_3(t)\frac{m_K-m_\pi}{m_K+m_\pi}\right)J_1^\lambda+\left(g_2(t)-g_3(t)\frac{4m_K m_\pi}{t-t_0}\right)J_2^\lambda\,,$$

or

$$V^\lambda=g_V(t)J_1^\lambda+g_S(t)J_2^\lambda\,,$$

or

$$V^\lambda=[f_+(t)(p_K+p_\pi)^\lambda+f_-(t)(p_K-p_\pi)^\lambda]\times(4m_\pi m_K)^{-1/2}\,, \tag{6}$$

one concludes[3] that the basic assumption of Fischbach *et al.*[1,2] that $g_S(t)$ should be regular at $t=t_0$ fails if $g_3(t)$ has no zero at $t=t_0$, or if $g_2(t)$ does not have a pole at $t=t_0$ with the exact residue $4m_K m_\pi g_3(t_0)$. (This conclusion is also true for vanishing g_2 or g_1 or both.) In Eq. (6), the connection between $f_\pm$ and g_V, g_S is given by

$$(4m_K m_\pi)^{1/2}f_+(t)=(m_K+m_\pi)g_V(t)\,,$$

$$(4m_K m_\pi)^{1/2}(m_K+m_\pi)f_-(t)=-(m_K^2-m_\pi^2)g_V(t)-(t-t_0)g_S(t)\,.$$

Now, the so-called scalar form factor $f_0(t)$ is the divergence of the matrix element of the boson current in K_{l3} decay. One readily verifies that

$$\partial_\lambda J_1^\lambda=\frac{m_K-m_\pi}{4m_K m_\pi}(t_0-t)\,, \tag{7}$$

$$\partial_\lambda J_2^\lambda=\frac{t(t_0-t)}{4m_K m_\pi(m_K+m_\pi)}\,, \tag{8}$$

$$\partial_\lambda J_3^\lambda=\frac{m_K+m_\pi}{4m_K m_\pi}[t-(m_K-m_\pi)^2]\,. \tag{9}$$

It then follows from (7), (8), and (6) that if one takes the (K,π) current as a linear combination of

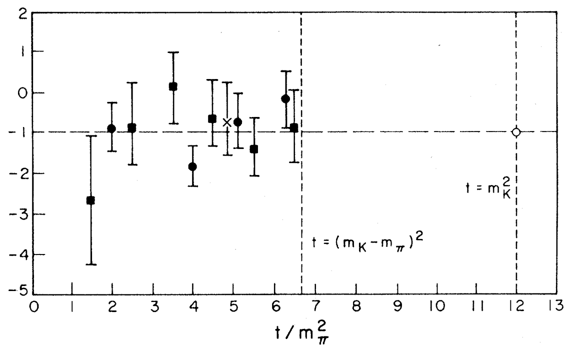

FIG. 1. Determination of ξ by the μ^+ polarization analysis as a function of t. The rectangular points are from Bettels *et al.* (Ref. 12), the circular points from Cutts *et al.* (Ref. 12), and the X-shaped point from Callahan *et al.* (Ref. 12).

J_1^λ and J_2^λ [Eqs. (1) and (2)] alone, then $f_0(t)$ has a zero at $t=t_0$. This is the argument of Ref. 1. But $\partial_\lambda J_3^\lambda \neq 0$ at $t=t_0$ and their argument fails, unless one can find a (dynamical) reason to exclude J_3^λ, or to give a zero to $g_3(t)$ in Eq. (5) at $t=t_0$.

Now we will impose the pion-gauge condition (PGC)

$$\lim_{p_\pi^\mu \to 0} M = 0, \qquad (10)$$

where M is the *invariant* amplitude for $K \to \pi l \nu$ decay. It is written in terms of the Klein-Gordon current in the usual form,

$$M = \frac{G \sin\theta_c}{\sqrt{2}} [(p_K + p_\pi)^\lambda f_+(t) + (p_K - p_\pi)^\lambda f_-(t)] \times [\bar{u}_l \gamma_\lambda (1-\gamma_5) u_\nu], \qquad (11)$$

and in terms of the Kemmer currents as follows:

$$M = \frac{G \sin\theta_c}{\sqrt{2}} (4 m_\pi m_K)^{1/2} V^\lambda [\bar{u}_l \gamma_\lambda (1-\gamma_5) u_\nu], \qquad (12)$$

where V^λ is given in Eq. (5) with J_i^λ's defined by Eqs. (1)–(3). Note that there is a factor $(4E_\pi E_K V^2)^{-1/2}$ in front of both of the amplitudes (11) and (12) (see Ref. 1), when the phase space is taken to be $\prod_i d^3 p_i$. For our invariant amplitudes (11) or (12) the phase space is $\prod_i d^3 p_i / 2 p_i^0$.

We now see immediately from the Kemmer form of the amplitude, Eq. (12), that, because V^λ is finite in the limit $p_\pi^\lambda \to 0$, the K_{l3}-decay invariant amplitude *does* indeed satisfy the PGC, Eq. (10), a result which cannot be seen directly from the Klein-Gordon form (11). On the other hand, if we now impose the PGC, Eq. (10), in Eq. (11) we obtain immediately the relation

$$f_+(t=m_K^2) + f_-(t=m_K^2) = 0 \qquad (13)$$

or

$$\xi(m_K^2) = f_-/f_+ |_{t=m_K^2} = -1. \qquad (13')$$

By substituting the result (13) into the so-called scalar form factor $f_0(t)$ we find that $f_0(t)$ vanishes in the limit $p_\pi^\lambda \to 0$. Thus, because $f_0(t)$ is proportional to the divergence of the matrix element of the boson current in K_{l3} decay, the boson current coupled to the lepton current is conserved at the (unphysical) point $p_\pi^\lambda \to 0$, i.e.,

$$\lim_{p_\pi^\lambda \to 0} g_i(t) \partial^\lambda J_{i\lambda} = 0, \quad i = 1, 2, 3. \qquad (14)$$

Equations (4)–(9), when compared with (14), yield the restrictions on the $g_i(t)$ imposed by the pion-gauge condition.

We should like to remark that the above considerations are essentially S-matrix considerations, the currents being used to write down the most general vertex. If, however, we postulate a field theory with an interaction Lagrangian using one of the currents in Eqs. (1)–(3), then, to lowest order, we can make predictions on the value of ξ, the form factors being constants in the lowest order:

Interaction current	ξ
J_1	-0.57
J_2	∞ ($f_+ \equiv 0$)
J_3	-1.75 .

An example of a perturbation theory of weak interactions based on J_3^λ alone was given some time ago in Ref. 9. The calculations with J_1^λ alone are reported elsewhere.[10]

The pion-gauge condition, variously derived or justified from different points of view,[5–7] and applied to several other processes,[8] is not generally used. It is, of course, different from and stronger than the usual soft-pion limit of current algebra. This is why the result (13) or (13′) is different from the Callan-Treiman-Suzuki-Mathur-Okubo-Pandit relation.[11] The apparent agreement of the result (13′) with experiment[12] (Fig. 1) may be considered as a support for the validity of the pion-gauge condition.

Finally we mention that some time ago Kang[13] postulated phenomenologically a zero in $f_0(t)$ between $(m_K - m_\pi)^2$ and $(m_K + m_\pi)^2$ in order to explain the experimental value $\xi(0) \cong -1$. The pion-gauge condition provides a theoretical basis for this assumption. Kang shows furthermore that the theoretical arguments based on the soft-pion limit of the SU(3)×SU(3) current algebra is not sufficient to conclude $\xi \cong -1$, i.e., the Callan-Treiman-

Suzuki-Mathur-Okubo-Pandit relation would not extrapolate smoothly to the on-mass-shell point; the relation (13) or (13′) does extrapolate smoothly to the physical point.

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Unified Explanation of the $K_L^0 \to 2\mu$ Puzzle, *CP* Nonconservation, and the Excess Muon Anomaly*

S. Pakvasa and S. F. Tuan

Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822

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We propose a common explanation of (a) the $K_L^0 \to \mu^+\mu^-$ puzzle, (b) *CP* nonconservation, and (c) the cosmic-ray muon anomaly of Utah. The model involves a pair of neutral bosons with pairwise strong interactions with known hadrons. Experimental implications are discussed.

It is the purpose of the present note to study the consequences of assuming that the following three outstanding puzzles have a unified explanation. These puzzles are

(a) the anomalously low branching ratio of $K_L^0 \to \mu^+\mu^-$,[1] viz., B. R. $(K_L^0 \to \mu^+\mu^-) \leq 1.8 \times 10^{-9}$ (90% confidence),

(b) the long-standing search for the mechanism leading to *CP* nonconservation in K_L^0 decays,

(c) the Utah effect[2] or the unexpected isotropic component in cosmic-ray muons for muon energies $\gtrsim 1$ TeV.

That (a) and (b) may be related was first stressed by Christ and Lee.[3] A specific model in terms of *CP*-violating neutral currents which synthesizes (a) and (b) has been suggested by Wolfenstein[4] recently. The proposition that (c) the X particle[5] responsible for the Utah effect[2] is also involved in resolution of (a) and (b) is of course a purely theoretical speculation. It is however consistent with existing experiments and can be tested experimentally.

THE MODEL

Wolfenstein[4] pointed out that the most economic way to understand the $K_L^0 \to 2\mu$ puzzle in terms of *CP* nonconservation [(a) and (b)] is to postulate that there exists a *CP*-noninvariant interaction H' involving a neutral muon current which allows the decay $K_1^0 \to \mu^+\mu^-$ but not the decay $K_2^0 \to \mu^+\mu^-$. A simple model for H' satisfying these constraints is

$$H' = G' \sin\theta\, A_\lambda^7 \bar{\psi}_\mu \gamma_\lambda \gamma_5 \psi_\mu\,, \tag{1}$$

$$A_\lambda^7 = (A_\lambda^K + A_\lambda^{\bar{K}})/\sqrt{2}\,, \tag{2}$$

where A_λ^7 is the seventh component of the hadronic