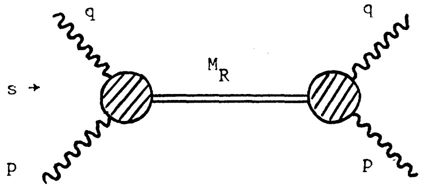

FIG. 1. Resonance production by photon-photon scattering.

The scaling limit is defined by $-q^2 \to \infty$ and $\nu W_2^\gamma(q^2, \nu) \to F_2^\gamma(\omega)$. It has been proposed that the photon and proton scale-invariant functions $F_2^\gamma(\omega)$ and $F_2^p(\omega)$ are proportional, that is,

$$F_2^\gamma(\omega) \simeq (\sigma_{\gamma p}/\sigma_{pp}) F_2^p(\omega) \simeq \tfrac{1}{300} F_2^p(\omega).$$ [1,6]

We choose the asymptotic value $F_2^p(\omega) \to 0.3$ and find from Eq. (5) that

$$|A(q^2, \nu_R)|^2 \simeq 10^{-3}\left(\frac{q^2 + 2\nu_R}{M_R{}^2 - q^2}\right). \tag{6}$$

The main result of this paper is our sum rule, Eq. (4), and we believe that there is a connection between a resonance and the scaling property of the photon structure function νW_2^γ.

[1] S. J. Brodsky, T. Kinoshita, and H. Terazawa, Phys. Rev. Letters 27, 280 (1971).
[2] R. W. Brown and I. J. Muzinich, Phys. Rev. D 4, 1496 (1971).
[3] C. L. Starke, Phys. Rev. D 5, 79 (1972).
[4] E. D. Bloom and F. J. Gilman, Phys. Rev. Letters 25, 1140 (1970).
[5] J. W. Moffat and V. G. Snell, Phys. Rev. D 4, 1452 (1971).
[6] R. L. Jaffe, Phys. Rev. D 4, 1507 (1971).

PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Multiple Counting in the Experimental Measurement of Diffractive Dissociation

G. F. Chew* and S. D. Ellis
National Accelerator Laboratory, Batavia, Illinois 60510
(Received 28 August 1972)

An experimentally measurable distribution function which serves to define double as well as single diffractive dissociation and yet avoids problems with multiple counting is defined and discussed.

An obstacle to unambiguous measurement of double diffractive dissociation, when both incident particles undergo "dissociation," has been the absence of a well-defined criterion by which produced particles may be divided into two "blobs," each of which is to be identified with one of the incident particles.[1] We here draw attention to a distribution function which is experimentally measurable in exclusive processes and whose definition is free from ambiguities. Further, assuming a factorizable Pomeranchukon, this distribution is expected to possess simple properties which give precise meaning to the general concept of diffractive dissociation. In particular one is led to a straightforward connection with single diffractive dissociation as studied in single-particle inclusive reactions. A striking feature of this distribution is that a single exclusive event may *legitimately* contribute to more than one region of the distribution space. Hence, within the present framework, there is no meaning for the notion of a "total" diffractive-dissociation cross section.

The distribution in question has been introduced by Abarbanel *et al.*,[2] with a specific model and application in mind. In particular, these authors were interested in the limit in which both "blob" masses become extremely large, a process with small probability occurring in a kinematic region which will presumably remain inaccessible to experiment for some time to come.[3] We here generalize the concept not only to arbitrary "blob" mass but also to arbitrary combinations of particles in each "blob," in the hope that there will be immediate experimental relevance.

In order to define the distribution of interest, suppose that the longitudinal rapidities of all the produced particles in a high-energy event have been measured and sequentially arranged in increasing order. Two criteria are usually invoked in order to classify an event as "diffractive dissociation." (1) Somewhere along the ordered rapidity chain there must occur a substantial gap. (2) The particle collections (blobs) on either side of the gap must each have the same quantum numbers as the initial particle whose rapidity lies on that same side. There are, however, ambiguities in these criteria: (a) The minimum acceptable rapidity gap is not sharply defined. (b) Even if a minimum gap can be agreed on, it is possible that more than one gap larger than this minimum may appear in the rapidity chain of a single event.

The distribution to which we are drawing attention in this paper is built by including *all* the ways that the ordered rapidity chain of final particles may be divided into two segments of appropriate quantum numbers. For a fixed center-of-mass energy squared s, our distribution depends on three continuous variables s_A, s_B, and t, whose meaning is displayed in Fig. 1, and certain discrete variables, e.g., the multiplicities $n_{A'}$ and $n_{B'}$, which specify the constituents of A' and B'. We shall speak of the triple-differential "cross section"

$$\frac{d\sigma_{AB\to A'B'}}{ds_A ds_B dt}, \tag{1}$$

but remind the reader that the distribution (1) is not a differential cross section in the usual sense, for if a sum is made over A', B', and integrations carried out over the continuous variables, the total cross section will be exceeded because of multiple counting.

Note that the rapidity gap between the blobs A' and B' grows in proportion to $\ln(s/s_A s_B)$, and we shall eventually be most interested in the region where the ratio $s/s_A s_B$ is large. There is, however, no need when constructing the distribution (1) to specify a definite lower limit for this ratio. Although an essential feature of the distribution (1) is that no "total" cross section can be constructed from it, this distribution still has a simple Regge asymptotic expansion in the limit as the ratio $s/s_A s_B$ becomes large. The logarithm of this ratio is essentially the rapidity gap between the "rightmost" particle in blob A and the "leftmost" particle in blob B. We take the essence of the Regge limit to be that Regge behavior obtains whenever the magnitude of this gap is large, and does not require gaps within the separate blobs to be small. Such an interpretation is consistent with the conventional applications of Regge behavior to multiparticle processes. Thus, if the Pomeranchukon is a simple factorizable Regge pole, then

$$s^2 \frac{d\sigma_{AB\to A'B'}}{ds_A ds_B dt} \underset{s/s_A s_B \to\infty}{\sim} A_{PA\to A'}(s_A, t) A_{PB\to B'}(s_B, t) \left[\frac{s}{[\lambda(s_A, m_A{}^2, t)\lambda(s_B, m_B{}^2, t)]^{1/2}}\right]^{2\alpha_P(t)}, \tag{2}$$

where

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$$

and $\alpha_P(t)$ is the Pomeranchuk trajectory. The coefficient $A_{PA\to A'}$ might be described as proportional to the "cross section" for Pomeranchukon plus particle A to produce A', although our normalization of this quantity has absorbed certain t-dependent factors that others might wish to exhibit explicitly. The coefficient $A_{PB\to B'}$ has a corresponding significance.

One may say that "measurement" of diffractive dissociation is achieved when the experimenter has succeeded in extracting from his data the coefficients $A_{PA\to A'}$ and $A_{PB\to B'}$. The problem here is the usual one in dealing with Regge behavior. The limit in (2) is never really achieved, no matter how high the ratio $s/s_A s_B$, so an extrapolation procedure is required. We shall not here pursue the question of the most effective procedure, which no doubt will vary with the specific experiment. Let us emphasize that the experimental results for $A_{PA\to A'}$ and $A_{PB\to B'}$ can be considered compelling only when they are independent of the s value of the experiment from which they are extracted. It should also be noted that the factorization property explicit in (2) should prove invaluable as a check on the results and for making the connection with single-particle inclusive reactions, i.e., single diffraction dissociation, as pointed out below.

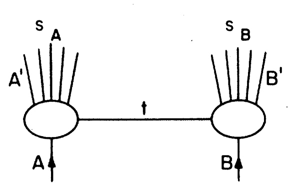

FIG. 1. Diagram defining the variables in the distribution (1).

The form of the limit (2) is maintained if one performs partial or total sums over the particle combinations in either or both of the two "blobs," the experimental statistics evidently being improved by such sums. If one could perform the *total* sum (difficult because of neutral-particle production) one would arrive at an especially interesting theoretical construct, $A_{PA}(s_A, t) = \sum_{A'} A_{PA\to A'}$,

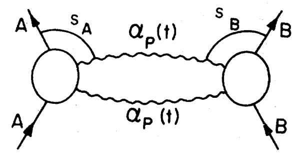

FIG. 2. Schematic representation of formula (2) when summed over all particle combinations in both "blobs."

the forward-direction absorptive part for the "elastic scattering" of a Pomeranchukon of mass squared t by particle A at total energy squared s_A. This quantity is proportional to the "total cross section" for a PA "collision," and is the same factor that occurs in the appropriate limit of single-particle inclusive cross sections.[4] In this latter case one is studying the diffractive dissociation of a single incident particle. The single-particle limits are in fact included in the above as the δ-function components of $A_{PA}(s_A, t)$ at $s_A = m_A{}^2$, or of $A_{PB}(s_B, t)$ at $s_B = m_B{}^2$. When s_A and/or s_B becomes large one may make a Regge expansion of the absorptive part in question and exhibit the dependence on s_A and/or s_B through triple-Regge vertices, as in Ref. 2, but even for moderate or small values of the "blob" masses and with less than a complete sum over particle combinations in the "blobs," formula (2) is deserving of experimental attention.

Formula (2), when summed over A' and B', is closely related to the formula for the discontinuity across the two-Pomeranchukon cut.[5] (See Fig. 2.) In fact it was by considering the multiple-counting aspect of this discontinuity formula that we were led to study the subject of the present note. It is indeed interesting that the structure of Regge asymptotic behavior should focus experimental attention on a type of distribution that heretofore might have been considered unnatural.

We would like to acknowledge extremely helpful discussions with M. Jacob. One of the authors (G.F.C.) wishes to thank R. R. Wilson for the hospitality extended to him at the National Accelerator Laboratory.

---

*Permanent address: University of California, Berkeley, Calif. 94720.

[1]An example of experimental data of the type relevant to our discussion may be found in W. Burdett *et al.*, Brookhaven report, 1972 (unpublished).

[2]H. D. I. Abarbanel *et al.*, Phys. Rev. Letters 26, 926 (1971).

[3]It should be noted that although such an event occurs only rarely, it may play an important role in determining the average multiplicity.

[4]The interested reader will note that the asymptotic limit in (2) is just such as to pick out Pomeranchukons with definite helicity [$\lambda = \pm \alpha_P(t)$] when looked at in the channel $\alpha_P + \alpha_P \to B + \bar{B}(A + \bar{A})$ of Fig. 2. This is exactly analogous to the situation in the single-particle inclusive reaction. For a more detailed discussion of the helicity question, see C. E. DeTar and J. H. Weis, Phys. Rev. D 4, 3141 (1971). For a discussion of Reggeon-particle scattering amplitudes, particularly as they appear in finite-energy sum rules for inclusive processes, see A. I. Sanda, Phys. Rev. D 6, 280 (1972); S. D. Ellis and A. I. Sanda, Phys. Letters 41B, 87 (1972); M. B. Einhorn, J. Ellis, and J. Finkelstein, Phys. Rev. D 5, 2063 (1972).

[5]The two most recent papers on the two-Reggeon discontinuity are A. R. White, Nucl. Phys. (to be published); and H. D. I. Abarbanel, Phys. Rev. D 6, 2788 (1972). These papers may be consulted for earlier references.