proached, the proposed experiment could provide a fairly high lower bound to the mass of any neutral IVB. In fact this bound may be very roughly estimated at $m_Z^{\text{limit}} = \sqrt{2}\, E/\sqrt{n}$ when effects are expected at an $n\%$ level. For $n = 2\%$ and $2E = 6$ BeV we would obtain $m_Z \geq 30$ BeV. In fact this aspect considerably increases the interest of the proposed experiment.

After completing this paper, we received preprints of similar work by Love[14] and by Chung, Mann, and Paschos.[15]

ACKNOWLEDGMENTS

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Diffractive Dissociation of Particles Through Collective Excitations*

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A model of diffractive dissociation is proposed in which these reactions are regarded as occurring through two stages: (i) coherent excitation of one (or both) of the incident particles as a whole, and (ii) subsequent decay of the excited states which occurs independently of (i). This assures the observed constancy of the average multiplicity of the dissociated systems $\langle n \rangle$ as a function of the four-momentum transfer t. With an additional assumption of an extended-particle eikonal approximation in (i) and a statistical hypothesis in (ii), the model describes satisfactorily all the essential features of diffractive dissociation.

I. INTRODUCTION

Experimental data have accumulated[1–3] in recent years indicating the importance of a class of production reactions,

$$a + b \to a + B \quad (\text{or } A + b) \tag{1}$$

$$\to A + B \tag{2}$$

(where A and B are systems of particles), with the following characteristics:

(i) b and B (and a and A) have the same intrinsic quantum numbers, such as charge, isospin, baryon number, strangeness, etc. Spin and parity seem to satisfy, whenever they can be determined, the selection rule that their changes are restricted to 0^+, 1^-, 2^+, ...[4]

(ii) The differential cross sections at high energies are strongly peaked in the small-angle region and seem to be slowly varying functions of energy. Actually, this is not quite well established yet, but in all cases for which we have measurements of $d^2\sigma/dMdt$, where $t=(p_a-p_A)^2$ [or $(p_b-p_B)^2$] and $M^2=p_B^2$ (or p_A^2), the shapes seem to be quite similar. Furthermore, the total cross section for single dissociation, Eq. (1), in which the multiplicity of B (or A) is fixed, is known to be constant.[5]

(iii) $d^2\sigma/dMdt$ exhibits a slope which is monotonically decreasing with M. This behavior is seen both for a definite system B (or A) as well as in the case when the multiplicity and the kind of particles in B (or A) are not fixed. For instance, the slope of $\pi^-p\to(2\pi^-\pi^+)p$ behaves, within the experimental indeterminacy, in the same way as that of $\pi^-p\to X_{\pi^-}p$, where X_{π^-} is the set of all systems of particles (except π^-) with the same total quantum number of π^- (see Fig. 1).

(iv) The mean multiplicity $\langle n\rangle$ of A (or B) does not depend on t and is a function only of M. Although just one measurement of this kind (π^-p at 16 GeV/c, ABBCHW Collaboration[3]) is known, we believe that it is a general feature of these reactions and we will assume that this is so.

As a function of M, $\langle n\rangle$ increases monotonically with a functional form which is, however, not well determined. For instance, the data on π^-p can be fitted by straight lines.

(v) In its own c.m. frame, the set of particles constituting the system B (or A) seems to be isotropically distributed[6,7]; the evidence for this statement, however, is not yet conclusive.

Properties (i) and (ii) are usually considered to be characteristics of a diffractive process, and since Good and Walker[8] have predicted their existence, much work on this subject has been done, both theoretical[9] and experimental.[1-3] The term diffractive dissociation, however, is, in general, used in a somewhat narrower sense, whereupon B and A in (i) and (ii) are actually taken to be some known resonances or else some unknown ones, which could, however, be classified in the usual way.[10] In the present paper, the term will be extended to include also a large part of the so-called "background," which is by no means negligible (about $\frac{2}{3}$ of all the events of π^-p at 8 GeV/c[1]). Thus, we will consider that the main part of $d^2\sigma/dMdt$ in a process of type (1) for small $|t|$ and small M, in which B is allowed to be any system having the characteristic stated by (i), is of diffractive origin. There may, indeed, exist a real background, but we believe that this is negligible in the M, t region of interest for us ($M\lesssim 3$ GeV, $|t|\lesssim 1.5$ GeV^2).

In the present paper, we shall fix our attention mostly on the points (iii)–(v), and we will try to understand the essential features of these processes without the use of any sophisticated calculations. Concretely, we propose a model for these reactions, which can be described as follows: Diffractive dissociation occurs through two stages. First, during the passage of the incident

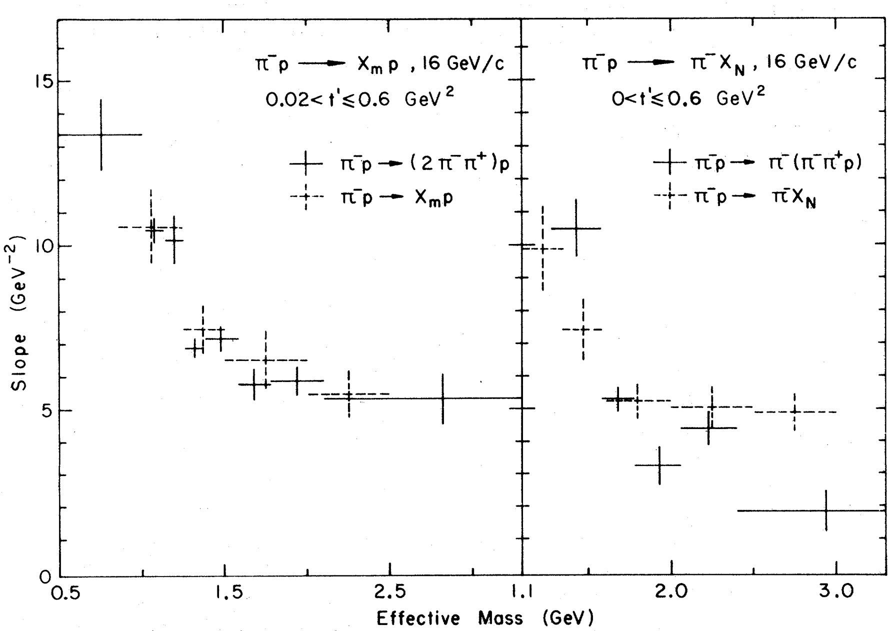


FIG. 1. Slope parameter A in $d^2\sigma/dMdt'\propto\exp(-At')$ at 16 GeV/c fitted over the $t'=|t-t_{\min}|$ region indicated, as a function of the invariant mass M. The data for $\pi^-p\to(2\pi^-\pi^+)p$ and $\pi^-p\to\pi^-(\pi^-\pi^+p)$ are taken from Ref. 2. Those for $\pi^-p\to X_m p$ and $\pi^-p\to\pi^-X_N$ are taken from Ref. 3.

particles through each other, one or both of them are excited collectively, the excitation depending on the structure of the particles involved and on the strength of the interaction. These excited states decay, then, producing the final particles whose distribution is independent of the first stage and is determined essentially by the phase space available. This model is clearly reminiscent of the "compound nucleus" model of nuclear physics. It is also very similar to that suggested by Takagi[11] a long time ago in analyzing cosmic-ray data. It bears also some resemblance to the fireball model,[12] although there are some differences, and we believe that our statement is more precise in that the domain of applicability is well defined here and is restricted just to diffractive processes. We believe also that our work is in the spirit of the "limiting fragmentation" hypothesis.[13] More recently, Hwa and Lam[14] and Jacob and Slansky[15] have proposed models which resemble the present one, although our emphasis is quite different from theirs.

In Sec. II the constancy of the mean multiplicity $\langle n \rangle$ as a function of t is discussed. An example of t distribution is given, to which simple heuristic arguments may lead, but which cannot account for the observed constancy of $\langle n \rangle$. Although this does not prove our model, we argue that the idea of collective excitation (such as we use it) is favored by this example when compared to the existing data.

In Sec. III a formulation of our model is given and, by fixing attention to the second stage of the reaction (decay of the excited states), the mean multiplicity $\langle n \rangle$ is calculated as a function of the effective mass M in a way very similar to Fermi's statistical model.[16] The consequences of this model are discussed by comparing our predictions with the existing data on particle production induced by $\pi^- p$.

A calculation along the lines of Chou and Yang[17] is performed in order to obtain $d^2\sigma/dMdt$ as well as the experimentally observed behavior of its slope with M. This is reported in Sec. IV.

Section V is devoted to discussions and comments about our model, especially regarding the so-called D resonances and the relation of our model to other existing models. Finally, in Sec. VI, a further outlook is given.

II. t DEPENDENCE OF THE AVERAGE MULTIPLICITY

Results of an analysis of the reactions

$$\pi^- p \to \pi^- + X_N , \tag{3}$$

$$\pi^- p \to p + X_m , \tag{4}$$

at 16 GeV/c of the incoming pion in the lab system have been reported[3] which show, besides the well-known behavior of the π^- (or p) angular distribution as a function of the invariant mass M, a remarkable constancy of $\langle n \rangle$, the average multiplicity of X_N (or X_m) with respect to $t' = |t - t_{\min}|$. Here X_N and X_m are, respectively, the set of systems of particles with the same quantum numbers of p and π^-. Figure 2 shows typical examples of the data. Indeed, $\langle n \rangle$ increases with increasing M, the invariant mass of X_N (or X_m), but the independence of $\langle n \rangle$ on t', so long as t' is not too large ($t' \lesssim 2$), contradicts the (naive) expectation that increasing the momentum transfer may produce

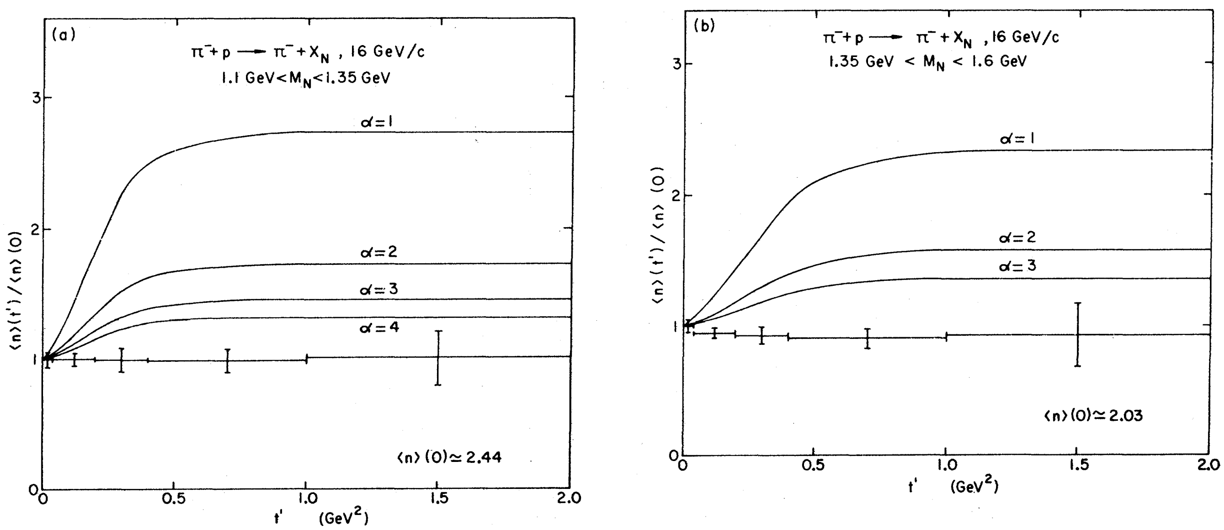


FIG. 2. Average multiplicities, normalized to 1 at $t'=0$, of the system X_N for two different mass intervals indicated. The experimental data (Ref. 3) for other mass intervals, as well as for the dissociation of $\pi^-(\pi^- p \to X_m p)$, are similar. The curves are those calculated by using Eq. (10), having fitted the data of Ref. 3 to Eq. (9). A comparison of Eq. (9) with Eq. (5) gives us a restriction on α, namely $\alpha \lesssim 3.5$ for (a) and $\alpha \lesssim 2.8$ for (b).

higher multiplicities (see, e.g., Ref. 13).

The above property shows, we believe, that it is unreasonable to assume that the angular distributions of these processes are made up of amplitudes corresponding to different multiplicities and having different shapes. In fact, it can be easily shown that distributions of the form[18]

$$\frac{d^2\sigma}{dMdt'}=\sum_{n=2}^{n_{\max}} c_n \exp\left(-\frac{at'}{n^\alpha}\right) \qquad (c_n\geq 0,\, a,\, \alpha>0) \tag{5}$$

are inconsistent with the experimental data. To show this, let us write the average multiplicity in terms of the distribution given by Eq. (5) as

$$\langle n\rangle=\frac{\sum n c_n \exp(-at'/n^\alpha)}{\sum c_n \exp(-at'/n^\alpha)}\equiv\frac{N(t')}{D(t')}\,. \tag{6}$$

Integrating the denominator of Eq. (6) from t' to ∞, over an appropriate transformation kernel, we get

$$\begin{aligned}\int_{t'}^{\infty} D(t_1)(t_1-t')^{1/\alpha-1}dt_1 &= \sum c_n \int_{t'}^{\infty}\exp\left(-\frac{at'}{n^\alpha}\right)(t_1-t')^{1/\alpha-1}dt_1\\ &=\sum c_n \exp\left(-\frac{at'}{n^\alpha}\right)\int_0^\infty \exp\left(-\frac{ax}{n^\alpha}\right)x^{1/\alpha-1}dx \qquad (x=t_1-t')\\ &=\sum c_n\exp\left(-\frac{at'}{n^\alpha}\right)\frac{\Gamma(1/\alpha)}{(a/n^\alpha)^{1/\alpha}}=\frac{\Gamma(1/\alpha)}{a^{1/\alpha}}\sum n c_n \exp\left(-\frac{at'}{n^\alpha}\right)\\ &=\frac{\Gamma(1/\alpha)}{a^{1/\alpha}}\,N(t')\,.\end{aligned} \tag{7}$$

Using Eqs. (5), (6), and (7) we have

$$\langle n\rangle=\frac{a^{1/\alpha}}{\Gamma(1/\alpha)}\int_{t'}^{\infty}\frac{d^2\sigma}{dMdt_1}(t_1-t')^{1/\alpha-1}dt_1\bigg/\frac{d^2\sigma}{dMdt'}\,. \tag{8}$$

If one takes the ratio $\langle n\rangle(t')/\langle n\rangle(0)$, the factor in front of the integral in Eq. (8) becomes irrelevant, so that the average multiplicity can be expressed in terms of $d^2\sigma/dMdt'$ and its integral, α being the only free parameter. In order to compare this with the experimental data, the angular distributions of Ref. 3 have been fitted by sums of exponentials as

$$\frac{d^2\sigma}{dMdt'}=\sum A_i\exp(-B_it') \qquad (B_i>0,\ A_i\geq 0)\,. \tag{9}$$

Putting this into Eq. (8), one has finally

$$\langle n\rangle\propto\frac{\sum (A_i/B_i^{1/\alpha})\exp(-B_it')}{\sum A_i\exp(-B_it')}\,. \tag{10}$$

In Fig. 2, the multiplicities thus obtained are compared with some of the experimental data. The constants A_i and B_i had been chosen in these cases as

$$A_1=7\text{ mb/GeV}^3,\quad B_1=12.8\text{ GeV}^{-2},$$
$$A_2=1.3\text{ mb/GeV}^3,\quad B_2=3.2\text{ GeV}^{-2}$$

for Fig. 2(a), and

$$A_1=8.4\text{ mb/GeV}^3,\quad B_1=10\text{ GeV}^{-2},$$
$$A_2=1.6\text{ mb/GeV}^3,\quad B_2=3.2\text{ GeV}^{-2}$$

for Fig. 2(b).

As it clearly appears, a good fit to the data could not be obtained in a reasonable range of values of the parameter α. Naturally the above argument does not prove the exact equality of the shape of the distributions for different multiplicities, for more and more sophisticated t distributions could be tried. We will rather regard it as an indication that, most likely, for diffractive processes the angular distributions are actually independent of the multiplicities, at least, so long as we are merely interested in their structure.

III. COLLECTIVE EXCITATION MODEL

A possible explanation of the results discussed in the preceding section would be to regard diffractive dissociation as a process occurring in two stages as illustrated by Fig. 3. First, during the interaction between the incident particles, one or both of them are *collectively excited*. In this process, states similar to the fireballs[12] are formed. The angular distribution $d^2\sigma/dMdt'$ of the leading particle in Eq. (1) depends only on how these excited states are formed (independent of the final multiplicity). In the second stage, these excited states decay, independently of how they have been formed. It is important to notice that our excited states are in general different from the usual resonances, which have definite decay modes giving definite (in general, low) multiplicities, but instead we think the decay of our excited states is determined essentially by the statistical weights for each final channel. The independence of the excitation and of the decay processes guarantees the constancy of $\langle n \rangle$ as a function of t.

The present model is meant to describe just diffractive dissociation, although it can be modified to also include processes involving quantum-number exchanges (this will be discussed in Sec. VI). Thus, in the first step when the incident particles interact, we are just making a statement that there is a finite probability that the particles are excited without the interchange of quantum numbers, but we do not plan to ascribe every possible reaction to such a mechanism. For instance, pionization is excluded from our model.

Another remark which would be appropriate at this point is that the model is designed just to describe the gross features of these reactions, and we cannot hope that it can account for the details of the data. For example, as a consequence of completely neglecting the usual resonances, the model cannot reproduce exactly their peaks. Nevertheless, in spite of the roughness of our approach, the agreement with the experiments is quite good.

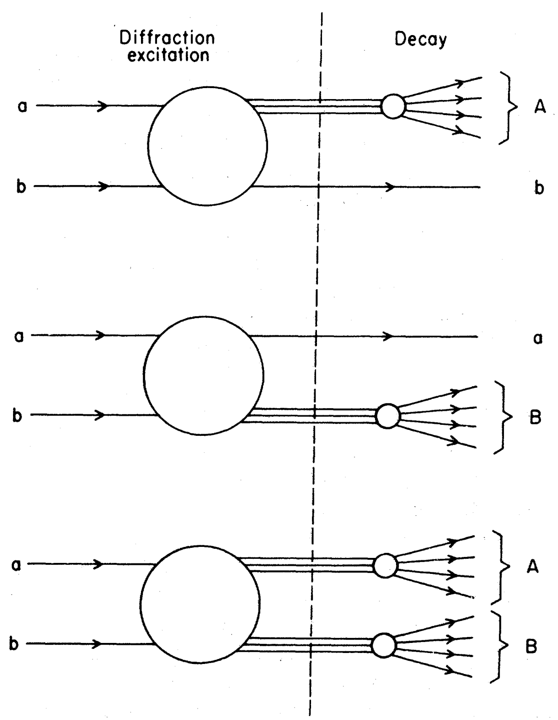


FIG. 3. Graphical representation of diffractive dissociation, in which the incident particle a, b, or both of them are dissociated.

In the remainder of this section, we will concern ourselves with the second stage of the reaction (decay of excited states) and, by using purely statistical arguments, try to obtain the experimentally measured invariant-mass dependence of the average multiplicity and relate different data among themselves.

A. Average Multiplicity as a Function of the Invariant Mass

In Sec. II, the constancy of the average multiplicity $\langle n \rangle$ of X_N and X_m as a function of t' has been discussed. As a function of the invariant mass M of X_N or X_m, however, $\langle n \rangle$ increases monotonically (see Fig. 4). We shall now try to show how this behavior of $\langle n \rangle$ can be accounted for.

Following Fermi's calculation,[16] we write the probability of X_N or X_m to decay into n particles of masses $m_1, m_2, \ldots, m_n$ as

$$P_n'(M; m_1, \ldots, m_n) \propto \left(\frac{\Omega}{8\pi^3}\right)^n (2m_1)(2m_2)\cdots(2m_n) \times R_n(M; m_1, m_2, \ldots, m_n), \tag{11}$$

where

$$\Omega = \tfrac{4}{3}\pi r^3 \tag{12}$$

is the volume occupied by the excited state and

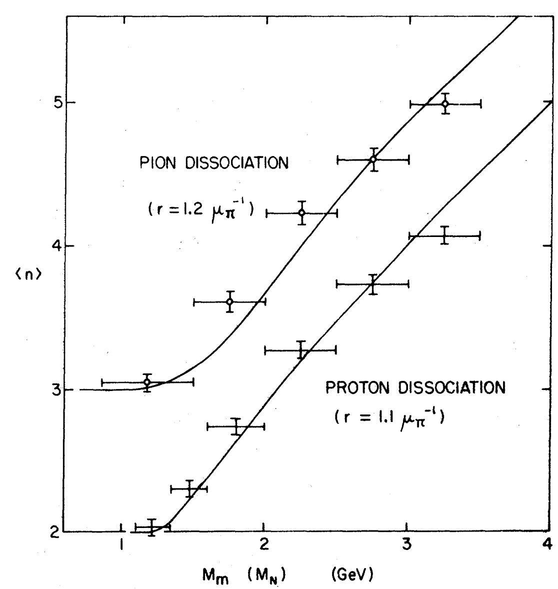


FIG. 4. Average multiplicity of X_m and X_N as a function of their invariant mass M_m and M_N. The experimental points are from Ref. 3. The curves have been calculated by a statistical model, taking the effective radius of the excited states equal to $1.2\ \mu_\pi^{-1}$ and $1.1\ \mu_\pi^{-1}$, respectively. See text for the details of calculation.

$$R_n(M; m_1, m_2, \ldots, m_n) = \int \cdots \int \prod_{i=1}^{n} \left[\frac{d^3p_i}{2(\vec{p}_i^{\,2} + m_i^2)^{1/2}}\right] \delta^3\Big(\sum_{j=1}^{n} \vec{p}_j\Big) \times \delta\Big(\sum_{k=1}^{n} E_k - M\Big), \tag{13}$$

with

$$E_k = (\vec{p}_k^{\,2} + m_k^2)^{1/2}, \tag{14}$$

is the invariant phase space of the n-particle system under consideration, with total invariant mass equal to M, expressed in terms of the c.m. variables.

The factor $(2m_1)(2m_2)\cdots(2m_n)$ has been put in Eq. (11) in order that the probability so calculated coincide with that obtained by Fermi in the low-energy limit. The main difference between our calculation and that of Fermi is, besides the use of the invariant phase space instead of the noninvariant one, the use of a constant (i.e., not Lorentz-contracting) volume Ω. This is because we are interested in the decay of a "particle" in its own center-of-mass system and therefore there is no contraction.

As is well known, the integration in Eq. (13) can be easily performed for $n=1$ and $n=2$, giving for the latter

$$R_2(M; m_1, m_2) = \frac{\pi}{2M^2}\{[M^2 - (m_1+m_2)^2][M^2 - (m_1 - m_2)^2]\}^{1/2}. \tag{15}$$

For higher multiplicities, use has been made of the recurrence formula

$$R_n(M; m_1, \ldots, m_n) = \int \frac{d^3p_n}{2E_n} \times R_{n-1}((M^2 + m_n^2 - 2ME_n)^{1/2}; m_1, \ldots, m_{n-1}), \tag{16}$$

which has been integrated directly with the help of a computer.

In order to get the decay probability of a definite isotopic-spin state into a number of particles with definite isotopic spins, we must still multiply p_n' as given by Eq. (11) by the isotopic-spin statistical weight, which in our case is[19]

$$F_{\nu n'}(S) = \frac{2S+1}{n'!}\sum_{i=0}^{n'} \frac{(-1)^{n'+i}}{2i+\nu+1}\binom{n'}{i}\binom{2i+\nu+1}{i+\frac{1}{2}\nu - S}, \quad \nu + n' = n \tag{17}$$

where ν, n', and S are, respectively, the number of isotopic spin-$\frac{1}{2}$ particles (nucleons), that of isotopic spin-1 particles (pions), and the total isotopic spin of the whole system. We are dealing with $S=\frac{1}{2}$ and $S=1$ systems and, in our calculation, only those final states consisting of one nucleon with pions or of an odd number of pions have been considered. The results are shown in Fig. 5, where the only parameter r has been appropriately chosen in order to reproduce the experimental $\langle n\rangle$ as explained below.

Once the relative probabilities

$$P_n(M; m_1, \ldots, m_n) = F_{\nu n'}(S)P_n'(M; m_1, \ldots, m_n) \tag{18}$$

have been determined, the average multiplicity is readily calculated as

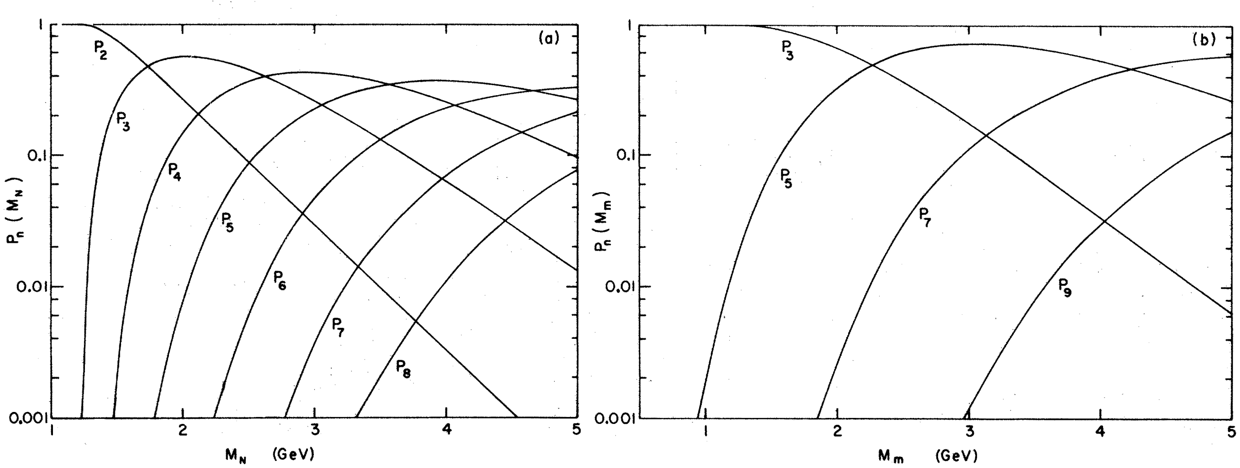


FIG. 5. The statistical prediction for the relative probability of dissociation of (a) a nucleon into a nucleon + $(n-1)$ pions and (b) a pion into n pions as function of the invariant mass of the dissociated systems. The parameter r has been taken as in Fig. 4.

$$\langle n\rangle = \sum_n P_n / \sum P_n \tag{19}$$

and this is plotted as a function of M in Fig. 4, where a comparison is given with the experimental points. It is seen that, despite the simplicity of the approach, the model can reproduce fairly well the existing data, with a physically reasonable choice of the parameter r.

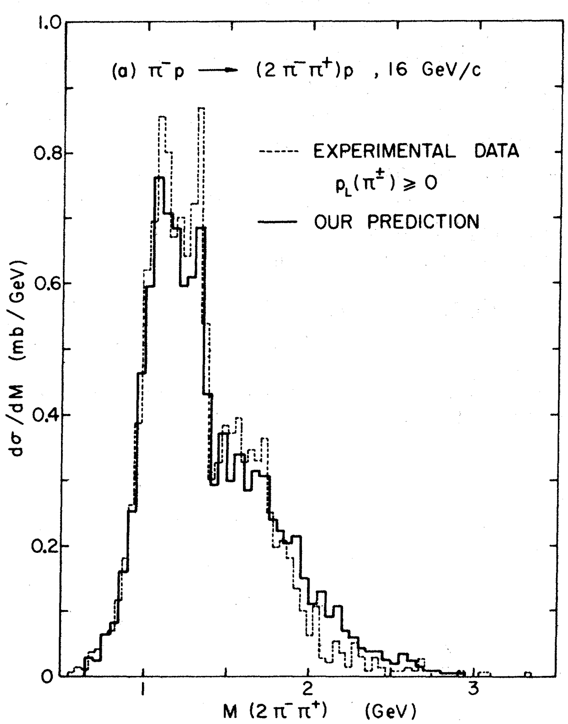
B. Effective-Mass Distributions of Dissociated Systems

Having obtained the multiplicity distribution as a function of M, we may ask ourselves what experimentally verifiable predictions can be given which make use of such a distribution. One such prediction concerns the effective-mass distributions of the dissociated systems for definite final states. Several experimental results have been reported on these kinds of distributions,[1,2,20] so that we have at our disposal an immediate check of the validity of our model.

If we assume that diffractive dissociation through the previous mechanism dominates a reaction of the type of Eq. (1), the invariant-mass distribution of a particular channel characterized by multiplicity n [say, e.g., $\pi^- p \to (3\pi)^- p$ with $n=3$] will be related to the total invariant-mass distribution [$\pi^- p \to \sum (n\pi) p$ in the preceding example] through

$$\frac{d\sigma_n}{dM} = P_n(M)\frac{d\sigma_{\text{tot}}}{dM}\,. \tag{20}$$

In Ref. 3, measurements of $d\sigma_{\text{tot}}/dM$ have been reported for both reactions (3) and (4). In both cases, the mass spectrum of X_n (or X_m) is given (i) for all events [curve (a) of Ref. 3], (ii) for events having a leading proton (or π^-) [curve (b) of Ref. 3], and (iii) for events in which the proton (or



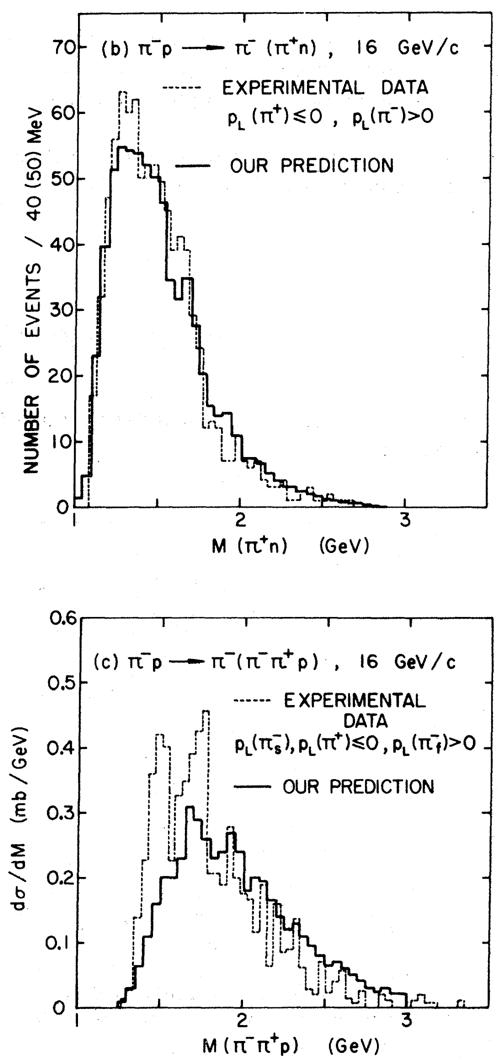


FIG. 6. Effective-mass distribution of (a) $(2\pi^-\pi^+)$, (b) $(\pi^+ n)$, and (c) $(\pi^-\pi^+ p)$ from the reactions $\pi^- p \to (2\pi^-\pi^+)p$, $\pi^-(\pi^+ n)$, and $\pi^-(\pi^-\pi^+ p)$, respectively, at 16 GeV/c. The experimental points (Ref. 2) are subjected to restrictions as indicated. The predictions are those given by Eq. (20) by using $d\sigma_{\text{tot}}/dM$ determined with 16 GeV/c events where the proton (or the π^-) is the only particle moving in the backward (or forward) direction (Ref. 3). For (a) and (c), the same relative normalization has been used, assuming just total isotopic-spin conservation, whereas for (b) this has been done arbitrarily because of the lack of information.

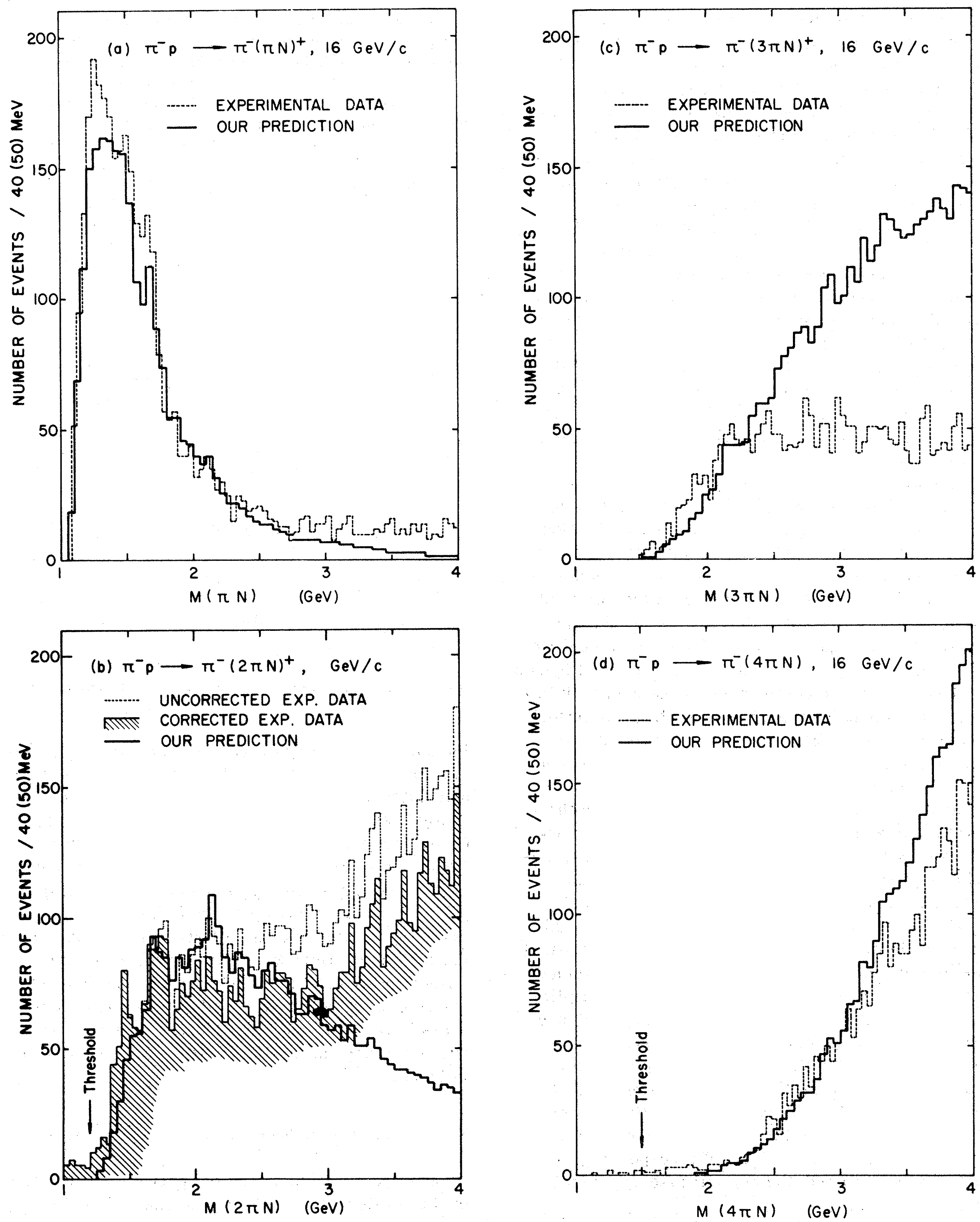


FIG. 7. Effective mass distribution of (a) $B\pi$, (b) $B2\pi$, (c) $B3\pi$, and (d) $B4\pi$ from the reactions $\pi^- p \to \pi^-(B\pi)$, $\pi^-(B2\pi)$, $\pi^-(B3\pi)$, and $\pi^-(B4\pi)$, respectively, at 16 GeV/c. The experimental data are from Ref. 20. When more than one π^- was produced, the one with the smallest $|t_{\pi^-\to\pi^-}|$ was selected as the leading particle. In (b) corrected data means $(2\pi N)^+_{\rm exp} - \frac{1}{3}(3\pi N)^+_{\rm exp} - \frac{1}{8}(4\pi N)^+_{\rm exp}$ (see text). The predictions are those given by Eq. (20), by using $d\sigma/dM$ at 16 GeV/c (Ref. 3) with leading π^-, and the normalization is arbitrary but the same for all the reactions.

π^-) is the only backward (or forward) particle [curve (c) of Ref. 3]. We can see there that for small M the events (iii) dominate, whereas even for quite large M values (~4 GeV) the events (ii) are dominant, thus confirming that diffractive dissociation dominates these reactions for small M.

Two sets of data on $d\sigma_n/dM$ for $\pi^- p$-induced reactions could be found which are useful for our comparison. The first set[2] refers to $\pi^- p \to \pi^-(\pi^-\pi^+ p)$ and $\to (2\pi^-\pi^+)p$ data at 11 and 16 GeV/c, and $\pi^- p \to \pi^-(\pi^+ n)$ and $\to \pi^-(\pi^0 p)$ data at 16 GeV/c. They are subjected to conditions which nearly (or exactly) correspond to case (iii), previously mentioned. Thus, in computing $d\sigma_n/dM$ with the help of Eq. (20), curve (c) of Ref. 3 has been used for comparison. Figure 6 shows some of these data together with the prediction of Eq. (20). The data at 11 GeV/c as well as those for $\pi^- p \to \pi^-(\pi^0 p)$ at 16 GeV/c are very similar to the ones presented here.

The other set of data is from the ABBCCHW collaboration,[20] where $(Bn\pi)$ effective-mass distributions have been measured from the reactions $\pi^- p \to \pi^-(Bn\pi)$ at 16 GeV/c. In contrast to the previous set of data, all the events with the leading π^- (which is defined here as one having the smallest $|t_{\pi^- \to \pi^-}|$), considered as the surviving incident particle, are regarded here. Thus, in computing $d\sigma_n/dM$, curve (b) of Ref. 3 has been used. Figures 7(a)–7(d) show the comparison of the data with the theoretical predictions. It is seen that our prediction for $\pi^- p \to \pi^-(2\pi N)^+$ is too low for $M \gtrsim 2.5$ as compared to the data of Ref. 20, and that for $\pi^- p \to \pi^-(3\pi N)^+$ it is too high in the same M region. However, in Ref. 20 it was assumed that for those events for which there was no kinematic fit there were only two neutral particles. This causes, obviously, an overestimation of the $\pi^- p \to \pi^-(2\pi N)^+$ cross section, whereas the $\pi^- p \to \pi^-(3\pi N)^+$ and $\to \pi^-(4\pi N)^+$ events would be underestimated. This fact is illustrated by Table I, where the statistical weights predicted by the isotopic-spin invariance[21] have been computed and tabulated up to multiplicity five. An attempt to account for this deficiency has led to a rough correction of the data, which is reported in Fig. 7(b). The correction consists of subtracting from the $(2\pi N)^+$ distribution $\frac{1}{3}$ and $\frac{1}{8}$, respectively, of the $(3\pi N)$ and $(4\pi N)$ distributions. As can be seen, this improves the agreement of our prediction with the data. Correspondingly, the data for $M(3\pi N)$ and $M(4\pi N)$ will rise approaching our prediction.

The conclusion which can be drawn from the analysis of Figs. 6 and 7 is that, despite its naiveness, the model provides a fairly good description of the existing data, as long as we confine ourselves to consideration of the general features of

TABLE I. Statistical weights, arising from the assumption of isotopic-spin invariance (Ref. 21), for the possible final states corresponding to the reactions indicated by Eq. (3) with lowest multiplicities. The events corresponding to those reactions indicated by arrows have been considered in Ref. 20 as $(2\pi N)$ events.

Final states	X_N		Statistical weights
$(\pi N)^+$	$p\pi^0$		$\frac{1}{3}$
	$n\pi^+$		$\frac{2}{3}$
$(2\pi N)^+$	$p\pi^0\pi^0$		$\frac{1}{3}$
	$p\pi^+\pi^-$		1
	$n\pi^+\pi^0$		$\frac{2}{3}$
$(3\pi N)^+$	$p\pi^0\pi^0\pi^0$	←	0.2
	$p\pi^0\pi^+\pi^-$		1.8
	$n\pi^0\pi^0\pi^+$	←	0.8
	$n\pi^+\pi^+\pi^-$		1.2
$(4\pi N)^+$	$p\pi^0\pi^0\pi^0\pi^0$	←	0.2
	$p\pi^0\pi^0\pi^+\pi^-$		2.8
	$p\pi^+\pi^+\pi^-\pi^-$		2
	$n\pi^0\pi^0\pi^0\pi^+$	←	0.8
	$n\pi^0\pi^+\pi^+\pi^-$		3.2

the data in the small-M region. The discrepancies occurring in the region of larger M values may be attributed to different causes, one of which has already been mentioned in the preceding paragraph. Other reasons would be, for instance, contributions coming from nondiffractive processes, such as pionization, to which the model cannot be applied, and the ambiguity in the definition of the leading particle; that is, different definitions of the leading particle would lead to different data.

Another aspect which the model cannot reproduce satisfactorily is, as expected, the detailed structures of the distributions, which are due to the "usual" resonances. This will be discussed later in Sec. V.

C. Particle Distribution Within the Dissociated Systems

Let us now consider how the particle distribution of the excited states would look like in their own c.m. system. Concerning the angular distribution, no constraint is provided by the model to allow any asymmetry, since in a strictly statistical approach a spherically symmetric distribution is expected. Although it would be possible to introduce additional constraints to relax this prediction, in the following we will work in the simplest scheme without introducing any new hypothesis.

Consider a system of n particles with total invariant mass M and assume that these particles are the decay product of an excited state, say B. According to our model, the probability of finding one of these particles, labeled 1, in a infinitesimal volume d^3p around $\vec{p}$ with energy $E=(\vec{p}^2+m_1^2)^{1/2}$ is in the center-of-mass frame of the whole system of particles proportional to

$$P(M,\vec{p})=\int\cdots\int\prod_{i=1}^{n}\left(\frac{d^3p_i}{2E_i}\right)\times\delta^3\left(\sum_{j=1}^{n}\vec{p}_j\right)\delta\left(\sum_{k=1}^{n}E_k-M\right)\delta^3(\vec{p}_1-\vec{p})$$
$$=\frac{1}{2E}R_{n-1}([(M-E)^2-\vec{p}^2]^{1/2};m_2,m_3,\ldots,m_n), \tag{21}$$

where the same notation of Eq. (13) has been used.

The over-all distribution of particle 1 as originated from the decay of the excited state B (in a reaction in which a fixed number of particles, n, is produced) is obtained by multiplying $P(M,\vec{p})$ by the differential cross section $d^2\sigma/dMdt'$ and integrating over the appropriate M and t' intervals.

The possible existence of identical particles will contribute just a proportionality factor. Thus, the probability of having particle 1 with momentum $\vec{p}$ is given by

$$P(\vec{p})=\int dM\int dt'P(M,\vec{p})\frac{d^2\sigma}{dMdt'}\ . \tag{22}$$

If there are no restrictions on t', aside from the trivial ones due to energy-momentum conservation, the above integral can immediately be integrated in t', giving

$$P(\vec{p})=\int dM\,P(M,\vec{p})\frac{d\sigma}{dM}\ . \tag{23}$$

From the above distributions, the longitudinal-momentum distribution is immediately obtained by projecting $\vec{p}$ on the incident direction and performing the $p_\perp$ integration:

$$P(p_\parallel)=2\pi\int P(\vec{p})p_\perp dp_\perp \tag{24a}$$

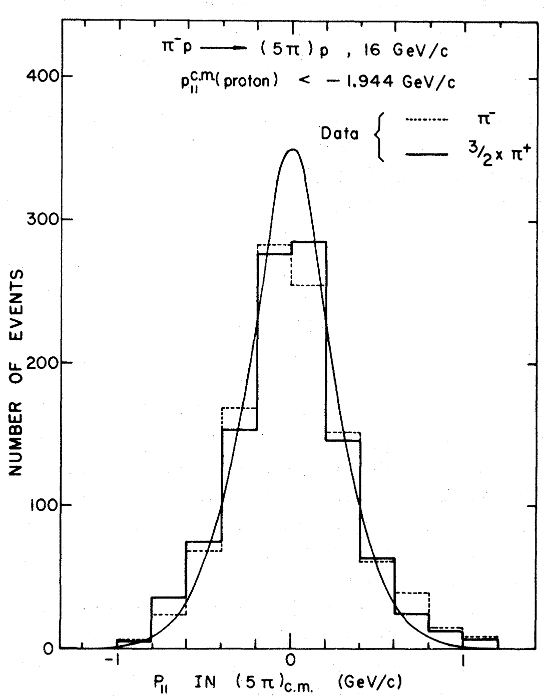


FIG. 8. Longitudinal-momentum distributions of π^+ and π^- produced in a subset of collisions $\pi^-p\to3\pi^-2\pi^+p$ at 16 GeV/c in the (5π) rest frame (Ref. 7). The subset is defined by $p_\parallel^{\text{c.m.}}$(proton) <-1.944 GeV/c. The curve is a prediction of our model, calculated by using Eqs. (23) and (24), where experimentally measured (Ref. 3) $d\sigma/dM$ has been used.

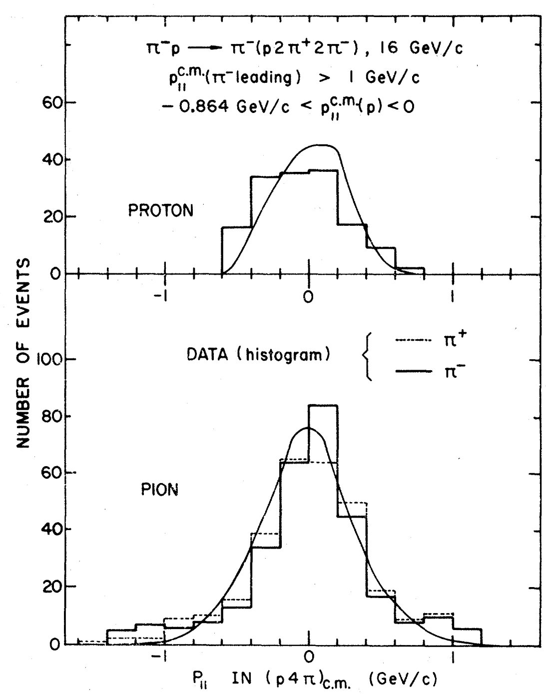


FIG. 9. Longitudinal-momentum distributions of p, π^+, and nonleading π^- produced in a subset of collisions $\pi^-p\to3\pi^-2\pi^+p$ at 16 GeV/c in the rest frame of $2\pi^-2\pi^+p$ (where π^- are the nonleading ones) (Ref. 7). The subset is defined by -0.864 GeV/$c<p_\parallel^{\text{c.m.}}$ (proton) <0 and $p_\parallel^{\text{c.m.}}>1$ GeV/c for the leading (i.e., most forward) negative pion, where $p_\parallel^{\text{c.m.}}$ denotes the longitudinal momentum in the over-all c.m. system. The curves are the predictions of the model.

$$= 2\pi \int dM \int dt' \frac{d^2\sigma}{dMdt'} \int P(M,\vec{p}) p_\perp dp_\perp \,. \tag{24b}$$

Let us look now at the experimental situation. Among the bulk of experimental data, we could find two sets of data of interest to us. One is the set of data on the eight-prong π^+p at 8 GeV/c,[6] where the angular distributions of pions have been measured in the c.m. frame of the π's.

The reactions which have been studied are

$$\pi^+p \to p4\pi^+3\pi^- \tag{25}$$

and

$$\pi^+p \to p4\pi^+3\pi^-\pi^0\,, \tag{26}$$

and the results show an approximate spherical symmetry in the distribution. These results, however, cannot be conclusive because, in a high-multiplicity and relatively low-energy experiment, most models favor spherically symmetric distributions. This becomes clear looking at another graph shown in Ref. 6, where the plot is given in the over-all c.m. system. Here too the distributions are nearly symmetric. Moreover, no separation of diffractive contribution from the remainder was given in Ref. 6.

The other set of data are those of the Aachen-Bonn-Berlin (Zeuthen)-CERN-Cracow-Warsaw Collaboration,[7] in which the reaction $\pi^-p \to 3\pi^-2\pi^+p$ at 16 GeV/c is studied. Using the longitudinal-phase-space technique, the diffractive events have been separated to give: (i) the π^- and π^+ distributions in the c.m. system of $(3\pi^-2\pi^+)$ (see Fig. 8), and, similarly, (ii) π^-, π^+, and p distributions in the c.m. system of $(p2\pi^-2\pi^+)$ (see Fig. 9). The angular distributions have not been measured, but what is important is that the (longitudinal) momentum has been measured. Thus, a test of our model can be more satisfactorily obtained by a comparison with these data. To this aim, one must perform the integration in Eq. (24a).

A few preliminary remarks, however, can be made immediately without any calculation. First, the forward-backward symmetry [especially in case (i)] of the particle distribution and the similarity of the π^- and π^+ distributions are features which can be expected from our model. Even the slight asymmetry which appears in the case (ii) is consistent with the experimental restrictions (on $p_\parallel$) imposed for these data.

In case (i), the condition $p_\parallel < -1.944$ GeV/c in the over-all c.m. system has been imposed on the proton, in order to guarantee diffractive dissociation of the π^-. From this constraint and from the threshold condition ($M \geq 5\mu_\pi$) the M interval could be determined and, by using Eqs. (23) and (24a), $P(p_\parallel)$ has been obtained. This is plotted in Fig. 8. For this calculation, $d\sigma/dM$ has been taken from the measurement at 16 GeV/c [curve (b) of Ref. 3], corrected near the upper limit of integration [consistently with the boundary condition $p_\parallel^{\text{c.m.}}$(proton) < -1.944 GeV/c], by assuming that in that domain the t' dependence is given by the experimentally measured $d^2\sigma/dMdt'$, in the interval 2 GeV $< M < 2.5$ GeV. This is illustrated by Fig. 10.

A similar calculation has been performed for the p, π^+ and π^- distributions in case (ii). Here the presence of the p and the experimentally imposed conditions -0.864 GeV/$c < p_\parallel^{\text{c.m.}}$ (proton) < 0 make the calculation more complex, although straightforward. In the preceding case, Eqs. (23) and (24a) have been used. Here, instead, Eq. (24b) has been used, where the M dependence of $d^2\sigma/dMdt'$ has been taken as $f(M)/P_5$, with $f(M)$ given by Fig. 11 and P_5 given by Eq. (18), and the t' dependence has been taken to be (throughout the entire M interval $2 \lesssim M < 4.45$ GeV) the one measured experimentally[3] in the interval 2.5–3.0 GeV, which was parametrized as

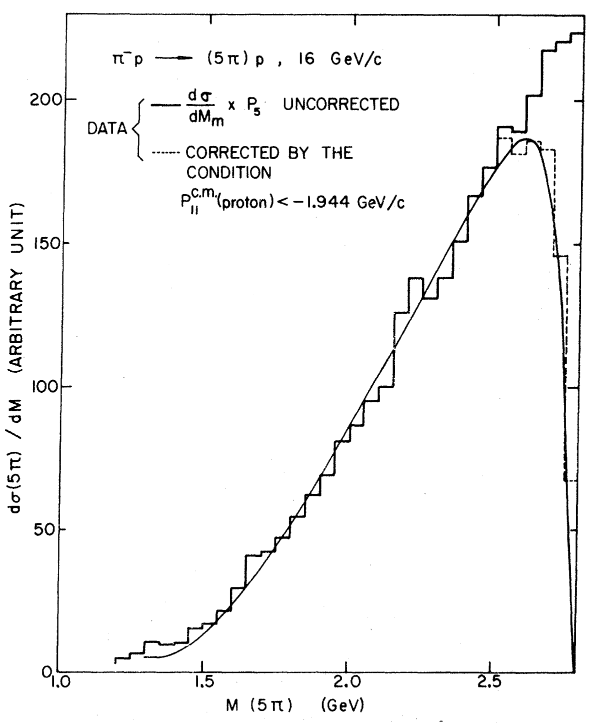


FIG. 10. Invariant mass distribution for the reaction $\pi^-p \to p(5\pi)$ at 16 GeV/c, which has been used in the calculation of $P(p_\parallel)$ for $\pi^\pm$. The curve is

$$f(M) = 100(1.9M + 100\,e^{-4M} - 2.97)\,(1 - e^{15.4(M-2.79)})\,.$$

Actually we have used $f(M)/P_5(M)$, where $P_5(M)$ is defined by Eq. (18), as $d\sigma/dM$ in Eq. (23). See text for details.

$$\frac{d^2\sigma}{dMdt'}(2.5\text{–}3.0 \text{ GeV}) \simeq 3.1\exp(-9.8t') + 3.2\exp(-3.8t') + 0.34\exp(-1.2t') . \quad (27)$$

Although this last assumption is not entirely justified by the experimental data (see Fig. 14), we believe that the error is quite small.

The results of above calculations have been plotted in Figs. 8 and 9 and compared with the experimental data. As is seen there, the agreement of our predictions with the data, in both cases, is excellent.

IV. ANGULAR DISTRIBUTIONS

In Sec. III some of the consequences of our model have been discussed regarding the second stage of the reactions, that is, the question "Once the excited states are formed, how do they decay into the final outgoing particles?" Let us now turn our attention to the first stage, that is, let us see how one can compute $d^2\sigma/dMdt'$ as a function of t' and M, for any given initial state.

As usual, the existence of a strong forward peak in $d^2\sigma/dMdt'$ [property (ii) of Sec. I] is attributed to the diffractive nature of the reaction. Accordingly, the structure of the angular distribution will be essentially determined by the extension and distribution of the hadronic matter inside the interacting particles. Instead of starting from a fundamental interaction between particles, we take the very simple approach of describing elementary particles by means of the extended-particle model, which was proposed by Yang and collaborators in connection with elastic[17] and charge-exchange scattering[22] and which has also been applied to diffractive dissociation.[9]

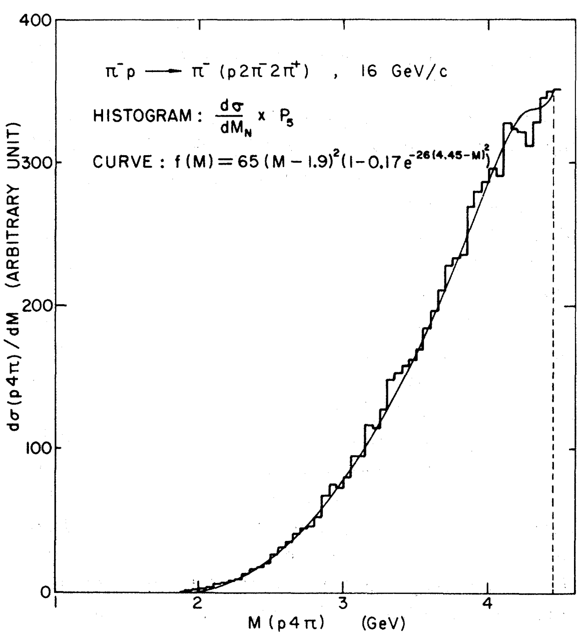


FIG. 11. Invariant mass distribution for the reaction $\pi^- p \to \pi^-(p4\pi)$ at 16 GeV/c, which has been used in the calculation of $P(p_{\parallel})$ for p and $\pi^{\pm}$. In order to facilitate numerical computation, $f(M)$ (represented by the curve) has been used instead of $d\sigma_5/dM_N$ shown by the histogram. However, we believe that the details of $f(M)$, such as the exact coefficients in the exponential, are irrelevant for the final distribution in Fig. 9. See text for details.

We argue that the existence of a sharp forward peak in $d^2\sigma/dMdt'$ which seems nearly energy-independent[5] is indicative of the possibility of coherent excitation of one or both of the interacting particles, with a finite probability even when $s \to \infty$.

Neglecting spin dependence, the differential cross section is written [for simplicity, we will concentrate on the case of single dissociation only, Eq. (1)] as

$$\frac{d^2\sigma}{dMdt'} = \pi |a(M,t')|^2 \tau(M) , \quad t' = |t - t_{\min}| \quad (28)$$

where

$$a(M,t') = \int_0^\infty a(M,b) J_0(b\sqrt{t'}) b\,db \quad (29)$$

and $\tau(M)$ is the final-state density. In Eq. (29), $a(M,b)$ represents the total amplitude for *coherently exciting* one of the particles (say, b) during the propagation of the other (say, a) with an impact parameter b. For simplicity, we consider only particles with spherical symmetry. Figure 12 illustrates the process, where for the sake of clarity one of the particles is taken to be pointlike.

As shown in Fig. 12, particle a may *coherently excite* b in one, two, ..., n steps, and all these "partial" amplitudes sum up coherently. Thus the total amplitude $a(M,b)$ may be written as a series, each term of which corresponds to a definite order of excitation:

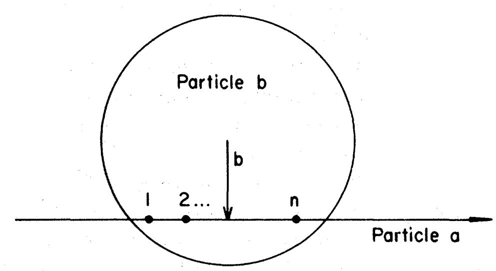


FIG. 12. Eikonal description of diffractive excitation. The points 1, 2, ..., n along the path of particle a represent multiple steps of coherent excitation of particle b.

$$a(M, b) = \sum_{n=1}^{\infty} a_n(M, b) . \tag{30}$$

In order to calculate each term of Eq. (30), we will follow the argument of Byers and Yang.[22] In Ref. 22, however, the excitation consisted in charge exchange, baryon-number exchange, etc. Here, instead, we are interested in excitations which change the mass of the interacting particles. We will assume, for simplicity, that the mass may vary continuously; that is, a continuous spectrum of excited states with a density $\tau(M)$ will be assumed. Another major difference between the present calculation and that of Ref. 22 is that in the latter paper only the first term of Eq. (30) was taken into account, whereas higher-order terms will be included here. These terms, in fact, will turn out to be important in the present case, as we will see below.

As in Ref. 22, we write for the first term of the series

$$a_1(M, b) = g_1(M)\chi(b)e^{-\chi(b)} , \tag{31}$$

where $\chi(b)$ is approximated by the elastic eikonal phase given by

$$\chi(b) \propto \int_{-\infty}^{\infty} dz \int d^3x' \times \rho_a(b_x - x', b_y - y', z)\rho_b(x', y', z') , \tag{32}$$

ρ_a and ρ_b being the hadronic matter densities of the two incident particles. Actually, instead of calculating $\chi(b)$ by assuming some distributions ρ (for instance, proportional to the charge distributions), we have used the experimentally measured elastic cross section

$$\frac{d\sigma}{dt'} = \pi |a_{\rm el}(t')|^2, \quad t' \equiv -t \tag{33}$$

where we have defined

$$a_{\rm el}(t') = \int_0^{\infty} (1 - e^{-\chi(b)}) J_0(b\sqrt{t'}) b\, db . \tag{34}$$

Assuming that $a_{\rm el}(t')$ is purely real (purely imaginary with the usual convention), we have computed $\chi(b)$ by inverting Eq. (34).

$g_1(M)$ in Eq. (31) is the probability amplitude per unit hadronic matter along the path for *coherently exciting* particle b into a state of mass M. When multiplied by $\chi(b)$ it will give the total probability amplitude at a fixed impact parameter b.

Probably, in the present case, to approximate $\chi(b)$ in Eq. (31) with the elastic eikonal phase is less justified than in the case of charge exchange. Nevertheless, in the limit of very high energy, the interaction time is so small that as a first approximation we may assume that the hadronic matter remains essentially constant during this short time.

Let us return to Eq. (30) and examine the other terms. It is easily realized that if the first term is given by Eq. (31), the second term will be given by

$$a_2(M, b) = \int_0^{\infty} f_1(x) f_1(\Delta M - x) \times \tau(M_0 + x) dx \frac{\chi^2(b)}{2!} e^{-\chi(b)} \equiv f_2(\Delta M) \frac{\chi^2(b)}{2!} e^{-\chi(b)} , \tag{35}$$

where M_0 is the mass of the particle b and

$$f_1(\Delta M) \equiv f_1(M - M_0) = g_1(M) . \tag{36}$$

The relative phase between $a_1(M, b)$ and $a_2(M, b)$ as well as those among the successive terms are not determined. As we will see later, however, a good fit with experiments can obtain if we choose $g_1(M)$ or $f_1(\Delta M)$ purely imaginary.

In general, the nth term of Eq. (30) is written as

$$a_n(M, b) = f_n(\Delta M) \frac{\chi^n(b)}{n!} e^{-\chi(b)} , \tag{37}$$

where

$$f_n(\Delta M) = \int_0^{\infty} f_{n-1}(x) f_1(\Delta M - x) \tau(M_0 + x) dx . \tag{38}$$

It is clear from Eqs. (28)–(31) and (37)–(38) that the model predicts a forward peak in $d^2\sigma/dMdt'$ and, moreover, if one chooses $g_1(M)$ [or $f_1(\Delta M)$] energy-independent (i.e., dependent only on M), we also have energy-independent angular distributions, in agreement with the experimental requirement (ii) mentioned in Sec. I.

Let us see now how property (iii) of Sec. I can be accounted for in the present model. Evidently, this will require an appropriate choice of the as yet undetermined functions $f_1(\Delta M)$ and $\tau(M)$.

In Sec. I we have simply stated that $d^2\sigma/dMdt'$ shows a slope monotonically decreasing with increasing M. It is now necessary to examine the experimental data somewhat more closely. Looking at Figs. 13 and 14, one notices immediately that, excluding the very small t' region (where there is a dip for π^- dissociation), $d^2\sigma/dMdt'$ shows a two-component structure which is especially evident in the small M intervals. One component which is much steeper than the other, with a slope parameter $A \sim 12\ \text{GeV}^{-2}$ (when fitted by $e^{-At'}$) for both π^- and p dissociation, dominates the small t' and M regions and seems to decrease as the effective mass increases. The other with $A \sim 3\ \text{GeV}^{-2}$ increases with increasing M and becomes dominating for sufficiently high M. The highest M data available show even a stronger flattening of the distribution.

Equations (28)–(38) do give multicomponent structure for $d^2\sigma/dMdt'$, and each term in Eq. (30) gives a flatter and flatter contribution to the differential cross section. To illustrate this, we have plotted in Figs. 13(a) and 14(a) the contributions coming from the first two terms, neglecting their interference with the remaining ones. It is interesting to note that the first term agrees quite well with the steeper component of $d^2\sigma/dMdt'$, whereas the second term reproduces the flatter one, and this is of course independent of the choice of $f_1(\Delta M)$ and $\tau(M)$. Thus, we would like to interpret these two components as due respectively to the first and the second orders of coherent excitation.

The function $f_1(\Delta M)$, i.e., the density of the probability amplitude of one of the particles for coherently exciting the other, must vanish with increasing ΔM:

$$f_1(\Delta M) \xrightarrow[\Delta M \to \infty]{} 0 . \tag{39}$$

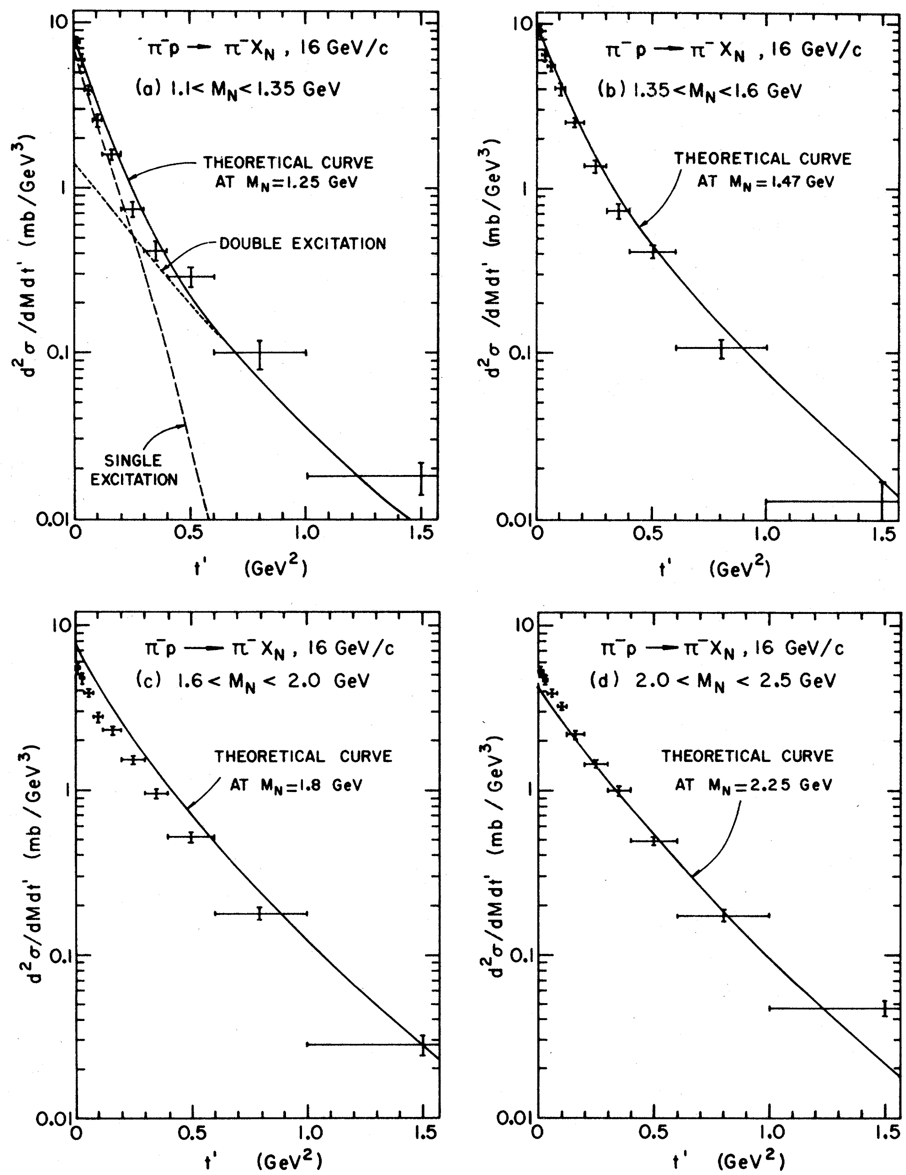


FIG. 13. Double differential cross sections $d^2\sigma/dMdt'$ for $\pi^- p \to \pi^- X_N$ at 16 GeV/c. The experimental data are from Ref. 3. In (a), the contribution from single excitation and that from double excitation are also shown.

The reason for this is that the larger the mass change, the harder it will be for the particle to maintain itself as a whole, without fragmenting. Here we will assume that

$$f_1(\Delta M) = i\beta e^{-\sigma^2 \Delta M^2}, \tag{40}$$

β and σ being two real parameters to be determined. (This exponential form is not necessarily expected to be valid for large ΔM but is a convenient way of cutting off the mass spectrum.) As mentioned earlier, the phase of $f_1(\Delta M)$ is not known *a priori*. We took it equal to $\pi/2$ just in order to agree with the experimental data on $d^2\sigma/dMdt'$. When the phase is zero, the interference between

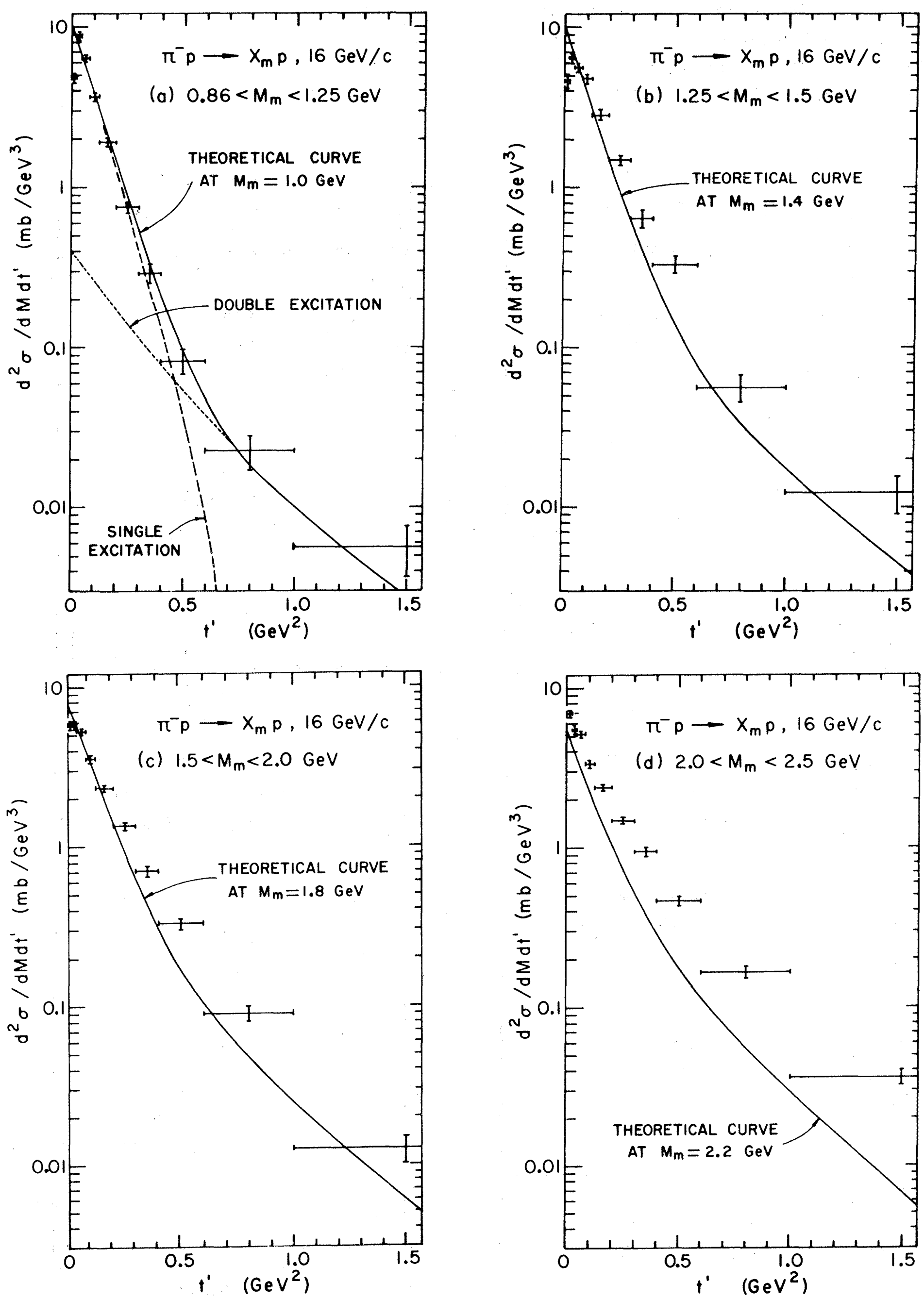


FIG. 14. Double differential cross sections $d^2\sigma/dMdt'$ for $\pi^-p \to X_m p$ at 16 GeV/c. The experimental data are from Ref. 3. In (a), the contribution from single excitation and that from double excitation are also shown.

the first- and second-order terms becomes important and it is not possible to reproduce the apparent two-component structure exhibited by the low-M distributions. Similarly, if the phase is equal to π, interference dips appear and, even neglecting these dips, the agreement becomes poorer.

The other function to be fixed is $\tau(M)$, the density of excited states as a function of M. This is certainly an increasing function of M, at least in the M range of several times the initial mass M_0. As we have no way to determine this function for the time being, we have chosen the simplest functional dependence,

$$\tau(\Delta M) = N\Delta M , \tag{41}$$

where N is some constant.

In fitting the experimental data with the above formalism, we have retained only the first two terms of Eq. (30), because our main purpose here is to show that a simple formalism such as that described above does work. Actually, the higher-order terms become too large for the present parametrization, but choosing some other parametrization, it may be possible to suppress these abnormally higher contributions. Also, we have not attempted the best fit of the data.

The free parameters in the present calculation are β, σ, and N, which had been chosen as follows:

$$\beta = 0.147, \quad \sigma = 1.2\ \text{GeV}^{-2}, \quad N = 57.6\ \text{GeV}^{-2}$$

for proton dissociation, and

$$\beta = 1.2, \quad \sigma = 0.5\ \text{GeV}^{-2}, \quad N = 0.5\ \text{GeV}^{-2}$$

for pion dissociation.

The results of these calculations are plotted in Figs. 13 and 14 together with the experimental data.

V. DISCUSSIONS

A. Summary of the Results Obtained

In the preceding sections, a model of diffractive dissociation has been proposed which reproduces quite well the existing experimental data. The model describes these processes as occurring through two stages: (i) coherent excitation of one or both of the interacting particles, and (ii) subsequent decay of the excited states. Stage (ii) is assumed to occur independently of stage (i). The main results obtained with these assumptions are as follows:

(a) The average multiplicity $\langle n \rangle$ of the dissociated system as a function of t is constant, at least in the region dominated by diffractive dissociation.

(b) A pure statistical calculation leads to $\langle n \rangle$ increasing with the effective mass of the dissociated system and, by fixing the only parameter $r \simeq 1.1$–$1.2\ \mu_\pi^{-1}$, it is possible to reproduce the experimental data on $\pi^- p$ at 16 GeV/c (Fig. 4).

(c) With the same assumption as in (b), the mass distribution for the dissociation of a particle into $2, 3, \ldots, n$ final particles can be related to the total $d\sigma/dM$ (Figs. 6 and 7). Given the crudeness of the model (based on statistical arguments only) the results are quite satisfactory (at least for $\pi^- p$ at 16 GeV/c).

(d) The particle distribution of the dissociated system is well reproduced in the reactions $\pi^- p \to p 5\pi$ (Figs. 8 and 9).

(e) An extended-particle model within an eikonal approximation describes appropriately the essential features of the coherent excitation and gives $d^2\sigma/dMdt'$, as a function of M and t', quite satisfactorily. The multicomponent structure of $d^2\sigma/dMdt'$ is interpreted as a manifestation of the different orders of excitation.

Although the comparison with the experimental data has been carried out, in practice, using $\pi^- p$ data at 16 GeV/c, we believe that the same conclusions will continue to hold at other energies as well as with other incident particles. Our belief is based on the similarity of the existing data for different experimental situations.

B. "D Resonances"

Recently Morrison has suggested the existence of the so-called "D resonances" which would be formed only through diffractive dissociation.[1] These are, e.g., A_1, A_3, Q, L, and $N^*(1300)$, and have so far been observed only in the effective-mass distribution of few-particle systems produced by diffractive dissociation. We think that at least part of these "resonances" are, actually, just kinematical effects due to the phase-space volume available for each final state [the one given by Eq. (13) (see Fig. 5)], combined with the dynamical difficulty of coherently exciting the incident particle as the mass increases [simulated in our calculation by the function $f_1(\Delta M)$]. Figures 6(a) and 6(b) show broad enhancements around 1.1 GeV and 1.3 GeV, corresponding, respectively, to A_1 and $N^*(1300)$. Although we have not performed any explicit calculation, we believe that also Q has a similar origin.

Some of these resonances (A_1 and Q) have also been found in reactions with various nuclei in which the incident π or K^- are diffractively dissociated into 3π (Ref. 23) or $(K\pi\pi)^-$ (Ref. 24). These results are consistent with the present model.

It must be emphasized that, in the above discussion, we are considering only broad enhancements in the mass distributions and not the narrow peaks which appear in some data. These may actually be resonances, but their contribution compared to the over-all number of events is small. No explanation is provided by the present model for A_3 and L.

C. Normal Resonances

The present model does not treat the usual resonances one by one. Rather, by statistical arguments we are treating them on the average. The model is intended to account also for the continuum (sometimes called simply background), which is by no means negligible, since it represents the main bulk of all the processes (about $\frac{2}{3}$ of the total in many cases[1]); for this purpose we think that our model is well suited. An additional inclusion of some resonances may evidently improve the results by giving the fine structure, but this is outside the scope of this paper.

It must be noted that some of these resonances have "wrong" quantum numbers, that is, different from those of the incident particles, or else the spin and parity do not satisfy the rule $\Delta J^{\Delta P} = 0^+, 1^-, \ldots$. This is the case, for instance, of Δ at $M \simeq 1250$ MeV in the $N\pi$ distribution [Fig. 6(b)], which should be removed before comparing with the theoretical predictions. Contributions coming from these resonances decrease with the incident energy.

D. $d^2\sigma/dMdt'$ in Diffractive Dissociation and Charge-Exchange Distribution

In Sec. V D, an eikonal formalism for diffractive dissociation has been developed, in close analogy with the one proposed for charge-exchange reactions.[22] Accordingly, the Bessel transform of the amplitude $a(M, t)$ has been approximated in the lowest order by Eq. (31). A consequence of this is the identity between the slopes of $d^2\sigma/dMdt'$ in diffractive dissociation, at relatively small M and t' values (say $\Delta M \lesssim 0.5$ GeV, where ΔM is the variation in the mass and $t' \lesssim 0.3$ GeV^2), and that of the charge-exchange reactions, initiated by the same particles. The validity of this prediction can be seen, for instance, by comparing the data on Figs. 13 and 14 with the corresponding ones on $\pi^- p \to \pi^0 n$ reaction.[25]

As a by-product, also the slopes corresponding to dissociation of particles a or b [Eq. (1)] must be the same in the M, t' regions specified above. The extent to which this is so follows by comparing Figs. 13 and 14.

E. Convergence of the Multiple-Excitation Series

In order that the expansion of the partial-wave amplitude in a series of multiple-excitation terms [Eq. (30)] have physical meaning, it is necessary that it converge. In Sec. IV, we have neither summed all of the series nor attempted to prove its convergence for our particular choice of the functions $f_1(\Delta M)$ and $\tau(M)$. We believe this is not essential in the present context, for both $f_1(\Delta M)$ and $\tau(M)$ are, apart from the properties we have discussed in Sec. IV, quite arbitrary. If a particular choice of these functions leads to a divergent series (especially for large t'), we think there will be many ways to make it convergent without modifying much the previous results. Physically, it is hard to imagine that higher-order terms give considerable contributions. Let us stress once again that the essential features such as the multicomponent structure of $d^2\sigma/dMdt'$, the value of the slope for $\Delta M \simeq 0$ and $t' \simeq 0$ ($A \simeq 12$ GeV^{-2}, $d^2\sigma/dMdt' \sim e^{-At'}$), and the decrease of the slope with M are in general independent of the choice of $f_1(\Delta M)$ and $\tau(M)$, provided they satisfy the property represented by Eq. (39) and that preceding Eq. (41).

F. Relations to Other Models

The present model is in the same spirit of the limiting-fragmentation hypothesis.[13] In fact, when the incident energy increases sufficiently, it leads to a constant particle distribution in the laboratory (or projectile) frame, consistent with the limiting-fragmentation hypothesis. We have picked up only those final states with the property (i) of Sec. I, but these are precisely the events which are expected to dominate the small-$p_\parallel$ distribution (in the lab system) at sufficiently high energies.

The model bears resemblances to the isobar model[11] (proposed for cosmic-ray jet analyses) as well as to the fireball model.[12] It is, however, more precise in the sense that, instead of applying the model to all high-energy reactions, as in Refs. 11 and 12 (which was probably due to the limited amount of experimental data), we restrict its applicability just to diffractive dissociation, which imposes a constraint on the quantum-number exchanges [condition (i) of Sec. I].

Other differences as compared with Ref. 12 are that, while only meson fireballs are considered in Ref. 12, nucleon excitation is also considered here and that in the present model the incident particles may or may not survive, but if one "fireball" is produced there will be just one survival, and if there are two "fireballs" there will be no survivals. This is not in contradiction with the small inelasticity observed at high energy, for we are considering just one class of reactions.

Starting from Glauber's[26] multiple-scattering model, Byers and Frautschi have discussed diffraction dissociation, and by considering, for instance, a reaction given by Eq. (1), have showed that[9]

$$a(M,t') \simeq \Delta \otimes \left(1 - \Delta_{el} + \frac{1}{2!}\Delta_{el}\otimes\Delta_{el} + \frac{1}{3!}\Delta_{el}\otimes\Delta_{el}\otimes\Delta_{el} - \dots\right), \tag{42}$$

where

$$\Delta = \mu F_{(b\to B)} F_a, \tag{43}$$

$$\Delta_{el} = \mu F_a F_b, \tag{44}$$

where F_a and F_b are the form factors of a and b, and $F_{(b\to B)}$ is the transition form factor from b to B. $\otimes$ denotes the convolution integral. The amplitude given by Eq. (42) corresponds to the first term of the expansion of our Eq. (30), which is certainly dominant, for small M, in the small-t' interval. The factor $\tau(M)$ in Eq. (28) is absent in Ref. 9, where only the transition to a particular resonance is considered, while we are concerned with a group of excitations. Furthermore, $F_{(b\to B)}$ has been approximated by const $\times F_b$ in the present work.

More recently, Hwa and Lam[14] and Jacob and Slansky[15] have proposed models which are very similar in spirit to ours, although our emphasis is quite different from theirs. While, by assuming the diffraction dominance, they are mainly interested in discussing properties such as the multiplicity and the inclusive cross section in a hadron-hadron collision considered as a whole, we are concerned with diffractive dissociations themselves. Thus a careful comparison to the experimental data has been done in our work, by taking the quantum number rules as well as limitations in the t and the M ranges into account. Although we agree with them that diffractive dissociations may dominate at high energies, we think it is by no means trivial that these processes themselves occur in the way we describe (as also assumed by them), especially when the final particles are not correlated in the usual way. We think the t independence of $\langle n\rangle$ is a very strong argument in favor of our description.

In Refs. 14 and 15, there has been no attempt to explain the t distribution, while in the present paper an intuitive and simple picture is given in terms of the particle dimensions, level density, and excitation probability.

Another difference between the present model and those of Refs. 14 and 15 concerns the way the particle distribution inside the dissociated system is treated. As seen in Sec. III C, we obtain this just by considering the phase space. The result is, as can be seen in Figs. 8 and 9 (there, the distribution has been integrated in M and $p_\perp$), an approximately Gaussian distribution.

In Ref. 15, this distribution is *assumed* to be effectively Gaussian, $\exp[-(p_\parallel^2+p_\perp^2)/K^2]$, with $K=300$ MeV (which has been fixed in order to give slope 9 for $p_\perp^2$ distribution). However, at least for $n=5$, this gives a $p_\parallel$ distribution which is too narrow, definitely in disagreement with the data in Fig. 8. It is true that when n increases, the phase-space calculation gives a narrower $p_\parallel$ distribution, and perhaps $n=5$ is too small to apply the statistical considerations they use.

In Ref. 14, moreover, besides the Gaussian distribution such as one used in Ref. 15, the phase space is explicitly taken into account, which causes an even narrower $p_\parallel$ distribution. We think that the parametrizations given by Refs. 14 and 15 can serve just to obtain quantities such as inclusive spectra, which are results of many averaging processes and, as observed in Ref. 14, are not very sensitive to the details of the particle distribution.

VI. FURTHER OUTLOOK

A. What Are the Excited States?

In the previous sections, a consistent description has been given of the main features of diffractive dissociation in terms of intermediate excited states, but just what these excited states are we have not yet discussed.

One could think they represent an average description of normal resonances, as their density increases and the width becomes larger. However, if one looks at the list of the presently known resonances, one concludes that these resonances are not sufficient to give all diffraction dissociation. As mentioned earlier, after subtracting the probable effects of these resonances, the "background" is still considerable [about $\frac{2}{3}$ of all events for $\pi^- p$ at 8 GeV/c (Ref. 1)]. Besides, these resonances decay in general into a rather small number of particles. Our excited states are thus more similar to fireballs than to the usual resonances.

One could also think that the excited states are just a language for describing the many-particle final states. But, in this case, the coherence of the survival particle (forward peak) becomes hard to understand.

Rather, it is tempting to say that they actually exist in the same sense as the collective excitations in nuclei, but with very short lifetimes. This, however, is not conclusive at our present state of knowledge. In short, the question of what the excited states are remains without a definite answer.

B. Momentum Spectra in Inclusive Reactions

Once $d^2\sigma/dMdt'$ and $P_n(M)$ are determined and spherical symmetry is assumed in the decay of the excited states, one can compute also the momentum spectra for different types of particles produced by diffractive dissociation. These correspond to parts of the total inclusive spectra, but it is expected that near the kinematical boundary ($x \simeq \pm 1$), where diffractive dissociations dominate, they coincide with the total spectra. With regard to the transverse-momentum distributions, a rough estimate may be made, by noticing that this is given as a superposition of distributions corresponding to different M and different multiplicities. Due to the rapid decrease of $d^2\sigma/dMdt'$ as a function of t' and the smallness of the pion mass (let us consider just the π distribution) compared to M, the centers of all these partial distributions are practically at the same point $x=0$. On the other hand, as discussed in Sec. V F and seen in Figs. 8 and 9, each distribution corresponding to a particular multiplicity is nearly Gaussian. Thus, we expect that the transverse-momentum distribution is roughly given by a sum of Gaussian distributions, which may account for the observed rapid decrease as $p_\perp$ increases. It will be interesting to verify it explicitly.

C. Double Dissociation

In the present paper, we are discussing both the reactions (1) and (2), but due to the lack of experimental data we have been mainly concerned with the simple dissociation in our comparison with the experiments.

Indeed, all the results discussed previously remain valid also in the case of double dissociation, with a remark regarding the double differential cross section $d^2\sigma/dMdt'$. The experimental data seem to indicate that double dissociation occurs much less frequently than simple dissociation. This suggests that double dissociation can be regarded as a succession of two simple dissociations, or in other words, the amplitude approximately factorizes. Accordingly, the second term in Eq. (30) is expected to dominate this class of reaction, giving a much flatter peak than in the simple dissociation case.

D. Extension of the Model to Other Reactions

Although the present paper is concerned with diffractive dissociation, some generalization of the model may be attempted. Consider, for instance, a reaction

$$a+b \to a^* + B, \tag{45}$$

where the leading particle in the right-hand side, a^*, differs from a by a charge exchange, and B is any system of particles with the appropriate quantum numbers. If our model is combined with that of Byers and Yang,[22] one expects that there is a finite probability of producing *coherently* the above reaction. If this happens, the relative probabilities of producing B with different multiplicities will be given by P_n of Eq. (18); if an argument similar to that in Sec. V C is used, one expects that $d^2\sigma/dMdt'$ is given by Eqs. (28)–(30) with dominant $a_2(M, b)$; and as $a_2(M, b)$ becomes dominant in this reaction, the effective-mass distribution of B is expected to become more important at higher M values.

We could not find data on $\pi^- p \to \pi^0 X_N$ (or $\to X_m n$) which could let us test these conclusions. However, charge-exchange K^0-producing reactions have been reported,[27] where the effective-mass distribution for different multiplicities of X^{++} and X^0 from

$$K^+p \to K^0 + X^{++} \quad \text{at} \quad 8.2 \text{ GeV}/c \tag{46}$$

and

$$K^-p \to \bar{K}^0 + X^0 \quad \text{at} \quad 10.1 \text{ GeV}/c \tag{47}$$

are given. These distributions seem to be approximately reproduced by P_n of Fig. 5(a). If we take $S=\frac{3}{2}$ for X^{++}, the agreement becomes, as expected, better.

The same paper reports $d^2\sigma/dMdt'$ for a number of M intervals. The one for the lowest M interval ($M<2$ GeV), for both the reactions, is consistent with the distribution given by the second term of Eq. (30).

Although the exponentially increasing $d\sigma/dM$, as reported in Ref. 28, could not be obtained, the second term of Eq. (30), with the same $f_1(\Delta M)$ and $\tau(M)$ used for proton dissociation in the present paper, reproduces on the average $d\sigma/dM$ for reactions given by Eqs. (46) and (47) up to $M \simeq 2$ GeV. The inclusion of higher-order terms of Eq. (30) may improve the results, but this will be attempted on another occasion.

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