Nonleptonic Hyperon Decays in the Quark Model

C. V. Sastry
Regional Engineering College, Rourkela-8, Orissa, India
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The quark model has been used to obtain s- and p-wave amplitudes for nonleptonic hyperon decays. While all the s-wave amplitudes could be related, we have only one sum rule for the p-wave amplitudes.

I. INTRODUCTION

The quark model[1] is fairly successful for the strong and electromagnetic processes in obtaining sum rules,[2] mass relations,[3] and decay amplitudes.[4] In obtaining the scattering amplitudes, where a meson is involved, it is usual to take the meson as a fundamental particle. But we have shown[5] that treating the meson also as a quark-antiquark composite is possible and indeed gives better results. Moreover, the inconsistency of treating some particles as composites and others as elementary is removed.

In this paper we calculate the s- and p-wave amplitudes of nonleptonic hyperon decays. We assume that a spurion which transforms as K^0 is responsible for the s-wave decay, and a spurion which transforms K^{*0} for the p-wave decay. It is equivalent to saying that the weak-interaction Hamiltonian transforms as the (8, 1) and (8, 3) of the $\underline{35}$ representation[6] of SU(6) for the s- and p-wave decays, respectively. The difficulty with the SU(6) calculation[7] of a similar nature is that, while correctly predicting $A(\Sigma^+_+)=0$, it also gives $B(\Sigma^+_+)=0$ (where A and B stand for the s- and p-wave amplitudes and Σ^+_+ refers to the decay $\Sigma^+\to n+\pi^+$), which is in very bad agreement with experiment. To avoid this, specific models are constructed which give $A(\Sigma^+_+)=0$ and $B(\Sigma^+_+)\neq 0$.

In the present calculation, we assume that the s-wave amplitudes are given by the interaction of an antiquark from the spurion with a λ quark of the decaying baryon. For the p-wave amplitudes, we assume that besides the quark-antiquark interaction, the quark-quark interaction also comes into play. This way $B(\Sigma^+_+)$ becomes nonzero. The above assumptions are not new to the quark model. Flamm and Majerotto[8] and Toda[9] have assumed that a single-quark (strange) transition is enough to explain the P.V. (parity-violating) amplitudes and a two-quark transition plays an important role for the P. C. (parity-conserving) amplitudes. But the essential difference between the present and the earlier work is (i) we have not assumed the mesons as elementary particles; (ii) the two-quark transition takes place between a quark from the baryon and the quark from the spurion.

II. s-WAVE AMPLITUDES

A spurion with strangeness one is responsible for the s-wave decays. We assume that it has the transformation properties of K^0 of the pseudoscalar octet. That means $A(\Lambda\to p+\pi^-)$ is proportional to the scattering amplitude of the process $K^0+\Lambda\to p+\pi^-$, the only relevant terms coming from the interaction of the λ quark of the initial baryon with the $\bar{\lambda}$ quark of the spurion. In the particle space, the most general form of the matrix element is given by[10]

$$M=\sum_{i=1}^{9} C_i' M_i\,,$$

where

$$M_1=\mathrm{Tr}(\bar{B}BPU)\,,$$
$$M_2=\mathrm{Tr}(\bar{B}UBP)\,,$$
$$M_3=\mathrm{Tr}(\bar{B}PUB)\,,$$
$$M_4=\mathrm{Tr}(\bar{B}PBU)\,,$$
$$M_5=\mathrm{Tr}(\bar{B}BUP)\,,$$
$$M_6=\mathrm{Tr}(\bar{B}UPB)\,,$$
$$M_7=\mathrm{Tr}(\bar{B}P)\,\mathrm{Tr}(BU)\,,$$
$$M_8=\mathrm{Tr}(\bar{B}U)\,\mathrm{Tr}(BP)\,,$$
$$M_9=\mathrm{Tr}(\bar{B}B)\,\mathrm{Tr}(PU)\,.$$

B and $\bar{B}$ stand for the baryon and antibaryon octets, and P and U stand for the pseudoscalar octet and a 3×3 matrix with $(U)^2_3=1$ and the rest of the elements equal to zero, respectively. With CP invariance of the weak interaction, the matrix elements of the P.V. and P.C. parts of the hadronic weak decays have, respectively, the following forms[10]:

$$a_1[\mathrm{Tr}(\bar{B}BPU)-\mathrm{Tr}(\bar{B}BUP)]+a_3[\mathrm{Tr}(\bar{B}PUB)-\mathrm{Tr}(\bar{B}UPB)]+a_7[\mathrm{Tr}(\bar{B}P)\,\mathrm{Tr}(BU)-\mathrm{Tr}(\bar{B}U)\,\mathrm{Tr}(BP)]$$

and

$$b_1[\mathrm{Tr}(\bar{B}\gamma_5 BPU)+\mathrm{Tr}(\bar{B}\gamma_5 UBP)] + b_3[\mathrm{Tr}(\bar{B}\gamma_5 PUB)+\mathrm{Tr}(\bar{B}\gamma_5 UPB)] + b_4\mathrm{Tr}(\bar{B}\gamma_5 PBU) + b_7[\mathrm{Tr}(\bar{B}P)\gamma_5\mathrm{Tr}(BU)+\mathrm{Tr}(\bar{B}U)\gamma_5\mathrm{Tr}(BP)]\,.$$

We now look at the matrix elements from the quark point of view. We observe that double-scattering effects contribute only to a_7, b_7, b_2, and b_4. $A(\Sigma_+^+)$ gets a contribution only from a_7 and hence we take a_7 to be equal to zero, since $A(\Sigma_+^+)=0$ experimentally. Hence it is reasonable to assume in general that double-scattering effects are negligible for the parity-violating amplitudes.

The quark-quark interaction Hamiltonian is characterized by the following equations[5]:

$$V|\mathcal{P}_+\mathfrak{N}_-\rangle = V_{dd}|\mathcal{P}_+\mathfrak{N}_-\rangle + V_{de}|\mathfrak{N}_+\mathcal{P}_-\rangle + V_{ed}|\mathcal{P}_-\mathfrak{N}_+\rangle + V_{ee}|\mathfrak{N}_-\mathcal{P}_+\rangle\,, \tag{1}$$

$$V|\mathcal{P}_+\lambda_-\rangle = V_{dd}^{(1)}|\mathcal{P}_+\lambda_-\rangle + V_{de}^{(1)}|\lambda_+\mathcal{P}_-\rangle + V_{ed}^{(1)}|\mathcal{P}_-\lambda_+\rangle + V_{ee}^{(1)}|\lambda_-\mathcal{P}_+\rangle\,. \tag{2}$$

The quark-antiquark interaction is characterized by[5]

$$\bar{V}|\mathcal{P}_+\bar{\mathfrak{N}}_-\rangle = \bar{V}_{dd}|\mathcal{P}_+\bar{\mathfrak{N}}_-\rangle + \bar{V}_{ed}|\mathcal{P}_-\bar{\mathfrak{N}}_+\rangle\,, \tag{3}$$

$$\bar{V}|\mathcal{P}_+\bar{\lambda}_-\rangle = \bar{V}_{dd}^{(1)}|\mathcal{P}_+\bar{\lambda}_-\rangle + \bar{V}_{ed}^{(1)}|\mathcal{P}_-\bar{\lambda}_+\rangle\,, \tag{4}$$

$$\begin{aligned}\bar{V}|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle &= \bar{V}_{dd}|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle + \bar{V}_{de}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathfrak{N}_+\bar{\mathfrak{N}}_-\rangle] \\ &\quad + \bar{V}_{de}^{(1)}|\lambda_+\bar{\lambda}_-\rangle + \bar{V}_{ed}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{P}}_+\rangle] \\ &\quad + \bar{V}_{ee}[|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{P}}_+\rangle - |\mathfrak{N}_+\bar{\mathfrak{N}}_-\rangle + |\mathfrak{N}_-\bar{\mathfrak{N}}_+\rangle] \\ &\quad + \bar{V}_{ee}^{(1)}[|\lambda_+\bar{\lambda}_-\rangle - |\lambda_-\bar{\lambda}_+\rangle]\,,\end{aligned} \tag{5}$$

where $\mathcal{P}$, $\mathfrak{N}$, and λ stand for the quarks. Now

$$A(\Lambda_0^0) \propto A(K^0+\Lambda \to n+\pi^0) \propto -\tfrac{1}{2}\sqrt{3}\,(\bar{V}_{de}^{(1)}+\tfrac{1}{2}\bar{V}_{ee}^{(1)})\,. \tag{6}$$

All the other s-wave amplitudes can be calculated similarly. We observe that there is only one constant $\bar{V}_{de}^{(1)}+\frac{1}{2}\bar{V}_{ee}^{(1)}$ for all the s-wave amplitudes. The $\Delta I=\frac{1}{2}$ relations follow immediately:

$$\sqrt{2}\,A(\Sigma_0^+)+A(\Sigma_+^+)=A(\Sigma_-^-)\,, \tag{7}$$

$$\sqrt{2}\,A(\Lambda_0^0)=-A(\Lambda_-^0)\,, \tag{8}$$

$$\sqrt{2}\,A(\Xi_0^0)=-A(\Xi_-^-)\,. \tag{9}$$

Besides the above relations we also obtain

$$A(\Lambda_-^0)=-\sqrt{3}\,A(\Sigma_0^+)=-A(\Xi_-^-) = \frac{1}{\sqrt{2}}\,A(\Omega_-^*)\,, \tag{10}$$

where Ω_-^* stands for the decay $\Omega^-\to\Xi^*+\pi^-$, and

$$\sqrt{3}\,A(\Sigma_0^+)-A(\Lambda_-^0)=2A(\Xi_-^-)\,, \tag{11}$$

where Eq. (11) is the Lee-Sugawara triangle relation.[11] Moreover relations like

$$\sqrt{2}\,A(\Omega_-^*)=\sqrt{3}\,A(\Sigma_0^+)+3A(\Lambda_-^0)\,, \tag{12}$$

$$\frac{\sqrt{3}}{\sqrt{2}}\,A(\Sigma_-^-)-A(\Lambda_-^0)=2A(\Xi_-^-)\,, \tag{13}$$

$$\frac{\sqrt{3}}{\sqrt{2}}\,A(\Sigma_0^+)+A(\Lambda_0^0)=2A(\Xi_0^0) \tag{14}$$

obtained from different models[12,13] are also seen to be satisfied.

III. p-WAVE AMPLITUDES

The spurion that is responsible for the p-wave decays transforms like K^{*0}. We assume that double scattering of the quarks should be taken into account. That is, besides the interaction of the $\bar{\lambda}$ quark of the spurion with the λ quark of the baryon, the quark in the spurion also interacts with the remaining quarks in the baryon. The effect of the double scattering is quite prominent. This can be seen from $B(\Sigma_+^+)$ which has contributions only from the simultaneous quark-antiquark and quark-quark scattering and we know that $B(\Sigma_+^+)$ is not small. All the p-wave amplitudes are described in terms of six independent amplitudes effectively in the exact SU(3) limit. They are $\bar{V}_{ee}^{(1)}$, $\bar{V}_{de}^{(1)}V_{ed}$, $\bar{V}_{de}^{(1)}V_{ee}$, $\bar{V}_{ee}^{(1)}V_{de}$, $\bar{V}_{ee}^{(1)}V_{ed}$, and $\bar{V}_{ee}^{(1)}V_{ee}$. For example, we obtain

$$B(\Lambda_-^0) \propto \frac{\sqrt{3}}{2\sqrt{2}}\,\bar{V}_{ee}^{(1)} - \frac{\sqrt{3}}{6\sqrt{2}}\,\bar{V}_{de}^{(1)}V_{ee} + \frac{\sqrt{3}}{2\sqrt{2}}\,\bar{V}_{ee}^{(1)}V_{de} + \frac{\sqrt{3}}{2\sqrt{2}}\,\bar{V}_{ee}^{(1)}V_{ed}\,, \tag{15}$$

and similarly for all other p-wave processes.

Besides the following $\Delta I=\frac{1}{2}$ sum rules,

$$\sqrt{2}\,B(\Sigma_0^+)+B(\Sigma_+^+)=B(\Sigma_-^-)\,, \tag{16}$$

$$\sqrt{2}\,B(\Lambda_0^0)=-B(\Lambda_-^0)\,, \tag{17}$$

$$\sqrt{2}\,B(\Xi_0^0)=-B(\Xi_-^-)\,, \tag{18}$$

we obtain only one more sum rule for the p-wave amplitudes:

$$\sqrt{2}\,B(\Xi_-^-)+B(\Omega_-^*)=-\frac{\sqrt{3}}{6\sqrt{2}}\,B(\Omega_K^-)\,, \tag{19}$$

which cannot be tested at present for lack of experimental data on Ω^- weak decays, with Ω_K^- standing for the decay $\Omega^-\to\Lambda+K^-$.

IV. DISCUSSION

We have only one constant to describe all the s-wave amplitudes, and the p-wave amplitudes are described by six independent constants. Whereas all the parity-violating amplitudes could be related, we have only one sum rule for the parity-conserving ones. Moreover, it is observed that SU(3)-symmetry breaking has no effect on the sum rules for the s wave. But the SU(3) violation changes the amplitudes for the p-wave decays of Ξ^- and Ω^- only. Therefore if SU(3) violation is taken into account, there is no change in the sum rules for the s-wave amplitudes, whereas no sum rule can be obtained for the p-wave amplitudes. However, if $\overline{V}_{ee}^{(1)} V_{ee} = 0$, we obtain the Lee-Sugawara relation for p-wave amplitudes also, under SU(3) symmetry. Nonleptonic hyperon decays were also considered by Misra and Dash[14] in a relativistic quark model using three constants to describe the s-wave as well as p-wave amplitudes. They were able to relate the p- and s-wave amplitudes and it is not possible in the model considered here.

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