be interpreted quite simply as due to the fact that forces in the $\bar{K}N$ system do play a significant role in determining the resonance position, as should be expected when the resonance is strongly coupled to $\bar{K}N$. The ambiguity in interpretation thus seems to reflect an ambiguity in the physics; the possibility of including a pole in the two-channel K matrix means that forces from higher-mass channels may play a role in forming the resonance, but the strong coupling to $\bar{K}N$ means that forces in this channel are also of significance.

A more stringent criterion for interpreting K-matrix poles has been proposed by Rajasekaran.[8] He argues that in order to ascribe a resonance to a K-matrix pole, $\det K^{-1}$ should pass through zero with a large negative slope at the pole position. One can easily demonstrate that this condition cannot be satisfied for any K matrix defined by Eq. (1) whose matrix elements are close to those of the usual constant K-matrix fits at some energy above the $\bar{K}N$ threshold. However, the argument for this test is based on a requirement that the resonance pole position have a negligible dependence on the threshold energy of the closed channel. This is certainly not the case for a resonance strongly coupled to that channel, as for the fits we have discussed here.

From a physical point of view, the nature of the $Y_0^*(1405)$ in both the constant K-matrix case, and the fits we have made using the Dalitz form, is essentially the same. In all the fits, the $\Sigma\pi$ amplitude below threshold is dominated by a pole whose position and residue is nearly model-independent. Thus, whether the resonance is regarded as a virtual bound state of the $\bar{K}N$ system, with negligible effects from couplings to other channels, or as having its origins in forces from higher-mass channels, it still appears to be strongly coupled to the $\bar{K}N$ system. The experimental difficulties associated with this coupling which were discussed earlier remain.

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Chiral Symmetry Breaking and the Cabibbo Angle

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The proposal of Oakes linking the pion mass and the strangeness-changing weak interactions is investigated in the (8,8) symmetry breaking of SU(3)⊗SU(3). This leads to a determination of the Cabibbo angle and the $\eta\rightarrow 3\pi$ decay rates which are in agreement with the earlier results in the $(3,\bar{3})\oplus(\bar{3},3)$ model. The connection between cancellations in the leading weak-interaction divergences and the Cabibbo angle, however, does not hold in the (8,8) scheme.

I. INTRODUCTION

Ever since Gell-Mann's[1] original suggestion that strong interactions are approximately invariant under SU(3)⊗SU(3) was put forward, considerable interest has centered around the following questions: (a) How does the symmetry-breaking Hamiltonian $\mathcal{H}'$ transform under SU(3)⊗SU(3), and (b) how is SU(3)⊗SU(3) broken down to SU(2)? The simplest assumption, that $\mathcal{H}'$ is part of a $(3,\bar{3})$

$\oplus(\bar{3},3)$ representation of SU(3)⊗SU(3), has been thoroughly investigated.[2] One finds that we are close to a world in which SU(3)⊗SU(3) is broken in the chain

$$SU(3)\otimes SU(3)\to SU(2)\otimes SU(2)\to SU(2).$$

This symmetry-breaking pattern led Oakes[3] to discuss the interesting possibility of a connection between the small amount of SU(2)⊗SU(2) breaking which is present in nature (i.e., the smallness of the pion mass m_π) and the Cabibbo angle θ. He proposed that the nonvanishing of both m_π and θ have a common origin, and thus obtained a prediction for the Cabibbo angle which is in agreement with experiment. Moreover, with the modification of $\mathcal{H}'$ suggested by his analysis, he was able to evaluate[4] the decays $\eta\to 3\pi$ and obtain decay rates in reasonable agreement with experiment. It has been shown[5-7] that this choice for $\mathcal{H}'$ leads to a cancellation of the leading weak and electromagnetic divergences.

In this paper we investigate this connection between m_π and θ in the (8,8) model of symmetry breaking. We show that the relation between m_π and θ obtained here, and the evaluation of the decays $\eta\to 3\pi$, are identical to those in the $(3,\bar{3})\oplus(\bar{3},3)$ model. However, in contrast to the latter model the cancellation between the leading divergences of the weak and electromagnetic interactions cannot occur.

We shall not discuss here the various motivations to abandon the $(3,\bar{3})\oplus(\bar{3},3)$ model. We simply note that there are sufficient apparent discrepancies[8] between this scheme and experiments to warrant a study of more complex models. If one abandons the $(3,\bar{3})\oplus(\bar{3},3)$ scheme, the next simplest possibility is the assumption that $\mathcal{H}'$ transforms under SU(3)⊗SU(3) as a member of an (8,8) representation. Many features of this model have already been discussed,[9-12] and in particular the symmetry-breaking parameter has also been obtained. Although SU(2)⊗SU(2) cannot be an exact symmetry limit of $\mathcal{H}'$ in the (8,8) model, we discuss an interesting analogy between the $(3,\bar{3})\oplus(\bar{3},3)$ and the (8,8) models. As a consequence one can understand why $m_\pi\approx 0$ in the (8,8). We begin with some definitions and specify our notation.

II. THE MODEL

The (8,8) model has 64 operators $S^{\alpha\beta}$, with $\alpha,\beta=1,\ldots,8$, transforming as

$$[Q^\alpha, S^{\beta\gamma}(x)] = i(f^{\alpha\beta\delta}S^{\delta\gamma}(x)+f^{\alpha\gamma\delta}S^{\beta\delta}(x)) \tag{2.1}$$

and

$$[Q_A^\alpha, S^{\beta\gamma}(x)] = i(-f^{\alpha\beta\delta}S^{\delta\gamma}(x)+f^{\alpha\gamma\delta}S^{\beta\delta}(x)), \tag{2.2}$$

where repeated indices have been summed over. It is convenient to introduce operators $S_S^{\alpha\beta}$ and $S_A^{\alpha\beta}$, defined by

$$S_S^{\alpha\beta}=S^{\alpha\beta}+S^{\beta\alpha} \tag{2.3}$$

and

$$S_A^{\alpha\beta}=S^{\alpha\beta}-S^{\beta\alpha}. \tag{2.4}$$

The symmetry-breaking Hamiltonian $\mathcal{H}'(x)$ may be written as

$$\mathcal{H}'(x)=\frac{A}{2\sqrt{8}}S_S^{\alpha\alpha}(x)+\frac{B\sqrt{3}}{2\sqrt{5}}d_{8\alpha\beta}S_S^{\alpha\beta}(x). \tag{2.5}$$

It will be convenient to define the symmetry-breaking parameter z by

$$z=\left(\frac{5}{8}\right)^{1/2}\left(\frac{A}{B}\right). \tag{2.6}$$

From Eqs. (2.5) and (2.6) the current divergences may now be computed[10] as linear forms in $S_A^{\alpha\beta}$. We present below the octet part of these expressions. These are projected out by contracting with the SU(3) structure constants $f_{\rho\alpha\beta}$, i.e., we write

$$3\mathcal{P}^\rho=f_{\rho\alpha\beta}S_A^{\alpha\beta}, \tag{2.7}$$

where $\mathcal{P}^\rho$ are the corresponding octet operators. One obtains

$$[\partial^\mu A_\mu^\pi]_8=+\frac{3B}{\sqrt{5}}(\tfrac{1}{2}+z)\mathcal{P}^\pi, \tag{2.8}$$

$$[\partial^\mu A_\mu^K]_8=+\frac{3B}{\sqrt{5}}(-\tfrac{1}{4}+z)\mathcal{P}^K, \tag{2.9}$$

and

$$[\partial^\mu A_\mu^\eta]_8=\frac{3B}{\sqrt{5}}(-\tfrac{1}{2}+z)\mathcal{P}^\eta. \tag{2.10}$$

We now compare Eqs. (2.7)–(2.10) with the corresponding expressions for the current divergences in the $(3,\bar{3})\oplus(\bar{3},3)$ model, which has

$$\mathcal{H}'=\epsilon_0u_0+\epsilon_8u_8 = \epsilon_0(u_0+cu_8), \tag{2.11}$$

where the u's are the scalar densities of the $(3,\bar{3})\oplus(\bar{3},3)$ representation of SU(3)⊗SU(3), and $c=\epsilon_8/\epsilon_0$. Using Eq. (2.11) the current divergences may be expressed in terms of the pseudoscalar densities v_α in the standard fashion:

$$\partial^\mu A_\mu^\pi=-\frac{\epsilon_0}{\sqrt{3}}(c+\sqrt{2})v_\pi, \tag{2.12}$$

$$\partial^\mu A_\mu^K=-\frac{\epsilon_0}{\sqrt{3}}(\sqrt{2}-\tfrac{1}{2}c)v_K, \tag{2.13}$$

and

$$\partial^\mu A_\mu^\eta = -\frac{\epsilon_0}{\sqrt{3}}[(\sqrt{2} - c)v_8 + c\sqrt{2}\, v_0]\,. \tag{2.14}$$

Comparing Eqs. (2.8)–(2.9) with Eqs. (2.12)–(2.13) we note that if we define

$$c = \frac{1}{z\sqrt{2}}\,, \tag{2.15}$$

$$\epsilon_0 = -\left(\frac{3\sqrt{3}}{2\sqrt{5}}\right)\frac{B}{c}\,, \tag{2.16}$$

the expressions in (2.8) and (2.9) are identical to those in (2.12) and (2.13) if we make the replacement $\mathcal{P}^\rho \to v^\rho$. In other words, the octet parts of the current divergences in (2.8) and (2.9) become identical to Eqs. (2.12) and (2.13). The same substitutions make Eq. (2.10) identical to Eq. (2.14), apart from the term proportional to v_0 in the latter equation. Such a term does not occur in Eq. (2.10), since $f_{0\alpha\beta}S_A^{\alpha\beta}$ vanishes identically. However, this difference is irrelevant for our purposes, since the analysis of Gell-Mann, Oakes, and Renner[2] suggests that

$$\langle\Omega|v_0|P^\alpha\rangle \approx 0\,, \tag{2.17}$$

if we neglect η-η' mixing, as we shall throughout. Using Eq. (2.15) and the value for c obtained in the $(3,\bar{3})\oplus(\bar{3},3)$ model, i.e.,

$$c = -2\sqrt{2}\left(\frac{m_K^2 - m_\pi^2}{2m_K^2 + m_\pi^2}\right), \tag{2.18}$$

we obtain

$$z = -\tfrac{1}{4}\left(\frac{2m_K^2 + m_\pi^2}{m_K^2 - m_\pi^2}\right), \tag{2.19}$$

in agreement with the value of z obtained in Refs. 9–12.

We may also note that the expressions for the octet part of the vector-current divergences in the (8, 8) model are also proportional to the corresponding expressions in the $(3,\bar{3})\oplus(\bar{3},3)$ model. The vector-current divergences are proportional to $S_S^{\alpha\beta}$, and their octet parts are readily projected out by contracting with the SU(3) coefficients $d_{\rho\alpha\beta}$.

III. SYMMETRY-BREAKING PATTERNS IN THE (8, 8) MODEL

Any model of chiral symmetry breaking has to cope with the existence of states transforming approximately covariantly under SU(3), and with the smallness of the pion mass. These two basic facts imply that $\partial^\mu A_\mu^{1,2,3}$ in the $(3,\bar{3})\oplus(\bar{3},3)$ or its octet part in the (8, 8) are related to v_π or $\mathcal{P}_\pi$, respectively, by a factor of order m_π^2/m_K^2. To see this it suffices to use Eqs. (2.8) and (2.12) and the values for c and z in Eqs. (2.18) and (2.19). These follow by making use of Eqs. (2.8) and (2.12) and by noting that approximate SU(3) for the vacuum and the pseudoscalar meson states $|P_\beta\rangle$ allows us to write

$$\langle\Omega|v_\alpha|P_\beta\rangle = r\delta_{\alpha\beta} \tag{3.1}$$

in the $(3,\bar{3})\oplus(\bar{3}.3)$ model, and

$$\langle\Omega|\mathcal{P}_\alpha|P_\beta\rangle = r\delta_{\alpha\beta} \tag{3.2}$$

or equivalently

$$\langle\Omega|[S_A^{\alpha\beta}]_{\underline{10}\oplus\underline{\overline{10}}}|P_\gamma\rangle = 0 \tag{3.3}$$

in the (8, 8) model. Since [once again from approximate SU(3)]

$$\frac{\langle\Omega|\partial^\mu A_\mu^3|\pi^3\rangle}{\langle\Omega|\partial^\mu A_\mu^4|K^4\rangle} \approx \frac{m_\pi^2}{m_K^2}\,, \tag{3.4}$$

we then obtain Eqs. (2.18) and (2.19). This implies the above assertion and shows furthermore that SU(3) is not a much better symmetry than SU(3) $\otimes$ SU(3). In other words, approximate SU(3) for the above states and the Hamiltonian would have implied in both models that SU(3) is also approximately valid for the particle masses, i.e., $m_\pi \approx m_K$.

This common state of affairs allows in the $(3,\bar{3})\oplus(\bar{3}.3)$ scheme a particular interpretation, since in that model the $\partial^\mu A_\mu^{1,2,3}$ are pure octet. Thus, the smallness of the octet part of this operator then implies that $\partial^\mu A_\mu^{1,2,3}$ itself almost vanishes. Therefore, since $\partial^\mu A_\mu^\pi = 0$ is the limit of SU(2) $\otimes$ SU(2), the smallness of m_π^2/m_K^2 is the consequence of an approximate symmetry in the $(3,\bar{3})\oplus(\bar{3},3)$ model. This then "explains" the smallness of m_π^2/m_K^2. It also follows that, apart from dynamical enhancements, the matrix elements of the σ terms $[Q_A^\pi, \partial^\mu A_\mu^\pi]$ have to be small and have to vanish in the limit $m_\pi = 0$. In particular, the ratio of the (πN) to the $(KN)_{I=1}$ σ terms has to vanish in the limit of vanishing pion mass, and is predicted to be of the order m_π^2/m_K^2. Furthermore, the $(\pi\Sigma)_{I=0,1}$ σ terms also vanish for $m_\pi \to 0$, and thus they are also predicted to be small in the $(3,\bar{3})\oplus(\bar{3},3)$. These experimentally testable predictions follow in the $(3,\bar{3})\oplus(\bar{3},3)$, since in that model the $\partial^\mu A_\mu^\pi$ are pure octet and vanish as $m_\pi \to 0$.

The situation is quite different in the (8, 8) scheme. In this model $\partial^\mu A_\mu^\pi$ has, in addition to the small octet parts, terms transforming as $\underline{10}$ and $\underline{\overline{10}}$ representations under SU(3) (which are projected out in the matrix elements $\langle\Omega|\partial^\mu A_\mu^\pi|\pi\rangle$). These terms will in general contribute to matrix elements of $[Q_A^\pi, \partial^\mu A_\mu^\pi]$. In fact, in actual calculations[12] the contribution from the octet part of $\partial^\mu A_\mu^\pi$ is frequently just a small correction to that from its $\underline{10}$ and $\underline{\overline{10}}$ parts. Thus, for example, not only are the (πN) and $(\pi\Sigma)_{0,1}$ σ terms nonvanishing as $m_\pi \to 0$ in the (8, 8), but, also, the $(\pi\Sigma)_{0,1}$ σ terms are found to be large in magnitude in such a limit.[12]

A study of the vacuum expectation value

$$\sigma_A(3) = \langle \Omega | [Q_A^3, \partial^\mu A_\mu^3] | \Omega \rangle$$

brings out yet another aspect. The conventional and appealing ideas of SU(3) for the states (in particular for the vacuum) and Goldstone symmetry breaking are also fulfilled in the (8, 8) scheme. Namely, for $\sigma_A(3)$ a saturation by the pion state should be correct according to the Goldstone nature of the symmetry breaking. This then projects out once again the octet parts of $\partial^\mu A_\mu^\pi$, and hence $\sigma_A(3)$ should be small. We can also saturate $\langle \Omega | [Q_A^K, \partial^\mu A_\mu^K] | \Omega \rangle$ and $\langle \Omega | [Q_A^8, \partial^\mu A_\mu^8] | \Omega \rangle$ by K and η states, respectively, and express all $\langle S_S^{\alpha\beta} \rangle_\Omega$ in terms of the above axial σ terms. In agreement with SU(3) symmetry for the vacuum state, expressions for $\langle [S_S^{\alpha\beta}]_{27} \rangle_\Omega$ and $\langle [S_S^{\alpha\beta}]_8 \rangle_\Omega$ are found to vanish in the SU(3) limit for f_K/f_π and f_η/f_π. Thus the conventional ideas on Goldstone symmetry breaking and the SU(3) symmetry of the vacuum hold in the (8, 8) model.

As a final comment we note that it is possible to define a small parameter corresponding to m_π^2/m_K^2, which may be used to do perturbation-theory calculations in the (8, 8) model. We may write $\mathcal{H}'$ as

$$\begin{aligned}\mathcal{H}' &= \epsilon_1 H_1 + \epsilon_2 H_2 \\ &= \epsilon_1 (S_S^{\alpha\alpha} - 2\sqrt{3}\, d_{8\alpha\beta} S_S^{\alpha\beta}) \\ &\quad + \epsilon_2 (\sqrt{2}\, S_S^{\alpha\alpha} + \sqrt{6}\, d_{8\alpha\beta} S_S^{\alpha\beta}),\end{aligned} \tag{3.5}$$

with

$$\begin{aligned}\epsilon_1 &= \tfrac{1}{3}(1 - \sqrt{2}\, c)\epsilon_0, \\ \epsilon_2 &= \tfrac{1}{3}(c + \sqrt{2})\epsilon_0,\end{aligned} \tag{3.6}$$

and ϵ_0 and c defined as in Eqs. (2.15) and (2.16). In Eq. (3.5) the term $\epsilon_1 H_1$ does not give rise to an octet part of $\partial^\mu A_\mu^\pi$, and thus $\epsilon_2 = 0$ would yield $m_\pi = 0$. The second term is responsible for the pion mass and tends to zero as $m_\pi \to 0$. Thus $\epsilon_2 \ll \epsilon_1$. The ratio ϵ_2/ϵ_1 may be conveniently used to do perturbation theory around the limit $m_\pi = 0$. To establish the analogy with the $(3,\bar{3}) \oplus (\bar{3},3)$ model we remind the reader of the possibility of writing $\mathcal{H}'$ in that model as

$$\mathcal{H}' = \tfrac{1}{3}(\sqrt{2} + c)(\sqrt{2}\, u_0 + u_8) + \tfrac{1}{3}(1 - \sqrt{2}\, c)(u_0 - \sqrt{2}\, u_8). \tag{3.7}$$

IV. CONNECTION BETWEEN THE PION MASS AND THE CABIBBO ANGLE

We now consider in the (8, 8) scheme the possible connection between m_π and the Cabibbo angle θ proposed by Oakes.[3,4]

The nonconservation of strangeness in the weak interactions can be thought of as arising from a rotation of the strangeness-conserving weak current through an angle 2θ about the 7th direction in SU(3) space. Moreover, it is an appealing idea that in the limit of vanishing pion mass the Cabibbo angle should vanish. Oakes proposed to link the two effects by demanding that the strong-interaction Hamiltonian for $m_\pi \neq 0$ be obtained by rotating the Hamiltonian with SU(2)⊗SU(2) symmetry by an angle 2θ about the 7th axis in SU(3) space and imposing strangness conservation on the result. Carrying out this rotation one obtains from Eq. (3.5) (with $c = -\sqrt{2}$)

$$\begin{aligned}e^{-2i\theta Q_7}\epsilon_1 H_1 e^{2i\theta Q_7}\big|_{\Delta S=0} \\ &= \epsilon_0 [S_S^{\alpha\alpha} - 2\sqrt{3}\,(1 - \tfrac{3}{2}\sin^2\theta) d_{8\alpha\beta} S_S^{\alpha\beta} \\ &\quad - 3\sin^2\theta\, d_{3\alpha\beta} S_S^{\alpha\beta}].\end{aligned} \tag{4.1}$$

The current divergences may now be evaluated from Eq. (4.1) in the standard fashion. By projecting the octet parts of the current divergences using Eq. (2.7) we obtain

$$[\partial^\mu A_\mu^\pi]_8 = 9\epsilon_0 \sin^2\theta \left(\mathcal{P}^\pi - \frac{1}{\sqrt{3}}\, \delta_{\pi 3} \mathcal{P}^8 \right), \tag{4.2}$$

$$[\partial^\mu A_\mu^K]_8 = 9\epsilon_0 \cos^2\theta\, \mathcal{P}^K, \tag{4.3}$$

and

$$[\partial^\mu A_\mu^\eta]_8 = 3\epsilon_0 [(4 - 3\sin^2\theta)\mathcal{P}^8 - \sqrt{3}\,\sin^2\theta\, \mathcal{P}^3], \tag{4.4}$$

which are once again identical to the corresponding expressions for the current divergences in the $(3,\bar{3}) \oplus (\bar{3},3)$ model if we make the replacement $\mathcal{P}^\alpha \to v^\alpha$. From Eqs. (4.2) and (4.3) we have

$$\begin{aligned}\frac{\langle \Omega | \partial^\mu A_\mu^\pi | \pi \rangle}{\langle \Omega | \partial^\mu A_\mu^K | K \rangle} &= \frac{f_\pi m_\pi^2}{f_K m_K^2} \\ &= \frac{\sin^2\theta}{\cos^2\theta} \frac{\langle \Omega | \mathcal{P}^\pi | \pi \rangle}{\langle \Omega | \mathcal{P}^K | K \rangle}.\end{aligned} \tag{4.5}$$

Thus, in the SU(3) limit for $\langle \Omega | \mathcal{P}^\pi | \pi \rangle / \langle \Omega | \mathcal{P}^K | K \rangle$ we have

$$\tan^2\theta = \frac{f_\pi}{f_K} \frac{m_\pi^2}{m_K^2}. \tag{4.6}$$

For $f_\pi/f_K = 1$ we obtain

$$\sin\theta = 0.28, \tag{4.7}$$

and for $f_\pi/f_K = (1.28)^{-1}$ we obtain

$$\sin\theta = 0.21. \tag{4.8}$$

The most recently reported value of $\sin\theta$ is 0.24.

Before concluding this section, note also that the tadpole term $-3\epsilon_0 \sin^2\theta\, d_{3\alpha\beta} S_S^{\alpha\beta}$ in general contributes to the electromagnetic mass difference.

For example, its contribution to the $(\Delta I=1)$ K^0, K^+ mass difference is

$$m_{K^0}{}^2 - m_{K^+}{}^2 \approx m_\pi{}^2 \approx m_{K^+}{}^2 \sin^2\theta, \qquad (4.9)$$

which is the same as in the $(3,\bar{3})\oplus(\bar{3},3)$ model. If desired, the tadpole contribution to the baryon masses may also be evaluated.

V. THE DECAY $\eta \to 3\pi$

Since the Hamiltonian in Eq. (4.1) contains a term $d_{3\alpha\beta}S_S^{\alpha\beta}$, there is now an additional isospin-breaking contribution from the strong interactions. Thus, for example, the $\eta\to 3\pi$ amplitudes no longer vanish in the soft-pion limit. We discuss this decay following the techniques of Refs. 4 and 13–15.

For $\eta\to\pi^+\pi^-\pi^0$, using partial conservation of the axial-vector current (PCAC), we obtain

$$\begin{aligned}\lim_{q_{\mu\pi^\pm}\to 0}\langle\pi^+\pi^-\pi^0|\mathcal{H}_S+H_{\rm EM}|\eta\rangle &= \lim_{q_{\pi^\pm}\to 0}[M(E_+,q_+{}^2;E_-,q_-{}^2;E_0,q_0{}^2)]\\ &=\frac{i}{f_\pi}(3\epsilon_0\sin^2\theta)\langle\pi^\mp\pi^0|[Q_A^{1\mp i2},d_{3\alpha\beta}S_S^{\alpha\beta}]|\eta\rangle\\ &=\frac{i}{f_\pi}(3\epsilon_0\sin^2\theta)\left(\mp\frac{2}{\sqrt{3}}\right)\langle\pi^\mp\pi^0|(S_A^{18}\mp iS_A^{28})|\eta\rangle,\end{aligned}\qquad (5.1)$$

where $H_{\rm EM}$, as usual, does not contribute to the decay in the soft-pion limit.[16]

Due to Bose symmetry for the pions and the s-wave form of the space-wave functions, the isospin of the final $\pi^\pm\pi^0$ state in Eq. (5.1) is $I=2$. On the other hand, $S_A^{(1\pm i2),8}$, which contains terms with SU(3) tensorial characters $\underline{8}\oplus\underline{10}\oplus\overline{\underline{10}}$, does not have $I=2$ parts. Thus in the soft-pion limits in the charged-pion momenta the decay amplitude $\eta\to\pi^+\pi^-\pi^0$ is zero:

$$\lim_{q_{\mu\pi^\pm}\to 0} M(E_+,q_+{}^2;E_-,q_-{}^2;E_0,q_0{}^2)=0\,. \qquad (5.2)$$

Precisely the same result obtains in the $(3,\bar{3})\oplus(\bar{3},3)$ model. However, in that scheme the commutator in Eq. (5.1) is *identically* zero, in contrast to the above $(8,8)$ scheme in which the *matrix elements* in Eq. (5.1) are zero.

In the limit $q_{\mu\pi^0}\to 0$ we have

$$M(\tfrac{1}{2}m_\eta,q_+{}^2;\tfrac{1}{2}m_\eta,q_-{}^2;0,0)=\frac{i\sqrt{2}}{f_\pi}(3\epsilon_0\sin^2\theta)\langle\pi^+\pi^-|i(S_A^{45}+S_A^{67})|\eta\rangle, \qquad (5.3)$$

which is not zero.

If we reduce out the pions in Eq. (5.3) and assume that the σ terms do not contribute, the q^2-dependent terms drop out and we have

$$\lim_{q_{\pi^+}\to 0;\,q_{\pi^-}\to 0}\{M(\tfrac{1}{2}m_\eta,q_+{}^2;\tfrac{1}{2}m_\eta,q_-{}^2;0,0)\}=\frac{2\sqrt{2}}{f_\pi{}^3}(3\epsilon_0\sin^2\theta)\langle\Omega|S_A^{45}+S_A^{67}|\eta\rangle\,. \qquad (5.4)$$

Using Eq. (4.4) to express $\langle\Omega|S_A^{45}+S_A^{67}|\eta\rangle$ in terms of matrix elements of the divergence of the axial-vector current we have

$$M(0,0;0,0;0,0)=\left(\frac{\sqrt{2}}{f_\pi{}^3}\right)\frac{2\sqrt{3}\,\sin^2\theta}{(4-3\sin^2\theta)}\langle\Omega|[\partial^\mu A_\mu^8]_8|\eta\rangle \qquad (5.5)$$

on noting $\langle\Omega|\mathcal{P}^3|\eta\rangle\approx 0$. In the $(3,\bar{3})\oplus(\bar{3},3)$ model one obtains the same result after using $\langle\Omega|v_3|\eta\rangle\approx 0$ and $\langle\Omega|v_0|\eta\rangle\approx 0$.

In a recent review of the $\eta\to 3\pi$ problem it has been shown by Mohapatra[15] that conditions (5.2) and (5.3) and the neglect of σ terms are sufficient to ensure that the experimentally observed slope in the Dalitz plot of the $\eta\to 3\pi$ decay spectrum be reproduced. On expanding the matrix element as

$$M(E_+,q_+{}^2;E_-,q_-{}^2;E_0,q_0{}^2)=A+BE_{\pi^0}+C(q_+{}^2+q_-{}^2)+Dq_0{}^2, \qquad (5.6)$$

the conditions (5.2) and (5.3) and the neglect of σ terms give

$$D=0, \qquad (5.7a)$$

$$B=-\frac{2}{m_\eta}A+2C\frac{m_\pi{}^2}{m_\eta}, \qquad (5.7b)$$

with

$$C \approx 0 \tag{5.8a}$$

and

$$A = M(0,0;0,0;0,0)\,. \tag{5.8b}$$

Then

$$M(E_+, q_+{}^2; E_-, q_-{}^2; E_0, q_0{}^2) = M(0)\left(1 - \frac{2E_{\pi^0}}{m_\eta}\right), \tag{5.9}$$

with the slope $-2/m_\eta$. The decay rate is given by[14,15]

$$\Gamma(\eta \to \pi^+\pi^-\pi^0) = |M(0)|^2 \frac{(m_\eta - 3m_\pi)^2}{3456\sqrt{3}\,\pi^2 m_\eta} \approx 0.81 \text{ keV}\,. \tag{5.10}$$

Also, a straightforward calculation shows that

$$\frac{\Gamma(\eta \to 3\pi^0)}{\Gamma(\eta \to \pi^+\pi^-\pi^0)} = \tfrac{3}{2}\,. \tag{5.11}$$

The experimental values[17] are

$$\Gamma(\eta \to \pi^+\pi^-\pi^0) = 0.63 \text{ keV}$$

and

$$\frac{\Gamma(\eta \to 3\pi^0)}{\Gamma(\eta \to \pi^+\pi^-\pi^0)} = 1.25\,.$$

VI. NONCANCELLATION OF THE LEADING DIVERGENCES IN WEAK AND ELECTROMAGNETIC INTERACTIONS

In Refs. 5–7 it has been pointed out that in the $(3,\bar{3}) \oplus (\bar{3},3)$ model an interesting cancellation mechanism occurs between the leading divergences of the weak and electromagnetic interactions when the symmetry-breaking parameters in the strong-interaction Hamiltonian are as given in the model of Oakes.[3]

To recapitulate, we note that the leading weak-interaction correction to the strong processes consists of adding a further symmetry-breaking term δH_W to the original breaking, and it is of the form

$$\delta H_W \sim G\Lambda^2 \{[Q^W, \partial^\mu J_\mu^{\dagger W}] + [Q^{\dagger W}, \partial^\mu J_\mu^W]\}, \tag{6.1}$$

where Λ is a cutoff.

On the other hand, the leading electromagnetic correction δH_{EM} [which cannot be readily calculated by a formula analogous to (6.1), as model-dependent terms would have to be included in this calculation] is expected to be a U-spin singlet. Requiring the cancellation of the leading weak and electromagnetic divergences, i.e.,

$$\delta H_W + \delta H_{EM} = 0\,, \tag{6.2}$$

and using the U-spin singlet nature of δH_{EM}, leads to an equation for the Cabibbo angle θ. This equation is identically satisfied in the $(3,\bar{3}) \oplus (\bar{3},3)$ model when the symmetry-breaking parameters in the strong-interaction Hamiltonian are as suggested by Oakes.[3]

We now show that this cancellation mechanism cannot occur in the $(8,8)$ model. In order to see this it suffices to consider the parity-conserving part of the leading weak correction δH_W [Eq. (6.1)] in the $(8,8)$ scheme. We have

$$\begin{aligned}\delta H_W \sim 2G\Lambda^2 \Big\{ &\Big[-\rho(S_S^{11} + S_S^{22} + 2S_S^{33}) - \sigma(S_S^{44} + S_S^{55} + S_S^{66} + S_S^{77}) + \frac{3B}{4\sqrt{5}}\sin^2\theta\Big(-S_S^{44} - S_S^{55} + S_S^{66} + S_S^{77} - \frac{8}{\sqrt{3}}S_S^{38}\Big)\Big]\cos^2\theta \\ &+ \Big[-\frac{3B}{4\sqrt{5}}\Big(S_S^{11} + S_S^{22} + S_S^{33} - S_S^{66} - S_S^{77} - S_S^{88} + \frac{2}{\sqrt{3}}S_S^{38}\Big) - \tau(S_S^{11} + S_S^{22} + S_S^{33} + S_S^{66} + S_S^{77}) \\ &\quad - \tfrac{1}{2}(\tau + \sqrt{3}\,\kappa)(S_S^{44} + S_S^{55}) - \sqrt{3}\,\kappa S_S^{88} - (\sqrt{3}\,\tau + \kappa)S_S^{38} \\ &\quad + \frac{3B}{4\sqrt{5}}\sin^2\theta\Big(-2S_S^{33} - 2S_S^{44} - 2S_S^{55} + S_S^{66} + S_S^{77} - 2S_S^{88} - \frac{8}{\sqrt{3}}S_S^{38}\Big)\Big]\sin^2\theta \Big\},\end{aligned} \tag{6.3}$$

where

$$\rho = \frac{2B}{\sqrt{5}}(1+z)\,, \tag{6.4}$$

$$\sigma = \frac{B}{\sqrt{5}}(-\tfrac{1}{2}+z)\,, \tag{6.5}$$

$$\tau = \frac{B}{\sqrt{5}}(\tfrac{1}{4}+z)\,, \tag{6.6}$$

and

$$\kappa = B\left(\frac{3}{5}\right)^{1/2}(-\tfrac{3}{4}+z)\,, \tag{6.7}$$

with B and z as in Eqs. (2.5) and (2.6). If Eq. (6.2) holds we can then use the U-spin property of the electromagnetic current to obtain

$$[Q_6, \delta H_W] = 0 , \tag{6.8}$$

since Q_6 is a generator of U-spin rotations. However, Eqs. (6.3), (6.8), and (6.4)–(6.7) then would imply

$$\cos^2\theta(\tfrac{5}{2}+z) = \sin^2\theta(-\tfrac{5}{4}+z) + \tfrac{3}{4}\sin^4\theta , \tag{6.9}$$

$$\cos^2\theta(\tfrac{3}{2}+z) = \sin^2\theta(\tfrac{1}{4}+z) - \tfrac{1}{2}\sin^4\theta , \tag{6.10}$$

and

$$\cos^2\theta(-\tfrac{1}{2}+z) = \sin^2\theta(-\tfrac{9}{4}+z) - \tfrac{3}{2}\sin^4\theta , \tag{6.11}$$

which are in clear contradiction. We therefore conclude that the compensation mechanism between the leading divergences of the weak and electromagnetic interactions proposed earlier[5-7] cannot occur in the (8, 8) model.

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