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Soft-Pion Emission in Proton-Antiproton Annihilation*

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We calculate the amplitude for the annihilation process $\bar{p}p \to K^+K^-\pi^+\pi^-$, and the differential rate for $\bar{p}p \to K^+K^-\pi^+\pi^-$, normalized to the $\bar{p}p \to K^+K^-$ rate, in the soft-pion limit, for annihilation in flight at vanishing momentum. The computation uses the formalism of current algebra and PCAC (partially conserved axial-vector current); of particular interest is the use of Low's soft-photon theorem to evaluate the isovector photon term. The final result is expressed as the $\bar{p}p \to K^+K^-\pi^+\pi^-$ rate, differential in all five independent kinematic variables, chosen for their symmetry and their utility in comparing with experimentally determined spectra. The integral of this expression yields the branching ratio $\bar{p}p \to K^+K^-\pi^+\pi^-/\bar{p}p \to K^+K^- = 2.25$, in reasonable agreement with experiment.

I. INTRODUCTION

It is well known that one can, in principle, express the amplitude of any process which involves emission of soft pions in terms of the amplitude of the same process without soft pions.[1] These theorems have been demonstrated at one time or another in the last few years within the framework of current algebra and PCAC (partially conserved axial-vector current),[2] chiral symmetry,[3] or phenomenological Lagrangians.[4] It has been shown that, whichever formalism one uses, the actual results of any computation are the same[5]; the literature is rich in reformulations of the general computational schemes. Applications to explicit calculations of specific processes were begun in the important early work of Weinberg[6] and Callan and Treiman[7] on K-decay form factors and pion scattering lengths. More recent applications of the current-algebra–PCAC methods have included pion production by two photons[8] and by electron-positron annihilation.[9]

We justify the presentation of yet another soft-pion calculation on the following two grounds. We apply these methods to annihilation of a baryon-antibaryon pair, an intrinsically interesting and inadequately understood process. Secondly, we present our result, before integration, in a form fully differential in all independent kinematic variables. We adhere strongly to the point of view, stressed especially by van Hove recently,[10] that, lacking a complete theory, one must present experimental information in the most differential form available and compare it with theoretical schemes that also predict an equally detailed differential form, rather than consider distributions with most, or all, of the independent variables integrated over, whereby most of the complicated behavior of transition amplitudes is lost.

Specifically, we have calculated the amplitude of the annihilation process, $\bar{p}p \to K^+K^-\pi^+\pi^-$, and from this the rate, differential in all five kinematic variables, normalized to the $\bar{p}p \to K^+K^-$ total rate, in the soft-pion limit for annihilation in flight at vanishingly low momentum. The integral of this is the branching ratio

$$\frac{\bar{p}p \to K^+K^-\pi^+\pi^-}{\bar{p}p \to K^+K^-} \;.$$

The calculation proceeds within the formalism of current algebra and PCAC. As is well known, such a soft-pion calculation involves three steps – deriving an identity that is valid at all four-momenta, taking the low four-momentum limit of this expression, and translating this unphysical result into a statement about pions with low physical four-momenta on the mass shell. We begin with the matrix element of the time-ordered product of two axial-vector currents. Contracting this twice with pion four-momenta results in an identity in which appear three new matrix elements. One is a σ term which we neglect. The second is a commutator of two axial-vector currents which, evaluated by current algebra, yields the matrix element for the emission of an isovector photon; an early theorem of Low[11] relates this intermediate result to the process without pion emission. The third term is the amplitude for the emission of two soft pions. The original term itself (matrix element of two axial-vector currents contracted with two pion momenta) vanishes in the soft-pion limit, except for those contributions which result from the attachment of the axial currents to external baryon lines.

We set up the above formalism in Sec. II. In Sec. III we deal with the isovector photon term by Low's soft-photon theorem, and in Sec. IV we compute the pole terms in the matrix element of the two axial-vector currents. When these separate parts are put together in Sec. V, we arrive at a relation between the amplitude of the process $\bar{p}p \to K^+K^-\pi^+\pi^-$ and of $\bar{p}p \to K^+K^-$. From this one obtains, after some kinematic manipulation, the final expression for the $\bar{p}p \to K^+K^-\pi^+\pi^-$ rate, differential in five appropriate kinematic variables, normalized to the $\bar{p}p \to K^+K^-$ rate, in the soft-pion limit, for annihilation at vanishing momentum. In Sec. VI we integrate over all kinematic variables and obtain the branching ratio

$$\frac{\bar{p}p \to K^+K^-\pi^+\pi^-}{\bar{p}p \to K^+K^-} .$$

In addition to the philosophical preference mentioned before, such an integration is highly problematic since it includes contributions from regions of high pion momentum, far from the "soft-pion region." Nevertheless, reasonable agreement with experiment is achieved.

II. SOFT-PION FORMALISM

Consider the reaction

$$i \to f + \pi^\alpha(k_1) + \pi^\beta(k_2),$$

where i and f are arbitrary multiparticle hadronic states, such as $\bar{p}p$ or K^+K^-. The pions have momenta k_1 and k_2, and carry isospin α and β.

Define the quantity

$$M_{\mu\nu}^{\alpha\beta} = i\int d^4x\, d^4y\, e^{-ik_1x} e^{-ik_2y} \times \langle f|T(A_\mu^\alpha(x), A_\nu^\beta(y))|i\rangle , \tag{2.1}$$

where the operators A are axial-vector currents with the specified Lorentz and isospin indices.

If we contract $M_{\mu\nu}^{\alpha\beta}$ with k_1^μ and k_2^ν, three kinds of terms result:

(i) The time-ordered product of two axial-vector current divergences,

$$\langle f|T(\partial^\mu A_\mu^\alpha(x), \partial^\nu A_\nu^\beta(y))|i\rangle .$$

We replace the divergences of the axial-vector currents by pion field operators through PCAC,

$$\partial^\mu A_\mu^\alpha(x) = c_\pi \phi_\pi^\alpha(x), \tag{2.2}$$

where

$$c_\pi = \frac{M_N \mu^2 g_A}{g_r K^{NN\pi}(0)} .$$

M_N is the nucleon mass; μ is the pion mass; g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r{}^2/4\pi \sim 14.6$); $K^{NN\pi}(0)$ is the pionic form factor of the nucleon [$K^{NN\pi}(-\mu^2) = 1$]; g_A is the renormalized axial-vector coupling constant ($g_A \sim 1.2$).

(ii) The commutator of the time component of an axial-vector current with the divergence of the axial-vector current; this is the σ term,

$$\delta(x_0 - y_0)\langle f|[A_0^\alpha(x), \partial^\nu A_\nu^\beta(y)]|i\rangle .$$

Following standard practice, we neglect this term.[12]

(iii) The commutator of two axial-vector currents,

$$\delta(x_0 - y_0)\langle f|[A_0^\alpha(x), A_\nu^\beta(y)]|i\rangle .$$

This is evaluated by the commutation relations of $SU(2)\times SU(2)$,

$$\delta(x_0 - y_0)[A_0^\alpha(x), A_\mu^\beta(y)] = i\delta^4(x-y)\epsilon_{\alpha\beta\gamma} V_\mu^\gamma(x) .$$

By repeating the contraction with the order of k_1 and k_2 reversed, combining with the original expression, and using the three results above, we have the following identity:

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = -ic_\pi{}^2\int d^4x\, d^4y\, e^{-ik_1x} e^{-ik_2y}\langle f|T(\phi_\pi^\alpha(x), \phi_\pi^\beta(y))|i\rangle + \tfrac{1}{2}i\epsilon_{\alpha\beta\gamma}(k_2 - k_1)^\lambda \int d^4x\, d^4y\, e^{-ik_1x} e^{-ik_2y}\delta^4(x-y)\langle f|V_\lambda^\gamma(x)|i\rangle . \tag{2.3}$$

We perform the trivial integration in the second term on the right-hand side and insert the Klein-Gordon operator in the first. Next we take the soft-pion limit of the expression, letting both k_1 and k_2 approach zero, and obtain the "soft-pion amplitude theorem,"

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = \frac{-ic_\pi{}^2}{\mu^4}\int d^4x\, d^4y\, e^{-ik_1x} e^{-ik_2y}(\mu^2 - \Box_x)(\mu^2 - \Box_y)\langle f|T(\phi_\pi^\alpha(x), \phi_\pi^\beta(y))|i\rangle + \tfrac{1}{2}i\epsilon_{\alpha\beta\gamma}(k_2 - k_1)^\lambda \int d^4x\, e^{-i(k_1+k_2)x}\langle f|V_\lambda^\gamma(x)|i\rangle . \tag{2.4}$$

In this expression the first term on the right-hand side is, to within a numerical factor,[13] the amplitude for the process $i \to f + \pi^\alpha(k_1) + \pi^\beta(k_2)$, and the second term is similarly related to the amplitude for the process $i \to f +$ isovector photon; the left-hand side does not vanish since $M_{\mu\nu}^{\alpha\beta}$ has contributions from terms of order k^{-2}. In the next two sections we see that both the isovector photon term and the term on the left-hand side can be expressed in terms of the $i \to f$ amplitude. As it stands, the entire expression is valid in the unphysical region $k_1^\mu = 0$ and $k_2^\nu = 0$, i.e., with the pions off the mass shell. It is assumed that the same relation remains valid if we extrapolate from zero pion four-momenta to low momenta on the mass shell.[14] Hence, we will arrive at an expression for the amplitude for $i \to f$ + two soft pions in terms of the amplitude for $i \to f$.

III. LOW-ENERGY THEOREM FOR PHOTON EMISSION

In the soft-pion limit, $k_1 \to 0$ and $k_2 \to 0$. The momentum of the isovector photon, discussed above, is $k = k_1 + k_2$; therefore, in this limit also $k \to 0$, and we may make use of any low-energy theorem for photon emission.

There exists, conveniently, just such a low-energy theorem, due to Low,[11] which relates the first two terms in the series expansion, in powers of the photon momentum, of the photon emission amplitude to the amplitude of the process without photon emission. We follow Low's procedure in a straightforward way, adapting it to the process we are discussing, the annihilation at vanishing momentum, $\bar{p}p \to K^+K^-$. (Until now we did not have to specify the initial and final particle states in the soft-pion amplitude theorem of Sec. II, but must do so now since the details of Low's procedure depend on the spins and charges of the particles.[11])

Consider the above annihilation accompanied by photon emission, $\bar{p}p \to K^+K^-$ + photon; the amplitude for this reaction can be divided into two parts:

$$M_\mu = M_\mu^{(1)} + M_\mu^{(2)} . \tag{3.1}$$

$M_\mu^{(1)}$ consists of the sum of all diagrams where the photon is emitted from an external line. In this reaction, photon emission can take place from the two charged K lines, but not from the initial proton-antiproton state, since this is a neutral state of two antiparticles at rest, with no net charge and no higher moments of the charge; hence, there is no net contribution to $M_\mu^{(1)}$ from $\bar{p}p$ to lowest order in photon momentum. See Fig. 1.

$M_\mu^{(2)}$ consists of all diagrams not in $M_\mu^{(1)}$, that is, diagrams where the photon emerges from the interaction "blob," rather than from the external lines. See Fig. 2.

As $k \to 0$, we see below that $M_\mu^{(1)} \sim k^{-1}$, and $M_\mu^{(2)} \sim$ constant.

Calculating $M_\mu^{(1)}$ from the diagrams of Fig. 1 (omitting temporarily the magnitude of the coupling constant), we have immediately

$$M^{\mu(1)}\epsilon_\mu = \frac{(2q_1+k)^\mu \epsilon_\mu}{(q_1+k)^2+\mu^2} \langle q_1+k\,,q_2|M|P\rangle - \langle q_1\,, q_2+k|M|P\rangle \frac{(2q_2+k)^\mu\epsilon_\mu}{(q_2+k)^2+\mu^2}\ , \tag{3.2}$$

where the matrix elements of M are the invariant amplitudes of the process without photon emission with the momenta of K^+ and K^- as shown in $\langle\ |$; P is the momentum of the $\bar{p}p$ system. Neglecting k^2, and noting the transversality condition, $k^\mu\epsilon_\mu = 0$, we have

$$M^{\mu(1)} = \frac{q_1^\mu}{q_1\cdot k}\langle q_1+k\,,\, q_2|M|P\rangle - \frac{q_2^\mu}{q_2\cdot k}\langle q_1\,, q_2+k|M|P\rangle . \tag{3.3}$$

Writing the invariant amplitude M as a function of relativistic invariants (in this case just the squares of the momenta),

$$M^{\mu(1)} = \frac{q_1^\mu}{q_1\cdot k} M(-4M_N{}^2,\, -m^2+2q_1\cdot k\,,\, -m^2) - \frac{q_2^\mu}{q_2\cdot k} M(-4M_N{}^2\,,\, -m^2,\, -m^2+2q_2\cdot k). \tag{3.4}$$

Now $M_\mu^{(1)}$ is not gauge-invariant, but M_μ is; hence,

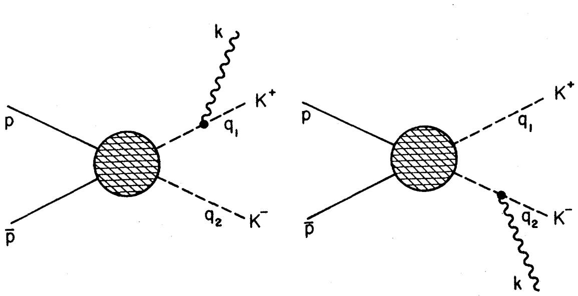

FIG. 1. Lowest-order diagrams contributing to $M_\mu^{(1)}$.

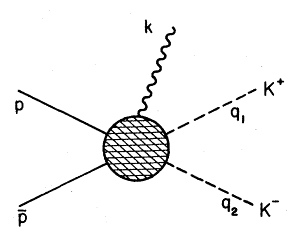

FIG. 2. Lowest-order diagram contributing to $M_\mu^{(2)}$.

$$k^\mu M_\mu = 0,$$

and (3.5)

$$k^\mu M_\mu^{(2)} = -k^\mu M_\mu^{(1)}.$$

Now

$$k^\mu M_\mu^{(1)} = \partial_2 M 2q_1 \cdot k - \partial_3 M 2q_2 \cdot k,$$

where $\partial_2 M$ and $\partial_3 M$ are the derivatives of M with respect to its second and third arguments, respectively. Since $M_\mu^{(2)}$ no has no singularity as $k \to 0$, one can combine the last two expressions and solve for $M_\mu^{(2)}$,

$$M^{\mu(2)} = -2q_1^\mu \partial_2 M + 2q_2^\mu \partial_3 M. \tag{3.6}$$

Expanding $M_\mu^{(1)}$ itself to zeroth order in k, and letting $k \to 0$ in the arguments of M, one finds

$$M^{\mu(1)} = \frac{q_1^\mu}{q_1 \cdot k} M(-4M_N^2, -m^2, -m^2) - \frac{q_2^\mu}{q_2 \cdot k} M(-4M_N^2, -m^2, -m^2) + \frac{q_1^\mu}{q_1 \cdot k} \partial_2 M 2q_1 \cdot k - \frac{q_2^\mu}{q_2 \cdot k} \partial_3 M 2q_2 \cdot k. \tag{3.7}$$

Adding $M_\mu^{(1)}$ and $M_\mu^{(2)}$, the derivative terms vanish, and one obtains

$$M^\mu = \left(\frac{q_1^\mu}{q_1 \cdot k} - \frac{q_2^\mu}{q_2 \cdot k}\right) M(-4M_N^2, -m^2, -m^2). \tag{3.8}$$

This is the result we sought; it relates the invariant amplitude[13] of the process $\bar{p}p \to K^+K^-$ ($\bar{p}p$ at rest) to the invariant amplitude, M_μ, of the same process accompanied by emission of a photon of vanishing momentum. We can insert the coupling constant now, noting that the coupling of the isovector current is to the isovector charges, which for K^+ and K^- are $+\frac{1}{2}$ and $-\frac{1}{2}$. Since Low's procedure refers to a physical, uncharged photon, we restrict the isospin index of the vector current in the soft-pion-amplitude theorem of Sec. II to that of a neutral photon, i.e., the isospin index is 3. Having noted these two points, as well as keeping track of numerical factors, the result above can now be used to evaluate the isovector term in the soft-pion-amplitude theorem, Eq. (2.4) of Sec. II.

IV. POLE TERMS

We must now consider the left-hand side of Eq. (2.4), namely, $k_1^\mu k_2^\mu M_{\mu\nu}^{\alpha\beta}$. This term will also be related to the $\bar{p}p \to K^+K^-$ amplitude in our approximation. Recalling the definition

$$M_{\mu\nu}^{\alpha\beta} = i\int d^4x\, d^4y\, e^{-ik_1x} e^{-ik_2y} \langle f | T(A_\mu^\alpha(x), A_\nu^\beta(y)) | i\rangle, \tag{4.1}$$

since we are calculating the amplitudes to zeroth order in pion momenta, it is necessary to identify and retain only those parts of $M_{\mu\nu}^{\alpha\beta}$ that go like k^{-2} for small k.

In the diagrams for $M_{\mu\nu}^{\alpha\beta}$ the axial-vector current vertices may be attached (i) to an internal line, (ii) to a terminating external pion line, and (iii) to a nonterminating external line. Only the terms of this last type are of order k^{-2}. Furthermore, the insertion of A into a pseudoscalar-meson line is forbidden by parity. Thus, for $\bar{p}p \to K^+K^-$, there are no insertions in the K lines, only in the $\bar{p}$ and p lines.

The diagrams to consider are shown in Fig. 3. We write down their contribution in a straightforward way, using Feynman rules; p_1 is the proton momentum, p_2 the antiproton momentum; k_1 and k_2 are the momenta carried by the axial-vector currents, and α and β their isospin indices. The central interaction we write out explicitly in terms of relativistic invariants, $\mathfrak{M} = A + B\gamma \cdot Q$, where $Q = q_1 - q_2$. Several points should be noted. Since we are dealing with annihilation at vanishing momentum, we retain only the large components of the Dirac spinors; for that reason the coefficient of A vanishes and only terms proportional to B re-

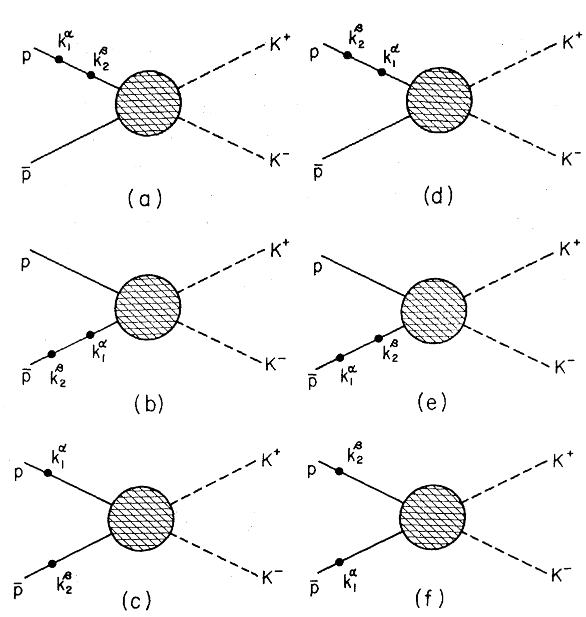

FIG. 3. Diagrams of order k^{-2} in $M_{\mu\nu}^{\alpha\beta}$. For isospins corresponding to emission of π^+ and π^-, only diagrams (a), (b), and (c) contribute.

main. We retain only the lowest-order terms in pion momenta in $k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta}$, that is, zeroth-order terms. We specialize to isospins corresponding to π^+ and π^- emission; for that case only the three diagrams of Figs. 3(a), 3(b), and 3(c) contribute. Evaluating in the $\bar{p}p$ center-of-mass frame, for definite spin states s and r, the sum of these three terms of Fig. 3 is

$$k_1^\mu k_2^\nu M_{\mu\nu}^{+-} = \frac{-2Bg_A{}^2\bar{v}^s(p_2)}{(k_1^0+k_2^0)k_1^0k_2^0}[k_1^0(\gamma\cdot k_2\gamma\cdot k_1\gamma\cdot Q-\gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1)+k_2^0(\gamma\cdot Q\gamma\cdot k_2\gamma\cdot k_1-\gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1)$$
$$+ik_1^0k_2^0(\gamma\cdot k_1\gamma\cdot Q-\gamma\cdot Q\gamma\cdot k_1+\gamma\cdot k_2\gamma\cdot Q-\gamma\cdot Q\gamma\cdot k_2)]u^r(p_1)\,. \tag{4.2}$$

This expression is proportional to B. Now, using $\mathfrak{M}=A+B\gamma\cdot Q$, the invariant amplitude for the annihilation $\bar{p}p\to K^+K^-$ at rest is itself also proportional to B,

$$M_{sr}=B\bar{v}^s(M_N)\gamma\cdot Qu^r(M_N)\,. \tag{4.3}$$

V. $\bar{p}p\to K^+K^-\pi^+\pi^-$ AMPLITUDE AND RATE

We have evaluated separately all the terms in the soft-pion-amplitude theorem, Eq. (2.4). Putting together all three terms, and keeping track of numerical factors in the various amplitudes,[13] we have the invariant amplitude for soft-pion emission in $\bar{p}p\to K^+K^-\pi^+\pi^-$ at vanishing momentum,

$$M_{sr}^{+-}=\frac{\mu^4}{c_\pi{}^2}B\bar{v}^s(M_N)\Big\{\frac{2g_A{}^2}{k_1^0k_2^0(k_1^0+k_2^0)}[k_1^0(\gamma\cdot k_2\gamma\cdot k_1\gamma\cdot Q-\gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1)$$
$$+k_2^0(\gamma\cdot Q\gamma\cdot k_2\gamma\cdot k_1-\gamma\cdot k_2\gamma\cdot Q\gamma\cdot k_1)$$
$$+ik_1^0k_2^0(\gamma\cdot k_1\gamma\cdot Q-\gamma\cdot Q\gamma\cdot k_1+\gamma\cdot k_2\gamma\cdot Q-\gamma\cdot Q\gamma\cdot k_2)]$$
$$+\frac{1}{4}\left[\frac{q_1\cdot(k_2-k_1)}{q_1\cdot(k_2+k_1)}-\frac{q_2\cdot(k_2-k_1)}{q_2\cdot(k_2+k_1)}\right]\gamma\cdot Q\Big\}u^r(M_N), \tag{5.1}$$

where B is the constant in the corresponding invariant amplitude for $\bar{p}p\to K^+K^-$ in Eq. (4.3), which has been explicitly used above. Equation (5.1) is an important intermediate result of our calculation, and forms the basis of all subsequent results.

Since we are dealing with the proton and antiproton both at vanishing momentum, we can introduce nonrelativistic Pauli spinor notation; in that notation we have

$$M_{sr}^{+-}=\frac{-\mu^4B}{c_\pi{}^2}\chi_s^\dagger\{4\eta[(k_1^0+k_2^0)(\vec{k}_1\times\vec{k}_2)\cdot\vec{Q}+i(k_1^0+k_2^0)\vec{k}_1\cdot\vec{k}_2\vec{\sigma}\cdot\vec{Q}$$
$$-ik_1^0\vec{k}_2\cdot\vec{Q}\vec{\sigma}\cdot\vec{k}_1-ik_2^0\vec{k}_1\cdot\vec{Q}\vec{\sigma}\cdot\vec{k}_2]+\tfrac{1}{4}if(q,k)\vec{\sigma}\cdot\vec{Q}\}\chi_r\,, \tag{5.2}$$

where

$$\eta=\frac{g_A{}^2}{k_1^0k_2^0(k_1^0+k_2^0)}\;;\quad f(q,k)=\frac{q_1\cdot(k_2-k_1)}{q_1\cdot(k_2+k_1)}-\frac{q_2\cdot(k_2-k_1)}{q_2\cdot(k_2+k_1)}\;.$$

From Eq. (5.2) we compute the absolute square of the amplitude, $|M|^2$, averaged over initial and summed over final spin states,

$$\langle|M|^2\rangle_{\rm av}=\frac{\mu^8B^2}{4c_\pi{}^4}(32\eta^2\{(k_1^0+k_2^0)^2(\vec{k}_1\times\vec{k}_2\cdot\vec{Q})^2+(k_1^0+k_2^0)^2(\vec{k}_1\cdot\vec{k}_2)^2\vec{Q}^2+(k_1^0)^2(\vec{k}_2\cdot\vec{Q})^2\vec{k}_1{}^2$$
$$+(k_2^0)^2(\vec{k}_1\cdot\vec{Q})^2\vec{k}_2{}^2-2[(k_1^0)^2+(k_2^0)^2+k_1^0k_2^0](\vec{k}_1\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{Q})(\vec{k}_2\cdot\vec{Q})\}$$
$$+4\eta f(k_1^0+k_2^0)[(\vec{k}_1\cdot\vec{k}_2)\vec{Q}^2-(\vec{k}_1\cdot\vec{Q})(\vec{k}_2\cdot\vec{Q})]+\tfrac{1}{8}f^2\vec{Q}^2)\,. \tag{5.3}$$

The differential rate is given in terms of $\langle|M|^2\rangle_{\rm av}$ by the usual expression,

$$d^{12}w=\frac{(2\pi)^4}{(2\pi)^{12}}\frac{M_N{}^2}{p_1^0p_2^0}\delta(P-k_1-k_2-q_1-q_2)\langle|M|^2\rangle_{\rm av}\frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\frac{d^3q_1}{2q_1^0}\frac{d^3q_2}{2q_2^0}\,. \tag{5.4}$$

Of the twelve kinematic variables for a four-particle final state, energy-momentum conservation eliminates four; the arbitrary direction of one vector and the arbitrary orientation of the entire system about this vector provide three additional trivial integrations. Hence, five independent variables remain. We choose the following particular set of five such variables that are useful for comparing with experimentally determined spectra:

$$m_{KK}^2,\quad m_{\pi\pi}^2,\quad \cos\theta_\pi,\quad \cos\theta_K,\ \phi,$$

where $m_{KK}^2=-R^2$ is the invariant mass of the dikaon system $(R=q_1+q_2)$; $m_{\pi\pi}^2=-K^2$ is the invariant mass of the dipion system $(K=k_1+k_2)$; θ_π is the angle in the dipion rest frame between pion 1 and the negative of the total momentum; θ_K is the angle in the dikaon rest frame between kaon 1 and the negative of the total momentum; ϕ is the relative azimuthal angle between the decay planes of the two-pion system and the two-kaon system. This set of variables possesses attractive symmetry, and makes calculation easier since the variables are not kinematically restricted, except in a simple way, i.e.,

$$2\mu+2m_K\le m_{\pi\pi}+m_{KK}\le M_{\bar p p},\quad 0<\theta_\pi<\pi,\quad 0<\theta_K<\pi,\quad 0<\phi<2\pi.$$

A straightforward calculation yields the expression for the rate, differential in the new set of variables,

$$d^5w=dm_{\pi\pi}^2\,dm_{KK}^2\,d(\cos\theta_\pi)d(\cos\theta_K)\,d\phi\,\frac{(2\pi)^4}{(2\pi)^{12}}\langle|M|^2\rangle_{\text{av}}\frac{\pi^2}{64M_{p\bar p}^2}$$

$$\times\frac{1}{m_{\pi\pi}m_{KK}}[(M_{p\bar p}^2+m_{\pi\pi}^2-m_{KK}^2)^2-4M_{p\bar p}^2m_{\pi\pi}^2)]^{1/2}[m_{\pi\pi}^2-4\mu^2]^{1/2}[m_{KK}^2-4m_K^2]^{1/2}. \tag{5.5}$$

In order to normalize our final answer to the $\bar pp\to K^+K^-$ rate, we must first calculate the $\bar pp\to K^+K^-$ rate. This is done easily; since

$$M_{sr}=B\bar v^s(M_N)\gamma\cdot Qu^r(M_N),$$

one finds

$$w(\bar pp\to K^+K^-)=\frac{B^2}{8\pi}\,\frac{(M_N^2-m_K^2)^{3/2}}{M_N}. \tag{5.6}$$

Thus, one obtains the final result, the differential rate of $\bar pp\to K^+K^-\pi^+\pi^-$, normalized to the total $\bar pp\to K^+K^-$ rate:

$$\frac{d^5w(\bar pp\to K^+K^-\pi^+\pi^-)}{w(\bar pp\to K^+K^-)}=dm_{\pi\pi}^2\;dm_{KK}^2\,d(\cos\theta_\pi)\,d(\cos\theta_K)\,d\phi$$

$$\times\frac{1}{m_{\pi\pi}m_{KK}}(m_{\pi\pi}^2-4\mu^2)^{1/2}(m_{KK}^2-4m_K^2)^{1/2}[(M_{p\bar p}^2+m_{\pi\pi}^2-m_{KK}^2)^2-4M_{p\bar p}^2m_{\pi\pi}^2]^{1/2}$$

$$\times\frac{1}{(4\pi)^5M_{p\bar p}^5(M_{p\bar p}^2-4m_K^2)^{3/2}}\left(\frac{g_rK^{NN\pi}(0)}{g_A}\right)^4|\tilde M|^2, \tag{5.7}$$

where $|\tilde M|^2$ is the bracketed part of $\langle|M|^2\rangle_{\text{av}}$ in Eq. (5.3); explicitly

$$|\tilde M|^2=32\eta^2\{(k_1^0+k_2^0)^2(\vec k_1\times\vec k_2\cdot\vec Q)^2+(k_1^0+k_2^0)^2(\vec k_1\cdot\vec k_2)^2\vec Q^2+(k_1^0)^2(\vec k_2\cdot\vec Q)^2\vec k_1^2$$
$$+(k_2^0)^2(\vec k_1\cdot\vec Q)^2\vec k_2^2-2[(k_1^0)^2+(k_2^0)^2+k_1^0k_2^0](\vec k_1\cdot\vec k_2)(\vec k_1\cdot\vec Q)(\vec k_2\cdot\vec Q)\}$$
$$+4\eta f(k_1^0+k_2^0)[(\vec k_1\cdot\vec k_2)\vec Q^2-(\vec k_1\cdot\vec Q)(\vec k_2\cdot\vec Q)]+\tfrac{1}{8}f^2\vec Q^2. \tag{5.8}$$

$|\tilde M|^2$ must of course be expressed in terms of the preferred set of variables, m_{KK}^2, $m_{\pi\pi}^2$, $\cos\theta_\pi$, $\cos\theta_K$, and ϕ. We express the variables appearing in Eq. (5.8) in terms of Lorentz-invariant variables, and then write these in turn in terms of the preferred set. These conversion formulas are tabulated in Eqs. (A3)–(A21) in the Appendix.

Our final expression, Eq. (5.7), is differential in all five independent kinematic variables. Although this is a relatively complicated expression, we feel it is important to work, if possible, with such fully differential expressions, before integrating to obtain the branching ratio.

VI. REMARKS

Certain aspects of our result deserve comment. The first specific point is the presence of the factor

$$\left(\frac{q_1\cdot(k_2-k_1)}{q_1\cdot(k_2+k_1)}-\frac{q_2\cdot(k_2-k_1)}{q_2\cdot(k_2+k_1)}\right)\times(\bar pp\to K^+K^-\text{ amplitude}),$$

which occurs in the Low procedure and subsequently in Eq. (5.1), which expresses the $\bar pp\to K^+K^-\pi^+\pi^-$ amplitude. This term is zeroth order in k_1 and k_2; yet it nonetheless depends on k_1 and k_2, i.e., on which pion is soft. This is reminiscent of a similar term in Weinberg's result for the K_{e4} form factor,[6] which is present because both pions are simultaneously taken off the mass shell in the calculation.

As we emphasized in the Introduction, although the resulting expressions are cumbersome, there is merit in computing theoretical expressions, and presenting experimental data, in fully differential form. Our final result, in its present form, includes a large quantity of information about the reaction, $\bar{p}p \to K^+K^-\pi^+\pi^-$, and predicts a spectrum in the five independent kinematic variables. The process of $\bar{p}p$ annihilation has had increasing experimental study,[15] and sufficient data are beginning to emerge to allow detailed comparisons of spectra, although at present one cannot yet make a full comparison in the five kinematic variables.

The simplest comparison of course is with the branching ratio; if one integrates over all five kinematic variables one obtains the branching ratio (B.R.)

$$\frac{\bar{p}p \to K^+K^-\pi^+\pi^-}{\bar{p}p \to K^+K^-}$$

for annihilation at vanishing momentum. This integration is highly problematic, since it includes all allowed pion momenta, including values up to 735 MeV/c, whereas the theory applies only for low momenta, ideally for unphysical zero four-momenta. Nevertheless, we obtain

$$\text{B.R.}\,\frac{\bar{p}p \to K^+K^-\pi^+\pi^-}{\bar{p}p \to K^+K^-} = 2.25$$

for annihilation at vanishing momentum,[16] in reasonable agreement with experiment.

Experimentally, $K\bar{K}(m\pi)$ final states are commonly observed for those events in which at least one kaon is neutral[17]; thus data are lacking for $\bar{p}p \to K^+K^-\pi^+\pi^-$. Typical branching ratios of specific final states, compared to all final states, are[18]

$$\text{B.R.}(\bar{p}p \to K^+K^-) = (1.1 \pm 0.1)\times 10^{-3},$$
$$\text{B.R.}(\bar{p}p \to K_1^0K_2^0) = (0.71 \pm 0.10)\times 10^{-3},$$
$$\text{B.R.}(\bar{p}p \to K_1^0K_2^0\pi^+\pi^-) = (2.79 \pm 0.42)\times 10^{-3},$$
$$\text{B.R.}(\bar{p}p \to K_1^0K_1^0\pi^+\pi^-) = (4.66 \pm 0.60)\times 10^{-3}.$$

We expect that B.R.$(\bar{p}p \to K^+K^-\pi^+\pi^-)$ is comparable to the last two numbers listed. For comparison, from the above numbers, one finds

$$\text{B.R.}\,\frac{\bar{p}p \to K_1^0K_2^0\pi^+\pi^-}{\bar{p}p \to K_1^0K_2^0} = 4.$$

The proton-antiproton annihilation reaction has lent itself to a calculation of soft-pion emission since even for annihilation at vanishing momentum there is enough energy to produce the desired final states; at the same time, in this limit useful computational simplifications occur.

The annihilation process itself has attracted relatively little theoretical interest, due to its complexity and lack of detailed experimental results. Some proposed models have included production and decay of two or three vector mesons,[19] rearrangement of three quarks and three antiquarks,[20] and direct-channel interference of massive bosons.[21] We have confined ourselves to a relatively straightforward application of soft-pion theorems to $\bar{p}p$ annihilation; a similar procedure is being applied to soft-pion emission in $\bar{p}p \to K\bar{K}K\bar{K}$.[22]

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APPENDIX

In Eq. (5.8) the term $|\tilde{M}|^2$ is given by the expression

$$|\tilde{M}|^2 = 32\eta^2\{(k_1^0+k_2^0)^2(\vec{k}_1\times\vec{k}_2\cdot\vec{Q})^2 + (k_1^0+k_2^0)^2(\vec{k}_1\cdot\vec{k}_2)^2\vec{Q}^2 + (k_1^0)^2(\vec{k}_2\cdot\vec{Q})^2\vec{k}_1^{\,2}$$
$$+ (k_2^0)^2(\vec{k}_1\cdot\vec{Q})^2\vec{k}_2^{\,2} - 2[(k_1^0)^2 + (k_2^0)^2 + k_1^0k_2^0](\vec{k}_1\cdot\vec{k}_2)(\vec{k}_1\cdot\vec{Q})(\vec{k}_2\cdot\vec{Q})\}$$
$$+ 4\eta f(k_1^0+k_2^0)[(\vec{k}_1\cdot\vec{k}_2)\vec{Q}^2 - (\vec{k}_1\cdot\vec{Q})(\vec{k}_2\cdot\vec{Q})] + \tfrac{1}{8}f^2\vec{Q}^2, \tag{A1}$$

where

$$\eta = \frac{g_A^{\,2}}{k_1^0k_2^0(k_1^0+k_2^0)}\,;\quad f(q,k) = \frac{q_1\cdot(k_2-k_1)}{q_1\cdot(k_2+k_1)} - \frac{q_2\cdot(k_2-k_1)}{q_2\cdot(k_2+k_1)}\ .$$

We express the variables occurring in $|\tilde{M}|^2$ first in terms of Lorentz-invariant quantities, and finally in terms of the preferred set, $m_{KK}^{\,2}$, $m_{\pi\pi}^{\,2}$, $\cos\theta_\pi$, $\cos\theta_K$, ϕ, defined in Sec. V. Define the following variables:

$$K = k_1 + k_2,\quad \Delta = k_1 - k_2,\quad R = q_1 + q_2,\quad Q = q_1 - q_2, \tag{A2}$$

and form the invariant quantities, $Q\cdot K$, Q^2, $Q\cdot\Delta$, R^2, $R\cdot K$, K^2, $R\cdot\Delta$, Δ^2. In terms of these, the variables in $|\tilde{M}|^2$ are given by

$$k_1^0=\frac{K^2+K\cdot R+\Delta\cdot R}{-2M_{p\bar{p}}}, \tag{A3}$$

$$k_2^0=\frac{K^2+K\cdot R-\Delta\cdot R}{-2M_{p\bar{p}}}, \tag{A4}$$

$$\vec{\mathrm{k}}_1{}^2=\tfrac{1}{4}(K+\Delta)^2+\frac{1}{4M_{p\bar{p}}{}^2}(K^2+K\cdot R+\Delta\cdot R)^2, \tag{A5}$$

$$\vec{\mathrm{k}}_2{}^2=\tfrac{1}{4}(K-\Delta)^2+\frac{1}{4M_{p\bar{p}}{}^2}(K^2+K\cdot R-\Delta\cdot R)^2, \tag{A6}$$

$$\vec{\mathrm{k}}_1\cdot\vec{\mathrm{k}}_2=\tfrac{1}{4}(K^2-\Delta^2)+\frac{1}{4M_{p\bar{p}}{}^2}(K^2+K\cdot R+\Delta\cdot R)(K^2+K\cdot R-\Delta\cdot R), \tag{A7}$$

$$\vec{\mathrm{Q}}^2=Q^2+\frac{(K\cdot Q)^2}{M_{p\bar{p}}{}^2}, \tag{A8}$$

$$\vec{\mathrm{k}}_1\cdot\vec{\mathrm{Q}}=\tfrac{1}{2}(K\cdot Q+\Delta\cdot Q)+\frac{K\cdot Q}{2M_{p\bar{p}}{}^2}(K^2+K\cdot R+\Delta\cdot R), \tag{A9}$$

$$\vec{\mathrm{k}}_2\cdot\vec{\mathrm{Q}}=\tfrac{1}{2}(K\cdot Q-\Delta\cdot Q)+\frac{K\cdot Q}{2M_{p\bar{p}}{}^2}(K^2+K\cdot R-\Delta\cdot R), \tag{A10}$$

$$(\vec{\mathrm{k}}_1\times\vec{\mathrm{k}}_2\cdot\vec{\mathrm{Q}})^2=\frac{1}{-4M_{p\bar{p}}{}^2}(\epsilon_{\mu\nu\lambda\sigma}\Delta^\mu K^\nu Q^\lambda R^\sigma)^2, \tag{A11}$$

$$f=2\frac{(R\cdot\Delta Q\cdot K-Q\cdot\Delta R\cdot K)}{(R\cdot K)^2-(Q\cdot K)^2}. \tag{A12}$$

In turn, the invariant quantities are expressed in terms of the preferred set through the following relations:

$$K^2=-m_{\pi\pi}{}^2, \tag{A13}$$

$$R^2=-m_{KK}{}^2, \tag{A14}$$

$$R\cdot K=\tfrac{1}{2}(m_{\pi\pi}{}^2+m_{KK}{}^2-M_{p\bar{p}}{}^2), \tag{A15}$$

$$Q^2=m_{KK}{}^2-4m_K{}^2, \tag{A16}$$

$$\Delta^2=m_{\pi\pi}{}^2-4\mu^2, \tag{A17}$$

$$\Delta\cdot R=-\frac{\cos\theta_\pi}{2m_{\pi\pi}}(m_{\pi\pi}{}^2-4\mu^2)^{1/2}[(m_{\pi\pi}{}^2+m_{KK}{}^2-M_{p\bar{p}}{}^2)^2-4m_{\pi\pi}{}^2m_{KK}{}^2]^{1/2}, \tag{A18}$$

$$Q\cdot K=-\frac{\cos\theta_\pi}{2m_{KK}}(m_{KK}{}^2-4m_K{}^2)^{1/2}[(m_{\pi\pi}{}^2+m_{KK}{}^2-M_{p\bar{p}}{}^2)^2-4m_{\pi\pi}{}^2m_{KK}{}^2]^{1/2}, \tag{A19}$$

$$\Delta\cdot Q=-(m_{\pi\pi}{}^2-4\mu^2)^{1/2}(m_{KK}{}^2-4m_K{}^2)^{1/2}\left[\cos\phi\,\sin\theta_\pi\,\sin\theta_K+\frac{m_{KK}{}^2+m_{\pi\pi}{}^2-M_{p\bar{p}}{}^2}{2(-m_{\pi\pi}{}^2)^{1/2}(-m_{KK}{}^2)^{1/2}}\cos\theta_\pi\cos\theta_K\right], \tag{A20}$$

$$(\epsilon_{\mu\nu\lambda\sigma}\Delta^\mu K^\nu Q^\lambda R^\sigma)^2=\tfrac{1}{4}(m_{\pi\pi}{}^2-4\mu^2)(m_{KK}{}^2-4m_K{}^2)[4m_{\pi\pi}{}^2m_{KK}{}^2-(m_{\pi\pi}{}^2+m_{KK}{}^2-M_{p\bar{p}}{}^2)^2]\sin^2\phi\,\sin^2\theta_\pi\sin^2\theta_K. \tag{A21}$$

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[1]S. Weinberg, Phys. Rev. Letters 16, 879 (1966); Y. Nambu and D. Lurié, Phys. Rev. 125, 1429 (1962); Y. Nambu and E. Shrauner, *ibid.* 128, 862 (1962).

[2]S. Weinberg, Ref. 1.

[3]Y. Nambu *et al.*, Ref. 1; R. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969).

[4]S. Weinberg, Phys. Rev. Letters 18, 188 (1967).

[5]R. Dashen and M. Weinstein, Ref. 3.

[6]S. Weinberg, Phys. Rev. Letters 17, 336 (1966); 17, 616 (1966).

[7]C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966).

[8]S. L. Adler, B. W. Lee, S. B. Treiman, and A. Zee, Phys. Rev. D 4, 3497 (1971); H. Terazawa, Phys. Rev. Letters 26, 1207 (1971).

[9]A. Pais and S. B. Treiman, Phys. Rev. Letters 25, 975 (1970).
[10]L. van Hove, Phys. Reports 1C, 347 (1971).
[11]F. E. Low, Phys. Rev. 110, 974 (1958); H. Chew, *ibid.* 123, 377 (1961).
[12]S. Weinberg, Phys. Rev. Letters 17, 336 (1966); H. D. I. Abarbanel, Phys. Rev. 153, 1547 (1966).
[13]In our notation, the Lorentz-invariant amplitude, M_{fi}, is defined by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta(p_i - p_f) M_{fi}/N,$$

where $N^2 = \prod_i N_i^2$; $N_i^2 = 2E$ for boson, $N_i^2 = E/m$ for fermions. The integrals in Eq. (2.4) are related to S_{fi} through the LSZ reduction formulas. In this way Eq. (2.4) can be expressed in terms of the Lorentz-invariant amplitudes, which are henceforth utilized.
[14]As is customary, PCAC is used in the role of a "smoothness condition" for the extrapolation matrix elements off the pion mass shell; the dependence of the matrix elements on pion momenta is "gentle" when $\partial A_\mu/\partial x_\mu$ is employed as the pion field. S. B. Treiman, Comments Nucl. Particle Phys. 1, 13 (1967); S. Weinberg, Refs. 2, 6.
[15]N. Barash, Ph.D. thesis, Columbia University (unpublished); C. Baltay *et al.*, Phys. Rev. Letters 15, 532 (1965); N. Barash *et al.*, Phys. Rev. 145, 1095 (1966); S. Devons *et al.*, Phys. Rev Letters 27, 1614 (1971); D. Cline, J. English, and D. D. Reeder, *ibid.* 27, 71 (1971); P. Espegat *et al.*, Nucl. Phys. B35, 93 (1972); R. Armenteros and B. French, in *High Energy Physics*, edited by E. H. S. Burhop (Academic, New York, 1969).
[16]Preliminary results for the branching ratio were reported in J. Roche and R. A. Uritam, Bull. Am. Phys. Soc. 17, 777 (1972).
[17]R. Armenteros and B. French, Ref. 15. At most energies, charged kaons cannot be distinguished from pions by ionization measurements.
[18]N. Barash, Ref. 15.
[19]J. J. Sakurai, Ann. Phys. (N.Y.) 11, 1 (1960).
[20]H. R. Rubinstein and H. Stern, Phys. Letters 21, 447 (1966).
[21]D. Cline and R. Rutz, Phys. Rev. D 5, 778 (1971).
[22]P. Nuthakki and R. A. Uritam (unpublished).

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Absorption Picture as a Consequence of Multi-Regge Iteration of Low-Energy Peripheral Resonances*

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Using the empirical fact that the dominant *low-energy* resonances are peripheral (i.e., they satisfy the relation $j_s + \frac{1}{2} = kR \approx \frac{1}{2}R\sqrt{s}$), the imaginary part, $A_0(s,t)$, of the low-energy amplitude is written in the factorized form $F(s)J_0(R\sqrt{-t})$. The resonance contribution to $F(s)$ near the resonance position is $\approx(2j_s+1)$, which is $\approx s^{1/2}$; and thus the well-known value for the vector-tensor trajectory intercept of $\approx\frac{1}{2}$ follows simply from peripherality. From finite-energy sum rules the residue of the trajectory is $\approx J_0(R\sqrt{-t})$. The quantity $A_0(s,t)$, which in the t channel corresponds to a fixed pole at $j_t = \frac{1}{2}$, through the multi-Regge iteration gives rise to a moving pole; and the imaginary part of the total $\pi\pi$ amplitude $A(s,t)$ with $I_t = 1$ satisfies the dual absorption result, $s^\alpha J_0(R\sqrt{-t})$. A crude estimate gives $\alpha(0) \approx \frac{1}{2} + (\Gamma_\rho/m_\rho)$ (= 0.6) and $\alpha'(0) \approx \frac{1}{4}(\Gamma_\rho/m_\rho)R^2$ (= 1.0 BeV^{-2} for $R = 1$ F). Thus, for non-diffractive processes, the multi-Regge (or multiperipheral) model is consistent with the absorption picture and as a consequence an approximate bootstrap solution is obtained. The physical interpretation in terms of j-plane cuts is discussed.

I. INTRODUCTION

It is an empirical fact that the dominant low-energy resonances are peripheral. This is true of meson as well as baryon resonances.[1] By peripheral we mean that the angular momentum j_s and the energy of the resonance in the s-channel satisfy the relation

$$j_s + \tfrac{1}{2} = kR \approx \tfrac{1}{2}R\sqrt{s}\,, \tag{1}$$

where k is the c.m. momentum and R is the radius parameter which turns out to be of the order of the radius of interaction, e.g., 1 F. The important consequence of this is that the Legendre polynomial, $P_j(\cos\theta)$, associated with individual resonances can be expressed as follows:

$$P_{j_s}(\cos\theta) \approx J_0((2j_s+1)\sin\tfrac{1}{2}\theta) \approx J_0(R\sqrt{-t})\,, \tag{2}$$

where $t\;[=-2k^2(1-\cos\theta)]$ is the momentum transfer. The contributions of the different resonances will, therefore, vanish at the same value of t. Of course, the lowest resonance (e.g., ρ meson) will have only the first zero; the next one, perhaps,