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(6, 6*) ⊕ (6*, 6) Representation of SU(3) ⊗ SU(3) and the Breaking of Chiral Symmetry*†

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The (6,6*) ⊕ (6*,6) representation of SU(3) ⊗ SU(3) is presented and its use in breaking chiral symmetry is discussed in terms of its contribution to meson masses, pion-pion scattering lengths, baryon masses, and the nucleon σ term. We include singlet, octet, and 27-plet SU(3) pieces in the symmetry-breaking Hamiltonian, and also discuss the possible SU(2) ⊗ SU(2) classifications of the Hamiltonian.

I. INTRODUCTION

Recent experimental evidence on the s-wave pion-pion scattering lengths[1,2] seems to indicate the need for a chiral-symmetry-breaking Hamiltonian which transforms in a way other than (3, 3*) ⊕ (3*, 3). In order to produce a large isospin-zero s-wave scattering length, the original Weinberg analysis[3] must be modified to include isospin-two contributions to the σ commutator. This in turn requires the symmetry-breaking Hamiltonian to contain pieces which belong to an SU(3) ⊗ SU(3) representation which has isospin-two components in its reduction to SU(3) and hence to SU(2). It is also possible that a large value of the nucleon σ term would require these other terms, but this conclusion is not definitely confirmed. Indirectly, a recent analysis of the hard-pion Ward identity approach to the pion-pion scattering problem[4] which enforces unitarity within certain smoothness approximations also requires isospin-two σ terms for the optimal solution. This result is, however, also rather uncertain because of the many assumptions involved.

Assuming that such additional pieces are necessary in the Hamiltonian, it is natural to investigate the consequences of the simplest possible choices. In order to have isospin two we require at least the 27-dimensional representation of SU(3). The two smallest SU(3)⊗SU(3) representations containing this are (8, 8) and (6, 6*)⊕(6*, 6), which reduce under parity and SU(3) as $1^+ \oplus 8^+ \oplus 27^+ \oplus 8^- \oplus 10^- \oplus \overline{10}^-$ and $1^+ \oplus 8^+ \oplus 27^+ \oplus 1^- \oplus 8^- \oplus 27^-$, respectively. The consequences of using the former have been discussed by several authors.[5,6,7] In this paper we shall explore the latter possibility.

Although both of the above symmetry-breaking mechanisms have been suggested on the basis of simplicity, no dynamical model has been proposed. If we use the quark model where the triplet belongs to (1, 3)⊕(3*, 1)⊕(1, 3*)⊕(3, 1), then a Fermi-like coupling could induce either of the above breaking mechanisms. For (8, 8) one could also have a three-point coupling to an octet of vector gluons. However, neither of these mechanisms is attractive from a theoretical standpoint.

We shall develop the (6, 6*)⊕(6*, 6) representation in analogy to the (3, 3*)⊕(3*, 3) case.[8] In Sec. II we review the (3, 3*)⊕(3*, 3) development, and then in Sec. III we present the (6, 6*)⊕(6*, 6) representation. Section IV is a discussion of the possible forms of the symmetry-breaking Hamiltonian in terms of its SU(3) and SU(2)⊗SU(2) properties. In Sec. V we apply this Hamiltonian to the calculation of the symmetry-breaking contribution to meson masses, pion-pion scattering lengths, baryon masses, and the nucleon σ term. We discuss these results in Sec. VI.

II. REVIEW OF (3, 3*)⊕(3*, 3)

The $\underline{3}$ and $\underline{3}^*$ representations of SU(3) are defined by the commutation relations

$$[F_\alpha, T_i] = \tfrac{1}{2} T_j \lambda^\alpha_{ji} \quad \text{for } \underline{3} \tag{1}$$

and

$$[F_\alpha, W_i] = -\tfrac{1}{2} \lambda^\alpha_{ij} W_j \quad \text{for } \underline{3}^*, \tag{2}$$

where the λ^α_{ij} are the eight 3×3 matrices of the three-dimensional representation of SU(3). (In this section Greek indices run from 1, ..., 8 and Latin from 1, ..., 3.) They satisfy

$$\lambda^{\alpha *}_{ij} = \lambda^\alpha_{ji},$$

$$[\lambda^\alpha, \lambda^\beta] = 2if_{\alpha\beta\gamma}\lambda^\gamma,$$

and

$$\{\lambda^\alpha, \lambda^\beta\} = 2d_{\alpha\beta\gamma}\lambda^\gamma + \tfrac{4}{3}\delta_{\alpha\beta} I.$$

From the above we write for (3, 3*) in SU(3)⊗SU(3)

$$\begin{aligned} [F^+_\alpha, T_{ij}] &= \tfrac{1}{2}\lambda^{\alpha *}_{ik} T_{kj}, \\ [F^-_\alpha, T_{ij}] &= \tfrac{1}{2}\lambda^\alpha_{jk} T_{ik}, \end{aligned} \tag{3}$$

and for (3*, 3)

$$[F^+_\alpha, W_{ij}] = -\tfrac{1}{2}\lambda^\alpha_{ik} W_{kj},$$

$$[F^-_\alpha, W_{ij}] = \tfrac{1}{2}\lambda^{\alpha *}_{jk} W_{ik},$$

where $F^+_\alpha = \frac{1}{2}(F_\alpha + F^5_\alpha)$ and $F^-_\alpha = \frac{1}{2}(F_\alpha - F^5_\alpha)$. Since $T^\dagger_{ij}$ transforms like (3*, 3) we can parity-double our decomposition by requiring

$$PT_{ij}P^{-1} = T^\dagger_{ji}$$

so that T_{ij} is now said to transform under (3, 3*)⊕(3*, 3).

In order to reduce this representation under parity, we define

$$P_{ij} = T_{ij} + T^\dagger_{ji} \tag{4}$$

and

$$M_{ij} = i(T_{ij} - T^\dagger_{ji}),$$

so that

$$PP_{ij}P^{-1} = P_{ij}, \quad P^\dagger_{ij} = P_{ji},$$

$$PM_{ij}P^{-1} = -M_{ij}, \quad M^\dagger_{ij} = M_{ji}.$$

The SU(3) content can be made manifest by writing

$$P_{ij} = \frac{1}{\sqrt{3}} U_0 \delta_{ji} + \frac{1}{\sqrt{2}} \lambda^\alpha_{ji} U_\alpha$$

and

$$M_{ij} = \frac{1}{\sqrt{3}} V_0 \delta_{ji} + \frac{1}{\sqrt{2}} \lambda^\alpha_{ji} V_\alpha.$$

We can invert these relations as

$$\begin{aligned} U_0 &= \frac{1}{\sqrt{3}} P_{ii}, \\ U_i &= \frac{1}{\sqrt{2}} \lambda^\alpha_{ij} P_{ij}, \\ V_0 &= \frac{1}{\sqrt{3}} M_{ii}, \\ V_\alpha &= \frac{1}{\sqrt{2}} \lambda^\alpha_{ij} M_{ij}, \end{aligned} \tag{5}$$

where the U's and V's are Hermitian scalar and pseudoscalar fields, respectively. They satisfy the well-known commutation relations[8] [from Eq. (3)]

$$[F_\alpha, U_0] = [F_i, V_0] = 0,$$

$$[F_\alpha, U_\beta] = if_{\alpha\beta\gamma} U_\gamma,$$

$$[F_\alpha, V_\beta] = if_{\alpha\beta\gamma} V_\gamma,$$

which identify U_0 and V_0 as SU(3) singlets and $\{U_\alpha\}$ and $\{V_\alpha\}$ as SU(3) octets. Also,

$$[F^5_\alpha, U_0] = -i(\tfrac{2}{3})^{1/2} V_\alpha,$$

$$[F^5_\alpha, V_0] = i(\tfrac{2}{3})^{1/2} U_\alpha,$$

$$[F^5_\alpha, U_\beta] = -id_{\alpha\beta\gamma} V_\gamma - i(\tfrac{2}{3})^{1/2}\delta_{\alpha\beta} V_0,$$

$$[F^5_\alpha, V_\beta] = id_{\alpha\beta\gamma} U_\gamma + i(\tfrac{2}{3})^{1/2}\delta_{\alpha\beta} U_0.$$

For calculations involving $(3,3^*)\oplus(3^*,3)$ it is customary to work directly with the U's and V's, since their commutation relations are simple and the properties of $f_{\alpha\beta\gamma}$ and $d_{\alpha\beta\gamma}$ are well tabulated.[9] However, as we shall see, for more complicated representations it proves simpler to work directly in terms of the analogs of the T_{ij}. Thus, for example, instead of writing the perturbing Hamiltonian as[10]

$$H^1 = C_{30}U_0 + C_{38}U_8$$

we could as well use

$$H^1 = \frac{1}{\sqrt{3}}C_{30}P_{ii} + \frac{1}{\sqrt{2}}C_{38}\lambda^8_{ij}P_{ij}.$$

This latter approach reduces calculations such as those in Sec. V to trace calculations with the $\{\lambda^\alpha\}$ matrices.

III. THE $(6,6^*)\oplus(6^*,6)$ REPRESENTATION

We develop this representation in analogy to Sec. II by writing the commutation relations

$$[F_\alpha, T_i] = \tfrac{1}{2}T_jS^\alpha_{ji} \quad \text{for } \underline{6}$$

and

$$[F_\alpha, W_i] = -\tfrac{1}{2}S^\alpha_{ij}W_j \quad \text{for } \underline{6}^*,$$

where the Latin indices now run from 1 to 6 rather than the 1 to 3 in Sec. II. The eight 6×6 matrices, $\{S^\alpha\}$, are the representation of the SU(3) generators in the $\underline{6}$ representation of SU(3). In the Appendix these matrices are explicitly presented using the phase conventions of Behrends *et al.*[11] They satisfy

$$S^{\alpha*}_{ij} = S^\alpha_{ji}$$

and

$$[S^\alpha, S^\beta] = 2if_{\alpha\beta\gamma}S^\gamma.$$

For $(6,6^*)$ we write

$$[F^+_\alpha, T_{ij}] = \tfrac{1}{2}S^{\alpha*}_{ik}T_{kj},$$
$$[F^-_\alpha, T_{ij}] = -\tfrac{1}{2}S^\alpha_{jk}T_{ik}, \qquad (6)$$

and extend this to $(6,6^*)\oplus(6^*,6)$ by introducing parity via $PT_{ij}P^{-1} = T^\dagger_{ji}$ as before. The parity content of this representation is reduced by

$$P_{ij} = T_{ij} + T^\dagger_{ji},$$
$$M_{ij} = i(T_{ij} - T^\dagger_{ji}),$$

as in Eq. (4).

Now, however, the SU(3) decomposition is slightly more complicated due to the presence of the 27-dimensional representation. We write

$$P_{ij} = \frac{1}{\sqrt{6}}\delta_{ji}U_0 + \frac{1}{\sqrt{10}}S^\alpha_{ji}U_\alpha + T^\Theta_{ji}U_\Theta,$$
$$M_{ij} = \frac{1}{\sqrt{6}}\delta_{ji}V_0 + \frac{1}{\sqrt{10}}S^\alpha_{ji}V_\alpha + T^\Theta_{ji}V_\Theta, \qquad (7)$$

where Θ is summed from 1 to 27. The matrices δ, $\{S^\alpha\}$, and $\{T^\Theta\}$ satisfy

$$\mathrm{Tr}(\delta\delta) = 6, \quad \mathrm{Tr}(\delta S^\alpha) = \mathrm{Tr}(\delta T^\Theta) = 0,$$
$$\mathrm{Tr}(S^\alpha S^\beta) = 10\delta_{\alpha\beta}, \quad \mathrm{Tr}(S^\alpha T^\Theta) = 0,$$
$$\mathrm{Tr}(T^\Theta T^{\Theta'}) = \delta_{\Theta\Theta'}.$$

We can explicitly construct the $\{T^\Theta\}$ by writing the Clebsch-Gordan series for $6\otimes6^* = 1\oplus8\oplus27$. We do not present the general result, since we shall need only T^{27}_{ij} (corresponding to the $I=Y=0$ member of $\underline{27}$) in our subsequent calculations. It is given by

$$T^{27}_{ij} = \frac{1}{\sqrt{30}}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}.$$

Using the trace relations, we invert Eqs. (7) as

$$U_0 = \frac{1}{\sqrt{6}}P_{ii}, \quad V_0 = \frac{1}{\sqrt{6}}M_{ii},$$
$$U_\alpha = \frac{1}{\sqrt{10}}S^\alpha_{ij}P_{ij}, \quad V_\alpha = \frac{1}{\sqrt{10}}S^\alpha_{ij}M_{ij}, \qquad (8)$$
$$U_\Theta = T^\Theta_{ij}P_{ij}, \quad V_\Theta = T^\Theta_{ij}M_{ij}.$$

The U's and V's are scalar and pseudoscalar fields, respectively; and $\{U_0\}$ and $\{V_0\}$, $\{U_\alpha\}$ and $\{V_\alpha\}$, and $\{U_\Theta\}$ and $\{V_\Theta\}$ transform as singlet, octet, and 27-plet representations of SU(3), respectively.

The commutation relations of the U's and V's are easily written down from Eq. (8) in analogy to Eq. (5). However, it is more convenient to use the relations for P_{ij} and M_{ij} directly. These are found from Eqs. (6) to be

$$[F_\alpha, P_{ij}] = \tfrac{1}{2}S^{\alpha*}_{ik}P_{kj} - \tfrac{1}{2}S^\alpha_{jk}P_{ik},$$
$$[F_\alpha, M_{ij}] = \tfrac{1}{2}S^{\alpha*}_{ik}M_{kj} - \tfrac{1}{2}S^\alpha_{jk}M_{ik},$$
$$[F^5_\alpha, P_{ij}] = -\tfrac{1}{2}iS^{\alpha*}_{ik}M_{kj} - \tfrac{1}{2}iS^\alpha_{jk}M_{ik}, \qquad (9)$$
$$[F^5_\alpha, M_{ij}] = \tfrac{1}{2}iS^{\alpha*}_{ik}P_{kj} + \tfrac{1}{2}iS^\alpha_{jk}P_{ik}.$$

IV. HAMILTONIAN FORMS

To construct a suitable symmetry-breaking Hamiltonian for the strong interactions, we wish to include terms which conserve parity, isospin, and hypercharge. Thus, we can include components proportional to U_0 (singlet), U_8 (from $\{U_\alpha\}$), and U_{27} (from $\{U_\Theta\}$), i.e., we write

$$H^1 = \frac{1}{\sqrt{6}} C_{60} P_{ii} + \frac{1}{\sqrt{10}} C_{68} S^8_{ij} P_{ij} + C_{6\ 27} T^{27}_{ij} P_{ij} . \qquad (10)$$

Two special cases of this general form may be of interest. If we set $C_{6\ 27}=0$, then H^1 contains only singlet and octet pieces and thus represents an octet-dominance-type breaking which will, for example, lead automatically to the Gell-Mann–Okubo mass formula for the meson and baryon states.

It is also interesting to ask how H^1 transforms under the subgroup $SU(2)\otimes SU(2)$, since the $(3,3^*)\oplus(3^*,3)$ symmetry-breaking scheme seems to indicate that the breaking term may be approximately in a $(0,0)$ representation.[10] [We use the conventional notation of labeling SU(3) representations by their dimension, but SU(2) representations by their spin content.] It is easy to see that if we are to induce isospin-two components in the σ commutator, then we have to include an $SU(2)\otimes SU(2)$ piece from the $(1,1)$ representation [the highest representation contained in $(6,6^*)\oplus(6^*,6)$]. Thus, the two most interesting cases are the $(0,0)$ and $(1,1)$ representations contained in $(6,6^*)\oplus(6^*,6)$.

Because we have parity doubling, we have both a $(0,0)^+$ and a $(0,0)^-$. Also, for $(1,1)$ we have two cases which reduce under SU(2) as $0^+\oplus\frac{1}{2}^-\oplus 1^+$ and $0^-\oplus\frac{1}{2}^+\oplus 1^-$, respectively. From these, clearly the $(0,0)^+$ state and the 0^+ member of a $(1,1)$ are the suitable candidates for forming H^1. By using a group-theoretic reduction or simply by examining the commutation relations directly, it is easy to see that P_{66} transforms like $(0,0)$ with respect to $SU(2)\otimes SU(2)$ and has positive parity, and $\sum_{i=1}^3 P_{ii}$ transforms like the 0^+ [SU(2)] member of a $(1,1)$ representation of $SU(2)\otimes SU(2)$. Examining the general form, Eq. (10), for H^1, we see that the choice

$$C_{60} : C_{68} : C_{6\ 27} \sim 1 : \frac{-4}{\sqrt{5}} : \frac{3}{\sqrt{5}} \qquad (11)$$

implies $H^1\sim(0,0)^+$ in $SU(2)\otimes SU(2)$, and

$$C_{60} : C_{68} : C_{6\ 27} \sim 1 : \frac{2}{\sqrt{5}} : \frac{1}{\sqrt{5}} \qquad (12)$$

implies $H^1\sim(1,1)$ in $SU(2)\otimes SU(2)$ (the $I=0^+$ member). Note that each of these separately requires a 27-plet piece in H^1, but the mixture, $C_{60}:C_{68}:C_{6\ 27}\sim 1:\sqrt{5}:0$, removes this dependence while maintaining a mixture of pure $(0,0)\oplus(1,1)$.

V. CALCULATIONS

In this section we shall employ the most general form of H^1 [Eq. (10)], and use it to find the $(6,6^*)\oplus(6^*,6)$ contributions to (A) meson masses, (B) pion-pion scattering lengths, (C) baryon masses, and (D) nucleon σ terms. In A and B we shall use the soft-meson approximation, but this is not needed for C and D. We shall also neglect the possible effects on the breaking of scale invariance by a scalar meson,[12,13] effectively assuming our symmetry-breaking H^1 to have dimension $l=3$. Additional factors due to such effects[14] can easily be included in our results. We shall use the simple assumption that H is given by H_0+H^1 where H_0 is invariant under $SU(3)\otimes SU(3)$, does not contribute to meson masses, and gives a uniform mass, M_0, to the baryon octet.

A. Meson Masses

Using the usual soft-meson reduction, the meson mass is given by

$$\langle\mu_\alpha|H^1|\mu_\alpha\rangle = -\frac{1}{F^2}\langle 0|[F_5^\alpha,[F_5^\alpha,H^1]]|0\rangle ,$$

where to this order we assume that the PCAC (partially conserved axial-vector current) constants are all equal,[15] $F_\pi=F_k=F_\eta=F$, i.e.,

$$\partial^\mu A_\mu^\alpha = m_\alpha{}^2 F\phi^\alpha .$$

We also write

$$\langle 0|P_{ij}|0\rangle = \frac{1}{\sqrt{8}}\delta_{ij}\langle 0|U_0|0\rangle ,$$

neglecting any possible contribution from $\langle 0|U_8|0\rangle$ and $\langle 0|U_{27}|0\rangle$. With these assumptions,

$$\langle\mu_\alpha|H^1|\mu_\alpha\rangle = -\frac{1}{F^2}\frac{\langle 0|U_0|0\rangle}{\sqrt{6}}\left[\frac{C_{60}}{\sqrt{6}}\mathrm{Tr}(S^\alpha S^\alpha)+\frac{C_{68}}{\sqrt{10}}\mathrm{Tr}(S^8 S^\alpha S^\alpha)+C_{6\ 27}\mathrm{Tr}(T^{27}S^\alpha S^\alpha)\right],$$

which yields

$$m_\alpha{}^2 = \frac{-5}{3F^2}\langle 0|U_0|0\rangle\left[C_{60}+\frac{7\sqrt{3}}{5\sqrt{5}}C_{68}d_{8\alpha\alpha}+\frac{2\sqrt{6}}{5}C_{6\ 27}\zeta^{27}_{\alpha\alpha}\right], \qquad (13)$$

where

$$d_{8\alpha\beta}=\frac{1}{2\sqrt{3}}\begin{pmatrix}2&0&0&0&0&0&0&0\\0&2&0&0&0&0&0&0\\0&0&2&0&0&0&0&0\\0&0&0&-1&0&0&0&0\\0&0&0&0&-1&0&0&0\\0&0&0&0&0&-1&0&0\\0&0&0&0&0&0&-1&0\\0&0&0&0&0&0&0&-2\end{pmatrix}$$

and

$$\zeta^{27}_{\alpha\beta}=\frac{1}{2\sqrt{30}}\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&-3&0&0&0&0\\0&0&0&0&-3&0&0&0\\0&0&0&0&0&-3&0&0\\0&0&0&0&0&0&-3&0\\0&0&0&0&0&0&0&9\end{pmatrix}.$$

Note that although the traces involved in finding Eq. (13) can be derived by using commutator identities, for the calculations involved here it is simpler to compute them by hand using the explicit form for $\{S^\alpha\}$ given in the Appendix. We also remark that $d_{8\alpha\beta}$ and $\zeta^{27}_{\alpha\beta}$ are the standard matrices obtained from coupling $8\otimes 8$ to $\underline{8}_{\text{sym}}$ $(I=Y=0)$ and 8×8 to $\underline{27}$ $(I=Y=0)$, respectively.[6]

B. Pion-Pion Scattering Lengths

In the soft-meson limit, the s-wave, isospin-zero scattering length is given by[16]

$$a_0^{(0)}=\frac{1}{(96\pi)}\;\frac{5A+16m_\pi^{\,2}/F^2}{m_\pi}$$

and

$$a_0^{(2)}=\tfrac{2}{5}\,a_0^{(0)}-\frac{3}{(20\pi)}\;\frac{m_\pi}{F^2},$$

where $a_0^{(2)}$ is the s-wave, isospin-two scattering length. In the case where the σ commutator has no isospin-two piece, $A=m_\pi^{\,2}/F^2$. In general it is given by

$$A=-\frac{1}{F^4}\langle 0|[F_5^i,[F_5^i,[F_5^i,[F_5^i,H^1]]]]|0\rangle,$$

where $i=1$ or 2 or 3 (no sum). A more general isospin decomposition of the fourfold commutator involves L_0 and L_2, which measure the relative isospin-zero and -two components of the σ commutator. In terms of these,

$$A=\tfrac{1}{3}L_0+\tfrac{2}{3}L_2,$$

but L_0 and L_2 are also constrained by a Jacobi-identity relation[17] which yields

$$2L_0-5L_2=6\,\frac{m_\pi^{\,2}}{F^2}.$$

Thus, $L_2=0$ implies $L_0=3\,m_\pi^{\,2}/F^2$, which yields $A=m_\pi^{\,2}/F^2$ for the pure isospin-zero case.

Using our form for H^1, we find

$$A=-\frac{1}{F^4}\,\frac{\langle 0|U_0|0\rangle}{\sqrt{6}}\left\{\frac{C_{60}}{\sqrt{6}}\,\mathrm{Tr}[(S^1)^4]+\frac{C_{68}}{\sqrt{10}}\,\mathrm{Tr}[S^8(S^1)^4]+C_{6\,27}\mathrm{Tr}[T^{27}(S^1)^4]\right\}$$

$$=-\frac{17}{3F^4}\langle 0|U_0|0\rangle\left(C_{60}+\frac{31}{17\sqrt{5}}\,C_{68}+\frac{13}{17\sqrt{5}}\,C_{6\,27}\right). \qquad (14)$$

C. Baryon Masses

With our neglect of scale-symmetry-breaking considerations, we write simply

$$M_\alpha = M_0 + \langle B_\alpha | H^1 | B_\alpha \rangle .$$

Now we must clearly keep $\langle N|U_0|N\rangle$, $\langle N|U_8|N\rangle$, and $\langle N|U_{27}|N\rangle$ all nonzero. We denote these by N_0, N_8, and N_{27}, respectively, and we let $D(F+D=1)$ be the mixing parameter in $\langle B_\alpha|U_8|B_\alpha\rangle$. In this notation

$$\langle N|U_0|N\rangle = \langle\Lambda|U_0|\Lambda\rangle = \langle\Sigma|U_0|\Sigma\rangle = \langle\Xi|U_0|\Xi\rangle = N_0 ,$$

$$\langle\Sigma|U_8|\Sigma\rangle = \frac{-\frac{2}{3}D}{(1-\frac{2}{3}D)} N_8 ,$$

$$\langle\Lambda|U_8|\Lambda\rangle = \frac{+\frac{2}{3}D}{(1-\frac{2}{3}D)} N_8 ,$$

$$\langle\Xi|U_8|\Xi\rangle = \frac{-(1-\frac{4}{3}D)}{(1-\frac{2}{3}D)} N_8 ,$$

$$\langle\Sigma|U_{27}|\Sigma\rangle = -\tfrac{1}{3} N_{27} ,$$

$$\langle\Lambda|U_{27}|\Lambda\rangle = -3N_{27} ,$$

$$\langle\Xi|U_{27}|\Xi\rangle = N_{27} .$$

Thus,

$$M_N = M_0 + C_{60}N_0 + C_{68}N_8 + C_{6\,27}N_{27} ,$$
$$M_\Sigma = M_0 + C_{60}N_0 - \frac{\frac{2}{3}D}{(1-\frac{2}{3}D)} C_{68}N_8 - \tfrac{1}{3} C_{6\,27}N_{27} , \qquad (15)$$

etc., with the obvious replacements for Λ and Ξ.

D. Nucleon σ Term

The nucleon σ term is given by

$$\sigma_N = \langle N|[F_5^i,[F_5^i,H^1]]|N\rangle ,$$

where $i=1$ or 2 or 3 (no sum). To evaluate this we again need N_0, N_8, and N_{27}. To perform the projection we write symbolically

$$\langle N|P_{ij}|N\rangle \to \frac{1}{\sqrt{6}}\delta_{ij}N_0 + \frac{1}{\sqrt{10}} S^8_{ji}N_8 + T^{27}_{ji}N_{27} ,$$

which yields

$$\begin{aligned}\sigma_N = {} & \frac{C_{60}}{\sqrt{6}}\left[\frac{N_0}{\sqrt{6}}\mathrm{Tr}(S^1S^1) + \frac{N_8}{\sqrt{10}}\mathrm{Tr}(S^8S^1S^1) + N_{27}\mathrm{Tr}(T^{27}S^1S^1)\right] \\ & + \frac{C_{68}}{\sqrt{10}}\left\{\frac{N_0}{\sqrt{6}}\mathrm{Tr}(S^8S^1S^1) + \frac{1}{2}\frac{N_8}{\sqrt{10}}[\mathrm{Tr}(S^8S^8S^1S^1) + \mathrm{Tr}(S^8S^1S^8S^1)] + \tfrac{1}{2}N_{27}[\mathrm{Tr}(S^8T^{27}S^1S^1) + \mathrm{Tr}(S^8S^1S^{27}S^1)]\right\} \\ & + C_{6\,27}\left\{\frac{N_0}{\sqrt{6}}\mathrm{Tr}(T^{27}S^1S^1) + \frac{1}{2}\frac{N_8}{\sqrt{10}}[\mathrm{Tr}(S^8T^{27}S^1S^1) + \mathrm{Tr}(S^8S^1S^{27}S^1)] + \tfrac{1}{2}N_{27}[\mathrm{Tr}(T^{27}T^{27}S^1S^1) + \mathrm{Tr}(T^{27}S^1T^{27}S^1)]\right\} \\ = {} & \tfrac{5}{3}N_0\left(C_{60} + \frac{7}{5\sqrt{5}}C_{68} + \frac{1}{5\sqrt{5}}C_{6\,27}\right) + \frac{7}{3\sqrt{5}}N_8\left(C_{60} + \frac{17}{7\sqrt{5}}C_{68} + \frac{11}{7\sqrt{5}}C_{6\,27}\right) \\ & + \frac{1}{3\sqrt{5}}N_{27}\left(C_{60} + \frac{11}{\sqrt{5}}C_{68} + \frac{13}{\sqrt{5}}C_{6\,27}\right) . \end{aligned} \qquad (16)$$

Note that even if H^1 has no 27-plet component ($C_{6\,27}=0$), Eq. (16) still will include contributions from N_{27}.

VI. DISCUSSION

Evidence from the K_{l4} decay[1,2] seems to favor a value of $a_0^{(0)}$ which is larger than the original Weinberg prediction[3] ($A=1$). For $A=1$, $a_0^{(0)} \simeq 0.16/m_\pi$ but increases rapidly with A. For example, if $A=10$, $a_0^{(0)} \simeq 0.5/m_\pi$. The experimental data indicate a value of $a_0^{(0)}$ of order $0.5/m_\pi$. Howeve, one must also realize that there are theoretical corrections to the soft-pion predictions. In a recent theoretical calculation which included unitarity corrections,[4] a value of $A \simeq \frac{1}{4}$ was found. However, in the same calculation, an effective value of $A \simeq \frac{4}{5}$ would be needed, in the simple soft-pion formula for $a_0^{(0)}$, to produce the calculated scattering length. Although this calculation may not be quantitatively reliable, it does indicate that unitarity corrections can enhance the effective value of A. In view of this we may conclude that even though the experimental evidence favors $A > 1$, it may not have to be as large as $A \sim 10$ which the simple soft-pion formula would indicate.

However, a simple estimate using the meson-mass formula [Eq. (13)] and our calculation of A [Eq. (14)] indicates that the use of (6, 6*) ⊕ (6*, 6) alone for H^1 is not reasonable. To make this estimate we set $C_{6\ 27} = 0$, although a similar result holds in any case. Let $N \equiv \langle 0|U_0|0\rangle/F^2$ and $\alpha \equiv C_{68}/C_{60}$. Then from Eq. (13)

$$m_\pi^2 = -\tfrac{5}{3} N\left(1 + \frac{7\alpha}{5\sqrt{5}}\right),$$

$$m_K^2 = -\tfrac{5}{3} N\left(1 - \frac{7\alpha}{10\sqrt{5}}\right),$$

from which we find

$$\alpha = -\frac{10\sqrt{5}}{7}\,\frac{(1 - m_\pi^2/m_K^2)}{(2 + m_\pi^2/m_K^2)}$$

and

$$N = -\tfrac{1}{5}(m_\pi^2 + 2m_K^2).$$

Using these we can calculate A from Eq. (14):

$$A = -\frac{(24m_K^2 - 143m_\pi^2)}{35F^2} \sim -5\,\frac{m_\pi^2}{F^2}.$$

Such a large negative value is clearly ruled out by the experimental data. We might note that pure (8, 8) breaking would also produce a negative value for A.

It is clear from the preceding remarks that (6, 6*) ⊕ (6*, 6) symmetry breaking cannot be the only contribution to H^1. On the other hand, since all of our results in Sec. V are linear in H^1, these calculations can be used to discuss more general schemes[7,18] which involve using (6, 6*) ⊕ (6*, 6) breaking in addition to some other contribution to H^1. Classification of H^1 pieces under the subgroup SU(2) ⊗ SU(2) (Ref. 18), as discussed in Sec. IV, may provide a tractable approach to this problem. We shall present several alternatives for combining (3, 3*) ⊕ (3*, 3), (6, 6*) ⊕ (6*, 6), and (8, 8) in a subsequent article.

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APPENDIX: THE 6-DIMENSIONAL REPRESENTATION OF SU(3)

We use the phase conventions and notation of Behrends *et al.*[11] to construct the representation of the eight generators of SU(3) on the six-dimensional representation. In terms of a spherical basis set, these are

$$H_1 = \frac{1}{\sqrt{3}}(|1\rangle\langle 1| - |3\rangle\langle 3|) + \frac{1}{2\sqrt{3}}(|4\rangle\langle 4| - |5\rangle\langle 5|),$$

$$H_2 = \tfrac{1}{3}(|1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|) - \tfrac{1}{6}(|4\rangle\langle 4| + |5\rangle\langle 5|) - \tfrac{2}{3}(|6\rangle\langle 6|),$$

$$E_1 = \frac{1}{\sqrt{3}}(|2\rangle\langle 3| + |1\rangle\langle 2|) + \frac{1}{\sqrt{6}}(|4\rangle\langle 5|),$$

$$E_2 = \frac{1}{\sqrt{3}}(|4\rangle\langle 6| + |1\rangle\langle 4|) + \frac{1}{\sqrt{6}}(|2\rangle\langle 5|),$$

$$E_3 = \frac{1}{\sqrt{3}}(|5\rangle\langle 6| + |3\rangle\langle 5|) + \frac{1}{\sqrt{6}}(|2\rangle\langle 4|),$$

and $E_{-i} = E_i^\dagger$. The states $\{|1\rangle, |2\rangle, |3\rangle\}$ form an isotriplet; $\{|4\rangle, |5\rangle\}$ form an isodoublet, and $\{|6\rangle\}$ is an isosinglet. We transform to a Cartesian basis by writing

$$S_1 = 6(E_1 + E_{-1}),$$
$$S_2 = -i\sqrt{6}\,(E_1 - E_{-1}),$$
$$S_3 = 2\sqrt{3}H_1,$$
$$S_4 = \sqrt{6}\,(E_2 + E_{-2}),$$
$$S_5 = -i\sqrt{6}\,(E_2 - E_{-2}),$$
$$S_6 = \sqrt{6}\,(E_3 + E_{-3}),$$
$$S_7 = -i\sqrt{6}\,(E_3 - E_{-3}),$$
$$S_8 = 2\sqrt{3}H_2,$$

such that $[S_i, S_j] = 2if_{ijk}S_k$. These matrices form the 6-dimensional representation of SU(3).

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Pion-Pion Scattering Based on Current Algebra, Analyticity, and Unitarity*

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Using SU(2) × SU(2) current algebra and pion-pole dominance, we derive from the Ward identities an exact crossing-symmetric expression for the $\pi\pi$ scattering amplitude. We make approximations which are suitable at low energy for those three- and four-point functions of the problem which cannot be determined from the constraints of current algebra. We parametrize these functions in terms of propagators and polynomials exhibiting the correct analyticity properties. Form factors, analytic in the cut plane, are expressed in effective-range form, and the s- and p-wave amplitudes are constructed in terms of them. The existence of resonances in the $\pi\pi$ system is not assumed, and soft-pion estimates are not used. Instead all the parameters are free to be varied. We determine all the free parameters of the problem self-consistently by imposing the constraints that follow from elastic unitarity. The scheme predicts all the features of low-energy $\pi\pi$ scattering, the only input parameters being m_π and F_π, the pion mass and decay constant. Among our principal results are the s- and p-wave scattering lengths, the corresponding phase shifts, and the determination of an important parameter which measures the isospin $T = 2$ component of the σ commutator, σ^{ab}. The details of the method predispose scattering lengths to be small. We find that unitarity prefers the $T = 2$ component of σ^{ab} to be small relative to the $T = 0$ component. As a consequence, our scattering lengths are in excellent agreement with those obtained by Weinberg. The $T = J = 1$ phase shift exhibits a ρ resonance around 915 MeV with a width of 210 MeV. The $T = 2$, $J = 0$ phase shift is small and in agreement with experimental results. The $T = J = 0$ phase shift displays acceptable behavior at low energy; we offer physical arguments to say that its higher-energy behavior is less reliable than that of the p wave at the same energies. We discuss our results and analyze the predictive power of the method presented. Finally, we suggest some improvements on our calculations, including possible applications to related problems.

I. INTRODUCTION

For more than a decade, the problem of determining the amplitude for $\pi\pi$ scattering has presented a challenge for theoretical physics to solve. In the absence of a fully developed theory of hadrons, an ultimate solution continues to be an overly ambitious goal. Many approaches to an approximate solution have evolved, and contributed to the unfolding of several features of the problem. The principles of S-matrix theory (including Lorentz invariance, analyticity, unitarity, and crossing symmetry) are cornerstones of hadron dynamics[1] and have long been advocated as the means by which a self-consistent solution to the $\pi\pi$ problem may be found. If hadronic theory is to include, in addition, the content of the algebra of vector and axial-vector currents,[2] then any treatments based purely on S-matrix theory are to be viewed as part of the prehistory of the problem. The low-energy