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Complex-Regge-Pole Model Consistent with the Dual Absorption Picture: Meson-Baryon Charge-Exchange Reactions*

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The reactions $\pi^- p \to \pi^0 n$, $\pi^- p \to \eta n$, $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$, where ρ and/or A_2 exchanges are involved, are considered within the complex-pole model with the amplitudes given by $\gamma_+ \xi_+ s^{\alpha_+} + \gamma_- \xi_- s^{\alpha_-}$. Our fits confirm the prediction of the dual absorption model in that we find the real part of the residue function, γ_R, to be $\sim J_n$ with $n = 0$, 1. For the imaginary part, γ_I, we find that since the j-plane cuts are strong for the spin-nonflip amplitude we have $\gamma_I \sim -\mathcal{K}_0$, the Struve function, whereas the cuts are weak for the spin-flip amplitude so that $\gamma_I \sim 0$. The contribution of γ_I to the amplitude always appears in the form $\alpha_I \gamma_I$, which is quite small. Good quantitative fits to all the data are obtained in terms of only four parameters for the exchange-degenerate ρ, A_2 residue if we take $J_0(R_0(-t)^{1/2}) \mp i\mathcal{K}_0(R_0'(-t)^{1/2})$ as the nonflip residue and $J_1(R_1(-t)^{1/2})/R_1(-t)^{1/2} \mp i\lambda\alpha_I$ as the flip residue with $R_0 \approx R_0' \approx R_1 \approx 1$ F and $\lambda < 1$. For the imaginary part of the trajectory function we find $\langle \alpha_I \rangle_\rho = 0.17$ and $\langle \alpha_I \rangle_A = 0.26$. The presence of nonzero α_I removes the need for any explicit ghost-eliminating factors. A detailed discussion of this model as well as a comparison with other models is made.

I. INTRODUCTION

There is, theoretically, little doubt that complex conjugate Regge poles exist for $t \leq 0$ if cuts in the j plane exist. The evidence comes from the general analytic structure as a result of pole-cut collisions,[1] t-channel unitarity,[2] Bethe-Salpeter equation,[3] multi-Regge models,[4] as well as K-matrix unitarization of the absorptive model.[5] The fact that the cuts themselves can be approximated by these poles is also very plausible if the imaginary part of the trajectory function, for $t \leq 0$, is reasonably small and energies are not asymptotic.[6] The complex-pole approximation thus provides an economical way of parametrizing amplitudes much in the same way as the Breit-Wigner form does in the energy plane. As we shall see below, it also provides a simple way of incorporating some of the significant results of the absorption model.

Phenomenologically, the model has been quite successful. It has been applied to πN charge-exchange scattering,[7] line-reversed reactions,[8] and πN backward scattering[9] as well as many other reactions.[10,11]

Even though it has strong theoretical backing, the model, as applied so far, nevertheless, has several parameters.[12] In essence one has effectively two poles now and, furthermore, the complex residues have been usually parametrized as polynomials, linear or quadratic.

It would be most desirable, therefore, if the residues and the trajectory functions in the complex-pole model can be described, at least *qualitatively*, by simple analytic functions (with very few parameters) which reproduce the main features of several different experiments. These simple functions would be obtained on some theoretical basis but by themselves need not, in general, fit the data *exactly*; the experiments are too accurate compared to any theoretical model that anyone can come up with. However, any deviation from these functions should be *small*.

The main features of nondiffractive reactions with vector and tensor exchanges are: shrinkage and power falloff in $d\sigma/dt$, nonzero polarizations, and crossover zeros and dips. The shrinkage and the power falloff in $d\sigma/dt$ is explained by the famous Regge s dependence $s^{\alpha(t)}$, but the existence of nonzero polarization needs cuts; hence complex poles. The crossover zeros and dips are understood well in terms of the absorption picture[13] and arise naturally if one assumes the dual absorption result,[14] $\mathrm{Im}T_n \sim J_n(R(-t)^{1/2})s^\alpha$, where n is the total helicity change in the s channel, and R is the radius parameter, expected to be of the order of 1 F. Thus in the complex-pole model, the imaginary part of the trajectory function α_I should be small and the real part, α_R, is essentially the same as the old (real) trajectory function. One can also show[15,16] on the basis of the dual absorption picture, that the real part of the residue function, γ_R, should be proportional to $J_n(R(-t)^{1/2})$. A crude estimate then shows that for the helicity-nonflip amplitude where the cuts are large, since the simple Regge-pole contribution is nonperipheral, $\gamma_I \sim -\mathcal{K}_0(R(-t)^{1/2})$, the Struve function. For the helicity-flip amplitude where the cuts are

weak, since the simple Regge-pole contribution is peripheral, $\gamma_I \sim 0$. It turns out that the contribution of γ_I to the total amplitude appears in the form $\alpha_I \gamma_I$ which is small.

We shall continue to fit the experiments with polynomial residues and trajectories but we anticipate that they will be described qualitatively by the above simple analytic functions. Thus, in a sense, we are carrying out an amplitude analysis of the different reactions but within the theoretical framework of complex poles.

We consider the following four charge-exchange reactions which involve ρ and/or A_2 exchange: $\pi^- p \to \pi^0 n$, $\pi^- p \to \eta n$, $K^- p \to \overline{K}^0 n$, and $K^+ n \to K^0 p$. We first analyze $\pi^- p \to \pi^0 n$ where considerable data are available. The data include $d\sigma/dt$, polarization, sum rules, and $\sigma_{\pi^- p} - \sigma_{\pi^+ p}$. This reaction was analyzed previously[7] but this time we go to larger $-t$'s including $t \approx -1.5$ where, in the simple Regge-pole model, there is a ghost (corresponding to $\alpha = -1$) which is usually eliminated by an explicit factor $(\alpha + 1)$ in the residue function. We find, however, that our γ_R is well described by J_0 for the nonflip, and J_1 for the flip amplitude. We confirm the previous result[7] that the phase of the residue corresponding to the pole $j = \alpha_+$ is negative. We find, as anticipated earlier, that $\gamma_I \sim -\mathcal{C}_0$ for the nonflip and $\gamma_I \sim 0$ for the flip amplitude. Fortunately, as mentioned earlier, the contribution of γ_I is small. We next go to $\pi^- p \to \eta n$. We keep γ_R and γ_I the *same* as in πN charge-exchange scattering except for the overall exponential falloff. We, however, take α_R and α_I different. We find that α_I is larger for the A_2 than for the ρ exchange: $\langle \alpha_I \rangle_\rho = 0.17$, $\langle \alpha_I \rangle_A = 0.26$. In $\pi^- p \to \eta n$, data are available for $d\sigma/dt$. We obtain the experimentally observed dip at $t \approx -1.5$ in $d\sigma/dt$. The polarization is predicted to be positive for low $-t$. We next go to $K^- p \to \overline{K}^0 n$ and $K^+ p \to K^0 n$. Here we keep the *same* γ's and α's determined previously for the ρ and A_2 exchanges, the only change being in the overall constants which are, however, determined from SU_3. Our predicted curves are compared with the experimental $d\sigma/dt$ and we find very good agreement.

An exact fit requires several parameters. However, the main features of all the four reactions can be qualitatively described, apart from the overall normalization factors and trajectory functions, by only two parameters if we fix R to be 1 F, the two parameters being the ones involved in the α_I of ρ and A_2. Good quantitative fits can be obtained if we also take the radii R_0 and R_0' in the nonflip residues

$$J_0(R_0(-t)^{1/2}) \mp i\mathcal{C}_0(R_0'(-t)^{1/2})$$

as parameters and the radius R_1 and λ in the flip residue

$$\frac{J_1(R_1(-t)^{1/2})}{R_1(-t)^{1/2}} \mp i\lambda\alpha_I$$

as parameters. The radii turn out to be quite close to 1 F; we find $R_0 = 1.2$ F, $R_0' = 0.9$ F, $R_1 = 1.1$ F, and $\lambda = 0.1$.

II. THE COMPLEX-POLE MODEL

As mentioned in the Introduction there is strong evidence in favor of complex conjugate poles for $t \leq 0$. If α_c is the position of the branch point in the complex j plane, then from the Mandelstam prescription we have $\alpha_c = \alpha$ at $t = 0$ where $\alpha(t)$ is the trajectory function. In the neighborhood of $t = 0$, the trajectory will, therefore, become complex.

For $t < 0$, one can write

$$A(s,t) = (\text{possible physical sheet poles}) + \frac{1}{\pi}\int_{-\infty}^{\alpha_c} dj\; s^j \,\mathrm{disc}A(t,j), \tag{1}$$

$$\mathrm{disc}A(t,j) = \frac{A^{\mathrm{I}} - A^{\mathrm{II}}}{2i},$$

where the superscripts I and II indicate the first and the second sheets, respectively. Since A^{I} and/or A^{II} will have complex poles, so will disc$A$. Thus disc$A$ will have a peak near the position of the poles. If we approximate the entire discontinuity by this peak (complex-pole approximation), then we can write[6]

$$A(s,t) = \gamma_+(t)\, s^{\alpha_+(t)} + \gamma_-(t) s^{\alpha_-(t)}, \tag{2}$$

where α_+ and α_- are the positions of the complex conjugate poles.

Three points need to be made regarding the above expression. First is that if there are physical sheet poles, then γ_+ contains their contribution as well as the contributions of the cut. That is, $\gamma_+(t)$ $[= \gamma_-^*(t)]$ will then be a sum of the residue of the physical sheet pole and the "residue" approximating the cut contribution. Secondly, whereas the residue of a physical sheet pole is strictly a function of t only, the "residue" of the pole approximating the cut may be a function of s also, hopefully a slowly varying function of s. Even though throughout the paper we will consider the total residues to be functions of t alone, this possibility should be kept in mind for possible future discussions. Finally, the complex-pole approximation will fail at asymptotic s where the s^j factor in (1) will dominate over the discontinuity function. What this asymptotic s will be is unknown, but from phenomenological considerations it appears that the presently available energies are within the region of complex-pole dominance.

Introducing crossing symmetry we get for the amplitude

$$T(s,t)=\gamma_+\left(\frac{\pm1-e^{-i\pi\alpha_+}}{\sin\pi\alpha_+}\right)s^{\alpha_+}+\gamma_-\left(\frac{\pm1-e^{-i\pi\alpha_-}}{\sin\pi\alpha_-}\right)s^{\alpha_-},$$
$$\alpha_+=\alpha_R+i\alpha_I=\alpha_-^*,$$
$$\gamma_+=\gamma_R+i\gamma_I=|\gamma|e^{i\phi}=\gamma_-^*, \tag{3}$$

where ±1 indicates odd or even signature. Thus,

$$\mathrm{Im}T(s,t)=\gamma_+ s^{\alpha_+}+\gamma_- s^{\alpha_-}=2|\gamma|s^{\alpha_R}\cos(\phi+\alpha_I\ln s), \tag{4}$$

$$\mathrm{Re}T(s,t)=\gamma_+\left(\frac{\pm1-\cos\pi\alpha_+}{\sin\pi\alpha_+}\right)s^{\alpha_+}+\gamma_-\left(\frac{\pm1-\cos\pi\alpha_-}{\sin\pi\alpha_-}\right)s^{\alpha_-}. \tag{5}$$

We note that when α_I is *small*

$$\mathrm{Im}T\simeq(\gamma_R+\alpha_I\gamma_I\ln s)s^{\alpha_R}. \tag{6}$$

A similar expression holds for ReT. Thus contributions of γ_I appear in the form of the combination $\alpha_I\gamma_I$. If α_I identically vanishes, then we do not have any contributions from γ_I either.

Furthermore, we note that for α_R near a negative integer, $\alpha_R\simeq-n$, the signature factor is given by

$$\frac{\pm1-e^{i\pi\alpha_+}}{\sin\pi\alpha_+}=\frac{\pm(-1)^n-1+\pi(\alpha_R+n+i\alpha_I)}{\pi(\alpha_R+n+i\alpha_I)}. \tag{7}$$

There is no peak at $\alpha_R=0,-2,\ldots$ for the odd-signature ρ trajectory, and at $\alpha_R=-1,-3,\ldots$ for the even-signature A_2 trajectory because the signature factor $(\pm1-e^{-i\pi\alpha_+})$ provides exact cancellation. However, at $\alpha_R=-1,-3,\ldots$ for ρ, and $\alpha_R=0,-2,\ldots$ for A_2, there will be a peak. Now there are two alternatives: either γ_+ is sufficiently rapidly falling in t so that the peak is "smoothed out" and no structure is seen in the amplitudes, or that γ_+ provides an *exact* cancellation by having explicit factors $(\alpha_++1)(\alpha_++3)\cdots$ etc. in ρ and $\alpha_+(\alpha_++2)\cdots$ in A_2.

In the simple Regge-pole model the latter alternative is chosen because the poles appear on the real axis and correspond to *infinities* in the amplitude. These are, of course, the so-called "ghosts" and the residues provide explicit ghost-eliminating factors. In the complex-pole picture we have complex ghosts (or tachyons) and since they do not correspond to infinities in the amplitude the former alternative mentioned above is entirely plausible as long as the ghost is "smoothed out" by falling γ's. The phenomenology seems to favor this alternative, but more data, especially on A_2 exchange, is needed before one can make a definite statement. We will return to this question later in this paper [Sec. VI F].

Having reviewed some of the important features of the complex-pole model, we now look at the characteristic features of nondiffractive processes.

III. CHARACTERISTICS OF NONDIFFRACTIVE PROCESSES AND ESTIMATED BEHAVIOR OF α_R, α_I, AND γ_R

A. Shrinkage and Power Falloff in $d\sigma/dt$: α_R and α_I

In the reactions with vector and tensor exchanges it is found that one can write

$$\frac{d\sigma}{dt}=|F(t)|^2 s^{2(\alpha_{\mathrm{eff}}-1)},$$

where $\alpha_{\mathrm{eff}}(t)$ is found to be linear for small negative t and when extrapolated to positive t gives roughly the correct spin values of the resonances. For large negative t it may deviate from linearity as a consequence of the presence of cuts.

In the complex-pole model, the energy dependence comes mainly from the factor s^{α_R}, but there is some energy dependence from the $\sin(\alpha_I\ln s)$ and $\cos(\alpha_I\ln s)$ factors also. If we assume at $t=0$

$$\alpha_I(0)=0,$$

then we anticipate that $\alpha_R(t)$ will be linear and consistent with the reasonances for positive t. Therefore, we expect

$$\left.\begin{aligned}&\alpha_R(t)=a+bt\\&\alpha_I(t)\ \text{small}\end{aligned}\right\}\quad t\leq0. \tag{8}$$

The smallness of α_I is dictated by the fact that the energy dependence predicted by the simple Regge theory with real trajectories is known to be roughly correct. It is also a desirable property from the complex-pole viewpoint since the smallness of α_I is the basis of the complex-pole approximation.

B. Nonzero Polarizations: The Phase of $\gamma_\pm$

The polarization, P, is given by

$$P\frac{d\sigma}{dt}\sim-\mathrm{Im}(A'B^*),$$

where A' and B are the nonflip and flip amplitudes. In the simple Regge theory the phases arise entirely from the signature factor and, consequently, for a given trajectory exchange, the phases of A' and B are identical, predicting a zero polarization. But it is known experimentally that in πN charge-exchange scattering, for instance, the polarization is nonzero. Thus the simple Regge-pole

theory which works extremely well as far as the s dependence is concerned fails in predicting the phases correctly.

In the complex-pole model the phases arise not only from the signature factor but also from the complex residues, $\gamma_\pm$. Since the residues for A' and B can have different phases, the polarization is nonzero. For small α_I, one can write (with ϕ_A, ϕ_B as the phases of the residues)[7]

$$P\frac{d\sigma}{dt}\sim \alpha_I \sin(\phi_B-\phi_A),$$

which clarifies our earlier statements. This particular aspect of the complex-pole model, as well as many others related to polarization, has already been discussed in the paper of Barik *et al.*[7] The important thing to note is that the *sign* of P puts restrictions on the *signs* of ϕ_A and ϕ_B.

C. Crossover Zeros and Dips: γ_R

Crossover zeros refer to the zeros in the amplitudes necessary to explain the change in sign in the difference $(d\sigma_{ab}/dt - d\sigma_{a\bar{b}}/dt)$ between particle and antiparticle differential cross sections. The evidence comes from $\pi^\pm p$, $K^\pm p$, and $\bar{p}p - pp$ reactions, and also from sum rules. The zeros seem to occur around $t=-0.2$ (BeV/c)^2 and in the nonflip amplitude. The dips refer to the dips in $d\sigma/dt$ which are associated with the zeros in the flip amplitude. For $\pi^- p \to \pi^0 n$ they occur around $t=-0.6$, and recent data indicate a dip in $\pi^- p \to \eta n$ around $t=-1.5$.

The absorption model[13] explains rather neatly and systematically the occurrence of these zeros. According to this model the low partial waves are strongly absorbed so that the partial-wave distribution is essentially peripheral. In the dual absorptive model proposed by Harari[14] it is argued that it is the *imaginary part* of the amplitude which is peripheral. The argument uses duality in addition to absorption. If one saturates the amplitude with *peripheral* s-channel resonances then it is most plausible that the imaginary part of the amplitude remains peripheral, but not necessarily the real part where the resonance contributions tend to alternate in sign.

The dual absorptive model then states that[14]

$$\mathrm{Im}T_n = C\, s^{\alpha(t)} J_n(R(-t)^{1/2}), \tag{9}$$

where n is the total s-channel helicity change and R is the radius parameter, expected to be of the order of 1 F. The above relation (9) should be understood in a qualitative sense, if not in a semiquantitative sense. However, it does describe very simply the characteristic zeros of the amplitude. We will use it to estimate the complex residue parameters.

A possible relation between the complex-pole model and the dual absorptive model was pointed out recently by Desai.[15,16] We will review here some of the important points.

If we assume that R in expression (9) is a constant, independent of energy, then expressions (4) and (9) are similar in the limit $\alpha_I \to 0$. In other words, one can think of the dual absorptive model as an approximate version of the complex-pole model. Equating the two in the limit $\alpha_I \to 0$, we obtain

$$\gamma_R = \tfrac{1}{2} C\, J_n(R(-t)^{1/2}). \tag{10a}$$

Thus we anticipate for the two amplitudes in question,

$$\gamma_R \sim J_0(R(-t)^{1/2}) \quad \text{(nonflip)},$$
$$\gamma_R \sim \frac{J_1(R(-t)^{1/2})}{R(-t)^{1/2}} \quad \text{(flip)}. \tag{10b}$$

Thus we see that although the characteristic features of nondiffractive processes provide a fair estimate of γ_R, α_R, the sign of the phase of the complex residues, and to some extent α_I, they do not apparently throw much light on the magnitudes of the imaginary part, γ_I, of the complex residues. This problem is pretty much similar to having a prescription for the imaginary part of the amplitude while the real part still remains unknown in the dual absorptive model. So we base our investigation of γ_I on some theoretical guesses.

IV. ESTIMATE OF γ_I

In our model the entire pole-cut structure is approximated by complex conjugate poles with residues and trajectories becoming complex for $t \leq 0$. Thus $\gamma_\pm(t)$ will have a (left-hand) cut in t from 0 to $-\infty$ in addition to the usual (right-hand) cut and subtraction terms. Thus

$$\gamma_R(t) = (\text{possible subtraction terms}) + \frac{P}{\pi}\int_{-\infty}^{0} \frac{dt'\,\gamma_I(t')}{t'-t} + \frac{P}{\pi}\int_{t_0}^{\infty} dt'\,\frac{\bar{\gamma}_I(t')}{t'-t}, \tag{11}$$

where t_0 (>0) is the t value at threshold.

We adopt the attitude inherent in the dual absorption picture,[14] namely, that if the imaginary part of an amplitude predicted by simple Regge poles has a *nonperipheral* partial-wave distribution, then the cuts in the j plane are *strong*. On the other hand, if the Regge poles give a *peripheral* amplitude then the cuts must be *weak*. The point is that the correct imaginary part of an amplitude *must* be peripheral so that the strength of the correction term due to cuts depends on whether the original amplitude, as predicted by simple Regge poles, is peripheral or not.

First let us consider the nonflip amplitude. From the last section (see III C) we concluded, on the basis of the dual absorptive model, that $\gamma_R \sim J_0(R(-t)^{1/2})$. For $R \sim 1$ F, this function has a zero at $t \approx -0.2$ (BeV/c)^2, which is, of course, the famous crossover zero. The amplitude predicted by simple Regge poles does not have this zero, and it is, therefore, not peripheral. Hence, for the nonflip amplitude, the cuts must be strong.[14]

As emphasized earlier, existence of cuts in the j plane implies the existence of complex poles for $t \leq 0$ and, therefore, of the left-hand cut in (11). Thus, if the cuts are strong then the contribution of the left-hand cut must be large. On the other hand, the right-hand cut (plus any subtractions) is present even when the cuts are absent and should, therefore, reflect more the property of the simple Regge-pole picture.

Evidently, our approach is somewhat perturbation-theoretic. However, since we are only interested in a crude estimate of γ_I, we feel justified in using this approach. Substituting $J_0(R(-t)^{1/2})$ for γ_R in (11), we obtain

$$J_0(R(-t)^{1/2}) = \frac{P}{\pi}\int_{-\infty}^{0} dt' \frac{\gamma_I(t')}{t'-t} + \text{small corrections}. \tag{12}$$

Now the following relation is known[16]:

$$\frac{J_n(R(-t)^{1/2})}{(R(-t)^{1/2})^n} = -\frac{P}{\pi}\int_{-\infty}^{0} \frac{dt' \mathcal{K}_n(R(-t')^{1/2})}{(R(-t')^{1/2})^n (t'-t)}, \tag{13}$$

where $\mathcal{K}_n(R(-t)^{1/2})$ is the so-called Struve function.

Comparing (12) and (13), we obtain $\gamma_I \approx -\mathcal{K}_0(R(-t)^{1/2})$. This was the estimate obtained in Refs. 15 and 16. However, we can take account of the small correction term in (12) to express

$$\gamma_I \approx -\mathcal{K}_0(R'(-t)^{1/2}), \quad R' \approx R \quad \text{(nonflip)}. \tag{14}$$

For phenomenological purposes, we can assume R' slightly different from R.

The flip amplitude predicted by simple Regge poles has the famous term $\Gamma(\alpha)^{-1}$ of which we will consider the first two factors, $\alpha(\alpha+1)$. Since this term already has the structure of J_1, the cuts must be weak. That is, substituting $J_1(R(-t)^{1/2})/R(-t)^{1/2}$ for γ_R in (11), we obtain

$$\frac{J_1(R(-t)^{1/2})}{R(-t)^{1/2}} = C_1\alpha(\alpha+1) + \frac{P}{\pi}\int_{-\infty}^{0} \frac{dt'\gamma_I(t')}{t'-t}, \tag{15}$$

where we have associated the term $C_1\alpha(\alpha+1)$ with the right-hand cut as it comes from the simple Regge-pole theory. The almost identical behavior of J_1 and $\alpha(\alpha+1)$ implies that

$$\gamma_I \approx 0 \quad \text{(flip)}. \tag{16}$$

We can assume $\gamma_I \approx -\lambda\alpha_I$ (flip) as a better estimate with $\lambda \leq 1$, since α_I is a small quantity.

In Refs. 15 and 16, the flip and the nonflip amplitudes were treated in the same footing. In other words, it was assumed that the J_1 structure in the flip amplitude came entirely from the j-plane cut and, therefore, the left-hand cut was the dominant contributor. For this case, we can follow the same procedure as in (12), (13), and (14) and obtain

$$\gamma_I \approx \frac{-\mathcal{K}_1(R(-t)^{1/2})}{R(-t)^{1/2}}. \tag{17}$$

We tend to favor (16) over (17) even though we are fully aware of the fact that thereby we are losing a certain amount of symmetry between the treatments of the nonflip and flip amplitudes. However, we feel that once the simple Regge poles have the factor $\alpha(\alpha+1)$, then it is inevitable that the right-hand cut (plus subtraction) will inherit that factor. We also point out that in potential theory where there are no cuts in the j plane the $\alpha(\alpha+1)$ factor is predicted and can be expressed in terms of the right-hand cut.

In summary, we anticipate

$$\begin{aligned} &\gamma_I \approx -\mathcal{K}_0(R'(-t)^{1/2}) \quad \text{(nonflip)}, \\ &\gamma_I \approx -\lambda\alpha_I \quad \text{(flip)}, \end{aligned} \tag{18}$$

where $R' \approx R$, $\lambda < 1$. We note that these expressions can only be estimates because rigorously speaking one cannot cleanly identify, as we have done, the left-hand cut as coming entirely from the j-plane cuts and the right-hand cut (plus subtraction) as due to the simple Regge poles.

In order to confirm our contention for the above estimates we make a phenomenological analysis of the experiments with complex residues as polynomials in t. Then we compare the residues we get from the fit with our estimates. We also attempt to get qualitative fits to the experimental data from these estimates.

V. ANALYSIS OF EXPERIMENTS AND THE BEHAVIOR OF $\alpha_\pm$ AND $\gamma_\pm$

If we assume SU_3 symmetry for the meson-vertex residue factors we get the following expressions for the charge-exchange amplitudes in general:

$$\begin{aligned} &T(\pi^- p \to \pi^0 n) = -\sqrt{2}\, T_\rho, \\ &T(\pi^- p \to \eta n) = \sqrt{\tfrac{2}{3}}\, T_{A_2}, \\ &T(K^- p \to \overline{K}^0 n) = T_\rho + T_{A_2}, \\ &T(K^+ n \to K^0 p) = -T_\rho + T_{A_2}. \end{aligned} \tag{19}$$

The expression for the differential cross section, polarization, etc. for the above processes can be given in terms of the conventional invariant ampli-

tudes A' and B as follows:

$$\frac{d\sigma}{dt}=\frac{m^2(2.57)}{16\pi s k^2}\left[\left(1-\frac{t}{4m^2}\right)|A'|^2 - \frac{t}{4m^2}\left(\frac{E_{\rm lab}{}^2-m^2+st/4m^2}{1-t/4m^2}\right)|B|^2\right],$$

$$P\frac{d\sigma}{dt}=\frac{\sin\theta}{16\pi\sqrt{s}}\,\mathrm{Im}(A'B^*),$$

$$\Delta\sigma(\pi^-p-\pi^+p)=\frac{2\,\mathrm{Im}A'_\rho}{P_{\rm lab}}, \tag{20}$$

$$S_0=\frac{1}{N}\int^N \mathrm{Im}A'^{(-)}d\nu,$$

$$S_1=\frac{1}{N}\int^N \mathrm{Im}(\nu B)^{(-)}d\nu.$$

We parametrize the amplitudes A' and B for the vector- and tensor-meson exchanges as follows:

$$\begin{aligned}A'_k(\nu,t)&=\gamma^A_{+k}(t)\xi(\alpha_{+k})(\nu/\nu_0)^{\alpha_{+k}}\\&\quad+\gamma^A_{-k}(t)\xi(\alpha_{-k})(\nu/\nu_0)^{\alpha_{-k}},\\B_k(\nu,t)&=\gamma^B_{+k}(t)\xi(\alpha_{+k})(\nu/\nu_0)^{\alpha_{+k}-1}\\&\quad+\gamma^B_{-k}(t)\xi(\alpha_{-k})(\nu/\nu_0)^{\alpha_{-k}-1},\end{aligned} \tag{21}$$

where

k stands for ρ or A_2 exchange,

$\nu=E_{\rm lab}+t/4m$,

$\nu_0=1$ BeV,

$\xi(\alpha_{\pm k})=\dfrac{\tau-e^{-i\pi\alpha_{\pm k}}}{\sin\pi\alpha_{\pm k}}$, $\tau=$ signature $=\pm1$.

The trajectories and the residues are parametrized in the following way:

$$\begin{aligned}\alpha_{+k}&=(a_k+b_k t)+ig_k(-t)^{1/2}\\&=\alpha^*_{-k},\\\gamma^A_{+k}&=h_0e^{h_k t}\left[(1+\gamma_0t+\gamma_1t^2)+i\gamma_2(-t)^{1/2}(1+\gamma_3t+\gamma_4t^2)\right]\\&=\gamma^{*A}_{-k},\\\gamma^B_{+k}&=d_0e^{d_k t}\left[(1+\lambda_0t+\lambda_1t^2)+i\lambda_2(-t)^{1/2}\right]\\&=\gamma^{*B}_{-k}.\end{aligned} \tag{22}$$

Hence we have 20 parameters as such. At the first instance it might appear that such a large number of parameters would give enough flexibility to fit the data. But it is not so. As a matter of fact the parameters introduced like h_0, d_0, γ_0, γ_1, λ_0, λ_1 and a_k and b_k can be approximately fixed in an obvious manner. As for example, from the knowledge of the sum-rule zeros of S_0 and the crossover zero we can fix γ_0 and γ_1, whereas the dips in $d\sigma/dt$ for $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta n$ and also the zero in S_1 would fix λ_0 and λ_1 uniquely. However, since one of our interests is in an unbiased search for the functional form of the amplitudes, we chose quadratic polynomials for the complex residues in spite of the fact that it led us to having too many parameters.

We have fitted the $d\sigma/dt$, $\Delta\sigma$, polarization, and FESR[17] (finite-energy sum rule) values for the process $\pi^-p\to\pi^0n$ as well as the differential cross-section data for $\pi^-p\to\eta n$. The data were obtained from Refs. 18 to 20. We have obtained an excellent fit to the above data with a χ^2 of 135 for 146 data points. The solid curves in Fig. 1 show our fits. The following are the trajectory and residue parameters obtained:

$$\alpha_{\pm\rho}=(0.57+0.91t)\pm i0.26(-t)^{1/2},$$

$$\alpha_{\pm A}=(0.48+0.8t)\pm i0.39(-t)^{1/2}, \tag{23}$$

$$\gamma^A_{\pm k}=1.29\,e^{h_k t}[(1+14.1t+15.2t^2) \mp 4.3i(-t)^{1/2}(1+0.27t-0.9t^2)], \tag{24}$$

$$\gamma^B_{\pm k}=15.19\,e^{d_k t}[(1+2.89t+1.43t^2)\mp i0.3(-t)^{1/2}], \tag{25}$$

where

$$h_\rho=2.245\ (\mathrm{BeV}/c)^{-2},\quad d_\rho=0.58\ (\mathrm{BeV}/c)^{-2},$$

$$h_A=2.71\ (\mathrm{BeV}/c)^{-2},\quad d_A=0.77\ (\mathrm{BeV}/c)^{-2}.$$

We note that α_I vanishes at $t=0$ and grows as $(-t)^{1/2}$ as $-t$ increases. To get a feeling for the average behavior of α_I over an interval t_0 we define

$$\langle\alpha_I\rangle_{t_0}=\frac{1}{t_0}\int_0^{t_0}dt\,\alpha_I(t).$$

Taking $t_0=-1$, we find for ρ and A_2 [see (23)],

$$\langle\alpha_I\rangle_\rho=0.17,\quad \langle\alpha_I\rangle_A=0.26.$$

The solid curves in Fig. 1 correspond to our fits. The dip structure in $d\sigma/dt$ and the sum rules S_0 and S_1 for πN charge exchange (CEX) are well reproduced. Here γ_R plays the important role. To get the correct sign of the polarization γ_I must be negative, and ϕ_A and ϕ_B (the phases of the residues γ_+ for the A' and B amplitudes) pass through $-\frac{1}{2}\pi$ at $t\approx-0.1$ and $t\approx-0.5$, respectively. This important observation was already made in the paper of Barik *et al.*[7] The difference between our work and that of Ref. 7 lies in the fact that we have obtained a larger α_I for ρ, the reason being that we have gone to much larger t's including the region near $\alpha_R=-1$ where in order to avoid a peak coming from $(\sin\pi\alpha_\pm)^{-1}$, we need a larger α_I. As far as γ_I is concerned, since it contributes very little to the amplitude, the contribution being $\alpha_I\gamma_I$ as compared to γ_R [see (14)], the only place where its value can be determined to a good accuracy would be those points at which $\gamma_R=0$. Therefore, for the same

value of $\alpha_I \gamma_I$ a larger α_I would mean a smaller γ_I. Thus our γ_I is smaller than given by Ref. 7. In fitting $d\sigma/dt$ for $\pi^- p \to \eta n$ where A_2 exchange occurs, we kept the *same* residue function as ρ except for the overall exponential constants. We obtain excellent fits to $d\sigma/dt$ including the dip near $\alpha = -1$. The available data on polarization in $\pi^- p \to \eta n$ are quite meager and have large errors; however, our predicted polarization is positive and consistent with the data.

The $d\sigma/dt$ for $K^- p \to \bar{K}^0 n$, and $K^+ n \to K^0 p$ are given in Fig. 2. Here we have $(A_2 \pm \rho)$ exchanges. The solid curves correspond to our *predicted* fits where no new parameters are introduced and the overall normalization constants are determined from SU_3. Our predicted curves agree quite well

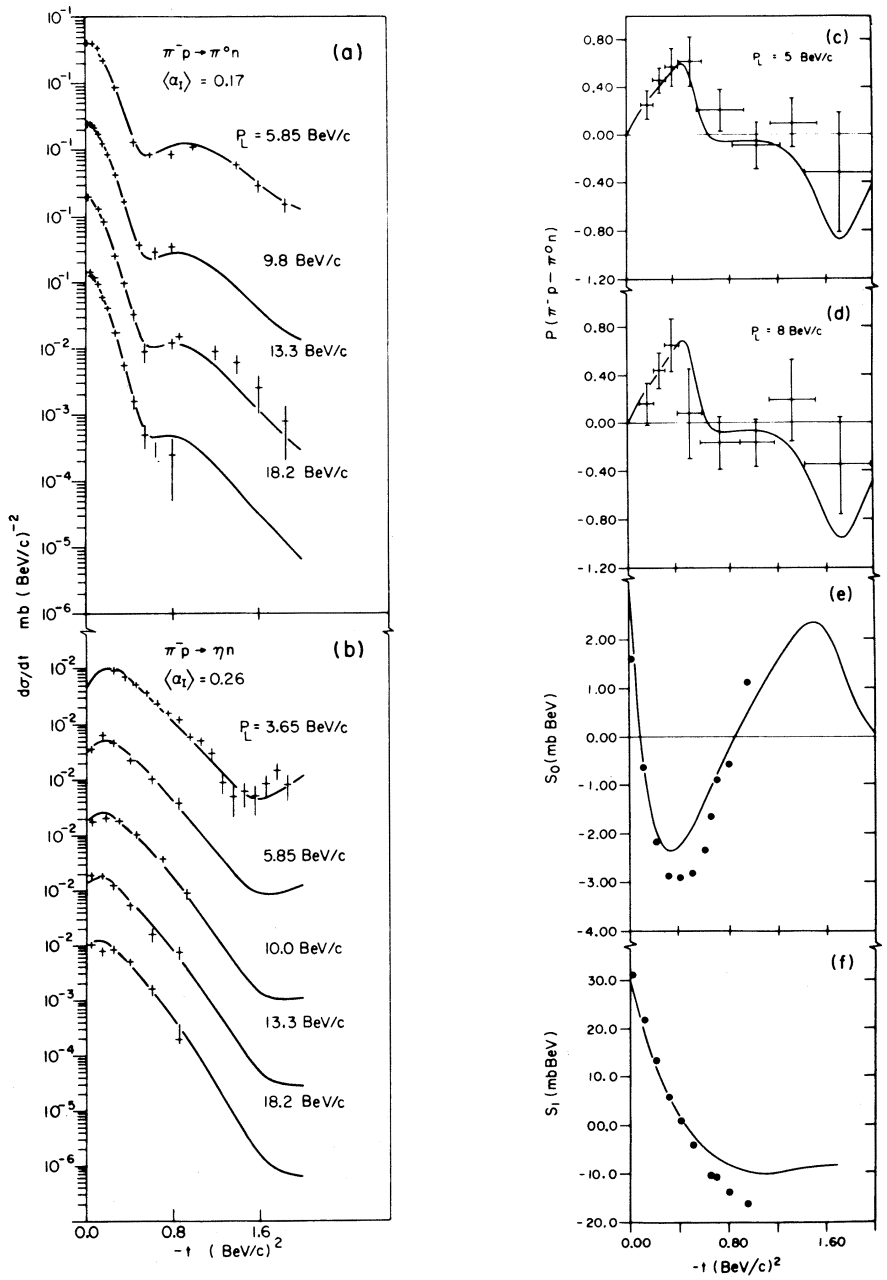

FIG. 1. The fits to the data: (a) $d\sigma/dt$ for $\pi^- p \to \pi^0 n$; (b) $d\sigma/dt$ for $\pi^- p \to \eta n$; (c) and (d) polarization P for $\pi^- p \to \pi^0 n$; (e) and (f) sum rule S_0 and S_1 respectively for $\pi^- p \to \pi^0 n$. The data are from Refs. 17–20.

with experiments. The experimental data are from Ref. 21.

The solid curves in Fig. 3 correspond to γ_R^A, γ_R^B, $\alpha_I\gamma_I^A$, and $\alpha_I\gamma_I^B$ that we obtained from our fits [see (24) and (25)]. The α_I corresponds to that of ρ. The dashed curves correspond to the Bessel-Struve functions which approximate these quantities. The reason for plotting $\alpha_I\gamma_I$ instead of γ_I is, as pointed out earlier [see (6)], the contribution of γ_I to the amplitude appears in the form $\alpha_I\gamma_I$.

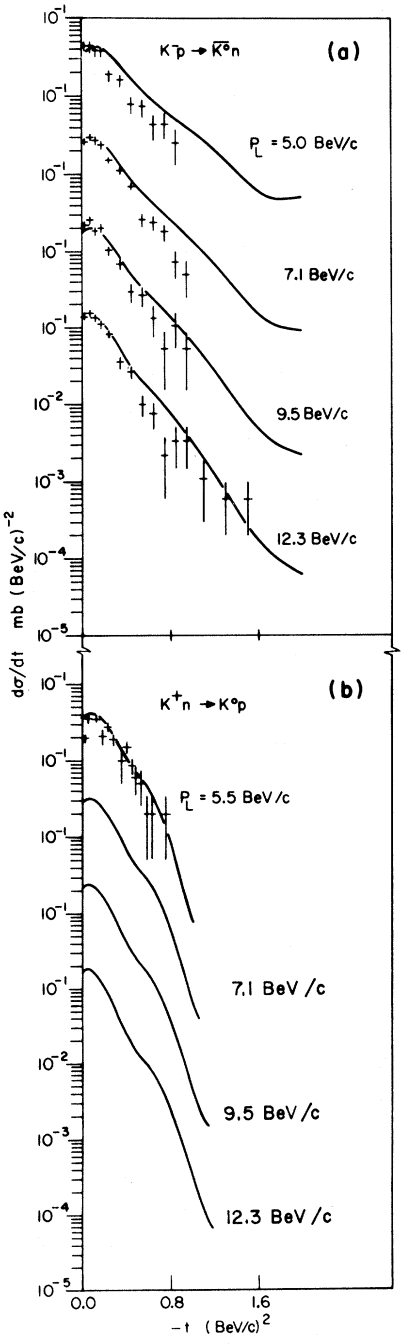

FIG. 2. The predicted $d\sigma/dt$ for (a) $K^-p \to \bar{K}^0 n$ and (b) $K^+n \to K^0p$ against the experimental data (Ref. 21).

The dashed curve in Fig. 3(a) corresponds to the J_0 function which approximates γ_R^A and is obtained by fitting the sum rule S_0. We find it to be of the form [t is in (BeV/c)^2]

$$\hat{J}_0 = 1.33\, e^{-0.7t} J_0(6.28(-t)^{1/2}) \text{ mb BeV}/c\,. \quad (26)$$

Notice that once the first zero in J_0 is known then all the subsequent zeros are determined. Therefore, instead of being coincident with the first zero of γ_R and, thereby, missing very badly the second zero, our J_0 function takes an in-between position.

The dashed curve in Fig. 3(b) corresponds to the $\mathcal{K}_0$ function which approximates γ_I^A and is given by

$$\alpha_I\hat{\mathcal{K}}_0 = -1.33\, e^{-0.7t}\alpha_I\mathcal{K}_0(4.50(-t)^{1/2}) \text{ mb BeV}/c\,. \quad (27)$$

Notice that the over-all normalization factor is the same as in (26), as it should be. It has a slightly different radius which presumably corresponds to small corrections due to the right-hand cut [see (14) and (18)]. The agreement with $\alpha_I\gamma_I^A$ is excellent.

In Fig. 3(a) we have the J_1 function which corresponds to γ_R^B,

$$\hat{J}_1 = 29.5\, e^{-0.85t}\,\frac{J_1(5.78(-t)^{1/2})}{5.78(-t)^{1/2}} \text{ mb}\,. \quad (28)$$

The agreement with γ_R^B is remarkably good.

In Fig. 3(c) we have plotted $\alpha_I\gamma_I^B$ which is extremely small. As pointed out in Sec. IV [see (16) and (18)] it was anticipated to be negligible. It is compared with $\lambda\alpha_I{}^2$ (dashed curve).

Thus, in terms of simple analytic functions our model would predict the following form for the amplitudes:

$$\begin{aligned} A' &\sim \{\hat{J}_0(R_0(-t)^{1/2}) - i\hat{\mathcal{K}}_0(R_0'(-t)^{1/2})\}\, s^{\alpha_+}\xi(\alpha_+) \\ &\quad + \{\hat{J}_0(R_0(-t)^{1/2}) + i\hat{\mathcal{K}}_0(R_0'(-t)^{1/2})\}\, s^{\alpha_-}\xi(\alpha_-)\,, \\ B &\sim \{\hat{J}_1(R_1(-t)^{1/2}) - i\lambda\alpha_I\}\, s^{\alpha_+ - 1}\xi(\alpha_+) \\ &\quad + \{\hat{J}_1(R_1(-t)^{1/2}) + i\lambda\alpha_I\}\, s^{\alpha_- - 1}\xi(\alpha_-)\,, \end{aligned} \quad (29)$$

where $R_0 = 6.28$ (BeV/c)$^{-1}$, $R_0' = 4.50$ (BeV/c)$^{-1}$, $R_1 = 5.78$ (BeV/c)$^{-1}$, and $\lambda = 0.1$. Note that, apart from the over-all normalization and trajectory functions, there are only five parameters, R_0, R_0', λ, and the two α_I's corresponding to ρ and A_2, the radius R_1 can be fixed *a priori* to be 1 F.

The dashed curves in Fig. 4 correspond to the approximation (29). The agreement is quite good. The negative sign for the residue in A' is neces-

sary to give positive polarization for $-t<0.6$ in πN CEX. The radius R_0 is somewhat larger than R_1 in order to fit the sum rule S_0. The R_0' is smaller than R_1 in order for polarization to be negative for $-t>0.6$.

We emphasize that a *qualitative* agreement with the main features of all the four reactions could be obtained in terms of only *two* parameters, i.e., the two α_I's with R_0, R_0', R_1 fixed *a priori* at 1 F and $\lambda=0$. The shrinkage and power falloff, the crossover zeros, and the dips, as well as positive polarization for low $-t$, can be understood.

In Figs. 5(a)–5(d) we have plotted the s-channel helicity amplitudes vs t (solid curves) and have compared them with the results obtained from the amplitude analysis at 6 BeV/c.[22] For the phase of the $I=0$ amplitude we have used the values obtained by Barger and Phillips.[22,23] The agreement

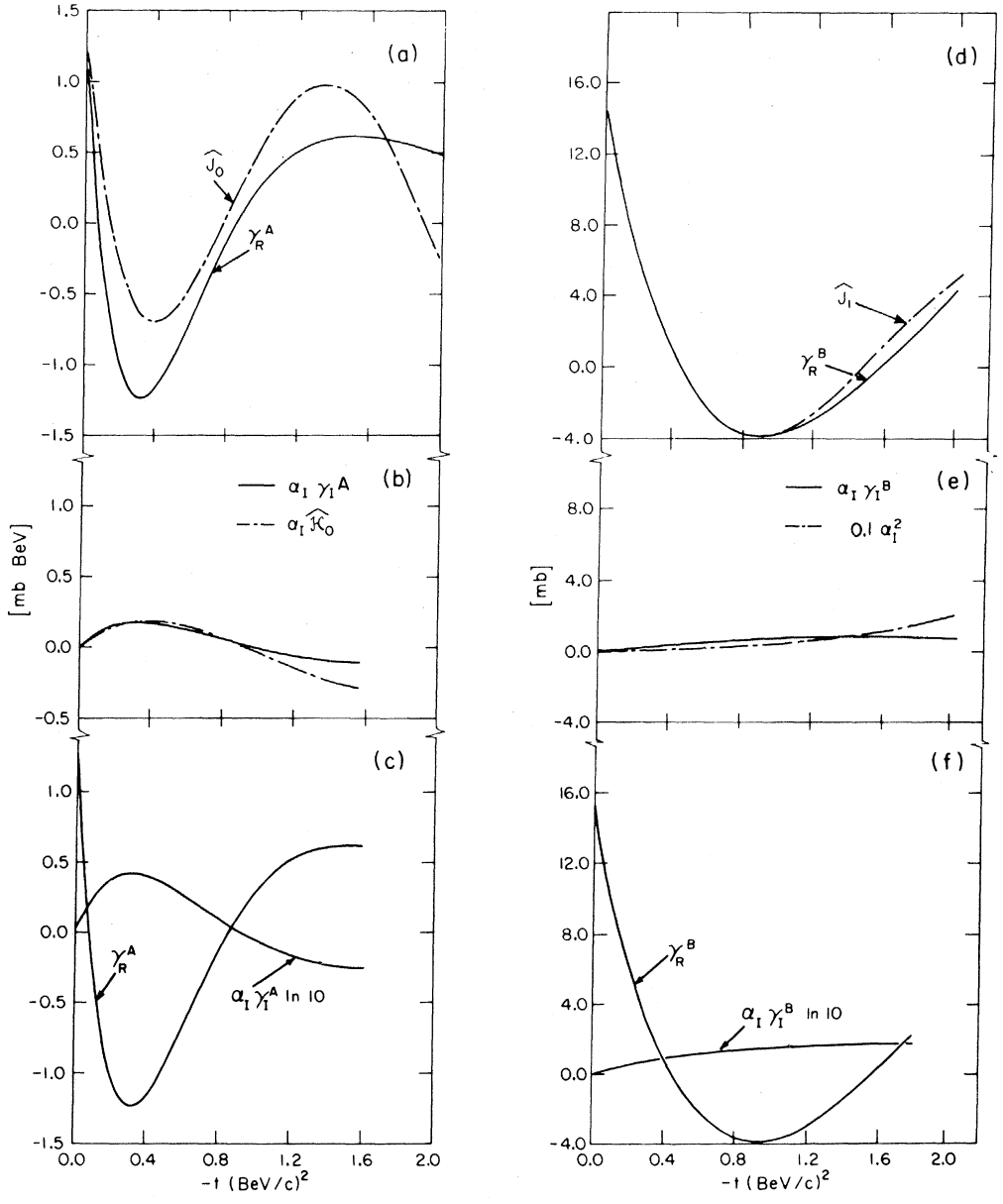

FIG. 3. Comparison of the real and imaginary part of the complex residue functions γ_R^A, γ_R^B, γ_I^A, and γ_I^B obtained from the fit (solid curves) with $\hat{J}_0$, $\hat{J}_1$, $\alpha_I\hat{\mathcal{K}}_0$, and $\lambda\alpha_I^2$, respectively (dashed curves). The quantities are defined in the text [see Eqs. (26)–(29) in the text]. Also compared are γ_R with $\alpha_I\gamma_I$ lns for a typical value of $s=10$ BeV^2. The negative signs for the imaginary parts of the residue functions are ignored.

is remarkably good. The zeros in $\mathrm{Im}F_{++}$ and $\mathrm{Im}F_{+-}$ arise naturally from the J_0- and J_1-type structure. The almost double zero in $\mathrm{Re}F_{+-}$ is because our F_{+-} is very much like a Regge pole with zeros coming from J_1 and the signature factor [see (29)]. In the SCRAM and weak-cut models (at least in their earlier versions[13]) $\mathrm{Re}F_{++}$ for ρ has a zero at a lower value of t than in $\mathrm{Im}F_{++}$. However, because of the negative phase of the γ_+ residue, the zero of $\mathrm{Re}F_{++}$ in our complex-pole

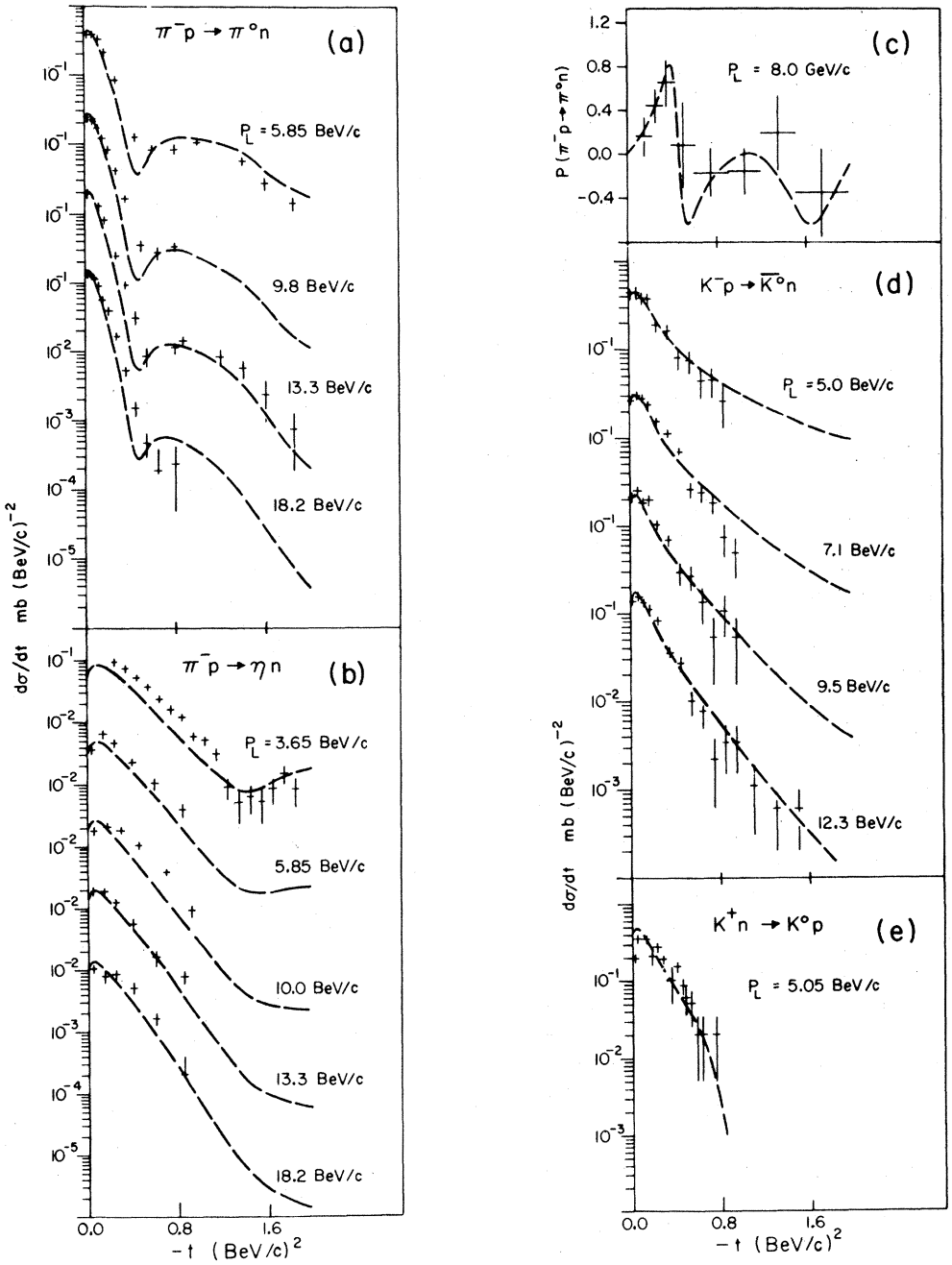

FIG. 4. A comparison of the results obtained (dashed curves) with the residues estimated in terms of Bessel-Struve functions, etc. [as explained in the text, see Eqs. (26)–(29)] with the experimental points (a) $d\sigma/dt$ for $\pi^-p \to \pi^0 n$, (b) $d\sigma/dt$ for $\pi^-p \to \eta n$, (c) polarization for $\pi^-p \to \pi^0 n$, (d) $d\sigma/dt$ for $K^-p \to \bar{K}^0 n$, and (e) $d\sigma/dt$ for $K^+n \to K^0 p$. In (b), (d), and (e) SU_3 coefficients are used for the over-all normalization [see Eq. (19)].

model is pushed out beyond that of $\mathrm{Im}F_{++}$.

VI. SOME OBSERVATIONS ON THE COMPLEX-POLE MODEL

A. $\alpha_I(t)$ near $t=0$

Throughout the paper we assumed α_I to vanish at $t=0$ and, in fact, took it to be of the form $g(-t)^{1/2}$. The point was that the singularity structure in t of $\alpha_\pm$ and $\gamma_\pm$ should be similar. Since the residues $\gamma_\pm$ are expected to be of the Bessel-Struve function type with argument $R(-t)^{1/2}$, it was logical to parametrize α_I as $g(-t)^{1/2}$. However, another similar form, e.g., $(-t)^{3/2}$, is also entirely plausible. If we do not take the Bessel-Struve function comparison seriously, then α_I need not be of the above forms. It can thus be a constant, or linear in t, etc. The present experiments do not distinguish between the different forms.

The data most sensitive to α_I (and also to γ_I) would be the shape of the polarization curve very near $t=0$. A careful measurement of this quantity near $t=0$ would throw considerable light on the behavior of α_I. It should be noted that the t-plane singularity of $\alpha_\pm(t)$ and $\gamma_\pm(t)$ is related to the j-plane singularity of the partial-wave amplitudes. A square-root-type singularity in t implies a square-root-type singularity in j, a log-type in t implies log-type singularity in the j plane, and so on.

B. $J_n(R(-t)^{1/2})$ as an "Effective" Residue

In potential scattering the residue of the leading trajectory has, generally, only the right-hand cut. From unitarity above threshold, $t>t_0$, the real part of the residue is proportional to $\mathrm{Im}\alpha$. Since $\mathrm{Im}\alpha$ is known to be a smoothly varying function of t the same will be the case for the residue. Its continuation to $t<0$ should also give a smooth function. On the other hand, nature seems to require an oscillating function with zeros along the negative t axis. The crossover zero is a case in point. In previous Regge-pole analyses such a zero was introduced in an *ad hoc* manner. In potential theory it corresponds to an "extinct" bound state which is impossible to achieve.[24] Even if one puts in an *ad hoc* zero in Regge residue, the real and imagniary part of the amplitude, within the simple Regge-pole picture, will vanish at the *same* point. This is, however, inconsistent with experiments.[22]

Clearly then "something else" is needed to provide a mechanism for oscillation and zeros in the residue for $t<0$. This could be in the form of an additional left-hand cut in the t plane. This is very plausible because from the property of the dispersion relations it is evident that if a function has only a right-hand cut then away from it (e.g., in the region $t\leq 0$) it will be a smoothly varying function.[16] Now a left-hand cut can arise from pole-pole collision or pole-cut collision. In either

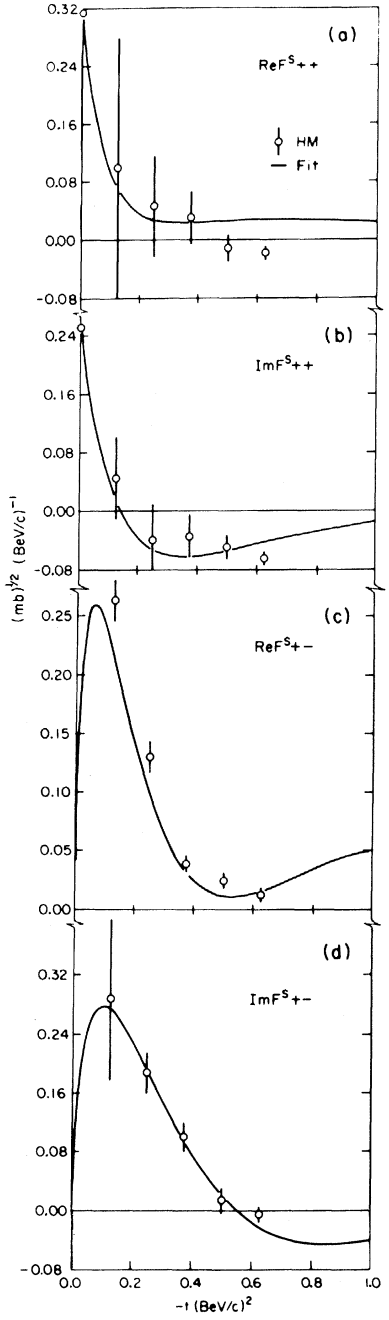

FIG. 5. Real and imaginary parts of the s-channel spin-flip and spin-nonflip amplitudes, shown against the points obtained by Halzen and Michael (Ref. 22). For the Pomeranchukon phases the values used are those given in Ref. 23.

case we have complex poles and (complex) residues with γ_I as the discontinuity of residue functions across the left-hand cut.

However, as we noted earlier the *contribution* of γ_I to the amplitude is small as long as α_I is small. Thus γ_R by itself can be considered as the "effective residue." From the dual absorption picture we have noticed that γ_R is essentially $J_n(R(-t)^{1/2})$. One of the remarkable facts is that even to the very low energies where prominent resonances exist and which are peripheral, $J_n(R(-t)^{1/2})$ continues to be the "effective" residue.

Finally, we note that even though the derivation of $J_n(R(-t)^{1/2})$ is couched in the s-channel language it can also be obtained from t-channel absorption. This is explained in Ref. 16.

C. The Negative Phase of γ_+ and the Shrinking Radius

It is pointed out in the earlier paper of Barik *et al.*[7] as well as in the present one that the phase of γ_+ must be *negative*, at least for the ρ trajectory. This is essential in order to have the positive sign for πN CEX polarization in the interval $-t = 0$–0.6. The question to ask is whether there is anything fundamental about this negative sign. It would certainly be nice if this were the case because then A_2, ω, etc., all vector and tensor trajectories will have a negative phase for the residues and there will be an enormous predictive power in our model. A negative phase for A_2 gives *positive* polarization for $\pi^- p \to \eta n$ for low $-t$. The data available at present are consistent with this but more data are clearly needed.

In Ref. 25 it is pointed out that a negative phase for the residue implies that the complex poles must be on the *unphysical* sheet. The arguments are not entirely rigorous and are based on a model, but if found true under rigorous examination then we are clearly dealing with a rather fundamental aspect of physics. The situation then becomes very similar to the Breit-Wigner poles which from causality are known to be on the unphysical sheet of the energy plane. We note that multi-Regge models also predict poles to be on the unphysical sheet.[4]

The existence of a negative phase also implies a "shrinking" radius. The point is that, within the complex-pole framework, we have [see (4)]

$$\mathrm{Im}T \sim s^{\alpha_R}\cos(\phi + \alpha_I \ln s)\,.$$

Since α_I, by definition, is positive, a negative phase, ϕ, implies that the first zero of $\mathrm{Im}T$ moves *out* in $|t|$ as s increases. This would correspond to a radius that is decreasing. Another way to put it would be to observe that the dual absorption picture can be expressed with a logarithmically dependent radius,[15] i.e.,

$$\mathrm{Im}T_n = s^{\alpha} J_n((R_0 - R_1 \ln s)(-t)^{1/2}),$$

where $R_0(-t)^{1/2}$ corresponds to ϕ and $R_1(-t)^{1/2}$ to α_I. This expression clearly shows the decrease in the radius. There is, of course, the characteristic radius $(\alpha' \ln s)^{-1}$ coming from s^{α} which remains unchanged. Now the radius involved in J_n shrinks but does so very slowly. Typically, R_0 is ~ 5 $(\mathrm{BeV}/c)^{-1}$ and, therefore, for R_1 between 0.1 and 0.5 corresponding to $\langle \alpha_I \rangle$ between 0.1 and 0.5 the radius shrinks to zero only for s between e^{50} and e^{10}. The shrinking of the radius is, therefore, too small to be of practical interest.

Conceptually, however, the shrinking implies that if the poles are on the unphysical sheet then at asymptotic energies the cut dominates for *all* t.[25] The zeros inherently associated with the complex poles by then have moved out.

D. The Radius R and α_I

One of the puzzling aspects of the complex-pole phenomenology is that whereas α_I is quite small, the residues $\gamma_\pm$ are large and rapidly varying. Another way of expressing the same thing would be to note that the radius R in the Bessel functions and α_I arise from the same source, namely, cuts in the j plane (or absorption effects), and yet R is more than an order of magnitude larger than α_I.

In Appendix B we attempt to answer this question and point out that the corrections to the residue function due to cuts are expected to be much larger than to the trajectory function. Even though we do not have any rigorous theory at the moment we anticipate that

$$R = f(\alpha_I)$$

such that as $\alpha_I \to 0$,

$$R \to 0$$

and

$$\gamma_R(t) \to \text{const}, \quad \gamma_I(t) \to 0\,.$$

We note, in passing, that as $R \to 0$,

$$\frac{J_n(R(-t)^{1/2})}{[R(-t)^{1/2}]^n} \to \text{const}\,,$$

$$\frac{\mathcal{K}_n(R(-t)^{1/2})}{[R(-t)^{1/2}]^n} \to 0\,.$$

E. Exchange Degeneracy

In our approach we tacitly assumed that exchange degeneracy was valid. That is, we assumed

$$\mathrm{Im}\,T_n \sim J_n(R(-t)^{1/2}) \qquad (30)$$

for both ρ and A_2 trajectories. Of course, α_I for ρ and A_2 are *different* and, in that sense, exchange degeneracy is broken. One might say that the presence of cuts breaks the exchange degeneracy.

As a possible model one might assume that, when cuts are "turned off," exchange degeneracy is exact and the amplitudes are described exactly by simple Regge poles. In that case we have

$$\mathrm{Im}\,A' \sim \alpha(\alpha+1)\cdots\left(\frac{\pm 1 - e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha} = \frac{1}{\Gamma(\alpha)}\left(\frac{\pm 1 - e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha},$$

$$\mathrm{Im}\,B \sim \alpha(\alpha+1)\cdots\left(\frac{\pm 1 - e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha} = \frac{1}{\Gamma(\alpha)}\left(\frac{\pm 1 - e^{-i\pi\alpha}}{\sin\pi\alpha}\right)s^{\alpha}$$

for ρ, A_2, etc. Notice that both amplitudes have the same α factors. This follows from exchange degeneracy as well as the ghost-eliminating mechanism. The factor $(\alpha+1)$, for instance, is needed to eliminate the ghost in ρ at $\alpha=-1$, and the factor α to eliminate the ghost in A_2 at $\alpha=0$. From exchange degeneracy then both ρ and A_2 have $\alpha(\alpha+1)$, etc.

When cuts are "turned on" the zero at $\alpha=0$ (i.e., at $t=-0.5$) in A' is moved in towards $t=-0.2$ (cuts are strong) whereas the same zero in B is not greatly disturbed (cuts are weak). Thus for ρ, A_2, etc. we have

$$A'\text{:}\quad \alpha(\alpha+1)\cdots \to J_0(R(-t)^{1/2}),$$

$$B\text{:}\quad \alpha(\alpha+1)\cdots \to J_1(R(-t)^{1/2}).$$

Exchange degeneracy within the A' amplitudes and B amplitudes still remains valid except for the breaking due to different α_Is. Our phenomenological approach suggests that the picture implied by (30) is roughly correct.

F. The Ghost Problem

The prescription

$$\mathrm{Im}\,T_n \sim J_n(R(-t)^{1/2})$$

cannot possibly be applied within the context of simple Regge poles. The problem is that of ghosts. For the nonflip amplitude, the function $J_0(R(-t)^{1/2})$ cannot eliminate the ghosts at negative integer values of α (odd for ρ, even for A_2), if R is taken to be the standard Fermi radius. Even the function J_1 for the flip amplitude cannot do the job. One can arrange $J_1(R(-t)^{1/2})$ to vanish at $\alpha=-1$ for ρ and at $\alpha=0$ for A_2 by choosing radii close enough to 1 F. But the ghosts next in line cannot be eliminated. This is so because we have for the distribution of zeros

$$J_1\text{:}\quad t_n \sim -n^2,$$

$$\frac{1}{\Gamma(\alpha)} \text{ or } \sin\pi\alpha\text{:}\quad t_n \sim -n.$$

Thus the zeros of J_1 are more widely spaced than those of $\sin\pi\alpha$ which are responsible for the ghosts in the amplitude.

One way of incorporating the relation $\mathrm{Im}\,T_n \sim J_n$ would be through the complex-pole formalism. In this formalism there are no infinities as α_R passes through negative integers.

Even though there are no infinities, the function $(\sin\pi\alpha_+)^{-1}$ will have the following form as $\alpha_R \to -n$:

$$\frac{1}{\sin\pi\alpha_+} \underset{\alpha_R \to -n}{\sim} \frac{1}{\pi[(\alpha_R+n) \pm i\alpha_I]}.$$

Thus it will have a peak of width $\pi\alpha_I$ as α_R passes through $-n$. The amplitudes will have additional factors multiplying the above function, namely, the residues $\gamma_\pm$, and the signature factor.

Near $\alpha_R=-1$ in the ρ trajectory, the B amplitude is dominant and its γ_R is proportional to $J_1(R(-t)^{1/2})$ (besides the exponential function) which is going rapidly through zero. Consequently there is no peak in B. The A' amplitude is extremely small compared to B and has an exponential function in the residue which is rapidly falling. Furthermore α_I at $t=-1.5$ (near $\alpha_R=-1$) is larger and the peak is "smoothed out." Thus the contribution of A' to $d\sigma/dt$ shows no peak. For A_2 the interesting region is near $\alpha_R=0$. Again B has a $J_1(R(-t)^{1/2})$ as the residue and has no peaks. The A' amplitude even though small compared to B is not negligible. However, α_I for A_2 is larger than for ρ and the peak is again "smoothed out." Thus the contribution of A' shows no peaks in $d\sigma/dt$.

We have arranged the residues so that phenomenologically at least there are no peaks in the $d\sigma/dt$ and in that sense we do not have a "problem" of the ghosts. Nevertheless, at negative integral values of α_R (odd for ρ, even for A_2) there *is* a pole in the complex j plane and $(\sin\pi\alpha_+)^{-1}$ *does* have a peak. In other words, when the cuts are absent the ghost poles are on the negative real axis and are totally eliminated by the corresponding zeros in the residues, but in the presence of cuts these poles are moved off into complex plane and are *not* totally eliminated by the residues.

The important conceptual question to ask is whether there are any basic physical arguments against the existence of these poles. We do not know of any, as long as the poles are in the complex plane. Furthermore, if the complex poles for $t \leq 0$ lie on the unphysical sheet in the j plane

for which there are good indications, then there should be even less objection against the ghost poles. We anticipate, of course, that the corresponding t-plane poles lie on the unphysical sheet of that plane.

One might wonder why one should not assume the presence of explicit ghost-eliminating factors even in the complex-pole formalism. This would be tantamount to the assumption

$$\mathrm{Im}T \sim (\alpha+1)(\alpha+3)\cdots J_n = \frac{1}{\Gamma(\frac{1}{2}(\alpha+1))} J_n(R(-t)^{1/2}) \quad \text{for } \rho$$

and

$$\mathrm{Im}T \sim \alpha(\alpha+2)\cdots J_n = \frac{1}{\Gamma(\frac{1}{2}\alpha)} J_n(R(-t)^{1/2}) \quad \text{for } A_2 .$$

The above expressions, besides looking ugly, are inconsistent with exchange degeneracy and, moreover, the significance of R as the radius of peripheral interaction is lost. Phenomenologically these expressions are *not* preferred. However, more information especially on A_2 is needed before the final word can be said in this matter.

VII. COMPARISON WITH OTHER MODELS

We compare our model with the following models:

A. Simple Regge-Pole Models

Essentially all the data available can be fitted by simple Regge poles provided we have enough of them.[23,26,27] The πN scattering data, both elastic and charge exchange, have been fitted by Barger and Phillips[23] using five poles. As a phenomenological fit it is a superb one and yet it is important to examine its theoretical significance in the light of the available information. We do know about the existence of P, P', and ρ trajectories but there is no direct evidence for their P'' and ρ' trajectories. To this we add the question we raised earlier about introduction of *ad hoc* crossover zeros in the nonflip residues. Since the publication of these fits there has been a considerable accumulation of knowledge regarding the j-plane structure. There is now increasing evidence that cuts exist and that absorption effects are important. Our complex-pole model can also be thought of as a two-pole model, ρ and ρ^* say; however, these are complex poles which take account of the cuts with residues which are capable of incorporating the absorption effects. To fit the data exactly we need many parameters but, as we pointed out earlier, the residues and trajectories can be approximated by simple analytic functions which reproduce all the main features of the reactions considered in this paper.

B. Veneziano Model

One of the characteristic properties of the Veneziano model is that, apart from the overall normalization constant, the entire amplitude is determined in terms of a single function, namely, the trajectory function. It is possible to incorporate complex poles in the model[28] but then the entire amplitude including the residues are determined in terms of the two parameters of the trajectory function, α_R and α_I. For simple Regge poles it is known that the model gives a factor $\Gamma(\alpha)^{-1}$ to both the nonflip and flip residues. In order to bring the zero in the nonflip amplitude from $t=-0.5$ (corresponding to $\alpha=0$) to $t=-0.1$, it is most likely that α_I will be found to be quite large. That is, if α_I is approximated as $g(-t)^{1/2}$ then one should expect $g\approx 1$. For the flip amplitude, on the other hand, the Veneziano representation already gives the correct first zero and therefore we should expect $g\approx 0$. Thus we are faced with two drastically different functional forms for the same trajectory. There may be ways to avoid this dilemma and this subject should certainly be pursued further.

C. Absorption Models

1. *The* SCRAM *Model*

This model along with the eikonal model[13] was responsible for bringing about a significant advance in our knowledge of high-energy phenomena involving, especially, the question of dips, crossover zeros, and the general ideas about absorption. The earlier version of the model has had difficulty in explaining the polarization in πN charge-exchange and elastic scattering as well as $d\sigma/dt$ in $\pi^- + p \to \eta + n$. Recently Ross has indicated that the absorption factor previously assumed to be purely real should have a significant phase corresponding to the nonzero real part in the $I=0$ amplitude.[29] It is not clear, at the moment, whether a complex absorption factor will solve all the difficulties facing the model.[30] However, it is obvious that the calculations in SCRAM are no longer simple. For instance, the multiplicative constant, λ, which took account of the additive contribution of diffraction-type amplitudes will now have to incorporate the phases of the respective amplitudes and in $\pi^- + p \to \eta + n$, one will have to worry about the phase of elastic $\eta + n \to \eta + n$, and so on. The Chu-Hendry model[31] is close in spirit to SCRAM but it is formulated differently. It has no specific built-in s dependence, but like SCRAM it assumes that both the real and imaginary parts of the partial-wave amplitudes are peripheral. However, indications from the

recent πN amplitude analysis at 6 BeV/c are that the real parts are not, generally, peripheral, though the imaginary parts are consistent with being peripheral.[22]

It is in the treatment of the real part of the nonflip amplitude, A', that the major difference lies between our complex-pole model and SCRAM. Whereas in SCRAM (at least in their earlier versions[13]) the ReA' for ρ has a zero in t before that of ImA', in our case the zero appears after because of the negative phase of γ_+. Consequently, in SCRAM the πN CEX polarization vanishes around $t=-0.2$, while in our case it vanishes around $t=-0.6$ in agreement with experiment. In ReB of ρ the difference between the two models is negligible because our λ parameter is negligibly small. Of course, ImA' and ImB of the two models are essentially the same.

2. *The Dual Absorption Model*

In this model, proposed by Harari,[14] it is assumed that only the imaginary part of the amplitude is peripheral. This picture was the basis of our present formulation of the complex-pole model. As emphasized earlier, complex poles provide the most logical framework for the purpose of incorporating the dual absorption idea. This is so because crossing symmetry in the complex-pole model brings in the signature factor and consequently both the real and imaginary parts of the amplitude are expressed simply. From the dual absorption model we estimate γ_R and then from dispersion relations, etc. (see Secs. III C and IV) we estimate γ_I and hence the real and imaginary parts of the amplitude [see (29)]. To this we add that we avoid the infinities due to "ghosts" on the negative real j axis (see Sec. VI F). Of course, we have an additional quantity α_I which is different for different trajectories.

VIII. CONCLUSION

Our complex-Regge-pole approximation in the j plane is very much akin to the Breit-Wigner approximation in the energy plane. Given the fact that there are cuts this is the most logical and economical way to express the partial-wave amplitudes. As long as energies are not extremely high, the total amplitude should also be well approximated by these poles.

Unlike the Breit-Wigner case where unitarity relates the residues directly to the imaginary part of the pole positions, in the complex Regge-pole case we are in the region $t \leq 0$ and no simple relation exists. The residues are, furthermore, complex. Here, however, the absorption model plays a crucial role.[13,14] The dual absorptive model[14] gives the correct estimate of the real part of the residue function, whereas from dispersion relations for the residues we obtain the correct estimate of the imaginary part.

Specifically, from the dual absorption model we find that the real part of the residue function, γ_R, should be proportional to the Bessel functions $J_n(R(-t)^{1/2})$ with $R \sim 1$ F. The magnitude of the imaginary part, γ_I, depends on the amplitude in question. For the spin-nonflip amplitude where cuts are large (in order to produce the crossover zeros) the left-hand cut in the residue function should be the dominant one, in which case γ_I is proportional to the Struve function $-\mathcal{H}_0(R(-t)^{1/2})$. The spin-flip amplitude given by the simple Regge poles is already consistent with absorption and, therefore, for complex poles the left-hand cut should be negligible. In either case the phase of the residue must be negative for small $-t$. The contribution of γ_I to the amplitude appears in the combination $\alpha_I\gamma_I$, which is small, and therefore its precise value is not crucial.

We have fitted the available data on $\pi^-p \to \pi^0 n$, $\pi^-p \to \eta n$, $K^-p \to \overline{K}^0 n$, and $K^+n \to K^0 p$. An exact fit requires many parameters. However, to a good approximation, we were able to express the trajectory and residue functions in terms of simple analytic functions with very few parameters. These functions being obtained on the basis of our present theoretical knowledge. The criterion for a successful model is not that it gives exact fits to a particular reaction, but rather that it describes, even if qualitatively, the main features of several different reactions. The main features that we have encountered are: shrinkage and power falloff in $d\sigma/dt$; dips and crossover zeros, and the signs of polarizations. In the four reactions mentioned above we have been able to explain the main features rather simply and to that extent our complex-pole model has certainly been successful. The project we would like next to embark upon is to analyze processes with K^*-K^{**} exchange, ω and (if possible) P' exchanges, as well as those with ρ and A_2 exchanges which we have not considered here.

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APPENDIX A: UNITARITY IN THE t CHANNEL, COMPLEX POLES, AND THE BREIT-WIGNER FORM

Let us give a simple argument in favor of complex Regge poles for $t \leq 0$ based on t-channel uni-

tarity. We first note that the S matrix which satisfies two-body elastic unitarity in the t channel can be written in the form

$$S(j,k)=\frac{D(j,-k)}{D(j,k)}\,, \tag{A1}$$

where $t=4(k^2+m^2)$. Because of the possibility of writing S in the above form it is clear that if S has a cut in the j plane then so must the D's. The two D's above differ only in the sign of k in the argument and it is difficult to imagine one of the D's having a cut and not the other.

The fact that $D(j,k)$ has a cut in j is important. We assume the cut is along the negative real axis between $-\infty$ and α_c. We consider now $t\leq 0$. For this t region $D(j,k)$ is *complex* for $j\leq\alpha_c$ and, therefore, it must have complex zeros. However, for the same t region D is *real* for $j\geq 1$. Therefore, from the Schwarz reflection principle the zeros for $j\leq\alpha_c$ must be complex conjugate.

The lowest elastic threshold in the t channel is at $t=4m_\pi{}^2$ which is quite close to $t\approx 0$ where the high-energy experiments are analyzed in terms of j-plane singularities. Thus complex Regge poles are quite relevant for high-energy reactions.

It is most instructive to write the D function and the amplitude in a Breit-Wigner-type (or effective-range-type) form:

$$\begin{aligned}D(j,k)&=j-\alpha_0(t)+\epsilon(t)(j-\alpha_c)^{1/2}\\&=j-\alpha_0(t)+i\epsilon(\alpha_c-j)^{1/2}\,,\end{aligned} \tag{A2}$$

where, for simplicity, we have assumed a square-root cut. This is an expansion around $\alpha_0(t)$ which we might think of as an "unperturbed" pole. It is similar to the expression in the *energy* plane or the t plane where, for *integral* j, one writes for $t>4m^2$

$$D(j,k)=t-m_0{}^2+i\tfrac{1}{2}\Gamma(t-4m^2)^{1/2}\,, \tag{A3}$$

where m_0 is the resonance position and Γ the width. In the absence of the energy-plane (t-plane) cuts ($\Gamma=0$), we have an "unperturbed" pole $m_0{}^2$, but in the presence of cuts ($\Gamma\neq 0$) we have complex poles, i.e., the Breit-Wigner poles.

Let us write the partial-wave amplitude $A(t,j)$,

$$\begin{aligned}A(t,j)&=\left(\frac{m}{k}\right)\frac{S(j,k)-1}{2i}\\&=\left(\frac{m}{k}\right)\frac{[D(j,-k)-D(j,k)]/2i}{D(j,k)}\\&=\left(\frac{m}{k}\right)\frac{\Delta D(j,k)}{D(j,k)}\,,\end{aligned}$$

where $\Delta D(j,k)$ is the discontinuity of D across the t-plane cut. From (A2) we have

$$A(t,j)=\left(\frac{m}{k}\right)\frac{\Delta\alpha_0-\Delta\epsilon(j-\alpha_c)^{1/2}}{j-\alpha_0+\epsilon(j-\alpha_c)^{1/2}}\,,$$

where $\Delta\alpha_0$ and $\Delta\epsilon$ are the discontinuities across the t-plane cut. Defining $m\,\Delta\alpha_0/k=\beta_0$ and $m\,\Delta\epsilon/k=-\epsilon'$, we get

$$A(t,j)=\frac{\beta_0+\epsilon'(j-\alpha_c)^{1/2}}{j-\alpha_0+\epsilon(j-\alpha_c)^{1/2}}\,. \tag{A4}$$

The important point to note is that the numerator *also* has a cut term. This should be contrasted with the form in the energy plane (i.e., the t plane) for integral $j\geq 1$,

$$A(t,j)=\frac{m\Gamma}{m_0{}^2-t-i\frac{1}{2}\Gamma(t-4m^2)^{1/2}}\,, \tag{A5}$$

where the numerator does *not* have a cut term. In fact, one can rigorously show that the numerator cannot have the cut term.

From (A4) we notice that the presence of the $(j-\alpha_c)^{1/2}$ term makes the poles of A complex for $j\leq\alpha_c$ with the imaginary part proportional to ϵ. What is important to notice is that the numerator is also complex. Consequently, the poles will have complex phases. Unlike (A5) where the numerator is simply related to the imaginary part of the denominator, there is no such simple relation in (A14).

Finally, we note that the zeros of (A3), the Breit-Wigner zeros, are known to be on the unphysical sheet. This follows from the fact that $\Gamma>0$. In (A2) the sign of ϵ is unknown but if $\epsilon>0$ ($\epsilon<0$) then the poles are in the unphysical (physical) sheet – the proof being the same as for (A3).

APPENDIX B: DEPENDENCE OF γ_R AND γ_I ON α_I FROM ABSORPTIVE MODEL

The usual absorption model predicts a logarithmic cut in the j plane. To simplify comparison with our earlier discussions we will, however, assume the cut to be of the square-root type (a more detailed treatment is given in Ref. 5). The statement of the absorption model is that there is a "Born" term given by the simple Regge poles with residues that do not contain any zeros and a "correction" term due to the cut. The cut term appears with a *negative* sign and has a slower fall-off compared to the pole term so that at some t there will be total cancellation and the amplitude will have a zero. Thus we write

$$A_0(t,j)=\frac{\beta_0}{j-\alpha_0}+\beta_c(j-\alpha_c)^{1/2} \tag{B1}$$

and

$$A_0(s,t)=\beta_0 s^{\alpha_0}-\frac{\beta_c}{2\sqrt{\pi}}\,\frac{s^{\alpha_c}}{(\ln s)^{3/2}}\,. \tag{B2}$$

Since complex poles arise from unitarization, we will convert the above expression into an expression which is consistent with unitarity. Let us use the K-matrix procedure, where the function

$$\frac{A_0}{1-i\rho A_0} \tag{B3}$$

is consistent with unitarity. The quantity ρ is $[(t-4m^2)/4m^2]^{1/2}$ which is $\approx i$ for small t. The unitarized amplitude, we call A, will then be given by

$$A(t,j)=\frac{\beta_0+\beta_c(j-\alpha_0)(j-\alpha_c)^{1/2}}{j-\alpha_0+\beta_0+\beta_c(j-\alpha_0)(j-\alpha_c)^{1/2}}\,. \tag{B4}$$

Since we are interested in poles of $A(t,j)$, we will use an iteration procedure and replace the factor $(j-\alpha_0)$ multiplying $(j-\alpha_c)^{1/2}$ by $-\beta_0$. This way we also facilitate comparison with (A4). We then have

$$A(t,j)\approx\frac{\beta_0-\beta_0\beta_c(j-\alpha_c)^{1/2}}{j-\alpha_0+\beta_0\beta_c(j-\alpha_c)^{1/2}}\,. \tag{B4'}$$

Now let us discuss how large β_0 and β_c should be in order to be consistent with the absorption picture. According to the absorption model there must be a zero at $t\approx -0.1$ in the amplitude given by (B2). For the typical value $s=10$ we find that

$$\beta_c\approx 10\,\beta_0\,. \tag{B5}$$

We have made the usual assumption that the slope of α_c is half that of α_0.

The quantity β_0 as we noted in Appendix A is simply $\Delta\alpha_0$, which in turn is $\simeq \mathrm{Im}\alpha$ along the *right-hand* cut in the t plane. Typically, for the $\pi\pi$ case for the ρ exchange, for instance, it is $\approx(m_\rho\Gamma_\rho)\alpha'$ ≈ 0.1. Thus

$$\beta_0\approx 0.1$$

and, therefore,

$$\beta_c\approx 1.0\,. \tag{B6}$$

Thus we find that for the j plane the imaginary part of α for $t\leq 0$ (i.e., α_I) is then given by

$$\langle\alpha_I\rangle\approx\beta_0\beta_c\approx 0.1\,. \tag{B7}$$

Hence $\langle\alpha_I\rangle$ is *small*. However, the correction term in the numerator of (B4), $\beta_0\beta_c$, is of the *same order* as β_0 itself.

In other words, even though in absolute magnitude the correction term in the numerator and the denominator of (B4) is small and of the same order, in the numerator the quantity being corrected, namely, β_0, is itself small. Thus it is entirely plausible to have $\langle\alpha_I\rangle$ small and yet have the residues and their phases quite large.

In order to be most general one should write instead of (A4) the following:

$$A(t,j)=\frac{\beta_0+\epsilon'(j-\alpha_c)^{1/2}+\epsilon''(j-\alpha_c)+\cdots}{j-\alpha_0+\epsilon(j-\alpha_c)^{1/2}}\,,$$

where the higher-order terms in the numerator are expected to be important just as the first-order term was. We write it compactly as follows:

$$A(j,t)=\frac{\beta_1(j,t)+(j-\alpha_c)^{1/2}\beta_2(j,t)}{j-\alpha_0+\epsilon(j-\alpha_c)^{1/2}}\,, \tag{B8}$$

where β_1 and β_2 do not have cuts in j. One can now take the Mellin transform of (B8) and then the resulting complex poles will have a real part of the residue, $\gamma_R\sim\beta_1$, and an imaginary part, $\gamma_I\sim\beta_2$. Each of these quantities will have considerable structure in t. A more detailed discussion of this will be given in Ref. 5.

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(6, 6*) ⊕ (6*, 6) Representation of SU(3) ⊗ SU(3) and the Breaking of Chiral Symmetry*†

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The (6,6*) ⊕ (6*,6) representation of SU(3) ⊗ SU(3) is presented and its use in breaking chiral symmetry is discussed in terms of its contribution to meson masses, pion-pion scattering lengths, baryon masses, and the nucleon σ term. We include singlet, octet, and 27-plet SU(3) pieces in the symmetry-breaking Hamiltonian, and also discuss the possible SU(2) ⊗ SU(2) classifications of the Hamiltonian.

I. INTRODUCTION

Recent experimental evidence on the s-wave pion-pion scattering lengths[1,2] seems to indicate the need for a chiral-symmetry-breaking Hamiltonian which transforms in a way other than (3, 3*) ⊕ (3*, 3). In order to produce a large isospin-zero s-wave scattering length, the original Weinberg analysis[3] must be modified to include isospin-two contributions to the σ commutator. This in turn requires the symmetry-breaking Hamiltonian to contain pieces which belong to an SU(3) ⊗ SU(3) representation which has isospin-two components in its reduction to SU(3) and hence to SU(2). It is also possible that a large value of the nucleon σ term would require these other terms, but this conclusion is not definitely confirmed. Indirectly, a recent analysis of the hard-pion Ward identity approach to the pion-pion scattering problem[4] which enforces unitarity within certain smoothness approximations also requires isospin-two σ terms for the optimal solution. This result is, however, also rather uncertain because of the many assumptions involved.

Assuming that such additional pieces are necessary in the Hamiltonian, it is natural to investigate