U(3) × U(3) breaking (Ref. 15), will be discussed elsewhere.

[15]F. Strocchi and R. Vergara Caffarelli, Phys. Letters 35B, 595 (1971).

[16]The advantage of using $A(\lambda)$ instead of $W(\lambda)$ is that $A(\lambda)$ will likely have a well-defined limit as $\epsilon_i(x) \to \epsilon_i$ = constant, whereas $W(\lambda)$ does not (Ref. 12). Moreover, the connections with the semiclassical approximation will be more apparent in terms of $A(\lambda)$ rather than $W(\lambda)$, as we will see below.

[17]In particular, the group-theoretical properties of $H(x)$ govern the divergences of the local currents and are connected with the PCAC equations.

[18]G. Cicogna, F. Strocchi, and R. Vergara Caffarelli, Phys. Rev. Letters 22, 497 (1969); Phys. Rev. D 1, 1197 (1970).

[19]The functional method proves to be very useful also in discussing the analyticity properties of the vacuum expectation values with respect to the breaking parameters ϵ_i. This will be discussed in a subsequent paper.

[20]R. Dashen, Phys. Rev. 183, 1245 (1969); L.-F. Li and H. Pagels, Phys. Rev. Letters 26, 1204 (1971); 27, 1089 (1971).

[21]Similar values have been obtained by Parisi and Testa, Ref. 12.

[22]These relations could be obtained also from the second-order equations (37)–(41). From a practical point of view, however, the present method automatically gives these formulas, whereas extracting them by direct elimination of ϵ and λ from Eqs. (37)–(41) can be a somewhat tedious task. This remark will appear even more relevant in the case $\epsilon_3 \neq 0$ where the second-order Ward identities are complicated by the occurrence of three mixing angles.

[23]S. L. Glashow, in *Hadrons and Their Interactions*, edited by A. Zichichi (Academic, New York, 1968), p. 83.

[24]N. N. Khuri, Phys. Rev. Letters 16, 75 (1966); 16, 601(E) (1966).

[25]Alternatively, one may use the following formula:

$$\tan\theta = -2\sqrt{2}\left(1-\frac{F_K}{F_\pi}\frac{\eta'-K}{\eta'-\pi}\right)\Big/\left(1-4\frac{F_K}{F_\pi}\frac{\eta'-K}{\eta'-\pi}\right),$$

which can be derived without making any approximation from Eqs. (59) and (60).

[26]This equation follows easily from Eq. (59) by putting $\lambda_8 = 0$, and is the standard GMO formula including mixing.

[27]Putting $m_{\eta'} = 1422$ MeV would lead to an angle of the order $-5°$. A further possibility, actually, cannot be excluded, viz., the occurrence of a mixing between X^0 and E. Such a mixing would in fact be consistent with our Eqs. (59) and (60).

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Quarks, Sum Rules, and Low-Energy Parameters in πN Scattering

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Finite-energy sum rules and current-algebra sum rules are shown to work at the quark level. Making use of these rules, and of a factorization assumption for the basic meson-quark amplitudes above threshold, some well-known SU(6) results are derived. Low-energy parameters in πQ and πN scattering are also evaluated. Using our model for the πN $a_{1\pm}^{(-)}$ p-wave scattering lengths, an inconsistency is found between the usual PCAC (partially conserved axial-vector current) or ρ-exchange-model treatments and dispersion relations. It can be removed if double counting of resonances and ρ-exchange terms is avoided. This provides good agreement with experiment.

I. INTRODUCTION

The quark model is usually seen as an easy way of applying unitary symmetries to hadronic interactions. The determination of coupling constants and widths of resonances at low energies and relations between cross sections at high energies are classic examples in which the quark model and the SU(6) symmetry scheme give the same results. However the quark model is not identical to SU(6), and the physics of hadrons is, on the other hand, much more complex than SU(6) or any other simple symmetry scheme. In this paper we exploit possible non-SU(6) [or $SU(6)_W$] aspects of the quark model.

Inverting the normal procedure, we start by going from hadrons to quarks rather than the other way round. The reason is that if we want more than SU(6), we have to use experimental information where it exists and where it has motivated and justified a large variety of theoretical approaches. That means that we have to incorporate the knowledge gained in hadron physics. We thus apply to quarks the well-established theoretical treatments of dispersion relations and current algebra, and successful concepts of duality and exchange degeneracy. This is the content of the next three sections. In Sec. II quark-level finite-energy sum rules (FESR) (i.e., for pseudoscalar-meson–quark scattering) are discussed. We show that duality, and in particular the connection between s-channel exotic states and t-channel exchange degeneracy, still holds. Low-energy and high-energy additivity are then unified in the additivity of quark-level duality diagrams to generate hadron-level ones. In Sec. III we consider current-algebra sum rules, and using a factorization assumption for the imaginary part of the amplitude over the entire physical region, we regain various SU(6) relations confirming the idea that the world of quarks is an SU(6) world. Section IV contains the treatment of the low-energy parameters in πQ scattering.

The advantage of working at the quark level before returning to hadrons is a gain in simplicity. The factorization assumption is justified by the absence of important resonances at the quark level and also by kinematic factors. This clarifies a contradiction between the means by which current algebra and the ρ-exchange model are applied to the determination of low-energy parameters, in contrast to dispersion-relation techniques. For current algebra, or in the ρ-exchange model, factorization is always applied to the real part of the amplitude, while in dispersion relations the only sensible factorization is of the imaginary part of the amplitude, and then only in the region beyond the strong resonance bumps. Current algebra or the ρ-exchange model overcounts the simultaneous contributions of direct- and exchange-channel terms. The overcounting is evident in the determination of the $a_{1^-}^{(-)}$ p-wave πN scattering amplitude, where direct-channel and exchange-channel contributions are both important. When only one of the terms is appreciable, for example in the $a_0^{(-)}$ s-wave scattering length, current algebra or ρ-exchange is adequate. We conclude, in agreement with the ideas of duality, that at a given energy we must include either resonances or t-channel exchanges but *not both*. These questions are discussed in Sec. IV and, as applied to πN scattering, in Sec. V. With a very crude model which handles correctly the s- and t-channel contributions we overcome the difficulties of the usual treatments and obtain good agreement with experiments for the $a_{1^-}^{(-)}$ πN p-wave scattering length.

Finally, in Sec. VI we conclude that it is possible to develop a quark model with quarks in the (s, t, u) plane as well as in the (IYB) plane.

II. FINITE-ENERGY SUM RULES AT QUARK LEVEL

Quark-model additivity[1] or exchange models with universality and factorization[2] give in general good predictions for high-energy scattering of hadrons. In the quark model the cross sections are determined by counting basic quark-quark (QQ or $Q\overline{Q}$) contributions. There are two types of such contributions: (1) those that are indifferent to the nature of the participant quarks and give an overall term proportional to the pairs of quark lines present in the process, and (2) those that involve $Q\overline{Q}$ annihilations. In the language of Regge poles, they correspond respectively to Pomeranchukon and particle exchanges.

We leave out the Pomeranchukon type of contributions and consider additivity in forward meson-baryon (MB) scattering. For πN and KN scattering one thus obtains the relations

$$f_{Nf_0} = f_{N\omega}, \quad (1)$$

$$f_{NA_2} = f_{N\rho}, \quad (2)$$

$$f_{N\omega} = 3f_{N\rho}, \quad (3)$$

where $f_{NV(T)}$ represents the MB amplitude corresponding to t-channel exchanges with the quantum numbers of the vector (tensor) meson V (T). Equations (1) and (2) are the usual relations for exchange degeneracy of opposite-parity vector and tensor mesons. Equation (3), a standard universality or SU(6) or quark model result, shows that exchange degeneracy does not occur for vector (or tensor) mesons of opposite G parity.

Now we look at additivity from a slightly modified point of view. Only the baryons are seen as made up of quarks ($Q \equiv \mathfrak{N}, \mathcal{P}, \lambda$) and we count basic MQ scattering amplitudes. Without Pomeranchukon terms one derives relations like

$$3(K^-\mathcal{P}) = \tfrac{1}{2}(f_{Nf_0} + f_{N\omega}) + \tfrac{3}{2}(f_{N\rho} + f_{NA_2}), \quad (4)$$

$$3(K^+\mathcal{P}) = \tfrac{1}{2}(f_{Nf_0} - f_{N\omega}) + \tfrac{3}{2}(-f_{N\rho} + f_{NA_2}), \quad (5)$$

$$3(K^+\mathfrak{N}) = \tfrac{1}{2}(f_{Nf_0} - f_{N\omega}) + \tfrac{3}{2}(f_{N\rho} - f_{NA_2}), \quad (6)$$

$$3(K^-\mathfrak{N}) = \tfrac{1}{2}(f_{Nf_0} + f_{N\omega}) - \tfrac{3}{2}(f_{N\rho} + f_{NA_2}), \quad (7)$$

$$3(\pi^+\mathcal{P}) = f_{Nf_0} - 3f_{N\rho}, \quad (8)$$

$$3(\pi^-\mathcal{P}) = f_{Nf_0} + 3f_{N\rho}, \quad (9)$$

where the left-hand sides refer to MQ amplitudes.

In relations (4)–(9), interpreted as finite-energy sum rules[3] (FESR's) with a resonance saturation approximation, it is easy to verify that the concept of duality[4] also works at the level of MQ scattering. We shall demonstrate this using the above examples.

The fictitious process $MQ \to MQ$ from the point of view of SU(3) has the decomposition

$$8 \otimes 3 = 3 \oplus 15 \oplus \bar{6}$$

and since we consider interactions with independent quarks, we treat the one-quark representation as nonexotic. The left-hand sides of sum rules (4) and (9) allow s-channel resonances; the right-hand sides also show a nonvanishing contribution. The left-hand sides of relations (5) and (6) are exotic: $(K^+\mathcal{P})$ and $(K^+\mathcal{N})$ have $S=+1$. The right-hand sides show vanishing contributions from the exchange-degeneracy relations (1) and (2). Note that the isospin-equivalent processes at the nucleon level, (K^+p) and (K^+n), are also exotic, and exchange degeneracy also makes the right-hand side of the corresponding sum rule vanish. The left-hand sides of Eqs. (7) and (8) are exotic: $(K^-\mathcal{N})$ has $I=1$ and $(\pi^+\mathcal{P})$ has $I=\frac{3}{2}$. The right-hand sides also vanish because of exchange degeneracy *and* relation (3). At the nucleon level, for (K^-n) and (π^+p) processes, the s channel is not exotic (Σ^- pole in one case, Δ resonances in the other), and the exchange contributions are also nonvanishing. The extra exchange degeneracy that occurs at quark level because of the equality of the quark couplings to the isoscalar and isovector particles is reflected in the absence of resonances in the $I=1$ and $I=\frac{3}{2}$ channels.

If we think in terms of s, t duality diagrams,[5] those that refer to forward scattering and can be related to additivity, can be drawn only for the nonvanishing sum rules (4) and (9) [Figs. 1(a) and 1(b)]. The resonant part of the forward MB amplitude can be obtained by summing duality MQ graphs in the presence of two quark lines (Fig. 2). Nonplanar diagrams, such as those for $\pi^-p \to K^+\Sigma^-$ and $\pi^-\Sigma^+ \to \pi^+\Sigma^-$, are in this way automatically excluded.

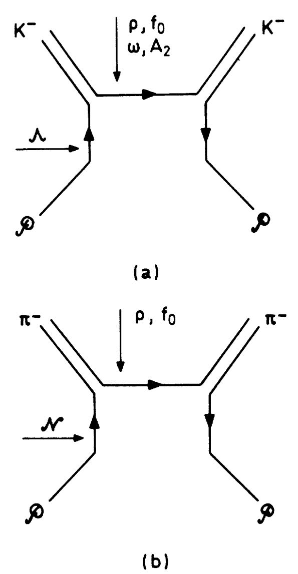

FIG. 1. Basic meson-quark (s, t) duality diagrams: (a) $K^-\mathcal{P} \to K^-\mathcal{P}$; (b) $\pi^-\mathcal{P} \to \pi^-\mathcal{P}$.

III. CURRENT-ALGEBRA SUM RULES AT THE QUARK LEVEL

Usually quark-model calculations can be interpreted as a means of applying SU(6) to the interactions of hadrons, and refined models with more sophisticated dynamical assumptions as a means of breaking that symmetry. However the classification of states into exotic and nonexotic is specific to the quark model and goes beyond SU(6), broken or unbroken. Further, that classification is related, via duality, to FESR's. The fact that duality and FESR's work in MQ scattering encourages us to go further and consider other types of sum rules, in particular current-algebra ones. As has been pointed out,[6] sum rules are the place where SU(6) can be used in the driving term in dynamical equations. These involve not only pole terms, where SU(6) acts, but dispersion integrals which are normally saturated by experimental resonances and high-energy fits. We will use the fact that some intermediate states ($I=1$, $I=\frac{3}{2}$) are absent in MQ scattering to make estimates of the dispersion integrals involved in the sum rules.

We need the basic MQ couplings which come from additivity[7] and which reproduce SU(6) relations at the baryon level. They are

$$f_Q = \tfrac{3}{5} f, \tag{10}$$

where f_Q (f) is the pseudovector πQ (πN) coupling constant;

$$f_Q^{\,2} = \frac{g_Q^{\,2}}{4\pi}\left(\frac{m_\pi}{2M}\right)^2, \quad f^2 = \frac{g^2}{4\pi}\left(\frac{m_\pi}{2m_N}\right)^2 = 0.081,$$

where g_Q (g) is the πQ (πN) pseudoscalar coupling

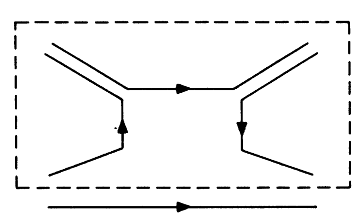

FIG. 2. Meson-baryon (s, t) duality diagram from additivity and meson-quark duality diagrams.

constant and M (m_N) is the quark (nucleon) mass; the vector-meson-dominance (VMD) relation

$$f_{VQ} = \tfrac{1}{2} f_\rho , \qquad (11)$$

where f_{VQ} is the QV electric coupling and f_ρ the $\gamma\rho$ coupling ($f_{\rho/4\pi}{}^2 \simeq 2$); the PCAC type of relation

$$\frac{g_{A_Q}}{f_Q} = \frac{g_A}{f} = (8\pi)^{1/2} \frac{F_\pi}{m_\pi} , \qquad (12)$$

where g_{A_Q} (g_A) is the axial-vector weak coupling of quarks (baryons) and F_π is the pion annihilation parameter. Another relation we need involves the anomalous magnetic moment and depends on assumptions about the mass of the interacting quarks. We can either use

$$M(\text{effective mass}) \simeq \tfrac{1}{3} M_B \simeq \tfrac{1}{2} M_M \text{ and } K^V = K^S = 0$$

or

$$M \gtrsim 5 \text{ GeV}$$

and in the ideal-mixing limit $\phi = \lambda\bar{\lambda}$ with $m_\rho \simeq m_\omega$,

$$K^V = -3K^S = \frac{1}{m_\rho} , \qquad (13)$$

where K^V (K^S) is the isovector (isoscalar) anomalous magnetic moment. The second hypothesis is the one consistent with the sum rules we consider next.

To generate current-algebra sum rules there exists a general procedure[8]: Sandwich the commutator of two operators (local or integrated operators) between two states, insert intermediate states, and separate Born terms (i.e., one-particle intermediate states) from physical-region contributions. In general, the sum rules have the structure

$$(\text{isospin factor}) = (\text{Born term}) + (\text{continuum}),$$

and the term (continuum) is an integral containing the imaginary part of an amplitude. This term can also be considered as the dispersion integral for the amplitude evaluated at $\nu = (s-u)/4M = \nu_{\text{Born}} = 0$.

In most of the sum rules at the nucleon level, the dispersion integral is saturated by low-energy resonances, and usually a fairly good approximation (70–100% of saturation of the sum rule) is achieved by simply inserting the first Δ resonance. When the sum rules are transposed to the quark level the $I = \frac{3}{2}$ resonances are exotic and we must necessarily abandon the philosophy of resonance saturation. The choice is to adopt some sort of exchange model dominating over all the continuum region. More specifically, apart from quark-model additivity relations, we will make use of high-energy experimental fits extrapolated down to the threshold, and thus relate different processes via factorization of the residues.

It is to be remarked that this extreme approximation on dispersion integrals is made on amplitudes whose Born term vanishes because $\nu_B = 0$ and thus corresponds to saying that at quark level and $\nu_B = 0$, FESR's are identically satisfied with any value for the cutoff.

As an example we consider now the Adler-Weisberger relation.[9,10] For simplicity and because we are not aiming at exact calculations, we neglect the pion mass inside the integrals.

We write the Adler-Weisberger sum rule in the standard form given above:

$$1 = g_{A_Q}{}^2 + F_\pi{}^2 G(0) \qquad (14)$$

with

$$G(0) = \frac{1}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} (\sigma_- - \sigma_+), \qquad (15)$$

where $F_\pi = 0.85 m_\pi$ is the annihilation parameter as given by the Goldberger-Treiman relation and $(\sigma_- - \sigma_+)$ is the difference of the $(\pi^- \mathcal{P})$, $(\pi^+ \mathcal{P})$ cross sections. Quark-model high-energy additivity tells us that [see (8) and (9)]

$$\sigma(\pi^- \mathcal{P}) - \sigma(\pi^+ \mathcal{P}) = \sigma(\pi^- p) - \sigma(\pi^+ p) \qquad (16)$$

and we evaluate (15), using an experimental fit of πN scattering[11] (units of m_π):

$$(\sigma_- - \sigma_+) = 1.70 \nu^{-0.7} . \qquad (17)$$

One then obtains, from (14), $g_{A_Q} \simeq 0.67$, to be compared to the additivity value $g_{A_Q} \simeq 0.707$ [from Eqs. (10) and (12) and numerical estimate of g_A]. We interpret these numbers as meaning that the approximation is not unreasonable and adjust $G(0)$ to reproduce the additivity result for g_{A_Q}:

$$G(0) = \frac{1}{2} \frac{1}{F_\pi{}^2} . \qquad (18)$$

If we consider now the Cabibbo-Radicati sum rule[12] with vector-meson dominance and additivity[13] (or ρ-exchange model at high energies) we obtain

$$K_1'^V(0) = \tfrac{1}{2}[K_2^V(0)]^2 + G(0)(1/f_\rho{}^2) \qquad (19)$$

with $K_1'^V(0) = 1/m_\rho{}^2$ and K_2^V given by (13); from (18) and (19) one derives the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation[14]

$$f_\rho F_\pi = m_\rho . \qquad (20)$$

The photoproduction sum rules of Fubini, Furlan, and Rossetti[6] say that

$$\tfrac{1}{2} K^V = \left(\frac{M}{g_Q}\right) A_1^{(+)}(0), \qquad (21)$$

$$\tfrac{1}{2}K^S=\left(\frac{M}{g_Q}\right)A_1^{(0)}(0), \qquad (22)$$

where $A_1^{(+)}$ and $A_1^{(0)}$ are Chew-Goldberger-Low-Nambu (CGLN)[15] amplitudes. Writing dispersion relations for $A_1^{(+)}(0)$ and $A_1^{(0)}(0)$ and considering again only the Regge type of contributions, the $A_1^{(+)}$ amplitude will be dominated by ω exchange and $A_1^{(10)}$ by ρ exchange. Coupling the photon via VMD the ratio of (21) to (22) immediately gives, from (13), for the ratios of the $\rho\gamma$ and $\omega\gamma$ coupling constants

$$f_\rho^{-1}/f_\omega^{-1}=-3, \qquad (23)$$

a typical VMD, SU(3) relation. If one relates $A_1(0)$ to $G(0)$, again using factorization, and assuming that the ratio of the residues of off-mass-shell particles is equal to that when they are on the mass shell, then we have

$$A_1^{(+)}(0)=\tfrac{1}{2}f_\rho^{-2}m_\rho^{2}g_{\rho\pi\omega}K^VG(0). \qquad (24)$$

From (21), (18), (12), and (20) and the additivity value of $g_{A_Q}=1/\sqrt{2}$ one then derives a well-known SU(6) result[16]

$$g_{\rho\omega\pi}=2m_\rho^{-1}f_\rho. \qquad (25)$$

Equations (20), (23), and (25) show that our treatment of the dispersion integrals is consistent with SU(6). It is probably not surprising that from sum rules one reaches SU(6) relations in a much more straightforward way working at quark level than at baryon level.

IV. QUARK-PION SCATTERING LENGTHS

Before going to low-energy πN scattering, we discuss from the same point of view low-energy πQ scattering in order to check the consistency of our model and to be able to reveal some inconsistencies in the usual PCAC or ρ-exchange-model determination of low-energy parameters.[17-20]

In the PCAC treatment one expresses the off-mass-shell amplitude as a sum of two terms, one corresponding to the scattering of axial-vector currents by on-mass-shell targets and the other being a commutator term. The first term is saturated by s-channel resonances and the second one is written in terms of electromagnetic form factors. Huang and Urani[20] improved the current-algebra treatment by including also a ρ particle in the first term and were able to show that this scheme is exactly the same as a full on-mass-shell treatment, with saturation of the amplitude by s-channel resonances *and* a ρ-exchange term.

Comparing the theoretical predictions of πN low-energy scattering parameters with dispersion-relation calculations based on detailed experimental information, one notes that they roughly agree when *either* one or the other of the above terms is dominant, but discrepancies occur when both give important contributions. An example where the commutator alone gives a good prediction is in the determination of the $a_0^{(-)}$ s-wave scattering length. The term with resonance saturation alone gives good predictions for the $a_{1-}^{(+)}$ and $a_{1+}^{(+)}$ p-wave scattering lengths. However, strong disagreement occurs in the $a_{1-}^{(-)}$ scattering length where both terms have contributions of the same order of magnitude. From our model we shall suggest that some sort of double counting between resonance and ρ-exchange terms occurs when both are added together.

We illustrate this first for πQ scattering. For the s-wave scattering lengths the commutator term gives the usual "universal" values for the scattering of pions from any target of isospin $\frac{1}{2}$.[17] For the (+) p-wave scattering lengths the resonance term is expected to give a good result, when saturated, according to our previous discussion, with the Born term alone (in the πN case the Born term and the first Δ resonance give a good approximation). It is

$$a_{1+}^{(+)}=\tfrac{1}{3}(a_{13}+2a_{33})=\tfrac{2}{3}f_Q^{\,2}, \qquad (26)$$

$$a_{1-}^{(+)}=\tfrac{1}{3}(a_{11}+2a_{31})=-\tfrac{4}{3}f_Q^{\,2}. \qquad (27)$$

If one uses the definition of the p-wave scattering lengths, neglects small s-wave contributions and terms of order m_π/M, and writes a dispersion relation for the $B_\pm$ amplitude for scattering of $\pi^\pm$ [see Hamilton and Woolcock (HW), Ref. 21], one derives

$$2(a_{33}-a_{31})+(a_{13}-a_{11})=6f_Q^{\,2}-\tfrac{3}{4}(I_++I_-), \qquad (28)$$

where

$$I_\pm=\frac{1}{4\pi^2M}\int_1^\infty d\nu\left[\frac{\mathrm{Im}B_\pm(\nu,0)}{\nu-1}-\frac{\mathrm{Im}B_\mp(\nu,0)}{\nu+1}\right].$$

Comparison of Eqs. (26) and (27) with (28) imposes

$$I_+=-I_-. \qquad (29)$$

This is a curious result which, in our approximation of evaluating dispersion integrals for πQ scattering, neglecting the pion mass, is automatically satisfied; but the important thing that Eq. (29) is telling us is that $\mathrm{Im}B_\pm$ are likely to be relatively smooth near threshold (as is an extrapolation from a high-energy fit) without strong oscillations or resonances whose relative contribution would change considerably in going from $\nu-1$ to $\nu+1$ in the denominators.

Also, from (29), the dispersion integrals for the $B^{(+)}\equiv\frac{1}{2}(B_++B_-)$ and $A^{(+)}\equiv\frac{1}{2}(A_++A_-)$ amplitudes (the latter being once subtracted at the origin) must vanish. This is what one requires to derive from the Adler condition[22] the relation $a_0^{(+)} \simeq 0$ for the (+) s-wave scattering lengths. But we do not here need to invoke the vanishing pion mass. At quark level, violations of this relation are thus expected to be even smaller than in πN scattering.

When the (−) p waves are also evaluated in the PCAC or ρ-exchange model two relations were obtained, which are easy to compare with dispersion relations. Using current algebra these relations can be derived, for instance, from Schnitzer's work[18] [Eqs. (26a) to (26d)], or using dispersion relations from HW[21] [Eqs. (4.28), (4.29), and (4.35)]. One relation, neglecting a small derivative contribution, is

$$\frac{(a_{13}-a_{33})+2f_Q^2}{(a_{31}-a_{33})+2f_Q^2}=\frac{k^V}{1+k^V}, \quad (30)$$

where $k^V = 2MK^V$. This result is consistent with dispersion relations, using our proposed ansatz for the imaginary part of the amplitude and with the same type of approximation. The other relation,

$$(a_{11}-a_{13})-(a_{31}-a_{33})=\frac{3}{8\pi M}(1+K^V)\left(\frac{f_\rho}{m_\rho}\right)^2, \quad (31)$$

is, however, in disagreement with the definition (neglecting small terms)

$$(a_{11}-a_{13})-(a_{31}-a_{33})=\frac{3}{8\pi M}\,\mathrm{Re}B^{(-)}(1,0), \quad (32)$$

if we disperse $B^{(-)}$ and we use our ansatz for $\mathrm{Im}B$. The reason is that while the ρ-exchange model uses a proportionality for the real part of the amplitudes

$$\mathrm{Re}B^{(-)}=(1+K^V)\,\mathrm{Re}A'^{(-)}, \quad (33)$$

we propose

$$\mathrm{Im}\nu B^{(-)}=(1+k^V)\,\mathrm{Im}A'^{(-)}, \quad (34)$$

where the factor $(1+k^V)$ now comes from ratios of coupling constants. Because of the Born terms, (33) cannot also be true; for $\mathrm{Re}A'^{(-)}$ there is an important Born-term contribution which is negligible in $\mathrm{Re}B^{(-)}$. In our πQ model,

$$\mathrm{Re}B^{(-)}(1,0)\simeq(1+k^V)G(0)=\tfrac{1}{2}(1+k^V)(1/F_\pi^2) \quad (35)$$

is *half of the quantity given by the exchange model for the real part of the amplitude*. Equations (26), (27), (30), and (32) [with the right-hand side given by (35)] form a soluble system from which the p-wave scattering lengths can be determined. In Eq. (30) we have used $k^V = 12$ but the precise value is not very important provided it is $\gg 1$. The resulting values are

$$a_{13}=-0.90f_Q^2,\quad a_{11}=-2.10f_Q^2,$$
$$a_{31}=-0.95f_Q^2,\quad a_{33}=1.45f_Q^2. \quad (36)$$

V. NUCLEON-PION SCATTERING LENGTHS

If one tries to transpose the model as it stands to nucleon-level interactions it obviously fails. In the low-energy region, resonance terms and extrapolated Regge terms disagree strongly. For instance, in πN scattering while $\mathrm{Im}A'^{(-)}_{\mathrm{Regge}}$ is always positive, $\mathrm{Im}A'^{(-)}_{\mathrm{Resonance}}$ is mostly negative because of the strong $\Delta(1236)$ contribution; as a consequence, while the contribution from the continuum in πQ case is positive and $g_{A_Q} < 1$, that contribution in πN scattering is negative and $g_A > 1$.

Following an idea of Gilman and Schnitzer,[13,23] what we propose in πN scattering is to extract the $\Delta(1236)$ pole from the continuum and treat the remainder as due to πQ scattering, keeping in mind (16). As the first Δ resonance is close to the threshold, we evaluate the Regge part as before with the threshold as a lower limit in the integral.

To check the validity of the model we take the Adler-Weisberger relation (14) for πN scattering. According to Adler's calculation the saturation of the continuum by the $\Delta(1236)$ alone would give $g_{A_\Delta} = 1.44$. In our model the Adler-Weisberger relation is then written as

$$g_A^2=(g_{A_\Delta})^2-F_\pi^2G(0), \quad (37)$$

and, using (18), one obtains $g_A = 1.23$ while Adler finds $g_A = 1.24$. In evaluating now the p-wave πN scattering lengths we write equations equivalent to (30) and (32), but having extracted first the $\Delta(1236)$ from the continuum. We have

$$\frac{\frac{1}{3}(a_{13}-a_{33})+\frac{2}{3}f^2-\Delta_1}{\frac{1}{3}(a_{31}-a_{33})+\frac{2}{3}f^2-\Delta_2}=\frac{k^V}{1+k^V} \quad (38)$$

and

$$\tfrac{1}{3}[(a_{11}-a_{13})-(a_{31}-a_{33})]=\Delta_3+\frac{1}{8\pi m_N}(1+k^V)G(0). \quad (39)$$

Equation (38) is still equivalent to the PCAC or ρ-exchange-model expression. A really different contribution comes from (39). For the (+) amplitudes previous calculations using PCAC or ρ-exchange models remain unchanged. We put $k^V = 3.7$ and take values for the Δ contributions from Ref. 19: $\Delta_1 = \Delta_2 = -0.028$ and $\Delta_3 = 0.048$. We summarize our result for the (−) p-wave scattering lengths in Table I and show for comparison the PCAC-ρ-exchange-model results and the results from dispersion relations using detailed experimental information.[21,24]

TABLE I. $a^{(-)}_{1\pm}$ p-wave scattering lengths for πN scattering. A comparison of PCAC or ρ-exchange-model calculations (Schnitzer,[18] Raman,[19] and Huang and Urani[20]) with detailed dispersion relations (HW[21] and Roper *et al.*[24]) and the present work.

	PCAC or ρ-exchange model			Dispersion relations		
	Schnitzer	Raman	Huang and Urani	HW	Roper *et al.*	Present work
$a^{(-)}_{1-}$	−0.005	+0.0123	−0.003	−0.021	−0.016	−0.016
$a^{(-)}_{1+}$	−0.075	−0.083	−0.089	−0.081	−0.081	−0.083

It should be noticed that the discrepancy found by previous authors in the $a^{(-)}_{1-}$ p-wave scattering length had resisted all attempts to explain it in a convincing way. The inclusion of an exchange term with $I_t=0$[19] does not, of course, affect the isospin antisymmetric part of the amplitude. Schnitzer's suggestion[18] of adding more contributions to the direct term beyond the $\Delta(1236)$ worsened the result rather than improving it, as can be seen in Raman's calculation.[19] It can still be argued[19] that if nonpole contributions are added in the direct-channel term, they may interfere with the excessively large ρ-exchange term, bringing the $a^{(-)}_{1-}$ p wave down to the experimental value. We think however that our simple explanation of the discrepancy is more appealing, and the best support for it is the good agreement obtained using an extremely crude model. The problem of finding a too strong ρ term in the B amplitude was also met in work with chiral Lagrangians,[25] where the unjustified suggestion was made of neglecting the magnetic ρNN coupling in order to decrease the over-all ρ contribution.

Our conclusion concerning the PCAC or ρ-exchange-model calculations of low-energy parameters is again that, in the way they are normally applied, they are in contradiction with dispersion relations; on the contrary, agreement is reached if, in the direct-channel term, one includes resonances below a given energy and, in the exchange-channel term, the remaining high-energy behavior. This is in total agreement with the current ideas of duality and the necessity of avoiding double counting of resonances and exchange terms.

VI. QUARK MODELS IN THE (s,t,u) PLANE

Coming back to the quark model we would like to draw attention to the following points:

(1) If the whole procedure of evaluating sum rules and dispersion relations in πQ scattering has any sense at all, a quark model with massive quarks ($M \gtrsim 5$ GeV) and large anomalous magnetic moment seems more fruitful than a model with light Dirac quarks.

(2) Apart from the absence of $I=\frac{3}{2}$ resonances, a kinematic argument can be given in favor of the extrapolation of the high-energy curve down to the threshold in πQ scattering. If some sort of quark excitations exist and lie on a Regge trajectory of universal slope, a resonance region for πQ scattering would be, in the s variable, of the same order of magnitude as in πN scattering. However in the ν variable that region in the πQ case is narrowed by a factor m_N/M and the Regge behavior is built up much earlier.

(3) Dispersion relations can be applied to determine the remaining low-energy πQ parameters, in particular, the s-wave effective ranges $b_0^{(\pm)}$. We use the $C^{(+)}$ and $C^{(-)}$ relations of HW, combined with an effective range expansion

$$3b_0^{(+)}=C^{(+)}, \tag{40}$$

where

$$C^{(+)}=\lim_{\nu'\to 1}\frac{3}{4\pi^2}\int_1^\infty \frac{\nu\,d\nu}{(\nu^2-1)^{1/2}}\frac{\sigma_-+\sigma_+}{\nu^2-\nu'^2}+\frac{3f_Q^{\,2}}{M} -(a_{11}+2a_{13}+2a_{31}+4a_{33}) \tag{41}$$

and

$$3b_0^{(-)}=\tfrac{1}{2}(a_{10}-a_{30})+C^{(-)}, \tag{42}$$

where

$$C^{(-)}=\lim_{\nu'\to 1}\frac{3}{4\pi^2}\int_1^\infty \frac{d\nu}{(\nu^2-1)^{1/2}}\frac{\sigma_--\sigma_+}{\nu^2-\nu'^2}-\sigma f_Q^{\,2} -(a_{11}+2a_{13}-a_{31}-2a_{33}), \tag{43}$$

and a_{10}, a_{30} are the s-wave scattering lengths. For $\sigma_-+\sigma_+$ we use the fits of Ref. 11 and the additivity relation

$$[\sigma_-+\sigma_+]^Q=\tfrac{1}{3}(\sigma_-+\sigma_+)^N=0.720+2.73\nu^{-0.7}. \tag{44}$$

Using also (10), (17), and (36), we obtain from (40) and (42)

$$b_0^{(-)}=+0.059,\qquad b_0^{(+)}=+0.065. \tag{45}$$

The current-algebra or ρ-exchange-model determinations of $b_0^{(\pm)}$ are much too model-dependent to allow an unambiguous comparison. The same parameters would be, in Schnitzer's[18] approach,

$$b_0^{(-)}=+0.058,\quad b_0^{(+)}=+0.144,$$

and with the Huang-Urani[20] prescription,

$$b_0^{(-)}=+0.003,\quad b_0^{(+)}=+0.038.$$

Some discrepancy appears in these results, but the same also happens when these methods are applied to πN scattering. However, there is over-all agreement with respect to the sign and the order of magnitude of the s-wave effective ranges. With the "universal" s-wave scattering lengths and Eqs. (36) and (45) we complete the usual set of low-energy parameters for πQ scattering.

It therefore seems possible to construct a quark model that includes, not only the language of unitary symmetries, but also makes use of other tools of strong-interaction physics, such as duality schemes and dispersion representations. Using Lipkin's image of the two planes (I, Y, B) and (s, t, u) for strong interaction physics, we tend to say that quarks and their interactions are not only objects belonging to the (I, Y, B) plane – as the usual criticism of more realistic quark models insists – but also to the (s, t, u) plane. This is at least what some of our results strongly suggest.

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