strengthen the constraints derived in Secs. III and IV.

[12]Terms like $\vec{\varphi}\cdot\bar{\psi}_l P_+ s$, which would lead to a neutrino mass, are not included.

[13]Cf. Ref. 4 and references cited there. A simple functional-integral derivation is given by A. Salam and J. Strathdee, Nuovo Cimento 11A, 397 (1972). See also Gross and Jackiw, Ref. 8.

[14]H. H. Chen and B. W. Lee [Phys. Rev. D 5, 1874 (1972)] have pointed out that $\nu e \to \nu e$ data imply $M_W > 65$ GeV in the Weinberg model.

[15]Here $a_\mu = [\frac{1}{2}(g-2)]_\mu = F_2^\mu(0)$. Theory and data are summarized by S. J. Brodsky, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

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[17]H. L. Anderson *et al.*, Phys. Rev. 187, 1565 (1969).

[18]M. S. Dixit *et al.*, Phys. Rev. Letters 27, 878 (1971).

[19]G. Backenstoss *et al.*, Phys. Letters 31B, 233 (1970).

[20]Cf. Jackiw and Weinberg, Ref. 9. In the SO(3) five-quark model of Georgi and Glashow, $\xi = (m_{q-} - m_p)/m_\mu$ for a proton quark, where m_{q-} is the mass of a heavy negatively charged quark. Thus ξ might be considerably larger than unity.

[21]If one takes seriously the discrepancies between the calculated and measured transition energies reported by Dixit *et al.*, Ref. 18, one may attempt to ascribe the discrepancies to V_ϕ. A fair fit to all the Dixit *et al.* measurements is obtained with $M_\phi = 0$ and $\xi\zeta = 2$ [cf. G. A. Rinker, Jr., and M. Rich, Phys. Rev. Letters 28, 640 (1972); $M_\phi = 0$ and $\xi\zeta = 2$ correspond to their c_{11}.] The largest value of M_ϕ permitted by these data is $M_\phi \approx 2$ MeV, with $\xi\zeta = 1.5$. A glance at Fig. 3 shows that these values are inconsistent with constraints obtained from other data. Thus a mechanism like V_ϕ cannot be responsible for the discrepancies noted by Dixit *et al.*

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[27]We thank Professor R. Jackiw for raising this point.

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Analysis of the Anomaly in the $\pi^+\pi^-$ System near the $K\bar{K}$ Threshold*

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An absorption-modified pion-exchange model with coupled-channel unitarity and analyticity for the $\pi^+\pi^-$ system near the $K\bar{K}$ threshold is introduced. Data-analysis techniques for the reactions $\pi + N \to$ meson + meson + baryon are discussed.

In recent experiments,[1] the $\pi^+\pi^-$ and K^+K^- systems produced in the reactions

$$\pi + N \to \pi^+ + \pi^- + \text{baryon}, \tag{1}$$

$$\pi + N \to K^+ + K^- + \text{baryon} \tag{2}$$

have been studied at high incident energies and small baryon momentum transfers in a region of $\pi\pi$ masses including the $K\bar{K}$ threshold point. It is found that the $\pi\pi$ "decay" angular distribution has a forward-backward asymmetry moment $\langle Y_1^0\rangle$ which exhibits a precipitous, almost discontinuous step from about 0.15 to zero in a 10-MeV interval in $\pi\pi$ mass at $m_{\pi\pi} \simeq 980$ MeV, along with a 50% drop in $d\sigma/dm_{\pi\pi}$. It has been duly pointed out that the discontinuity in $\langle Y_1^0\rangle$ could be due to the strong onset of the S-wave $K\bar{K}$ channel, which absorbs some of the $\pi\pi$ S-wave probability, thereby reducing the $\pi\pi$ SP-wave interference in $\langle Y_1^0\rangle$. The effect is actually complicated and difficult to study in terms of $\pi\pi$ scattering amplitudes or phase shifts. However, we find the effect should be susceptible to careful analysis of the "decays" and the production mechanisms for reactions (1) and (2). In this paper we argue that the analysis should be done, and we discuss the techniques necessary to do it properly. The yield could be very interesting information on $\pi\pi$ and even $K\bar{K}$ elastic scattering amplitudes. The technique to be described includes absorption-modified one-pion exchange together with a K-matrix approach for the coupled-channel meson-meson scattering problem. It is clear that coupled-channel unitarity and analyticity near the $K\bar{K}$ threshold are required in the analysis and will be crucial in resolving at least part of the isoscalar S-wave $\pi\pi$ phaseshift ambiguity in the region 750–1000 MeV.[2]

The reduction of the data for reactions (1) and (2) will yield for example from the meson-rest-frame canonical "decay" angular distribution all the spherical average moments $\langle \mathrm{Re} Y_L^m \rangle$ as functions of t and $m_{\pi\pi}$. The moments are linearly related to the production density-matrix elements for the mesonic system $\rho^{nn'}_{\mu\mu'}(t, m_{\pi\pi})$. If the production mechanism is pure one-pion exchange, all of the moments $\langle \mathrm{Re} Y_L^m \rangle$ with $m \neq 0$ will vanish, so that the angular distribution for decay of a mesonic system with spin l is the square of the Legendre polynomial $|P_l|^2$. Of course this is not observed, and the most commonly accepted explanation is that absorptive corrections "depolarize" the mesonic system from the $m = 0$ state through initial- and final-state elastic rescattering. Absorptive effects have been verified qualitatively in a variety of experiments, but most convincingly for *small t* $[t \leq 0.15\ (\mathrm{GeV}/c)^2]$ by the experiment at SLAC of Baillon, *et al.*[3]

In order to study the actual processes $\pi + \pi \to \pi + \pi$ and $\pi + \pi \to K + \bar{K}$ from reactions (1) and (2) with all particles on mass shell it is necessary to extrapolate the moments from values of $t < 0$ to the point $t = m_\pi^2$. Because absorptive corrections are important for small t ($t < 0$), the Chew-Low extrapolation is difficult (pure pion exchange does not overwhelmingly dominate).[4] In particular, the moments $\langle Y_L^m \rangle$ are affected. It is necessary to incorporate absorptive effects into the extrapolation procedure, either by using sufficiently high-order series expansions of the quantities to be extrapolated or by using models which reduce the number of free parameters.

The model we will describe is simple and (so far) successful.[5] It gives useful expressions for the density-matrix elements $\rho^{nn'}_{\mu\mu'}$ at high energy and small t. It was used with $l, l' \leq 1$ to describe all of the decay data of the experiment of Baillon *et al.* in the ρ-meson region, giving very good fits with only two parameters which are related to on-shell $\pi\pi$ phase shifts.[3] The model gives helicity amplitudes at high energy and small t for the production of a mesonic particle with spin l and helicity μ in reactions (1) and (2) as

$$T \propto (-t)^{n/2} P(m_\pi^2) e^{A(t-m_\pi^2)}/(t - m_\pi^2), \qquad (3)$$

with the proportionality independent of helicity, and

$$(-t)^{n/2} P(t) \propto (2l+1)^{1/2} a_l d^l_{\mu,0}(\psi) \quad \text{(baryon vertex)}. \qquad (4)$$

The factor $(-t)^{n/2}$ is the minimal t dependence required by angular momentum conservation, where n is the net helicity flip for the reaction. The rotation function $d^l_{\mu,0}(\psi)$ for a mesonic system with spin l and helicity μ has argument ψ, which is the crossing angle of Trueman and Wick.[5] The factor $P(t)$ in Eq. (4) represents a helicity-dependent polynomial in t which is evaluated at the pion pole $t = m_\pi^2$ in Eq. (3). This procedure eliminates s-channel low-partial-wave anomalous terms, which are offensive to unitary limits,[5,6] and makes the proportionality independent of l.

The exponential factor in Eq. (3) represents helicity-independent collimation due to "long-range" absorption (which may depend on net helicity flip n). Finally, the quantity a_l is the partial-wave amplitude for $\pi\pi \to \pi\pi$ ($\pi\pi \to K\bar{K}$), referring to reaction (1) [(2)], normalized to $\exp(i\delta_l)\sin\delta_l$ for elastic scattering in a pure isospin state, where δ_l is the on-shell $\pi\pi$ phase shift. The baryon vertex factors are given explicitly in Ref. 5.

The model in Eqs. (3) and (4) enables one to calculate the density matrix elements. These, in various combinations, can be fitted to the moments $\langle Y_L^m \rangle$ obtained from the data, with for example the ratios $\mathrm{Re}(a_l^* a_{l'})/|a_0|^2$ appearing as parameters. For the assumption $l, l' \leq 1$, appropriate for the ρ-meson region, there are only two such parameters, while for $l, l' \leq 2$ there are five.

The remaining step is to treat the data on reactions (1) and (2) simultaneously. Probably the best (tried and true) procedure in the vicinity of a strong inelastic threshold is to use the K-matrix approach for the partial-wave amplitudes a_l.[7] This approach gives a unitary representation of the partial-wave reaction amplitudes with correct analyticity near threshold. We review it and its limitations for the present problem in the following.

The K-matrix approach has been used extensively by many authors working on baryonic systems. In this approach, the amplitude matrix $[a_l] \equiv K[1 - iK]^{-1}$, where K is a $n \times n$ matrix that is Hermitian, so that $[a_l]$ automatically satisfies unitarity. The K matrix defines a matrix M by $M \equiv k^{l+1/2}[K]^{-1}k^{l+1/2}$, where k is a diagonal matrix of center-of-mass channel momenta. The matrix M is analytic in the square of a channel momentum k^2 near its threshold ($k^2 = 0$), so it is expanded to first order as $M \cong M_0 - \frac{1}{2}R_0 k^2$. In terms of M,

$$[a_l] = k^{l+1/2}(M - i k^{2l+1})^{-1} k^{l+1/2},$$

and for a single-channel problem, $M = k^{2l+1}\cot\delta_l$. In a multichannel situation, a useful approximation is to assume that R_0 is diagonal. Both M_0 and R_0 are real and symmetric, and so is K if all channels are open. Thus a simple two-channel situation is described by five real parameters over a small range of k^2. This is appropriate for the $\pi\pi$, $K\bar{K}$ problem in the vicinity of 980 MeV, if the contributions from, e.g., the 4π channel can be safely ignored. It is worth pointing out here that the $K\bar{K}$ elastic reaction (with $G = +$) cannot be studied easily, as it would have to be reached through $\bar{K}N$

$\rightarrow \bar{K}KY$ with a much longer extrapolation to the kaon pole and with concomitant difficulties arising from K^* exchange. Thus it is altogether a much harder problem. On the other hand, we can get some information via the unitary K-matrix approach, since we can (almost) measure $\pi\pi \rightarrow \pi\pi$, $K\bar{K}$ from reactions (1) and (2). To illustrate this, we can write the two-channel Hermitian K matrix as

$$K = \begin{pmatrix} \alpha & \beta \\ \beta^+ & \gamma \end{pmatrix},$$

with α (γ) referring to the $\pi\pi$ ($K\bar{K}$) channel. For S waves near the $K\bar{K}$ threshold, in terms of the $K\bar{K}$ complex scattering length $A \equiv a + ib$, the parameters β, γ can be eliminated, and we can write[7]

$$a_0(\pi\pi \rightarrow \pi\pi) = \frac{\alpha}{1 - i\alpha} + \frac{ikb}{1 - ikA}\left(\frac{1 + i\alpha}{1 - i\alpha}\right), \tag{5a}$$

$$a_0(\pi\pi \rightarrow K\bar{K}) = \frac{\sqrt{kb}}{1 - ikA}\left(\frac{1 + i\alpha}{1 - i\alpha}\right)^{1/2}, \tag{5b}$$

$$a_0(K\bar{K} \rightarrow K\bar{K}) = \frac{kA}{1 - ikA}, \tag{5c}$$

where k is the $K\bar{K}$ center-of-mass momentum. In this "zero-range" approximation, the three parameters α, a, b are real and essentially constant. Through measurement of the magnitude and phase of $a_0(\pi\pi \rightarrow \pi\pi)$ and $a_0(\pi\pi \rightarrow K\bar{K})$, the low-energy behavior of $a_0(K\bar{K} \rightarrow K\bar{K})$ can be determined. It is clear that any contribution from the 4π channel would weaken this determination, so ignoring 4π, although perhaps warranted by the data, is in fact a fairly strong assumption. We will return to this point later.

Let us further assume, for the moment, that $l, l' \leq 1$, and that the P-wave $K\bar{K}$ scattering length is small. We can estimate the value of A necessary to produce the cusp in $\langle Y_1^0 \rangle$ for values of α corresponding to the ambiguous "up" or "down" isoscalar S-wave $\pi\pi$ phase shifts in the 750–950-MeV region.[2,6] It is convenient to write Eq. (5a), which corresponds to *isoscalar*, since $K\bar{K}$ does not affect isotensor, as $a_0(\pi\pi \rightarrow \pi\pi) = \alpha_R/(1 - i\alpha_R)$, where the reduced K-matrix element α_R is given by

$$\alpha_R = \alpha + \frac{ikb(1 + \alpha^2)}{1 - ik(a + \alpha b)}. \tag{6}$$

Below threshold, $k \rightarrow i|k|$, so it is apparent that $\alpha_R \rightarrow$ real, and $a_0(\pi\pi \rightarrow \pi\pi)$ satisfies elastic unitarity. As pointed out in Ref. 2, it is possible that the isoscalar phase shift δ_0^0 makes a rapid excursion from the "down" region ($\delta_0^0 \approx 65°–75°$) to the "up" region ($\delta_0^0 \approx 150°–170°$) near $k^2 = 0$. From Eqs. (6) and (5b) it is apparent that for this to happen, and at the same time to have $\sigma_0(\pi\pi \rightarrow K\bar{K})$ be large, would probably require that the parameter a be large and

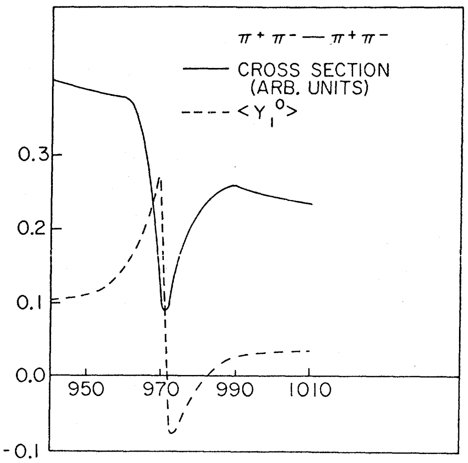

FIG. 1. The calculated on-shell forward-backward asymmetry moment $\langle Y_1^0 \rangle$ and the cross section for $\pi^+\pi^-$ scattering near the $K\bar{K}$ threshold are plotted vs $\pi\pi$ effective mass in MeV/c^2.

negative. This in turn implies the existence of a *virtual bound state*[7] of $K\bar{K}$, which, of course, shows up as a resonance in $\pi\pi$ scattering just below the $K\bar{K}$ threshold (k^2 negative and small).

The results of a simple two-channel on-shell calculation are shown in Fig. 1. The moment $\langle Y_1^0 \rangle$, and $\sigma_{\pi\pi}$ as functions of $m_{\pi\pi}$, are calculated with $a = -15.0$ (GeV/c)$^{-1}$ and $b = 4.0$ (GeV/c)$^{-1}$ in the "down" case $\alpha = \tan(1.2$ rad), with the p-wave amplitude given by the ρ-meson elastic Breit-Wigner tail ($m_\rho = 0.765$ GeV/c^2, $\Gamma_\rho = 0.120$ GeV/c^2). The S-wave isotensor phase shift was taken to be -0.5 rad.[8] The isoscalar parameters correspond to a virtual bound state (resonance) at $m_{\pi\pi} \approx 982$ MeV/c^2 of width < 10 MeV/c^2. The rapid variation of $\langle Y_1^0 \rangle$ from a rather constant value ~0.15 to a much smaller constant value ~0.06 within a 10-MeV/c^2 interval is reproduced. Note that $\sigma_{\pi\pi}$ shows a narrow *dip* at resonance, instead of a peak, and that $\sigma_{\pi\pi}$ also undergoes a rapid overall decrease by about 30% as one goes from the region below 970 MeV/c^2 to the region above 990 MeV/c^2 ($K\bar{K}$ threshold). The gross features in Fig. 1 are consistent with those of the data of Alston-Garnjost *et al.*[1], which are binned in 10-MeV/c^2 intervals. By contrast, if we started from the "up" case with $\alpha = \tan(2.5$ rad), $\sigma_{\pi\pi}$ shows a peak and $\langle Y_1^0 \rangle$ a dip (not a step), so in this case no virtual bound state could be nearby if *rapid* changes take place as observed.

Finally, we wish to stress the point that the data, of course, are off-shell. The extrapolation of the data to the pion pole must be done literally, or in terms of a model. Details of the production mechanism may affect the extrapolation procedure. For this reason, we have given a model so that the on-shell parameters a, b, α, etc. can be fitted to off-shell data.

We note that the first multichannel analysis of this region has been attempted,[2] but this analysis has not included absorptive effects in the production mechanism. The extrapolation to the pion pole is therefore somewhat suspect,[9] but the possibility of a resonance just below the $K\bar{K}$ threshold is clearly present.

We conclude with a comment on the assumptions that the 4π channel is negligible and that the isotensor phase shift is slowly varying. The analysis of Ref. 8 indicates some increase of δ_0^2 in this region. A significant variation might result from the 4π channel, which is important above ~1 GeV/c.[2] Consequently, the 4π channel and a nonconstant δ_0^2 might have to be considered in a full investigation of this region. This opens the door to considering other effects which we have neglected here. For example, D waves could be easily included along with P- and D-wave inelasticity in the present multichannel formulation. This is indeed a very rich region for studying multichannel effects with spin-zero particles.

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[1]M. Alston-Garnjost *et al*. [Phys. Letters 36B, 152 (1971)] discuss $\pi^+ + p \rightarrow \pi^+ + \pi^- + \Delta^{++}$ at 7 GeV. There had been previous indications of such an effect in several experiments on $\pi^- + p \rightarrow \pi^+ + \pi^- + n$, but until recently none were of compelling statistical significance. A recent counter experiment also shows the effect quite dramatically: G. Grayer *et al*., in *Experimental Meson Spectroscopy–1972,* edited by A. H. Rosenfeld and K. W. Lai (American Institute of Physics, New York, 1972).

[2]S. Flattê *et al*., Phys. Letters 38B, 232 (1972); S. Protopopescu *et al*., in *Experimental Meson Spectroscopy–1972*, Ref. 1.

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