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Muon g–2 and Other Constraints on a Model of Weak and Electromagnetic Interactions Without Neutral Currents*

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A simple spontaneously broken gauge model of electromagnetic and weak interactions without neutral currents has recently been constructed by Georgi and Glashow. Models of this sort characteristically contain a variety of hypothetical particles: intermediate vector bosons, one or more massive scalars, heavy leptons, "charmed" hadrons. In this paper we show that the agreement between the conventional calculation of muon $g-2$ and experimental data imposes relations among the masses of the intermediate vector meson, the heavy leptons associated with the muon, and the Higgs scalar meson, in the Georgi-Glashow model. We also deduce additional constraints on models of this sort from muonic atom data, and we briefly discuss scattering phenomena involving the presently unobserved particles of the Georgi-Glashow model.

I. INTRODUCTION

Elegant models of electromagnetic and weak interactions can be constructed by starting from a Lagrangian in which vector fields possessing gauge symmetry are coupled minimally to conserved fermion currents, and then allowing the gauge symmetry to be broken spontaneously in such a way that only the photon and neutrinos remain massless.[1] Formal arguments[2,3] and explicit calculations[4,5,6] have indicated that such spontaneously broken gauge theories are renormalizable.

Several years ago, Weinberg[1] presented a spontaneously broken gauge model for the electromagnetic and weak interactions of leptons in which the underlying symmetry is $SU(2)_L \times U(1)_R$. The model thus has two charged and one neutral massive intermediate vector mesons in addition to the photon. More recently, Georgi and Glashow[7] have exhibited another such model in which the initial symmetry is SO(3), and there is no neutral intermediate vector meson. They are able to eliminate neutral weak currents only at the price of introducing new fermion fields. Additional fermion fields are also necessary in the Weinberg model, if anomalous Ward identities are to be avoided and renormalizability preserved.[8] Thus the two models are about equally "simple," and the SO(3) model avoids the prediction of unobserved neutral processes in lowest order.

The purpose of the present paper is to show that the requirement that the muon magnetic moment be consistent with experiment places significant constraints on models of the sort considered by Georgi and Glashow, although not on the Weinberg model.[9] Muonic atom data and scattering data also provide constraints on the parameters of the SO(3) model.

The paper is organized as follows. In the next section we review the salient features of the SO(3) model. Section III presents the $g-2$ calculation. In Sec. IV we consider muonic atoms and deduce further constraints on the parameters of the model. Finally, in Sec. V we briefly discuss various other experiments and summarize our conclusions.

II. THE SO(3) MODEL

In the model considered by Georgi and Glashow,[7] the electromagnetic charge Q_E and the weak charges Q_W and $Q_{\overline{W}}$ generate the group SO(3). A pair of heavy leptons X^0, X^+ are added to the electron and its neutrino (ν) to form an SO(3) triplet $\vec{\psi}_e$ and singlet s_e, and another pair of heavy leptons Y^0, Y^+ are analogously associated with the muon and its neutrino (ν'). The physical neutrino ν_L, and $X^0_L = P_- X^0$,[10] are assumed to be mixed, with angle β; ν'_L and Y^0_L are assumed to be mixed with the same angle. The coupling to the SO(3) vector gauge field $\vec{W}_\mu$ is

$$-\mathcal{L}_{W-l} = \sum_{l=e,\mu} eW^a_\alpha(\bar{\vec{\psi}}_l T^a \gamma^\alpha \vec{\psi}_l), \tag{1}$$

where $(T^a)_{bc} = -i\epsilon_{abc}$ is the usual isospin matrix. Since this coupling is "vectorlike" (i.e., γ_5 does not appear explicitly), the currents will remain conserved in the presence of symmetric mass terms[11]:

$$-\mathcal{L}_{m^0}=m_e^0\overline{\vec{\psi}}_e\cdot\vec{\psi}_e+m_\mu^0\overline{\vec{\psi}}_\mu\cdot\vec{\psi}_\mu\,. \tag{2}$$

Finally, a scalar isovector field $\vec{\varphi}$ is introduced into the Lagrangian

$$\mathcal{L}=\mathcal{L}_{\text{kinetic energy}}+\mathcal{L}_{m^0}+\mathcal{L}_{W-l}+\mathcal{L}_\varphi+\mathcal{L}_W \tag{3}$$

and is coupled to almost[12] everything:

$$-\mathcal{L}_\varphi=\sum_{l=e,\mu}\left[g_l\varphi^a(\overline{\vec{\psi}}_lT^a\vec{\psi}_l)+g_l'\vec{\varphi}\cdot(\overline{\vec{\psi}}_lP_-s_l+\text{H.c.})\right]+\left|(i\partial_\alpha-e\vec{W}_\alpha\cdot\vec{T})\vec{\varphi}\right|^2+M_0{}^2\vec{\varphi}\cdot\vec{\varphi}+h(\vec{\varphi}\cdot\vec{\varphi})^2\,. \tag{4}$$

In the now-familiar manner,[1] $\varphi^\pm$ are eliminated by a gauge transformation, the gauge group is spontaneously broken by giving the remaining neutral component a nonvanishing vacuum expectation value λ, and a shifted field ϕ is defined, with $\langle\phi\rangle_0=0$. As a result of the spontaneous symmetry breaking, the fields W_α^+, W_α^- acquire a mass

$$M_W=e\lambda\,, \tag{5}$$

while the photon $A_\alpha=W_\alpha^0$ remains massless. The lepton masses are also split by the symmetry breaking. Determining g_l' in order to eliminate $X^0-\nu$ and $Y^0-\nu'$ direct interactions, one can solve for m_l^0 and g_l in terms of lepton masses[7]:

$$\begin{aligned}&2m_e^0=2m_{X^0}\cos\beta=m_{X^+}+m_e\,,\quad 2\lambda g_e=m_{X^+}-m_e\\&2m_\mu^0=2m_{Y^0}\cos\beta=m_{Y^+}+m_e\,,\quad 2\lambda g_\mu=m_{Y^+}-m_e\,.\end{aligned} \tag{6}$$

The mass of the neutral scalar field ϕ remains arbitrary.

The interaction terms in the Lagrangian are now completely determined, except for the ϕ self-interaction. Explicitly,

$$\begin{aligned}-\mathcal{L}_{\text{int}}=\;&eA_\alpha(\overline{X}^+\gamma^\alpha X^+-\overline{e}^-\gamma^\alpha e^-)\\&+eW_\alpha^+(\sin\beta\,\overline{\nu}\gamma^\alpha P_-e^-+\cos\beta\,\overline{X}^0\gamma^\alpha P_-e^-+\overline{X}^0\gamma^\alpha P_+e^-)+\text{H.c.}\\&+eW_\alpha^+(\sin\beta\,\overline{X}^+\gamma^\alpha P_-\nu+\cos\beta\,\overline{X}^+\gamma^\alpha P_-X^0+\overline{X}^+\gamma^\alpha P_+X^0)+\text{H.c.}\\&+(2eM_W\phi+e^2\phi^2)[(W_\alpha^+)^2+(W_\alpha^-)^2]+e\frac{m_{X^+}-m_e}{2M_W}\phi(\overline{X}^+X^+-\overline{e}^-e^-)\\&+e\frac{m_{X^+}+m_e}{2M_W}\sin\beta[\tan\beta\,\phi\overline{X}^0X^0+\phi(\overline{X}^0P_-\nu+\overline{\nu}P_+X^0)]+\{e\to\mu,\ X\to Y,\ \text{etc.}\}+F(\phi)+\mathcal{L}_W\,,\end{aligned} \tag{7}$$

where $F(\phi)$ is a quartic polynomial in ϕ chosen so that $\langle\phi\rangle_0=0$. The effective interaction Hamiltonian differs from the above expression only by the presence of the term

$$6i\delta^4(0)\ln(1+\phi/\lambda)$$

which arises because of the elimination of $\varphi^\pm$ and the shift performed in introducing ϕ.[13] The perturbation expansion is in powers of $e^2=4\pi\alpha$.

By comparing the lowest-order amplitude for μ decay obtained from Eq. (7) with experiment, one obtains

$$\frac{e^2\sin^2\beta}{4M_W{}^2}=\frac{G}{\sqrt{2}},$$

or

$$M_W=\sin\beta\sqrt{2}\ (37.3\ \text{GeV}/c^2)\,.$$

In Weinberg's model[1,4], 37.3 GeV/c^2 is the lower limit on the $W^\pm$ mass, and the lower limit on the mass of the neutral intermediate vector boson is twice as large.[14] In the SO(3) model, by contrast, 52.6 GeV/c^2 is the *upper* limit on M_W, and M_W could be much smaller. In particular, Georgi and Glashow,[7] in considering extensions of their SO(3) lepton model to include hadrons, have noted that in the simplest such models, K_1-K_2 mass splitting and $K_2^0\to\overline{\mu}\mu$ will be of order $G(GM_W{}^2)$. They conclude that $M_W\leq 4$ GeV/c^2 in such models, or $\sin\beta<0.1$. (We regard this conclusion as somewhat speculative, in the absence of more complete calculations.)

In this paper, we will determine the conditions placed on the parameters of the SO(3) model by various leptonic data, without making any detailed assumptions about how the model may be extended to include hadrons. Two features of the interaction Lagrangian make these limits stronger than might be initially supposed. First, although the ordinary weak coupling $W_\alpha^+\overline{\nu}\gamma^\alpha P_-e$ is suppressed by a possibly small factor of $\sin\beta$, the coupling to the heavy leptons X^0, X^+ is not. Second, the coupling of ϕ to leptons is of order em_{X^+}/M_W or em_{Y^+}/M_W.

The masses m_{X^+} and m_{Y^+} are not necessarily small compared to M_W.

III. $g-2$

The four Feynman graphs which contribute to muon $g-2$ in lowest order in the SO(3) model are drawn in Fig. 1. The first graph, Fig. 1(a), is just the familiar electromagnetic correction. This term and the higher-order electromagnetic corrections add up to[15]

$$(a_\mu)_{\mathrm{QED}} = \frac{\alpha}{2\pi} + 0.765\,78\left(\frac{\alpha}{\pi}\right)^2 + (21.8 \pm 1.3)\left(\frac{\alpha}{\pi}\right)^3 + \cdots = (11\,658.1 \pm 0.2) \times 10^{-7}.$$

The strong corrections to the QED (quantum electrodynamics) calculation are estimated[15] to be $(0.65 \pm 0.1) \times 10^{-7}$. These numbers are to be compared with the experimental value[15]

$$(a_\mu)_{\mathrm{exp}} = (11\,661.6 \pm 3.1) \times 10^{-7}.$$

The difference is

$$(a_\mu)_{\mathrm{exp}} - (a_\mu)_{\mathrm{QED}} - (a_\mu)_{\text{strong connections}} = (2.8 \pm 3.1) \times 10^{-7}.$$

It thus seems reasonable to conclude that

$$-3 \times 10^{-7} \leq (a_\mu)_{\mathrm{weak}} \leq 9 \times 10^{-7}, \tag{8}$$

allowing for a discrepancy of two standard deviations.

The second graph, Fig. 1(b), has been calculated by several authors.[16] It is logarithmically divergent unless $g_W = 2$. Since $g_W = 2$ in models where the vector mesons have Yang-Mills coupling, there is no divergence in our case and

$$(a_\mu)_b = \frac{\alpha \sin^2\beta}{8\pi} \frac{m_\mu^2}{M_W^2} \frac{10}{3} \approx 3 \times 10^{-9},$$

which is negligible.

The next graph, Fig. 1(c), is evaluated in the Appendix. Dropping terms of order m_μ/m_{Y^0}, we find ($r \equiv m_{Y^0}^2/M_W^2$)

$$(a_\mu)_c = \frac{-\alpha}{2\pi} \frac{m_\mu m_{Y^0} \cos\beta}{M_W^2}\left(3 \int_0^1 \frac{dz\, z^2}{z + (1-z)r} + \frac{1}{2}\right)$$
$$= \frac{-\alpha}{8\pi} \frac{m_\mu (m_{Y^+} + m_\mu)}{M_W^2} \times \left[\frac{3}{(1-r)^2}\left(1 - 3r - \frac{2r^2}{1-r}\ln r\right) + 1\right]. \tag{9}$$

The graph 1(d) was evaluated by Jackiw and Weinberg[9] in the SU(2)×U(1) model and found to be $\lesssim 10^{-9}$. For the SO(3) model it is considerably

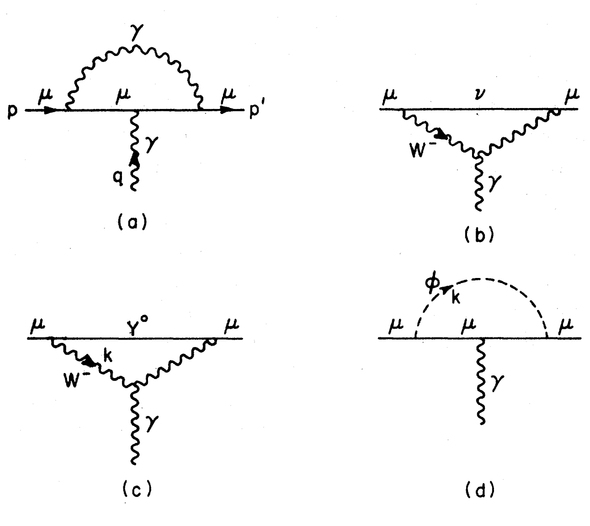

FIG. 1. The Feynman graphs which contribute to $(g-2)$ in order e^2.

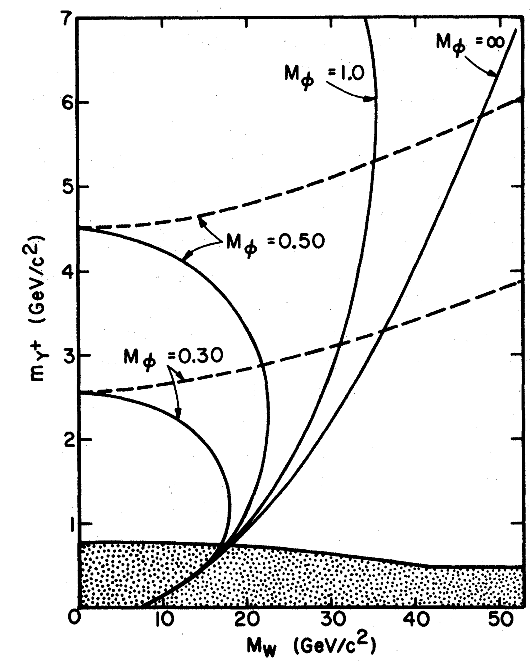

FIG. 2. Constraints on the vector and heavy lepton masses (M_W and m_{Y^+}) associated with muon $(g-2)$. Experimental agreement with QED requires $-3 \times 10^{-7} \leq (a_\mu)_{\mathrm{weak}} \leq 9 \times 10^{-7}$ (allowing for two standard deviations), where $(a_\mu)_{\mathrm{weak}} = (a_\mu)_b + (a_\mu)_c + (a_\mu)_d \simeq (a_\mu)_c + (a_\mu)_d$. Since $(a_\mu)_d$, Eq. (10), depends upon M_ϕ, curves are presented for several values of M_ϕ. The solid and dashed curves for each value of M_ϕ correspond respectively to the lower and upper limits on $(a_\mu)_{\mathrm{weak}}$; the allowed region lies to the right of both curves. The shaded region at the bottom of the figure is excluded by the observation that Y^+ and Y^0 are not decay products of K^+. The upper limit $M_W = 52.7$ GeV/c^2 is imposed by the model.

larger:

$$(a_\mu)_d = \frac{\alpha}{8\pi}\frac{(m_{Y^+}-m_\mu)^2}{M_W^2}\frac{m_\mu^2}{M_\phi^2}\int_0^1 dz\,\frac{2z^2-z^3}{z^2\rho-z+1},\qquad \rho\equiv m_\mu^2/M_\phi^2\,. \quad (10)$$

In Fig. 2 we present graphically the constraints on M_W and m_{Y^+} $(=2m_{Y^0}\cos\beta - m_\mu)$ following from Eq. (8). If M_ϕ is very large, then $(a_\mu)_d$ is negligible unless m_{Y^+} is comparably large; in this case we can conclude that $M_W \gtrsim 18$ GeV$/c^2$. However, in the absence of any further information on m_{Y^+} and M_ϕ, we cannot rule out the possibility of a cancellation between $(a_\mu)_c$ and $(a_\mu)_d$, in which case M_W can be small without violating Eq. (8). On the other hand, if M_ϕ is small then m_{Y^+} is bounded above by the requirement that $(a_\mu)_d$ not grow too large.

Since Georgi and Glashow's[7] simple five-quark extension of their model to include hadrons requires $M_W \lesssim 4$ GeV$/c^2$, this can only be consistent with muon $g-2$ if there is a cancellation between $(a_\mu)_c$ and $(a_\mu)_d$. While this cancellation cannot be ruled out on the basis of the discussion thus far, if we make the reasonable estimate that $M_\phi > \frac{1}{2}$ GeV$/c^2$ – based on the fact that the ϕ has not been seen in K decay or e^+e^- scattering or (though its virtual effects) in muonic atoms, as discussed further below – then the cancellation can occur only for a Y^+ mass greater than 4.5 GeV$/c^2$.

The electron $\frac{1}{2}(g-2)$ is measured to an accuracy of about 3×10^{-9}, and agrees with QED to within the quoted errors.[15] There are weak contributions to electron $g-2$ from graphs analogous to Figs. 1(b)–1(d), but these contributions are smaller for the electron by the factor (m_e/m_μ) for (c) and $(m_e/m_\mu)^2$ for (b) and (d). Thus the lower bound on M_W obtained by considering just graph (c) for electron $g-2$ is about half that obtained previously from muon $g-2$, assuming $M_\phi=\infty$. Since graph (d) is much smaller for the electron, however, the cancellation discussed above for the muon for relatively small M_ϕ is a negligible possibility for the electron.

We conclude that

$$M_W \gtrsim 10 \text{ GeV}/c^2$$

from electron $g-2$, with a possible factor of two increase in this lower bound from consideration of muon $g-2$ if m_{Y^+}/M_ϕ^2 is small so that graphs (c) and (d) cannot cancel.

IV. MUONIC ATOMS

Extremely accurate measurements of muonic energy levels have recently become available through the work of Anderson, Dixit, and co-workers[17,18] and Backenstoss *et al.*[19] As Jackiw and Weinberg[9] have observed, the main effect on muonic atoms in the models we are discussing arises from ϕ exchange between the μ and the nucleus, which in lowest order and static approximation generates a potential

$$V_\phi(r) = \sqrt{2}\, m_\mu^2 G A \xi\zeta\, e^{-rM_\phi}/r\,,$$

$$\zeta = \frac{1}{\sin^2\beta}\,\frac{m_{Y^+}-m_\mu}{m_\mu}\,.$$

In writing this expression, we have followed Jackiw and Weinberg[9] in assuming that the coupling of ϕ to the nucleons is given by $(\sqrt{2}G)^{1/2}\phi\delta\mathcal{L}$, where the one-nucleon matrix elements of $\delta\mathcal{L}$ are estimated to equal ξm_μ, with $0.1 \lesssim \xi \lesssim 10$. In the Georgi-Glashow SO(3) five-quark model there are definite expressions for the ϕ-quark coupling in terms of quark masses, but we trust that the above will be an adequate physical parametrization.[20]

Measurements of a large number of transitions in muonic ^{206}Pb were used by Anderson *et al.*[17] to determine the two parameters of the nuclear charge distribution; then the Dirac equation was solved numerically, and radiative and other corrections included, in order to calculate the theoretical energy levels. The largest source of error in the calculated energy levels is the nuclear polarization by the muon; the uncertainty is estimated to be ±3 keV for the $1s$ level, ±1 keV for $2s$, and negligible for all other levels. All of the measured transition energies agree with these calculations to within the experimental errors, which are typically 0.1 to 0.5 keV. Additional measurements of the $4f$-$3d$ and $5g$-$4f$ transitions in Pb by Backenstoss *et al.*[19] also agree with calculations to within experimental errors, which are 0.10 and 0.04 keV, respectively. The most recent measurements are by Dixit *et al.*[18] These measurements, which have quoted experimental uncertainties of less than ±0.02 keV, are in disagreement with the calculated transition energies for $5g$-$4f$ transitions in Pb, and with the Backenstoss *et al.* measurements, by about 0.10 keV. Dixit *et al.* find similar but smaller discrepancies in lighter elements. We conclude that for Pb there is room for shifts ranging from $\lesssim 1$ keV for $2s$ to $\lesssim 0.1$ keV for the $3d$ or $4f$ levels, without contradicting these experiments.

Noting that the nuclear radius of Pb is about 18 fm, and the μ atomic radius is (30 fm)n^2, we conclude that it is not unreasonable to estimate the effect of V_ϕ for $n \geq 2$ by using hydrogenic (point-nucleus) wave functions in lowest-order perturbation theory. For states with $l=n-1$, we obtain

$$\langle V_\phi\rangle_{n,l=n-1} = (1.56\times10^{-7})A\xi\zeta \times\frac{1}{n^2}\left(1+\frac{M_\phi}{2z/na_0}\right)^{-2n}\frac{Z}{a_0}\hbar c\,,$$

where

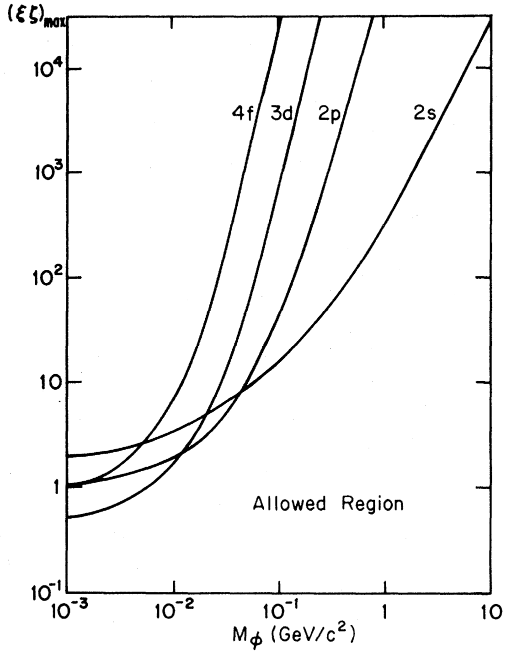

FIG. 3. The muonic-atom data constrain $\xi\zeta$ to lie below these curves, where ξ parametrizes the strength of the ϕ coupling to nucleons, and $\zeta=(M_W/53\ \mathrm{GeV}/c^2)^{-2}\times(m_{Y^+}-m_\mu)/m_\mu$. In obtaining these curves, we have used $V_\phi<1$ keV for $2s$, <0.5 keV for $2p$, and <0.1 keV for $3d$ and $4f$ energy levels in muonic Pb. (A decrease in the value of V_ϕ allowed by the data for any level would correspond to a rescaling of $(\xi\zeta)_{\max}$ by the same fraction).

$$a_0=\hbar/\alpha m_\mu c\,.$$

We will also need

$$\langle V_\phi\rangle_{2s}=(1.56\times10^{-7})A\xi\zeta\left(\frac{Z}{a_0}\right)\frac{\hbar c}{2p^2}\left(1-\frac{2}{p}+\frac{3}{2p^2}\right),$$

$$p\equiv\left(1+\frac{M_\phi}{Z/a_0}\right).$$

For Pb, $Z\hbar c/a_0=63$ MeV and $(1.56\times10^{-7})AZ\hbar c/a_0=2.02$ keV.

In Fig. 3 we present the constraints on the Y^+, W, and ϕ masses implied by the Pb muonic atom data. Since $m_{Y^+}/m_\mu>5$, and ξ is thought[20] to be at least 1, if $M_W=10$ GeV/c^2 it follows that $\xi\zeta\approx125$; this implies $M_\phi\gtrsim500$ MeV. A small, but not impossible, value for $\xi\zeta$ is 5; then $M_\phi\gtrsim30$ MeV. Without more information, it is difficult to say more.[21]

V. DISCUSSION AND SUMMARY

Spontaneously broken gauge models offer the possibility of unifying in a natural manner electromagnetic and weak interactions of the leptons, and possibly also those of the hadrons. In addition, these models appear to be renormalizable if they are treated sufficiently carefully. These attractive features are obtained at the cost of introducing several classes of presently unobserved particles: (a) massive vector bosons (here, $W^\pm$), (b) one or more massive scalar bosons (ϕ), (c) heavy leptons, and possibly also (d) heavy "charmed" hadrons.

Weinberg's simple $\mathrm{SU}(2)_L\times\mathrm{U}(1)_R$ model does not explicitly include unobserved heavy leptons (although additional fermion fields are necessary in this model to avoid anomalous Ward identities, as we have mentioned). However, models which include only the observed leptons possess neutral weak leptonic currents. There is no experimental evidence favoring the existence of such currents, and an unsuccessful search for reactions of the form $\nu_\mu+N\to\nu_\mu+N+\pi^0$ ($N=p,n$) in optical-spark-chamber data has recently provided strong evidence[22] against the existence of a neutral neutrino current, and thus against the $\mathrm{SU}(2)_L\times\mathrm{U}(1)_R$ model.[23]

It is possible to construct limitless numbers of spontaneously broken gauge models consistent with the absence of neutral currents and also with the absence of triangle-graph anomalies. These models will in general have various numbers of hypothetical mesons and leptons. It is difficult to choose the most attractive among this multitude of possibilities. Obviously, it would be exceedingly helpful if even one of the hypothetical bosons or fermions were to be observed.

In the absence of such direct information, we have chosen in this paper to discuss the Georgi-Glashow SO(3) model, both because of its aesthetic simplicity and because it is constructed in such a way that it can be strongly constrained by experiment. Specifically, the intermediate vector boson $W^\pm$ cannot be heavier than 53 GeV/c^2 and can weigh much less, and the scalar meson and heavy leptons may be coupled almost with electromagnetic strength to the normal leptons and to the vector bosons.

In Sec. III of this paper we have considered the contributions to the muon $g-2$ in this model. The requirement that these be within the very good experimental limits provides the relations among the masses of the vector and scalar mesons and the heavy leptons shown in Fig. 2. Making the not unreasonable assumption that the ϕ mass is greater than 0.5 GeV/c^2, we find that $M_W>18$ GeV/c^2 unless the mass of the Y^+ is such as to cause a cancellation between the contributions from the Feynman graphs of Figs. 1(c) and 1(d). Such cancellations would require a Y^+ mass on the order of ten times the ϕ mass. This cancellation is a negligible possibility for the electron, so a firm lower bound on the vector-meson mass of $M_W\gtrsim10$ GeV/c^2 can be obtained by considering electron $g-2$.

In Sec. IV we considered muonic-atom data and showed that if M_W is as light as 10 GeV/c^2, then even if the scalar meson ϕ couples no more strongly to nucleons than it does to leptons, $M_\phi \gtrsim 500$ MeV/c^2. The scalar mass in this model could be smaller than this if $\sin\beta = M_W/(53\ \text{GeV}/c^2) \approx 1$ and m_{Y^+} were not too large; however, if M_ϕ were much less than M_K, it would doubtless have been seen in K decay. Thus we conclude that in this model $M_\phi \gtrsim 500$ MeV/c^2.

It will perhaps be of some interest to mention briefly some additional properties of the ϕ and the heavy leptons in the Georgi-Glashow model.

Scalar meson (ϕ). The ϕ will decay into e^+e^- and $\mu^+\mu^-$ in roughly equal proportions if $m_{X^+} \approx m_{Y^+}$. The partial decay rates are

$$\Gamma_{\phi \to e^+e^-} = \tfrac{1}{8}\alpha\left(\frac{m_{X^+}}{M_W}\right)^2 M_\phi ,$$

$$\Gamma_{\phi \to \mu^+\mu^-} = \tfrac{1}{8}\alpha\left(\frac{m_{Y^+}}{M_W}\right)^2 M_\phi .$$

Thus the ϕ is relatively short-lived: A ϕ of mass 500 MeV/c^2 would have a half-life of less than 1.5 $\times 10^{-17}$ sec. If it is sufficiently massive, which it presumably is, the ϕ will also have hadronic decay modes.

The ϕ can be produced by "bremsstrahlung" in sufficiently energetic scattering events. It can also be exchanged virtually in many scattering processes, and may contribute significantly to certain processes which are regarded as tests of QED. In particular, it can be searched for in e^+e^- colliding-beam experiments, where it would produce a large but narrow peak. For example, in $e^+e^- \to \mu^+\mu^-$, for a bin of width $\Delta s = \eta M_\phi^2$ centered at M_ϕ^2, one would see an enormous cross-section enhancement

$$\frac{\sigma}{\sigma_{\text{QED}}} = 1 + \tfrac{3}{8}\pi\left(\frac{m}{M_W}\right)^2 \frac{1}{\alpha\eta},$$

where m equals the smaller of m_{X^+}, m_{Y^+} and where $\alpha = \frac{1}{137}$.

Massive Leptons.[24] The heavy μ-type lepton Y^+ will decay weakly, analogously to μ decay, except that it is sufficiently massive that the decay will be much faster; also, hadrons can appear in the final state. The Y^0 has similar decay modes, for example $Y^0 \to \mu^- e^+ \nu_e$ or $Y^0 \to \mu^- +$ hadrons. Such decays would obviously be very striking to observe. These remarks also apply, *mutatis mutandis,* to the e-type leptons X^+, X^0.

These heavy leptons should be produced in reactions of the sort $\nu_\mu p \to Y^+ +$ (hadrons) at rates comparable to that of $\nu_\mu p \to \mu^- +$ (hadrons) as soon as sufficient energy is available. It should thus be possible at NAL either to find these particles, or else to put rather high lower limits on their mass – which would also increase the lower limit on the W mass in the Georgi-Glashow SO(3) model (cf. Fig. 2). The charged leptons will also be pair-produced electromagnetically as soon as sufficient energy is available in colliding beam machines. There is obviously a rich variety of ways to seek these hypothetical particles.

Despite the obviously attractive features of the spontaneously-broken-gauge-symmetry approach toward constructing a theory of weak interactions, the models of this sort which have been constructed thus far possess evident shortcomings. In the absence of strong theoretical as well as experimental constraints, these models all appear rather artificial and *ad hoc*. Restricting our attention to models of the leptons and neglecting strong interactions, we would like our models to give at least a little insight into the role of the muon and the origin of the μ-e mass difference. Instead, additional massive leptons are introduced, and they possess an even more puzzling mass spectrum. Particularly curious in the SO(3) model considered here is the introduction of leptons with masses exceeding 500 MeV/c^2 in the same multiplet as the electron. The large mass of the W, and the consequent weakness of the weak interactions, also remains unexplained.

Note added in proof. Zakharov has kindly informed us of a forthcoming paper by himself and Okun [V. I. Zakharov and L. B. Okun, Zh. Eksp. Teor. Fiz. (to be published)] on the subject of detecting anomalous muon-nucleon interactions by analysis of muonic atom data. They conclude that short-range interactions can be mostly absorbed into a readjustment of the nuclear charge radius. If this applies to the potential generated by ϕ exchange, our lower bounds on M_ϕ, obtained by considering muonic-atom data, should be relaxed.

In any case, $M_\phi \gtrsim \frac{1}{2}$ GeV/c^2 since ϕ is not seen in K decay. A much more stringent lower bound on M_ϕ can be obtained in the Georgi-Glashow 5-quark model by considering $K^+ \to \pi^+ e^+ e^-$. However, the 5-quark model predicts values for $\Gamma(K_L \to \bar{\mu}\mu)$ and $m_{K_1} - m_{K_2}$ which are independent of M_W and much larger than experiment, so this model must be abandoned. The 8-quark variant of this model appears to be free of such difficulties, however. [See B. W. Lee, J. R. Primack, and S. B. Treiman, Phys. Rev. D (to be published).] It should be understood that the calculations in this present paper are independent of which model is chosen for hadrons.

ACKNOWLEDGMENTS

We would like to thank Howard Georgi and Shelley Glashow for much enlightenment and encouragement. We are also indebted to Ben Lee for pointing out to us an error in the previous version of this paper, as mentioned in the Appendix.

APPENDIX

We will evaluate the leading term in $g-2$, of order $m_\mu m_{Y^0}/M_W^2$, coming from the Feynman graph Fig. 1(c). We work in the "U formalism," in which the W propagator has the canonical form, and obtain

$$\Lambda_\gamma = \frac{ie^2}{(2\pi)^4}\int d^4k\bar{u}(p')\gamma_{\alpha'}[m_{Y^0}\cos\beta + \tfrac{1}{2}(\not{p}-\not{k})(1+\cos^2\beta-\gamma_5\sin^2\beta)]\gamma_{\beta'}u(p)$$
$$\times\frac{(k+q)_\alpha(k+q)_{\alpha'}/M_W^2 - g_{\alpha\alpha'}}{(k+q)^2-M_W^2}\,\frac{k_\beta k_{\beta'}/M_W^2 - g_{\beta\beta'}}{k^2-M_W^2}\,\frac{1}{(p-k)^2-m_{Y^0}^2}$$
$$\times[(k-q)_\alpha g_{\beta\gamma} + (2q+k)_\beta g_{\alpha\gamma} - (2k+q)_\gamma g_{\alpha\beta}]. \tag{A1}$$

We will calculate only the leading term in $g-2$, of order $m_\mu m_{Y^0}/M_W^2$; we thus keep only the $m_{Y^0}\cos\beta$ term in the first bracket in the above expression. The product of the numerators of the W propagators gives four terms. The $(k+q)_\alpha k_\beta$ term is divergent, but it does not contribute to $g-2$ since it vanishes at $q=0$. The divergent parts of the mixed $k-g$ terms cancel if treated naively (this remains true for any choice of the internal momentum routing), and the remainder is negligible. However, a more careful treatment of this divergent part is required.

It is particularly simple to use for this purpose the gauge-invariant regulation procedure of 't Hooft and Veltman,[25] in which one continues possibly divergent integrals away from $n=4$ using, for example,

$$\int d^nk\frac{1}{(k^2+2k\cdot l+m^2)^\alpha} = \frac{i\pi^{n/2}}{(m^2-l^2)^{\alpha-n/2}}\,\frac{\Gamma(\alpha-\frac{1}{2}n)}{\Gamma(\alpha)}; \tag{A2}$$

similarly, symmetric integration takes the form

$$k_\mu k_\nu \to k^2 g_{\mu\nu}/n.$$

The integral in question is of the form

$$\int d^4k\frac{4k\cdot pk_\mu - k^2p_\mu}{(k^2+a)^3} = \lim_{n\to4}\int d^nk\frac{k^2p_\mu(4/n-1)}{(k^2+a)^3}$$
$$= \lim_{n\to4} i\pi^2\tfrac{1}{2}n\frac{\Gamma(2-n/2)}{\Gamma(3)}p_\mu(4/n-1)$$
$$= \tfrac{1}{2}i\pi^2p_\mu. \tag{A3}$$

The second term in the bracket in Eq. (9) arises in this way if the 't Hooft-Veltman regulation is used. The first term in the bracket is just the contribution of the $g_{\alpha\alpha'}g_{\beta\beta'}$ term in Eq. (A1).

An alternative evaluation of this Feynman graph is presented in a paper by Fujikawa, Lee, and Sanda,[26] who introduce a "generalized renormalizable gauge." We wish to thank Professor B. Lee for telling us of this work and emphasizing to us the importance of using a gauge-invariant regulator procedure; we had initially failed to do so and thereby missed the second term in Eq. (9). It should perhaps be noted that the naive result (without this term) is routing-independent and satisfies the relevant Ward identity.[27] We believe Eq. (9) is the correct result because the regularization procedures which lead to it appear to preserve renormalizability in all orders.[25,26]

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[1]S. Weinberg, Phys. Rev. Letters 19, 1264 (1967).

[2]G. 't Hooft, Nucl. Phys. B35, 167 (1972).

[3]B. W. Lee, Phys. Rev. D 5, 823 (1972); B. W. Lee and J. Zinn-Justin, *ibid.* 5, 3121 (1972); 5, 3137 (1972); 5, 3155 (1972).

[4]S. Weinberg, Phys. Rev. Letters 27, 1688 (1971).

[5]T. W. Appelquist and H. R. Quinn, Phys. Letters, 39B, 229 (1972).

[6]T. W. Appelquist, J. R. Primack, and H. R. Quinn, Phys. Rev. D 6, 2998 (1972).

[7]H. Georgi and S. L. Glashow, Phys. Rev. Letters 28, 1494 (1972). J. D. Bjorken has independently constructed and analyzed similar models (private communication).

[8]C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Letters 38B, 519 (1972); D. J. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972); H. Georgi and S. Glashow, *ibid.* 6, 429 (1972). The fact that additional fermion fields besides the leptons are required in Weinberg's model (or similar models) may perhaps be a theoretical advantage, providing a constraint on the construction of the full Lagrangian including hadrons in order that the complete theory be anomaly-free.

[9]R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2396 (1972); I. Bars and M. Yoshimura, *ibid.* 6, 374 (1972).

[10]Here, $P_\pm = \frac{1}{2}(1\pm\gamma_5)$. We use the metric and notation of J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[11]One could also generate such terms by introducing a scalar singlet with nonzero vacuum expectation value. (This was done by Bjorken, Ref. 7.) The additional contributions from such a scalar singlet would only

strengthen the constraints derived in Secs. III and IV.

[12]Terms like $\vec{\varphi}\cdot\bar{\vec{\psi}}_l P_+ s$, which would lead to a neutrino mass, are not included.

[13]Cf. Ref. 4 and references cited there. A simple functional-integral derivation is given by A. Salam and J. Strathdee, Nuovo Cimento 11A, 397 (1972). See also Gross and Jackiw, Ref. 8.

[14]H. H. Chen and B. W. Lee [Phys. Rev. D 5, 1874 (1972)] have pointed out that $\nu e \to \nu e$ data imply $M_W > 65$ GeV in the Weinberg model.

[15]Here $a_\mu = [\frac{1}{2}(g-2)]_\mu = F_2^\mu(0)$. Theory and data are summarized by S. J. Brodsky, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972).

[16]T. Burnett and M. J. Levine, Phys. Letters 24, 467 (1967); S. J. Brodsky and J. Sullivan, Phys. Rev. 156, 1644 (1967).

[17]H. L. Anderson *et al.*, Phys. Rev. 187, 1565 (1969).

[18]M. S. Dixit *et al.*, Phys. Rev. Letters 27, 878 (1971).

[19]G. Backenstoss *et al.*, Phys. Letters 31B, 233 (1970).

[20]Cf. Jackiw and Weinberg, Ref. 9. In the SO(3) five-quark model of Georgi and Glashow, $\xi = (m_{q-} - m_p)/m_\mu$ for a proton quark, where m_{q-} is the mass of a heavy negatively charged quark. Thus ξ might be considerably larger than unity.

[21]If one takes seriously the discrepancies between the calculated and measured transition energies reported by Dixit *et al.*, Ref. 18, one may attempt to ascribe the discrepancies to V_ϕ. A fair fit to all the Dixit *et al.* measurements is obtained with $M_\phi = 0$ and $\xi\zeta = 2$ [cf. G. A. Rinker, Jr., and M. Rich, Phys. Rev. Letters 28, 640 (1972); $M_\phi = 0$ and $\xi\zeta = 2$ correspond to their c_{11}.] The largest value of M_ϕ permitted by these data is $M_\phi \approx 2$ MeV, with $\xi\zeta = 1.5$. A glance at Fig. 3 shows that these values are inconsistent with constraints obtained from other data. Thus a mechanism like V_ϕ cannot be responsible for the discrepancies noted by Dixit *et al.*

[22]Wonyong Lee, Phys. Letters 40B, 423 (1972).

[23]B. Lee, Phys. Letters 40B, 420 (1972).

[24]C. H. Llewellyn Smith and J. D. Bjorken have investigated the various modes of production and decay of massive leptons of the sort that populate hypothetical models of weak interactions. We would like to thank them for helpful conversations.

[25]G. 't Hooft and M. Veltman, Nucl. Phys. B44, 189 (1972); W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. B46, 319 (1972).

[26]K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D 6, 2923 (1972), and references contained therein.

[27]We thank Professor R. Jackiw for raising this point.

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Analysis of the Anomaly in the $\pi^+\pi^-$ System near the $K\bar{K}$ Threshold*

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An absorption-modified pion-exchange model with coupled-channel unitarity and analyticity for the $\pi^+\pi^-$ system near the $K\bar{K}$ threshold is introduced. Data-analysis techniques for the reactions $\pi + N \to$ meson + meson + baryon are discussed.

In recent experiments,[1] the $\pi^+\pi^-$ and K^+K^- systems produced in the reactions

$$\pi + N \to \pi^+ + \pi^- + \text{baryon}, \tag{1}$$

$$\pi + N \to K^+ + K^- + \text{baryon} \tag{2}$$

have been studied at high incident energies and small baryon momentum transfers in a region of $\pi\pi$ masses including the $K\bar{K}$ threshold point. It is found that the $\pi\pi$ "decay" angular distribution has a forward-backward asymmetry moment $\langle Y_1^0 \rangle$ which exhibits a precipitous, almost discontinuous step from about 0.15 to zero in a 10-MeV interval in $\pi\pi$ mass at $m_{\pi\pi} \simeq 980$ MeV, along with a 50% drop in $d\sigma/dm_{\pi\pi}$. It has been duly pointed out that the discontinuity in $\langle Y_1^0 \rangle$ could be due to the strong onset of the S-wave $K\bar{K}$ channel, which absorbs some of the $\pi\pi$ S-wave probability, thereby reducing the $\pi\pi$ SP-wave interference in $\langle Y_1^0 \rangle$. The effect is actually complicated and difficult to study in terms of $\pi\pi$ scattering amplitudes or phase shifts. However, we find the effect should be susceptible to careful analysis of the "decays" and the production mechanisms for reactions (1) and (2). In this paper we argue that the analysis should be done, and we discuss the techniques necessary to do it properly. The yield could be very interesting information on $\pi\pi$ and even $K\bar{K}$ elastic scattering amplitudes. The technique to be described includes absorption-modified one-pion exchange together with a K-matrix approach for the coupled-channel meson-meson scattering problem. It is clear that coupled-channel unitarity and analyticity near the $K\bar{K}$ threshold are required in the analysis and will be crucial in resolving at least part of the isoscalar S-wave $\pi\pi$ phaseshift ambiguity in the region 750–1000 MeV.[2]

The reduction of the data for reactions (1) and (2)