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PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Double-Pole Model for the K_{l3} Form Factors

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(Received 17 July 1972)

The model of Gerstein, Gottfried, and Huang for the pion electromagnetic form factor is extended to K_{l3} decays. One characteristic of the model is the appearance of double poles which enhance the slopes of form factors. One gets $\lambda_+ \simeq 0.05$ and $\xi(0) \simeq -0.32$.

I. INTRODUCTION

Conventional theories[1] fail to explain why the $\xi(0)$ parameter in K_{l3} decays is nearly equal to -1 as reported in several recent experiments.[2] This discrepancy follows from the fact that the slope λ_0 of the scalar form factor $f_0(t)$ is generally predicted to be small and positive ($\lambda_0 \simeq 0.01-0.03$) and the slope λ_+ of the vector form factor $f_+(t)$ is usually assumed to be determined by a simple K^* pole ($\lambda_+ \simeq 0.023$). Since $\xi(0)$ is proportional to the difference between λ_0 and λ_+, whenever these parameters have the same sign and approximately the same magnitude, $\xi(0)$ cannot reach the value -1 as determined by experiment.

In order to account for the experimental data, one is forced to assume some mechanisms which either modify λ_0 in such a way that it becomes negative (unconventional symmetry breaking,[3] zero in the scalar form factor,[4] or second-class currents[5]) or to enhance λ_+ (K^* dominance through a dipole[6] or second-class currents[5]). The effective dipole form factor, known to describe the nucleon electromagnetic interactions, might be a universal feature indicating that vector-meson dominance should be expressed through second-order poles. Gerstein, Gottfried, and Huang[7] (hereafter referred to as GGH) recently considered a model for the pion electromagnetic form factor that incorporates both the constraints given by current algebra and the duality property of strong interactions and that gives rise to double poles in a natural way. The model has some difficulties already pointed out by Grisaru and Visinescu[8]: (i) Contribution of the diffractive part of the pion-pion scattering amplitude has to be treated separately. (ii) The disconnected graph and the diffractive part must conspire in such a way as to make the form factor vanish at $t=\infty$. (iii) Only elastic scattering is considered in the intermediate states.

In the present note, the model of GGH is extended to the study of the scalar and vector K_{l3} form factors. One assumes, as a working hypothesis, that the W meson (in analogy to the photon) interacts with hadrons through pair creation as in field theory and that the t dependence of the form factors arises from the lowest-order πK final-state interaction which is assumed to be a Veneziano type. Since the $\xi(0)$ parameter is proportional to the difference between the scalar and vector form factors, it is reasonable to believe that in evaluating this parameter from the model, the effect of the above-mentioned difficulties may be attenuated. At first sight, one might guess that since the Veneziano amplitude is used, $f_+(t)$ and $f_0(t)$ will both have double poles at positions determined by the K^* trajectory, and their slopes will be equal, yielding $\xi(0)=0$. However, as we shall see in more detail, the residues of the poles are different and although this difference leads to a negative value of $\xi(0)$, it does not appear to be great enough to reproduce the experimental value −1.

A somewhat similar approach has already been given by Yamada[9] who considers a scattering of K, π with spinless quarks (λ, $\mathcal{P}$). The amplitude has a pure Regge form (even in the low-energy region). Double poles then also arise explicitly for the form factors. However, the model makes use of some unknown parameters (Regge residues, quark mass) hence the slope λ_+ cannot be predicted.

II. SCALAR AND VECTOR FORM FACTORS

Usually, the matrix element for the decay $K^0 \to \pi^- l^+ \nu$ is written in terms of two invariant form factors $f_+(t)$ and $f_-(t)$ defined by (apart from the conventional normalizations)

$$\langle \pi^-(p_2)| V_\mu^{\Delta S=1} | K^0(p_1)\rangle = (p_1+p_2)_\mu f_+(t) + (p_1-p_2)_\mu f_-(t)$$

with $t=(p_1-p_2)^2$. The dispersion relation for $f_+(t)$ receives contributions only from $J=1$ intermediate states [hence $f_+(t)$ is called the vector form factor]. On the other hand, $f_-(t)$ is not dynamically independent and it is convenient to introduce a third form factor $f_0(t)$ which is proportional to the matrix element of the divergence $\partial_\mu V_\mu^{\Delta S=1}$ which obviously receives contributions only from s-wave intermediate states:

$$(m^2-\mu^2) f_0(t) \equiv \langle \pi^-(p_2)| \partial_\mu V_\mu^{\Delta S=1} | K^0(p_1)\rangle = (m^2-\mu^2) f_+(t) + t f_-(t),$$

where m and μ denote the K and π masses. The scalar form factor

$$f_0(t) = f_+(t) + \frac{t}{m^2-\mu^2} f_-(t)$$

and the vector form factor $f_+(t)$ are dynamically independent.

If we define λ_+ and λ_0 as the logarithmic derivatives of $f_+(t)$ and $f_0(t)$, respectively, at $t=0$, i.e.,

$$\lambda_+ = \frac{1}{\mu^2}\frac{f_+'(0)}{f_+(0)}, \quad \lambda_0 = \frac{1}{\mu^2}\frac{f_0'(0)}{f_0(0)},$$

then $\xi(0) \equiv f_-(0)/f_+(0)$ is related to λ_+ and λ_0 by

$$\xi(0) = \frac{m^2-\mu^2}{\mu^2}(\lambda_0 - \lambda_+). \quad (1)$$

Now, following GGH, the K_{l3} form factors are obtained through the Feynman graphs of Fig. 1 which give the K_{l3} matrix element proportional to

$$p_\mu - \frac{i}{(2\pi)^4}\int d^4q \frac{(2q-p)_\mu A(t, s=q^2, u)}{[(q-p_1)^2-m^2][(q-p_2)^2-\mu^2]}, \quad (2)$$

where $p_\mu = (p_1+p_2)_\mu$, $t=(p_1-p_2)^2$, $u=(p-q)^2$, and $A(t, s=q^2, u)$ is the invariant amplitude for $K\pi$ scattering in the isospin-$\frac{1}{2}$ state with squared c.m. energy t and squared momentum transfer $s=q^2$. We assume that the $\Delta I = \frac{1}{2}$ rule is quite adequate in semileptonic strangeness-changing weak interactions, as is well confirmed by experiments. The scalar form factor $f_0(t)$ is obtained from Eq. (2) by multiplying (2) by $k_\mu = (p_1-p_2)_\mu$. Thus we have

$$f_0(t) = 1 + \frac{i}{(2\pi)^4}\int d^4q \frac{A(t,q^2,u)}{(q^2-2q\cdot p_1)(q^2-2q\cdot p_2)} \times \left(1 - \frac{2k\cdot q}{m^2-\mu^2}\right). \quad (3)$$

By multiplying Eq. (2) by the projection operator $D_\mu = tp_\mu - (m^2-\mu^2)k_\mu$, we obtain for the vector form factor

$$f_+(t) = 1 + \frac{i}{(2\pi)^4}\int d^4q \frac{A(t,q^2,u)}{(q^2-2q\cdot p_1)(q^2-2q\cdot p_2)} \times \left(1 - \frac{2D\cdot q}{D\cdot p}\right). \quad (4)$$

The scattering amplitude $A(t, q^2, u)$ which enters in Eqs. (3) and (4) is assumed to contain the Veneziano terms (narrow resonances with Regge asymptotic behavior) plus a diffractive part containing the Pomeranchukon. Correspondingly, the form factors $f_+(t)$ and $f_0(t)$ can be decomposed as

$$f_+(t) = 1 + f_+^V(t) + f_+^P(t),$$
$$f_0(t) = 1 + f_0^V(t) + f_0^P(t).$$

It is plausible that for small t, $f_+^P(t) \simeq f_0^P(t)$, since GGH have argued that the diffractive contribution to the pion form factor represents a small correction except at large t. For the K_{l3} decays, where we are only interested in the domain of small values of t, it is not unreasonable to assume that the difference $f_+(t) - f_0(t)$ is essentially given by the difference $f_+^V(t) - f_0^V(t)$. Also when we consider, not separately the form factors, but their differ-

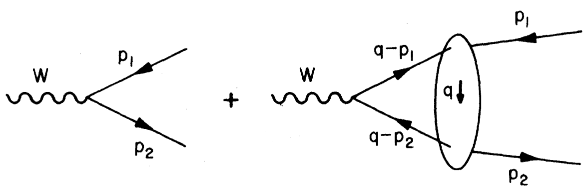

FIG. 1. Feynman graph for K_{l3} matrix element through $K\pi \to K\pi$ interaction.

ence, the contact term (which comes from the disconnected graph) does not intervene and by studying the difference $f_+(t)-f_0(t)$ it is hoped that the previously mentioned difficulties of the GGH model can be mitigated. From Eqs. (3) and (4), neglecting μ^2 with respect to m^2, we have finally

$$f_+(t)-f_0(t)=\frac{2it}{(2\pi)^4(t-m^2)^2}\int d^4q\frac{A^V(t,q^2,u)}{(q^2-2q\cdot p_1)(q^2-2q\cdot p_2)}\left[p\cdot q-\left(2-\frac{t}{m^2}\right)k\cdot q\right]. \quad (5)$$

Here in Eq. (5) we consider only the nondiffractive part (or Veneziano terms) of the scattering amplitude, that we denote by $A^V(t,q^2,u)$. Next, we examine the nondiffractive part of the $K\pi$ scattering amplitude. Using the standard notation

$$A(s,t,u)=\delta_{\alpha\beta}A^+(s,t,u)+\tfrac{1}{2}[\tau_\alpha,\tau_\beta]A^-(s,t,u),$$

where α,β denotes the π-meson isospin, the $K\pi$ amplitude is given by

$$A^{I=3/2}=A^+-A^-,$$

$$A^{I=1/2}=A^++2A^-.$$

Following,[10] let us write down $A^\pm$ in the Veneziano form:

$$A^\pm(s,t,u)=-\tfrac{1}{2}\beta[V(\alpha_{K^*}(s),\alpha_\rho(t))\pm V(\alpha_{K^*}(u),\alpha_\rho(t))],$$

where $\beta=f_{K^*K\pi}{}^2\simeq 8\pi$ and $V(x,y)$ is defined by

$$V(x,y)=\frac{\Gamma(1-x)\Gamma(1-y)}{\Gamma(1-x-y)}.$$

The scattering amplitude $A^V(t,q^2,u)$ in Eq. (5) is then

$$A^V(t,q^2,u)=-\tfrac{1}{2}\beta[3V(\alpha_{K^*}(t),\alpha_\rho(q^2)) - V(\alpha_{K^*}(u),\alpha_\rho(q^2))]. \quad (6)$$

When we expand the first term $V(\alpha_{K^*}(t),\alpha_\rho(q^2))$ of the right-hand side of Eq. (6) as a sum of q^2 poles and put them in Eq. (5), we thus obtain an infinite sum of triangular graphs, each of which can be evaluated by using the Feynman technique of auxiliary variables. After that, the results can be resummed to yield as expression involving $V(\alpha_{K^*}(u),\alpha_\rho(q^2))$ itself [see Eqs. (7) and (8)]. As for the second term $V(\alpha_{K^*}(u),\alpha_\rho(q^2))$, this technique cannot be carried out since here the residues at the q^2 poles (contrary to the first term) are no longer independent of q^2 (for off-mass-shell particles, $t+q^2+u$ is not constant). However, if we proceed in the spirit of GGH, i.e., if we assume that the Veneziano formula is also valid for off-mass-shell particles, then we find out that the contribution of the second term $V(\alpha_{K^*}(u),\alpha_\rho(q^2))$ to the integral (5) can be neglected with respect to that of the first term $V(\alpha_{K^*}(t),\alpha_\rho(q^2))$.[11]

After rather lengthy calculations, we get

$$f_+(t)-f_0(t)=\frac{3}{2}\frac{\beta t}{16\pi^2m^2}\int_0^\infty dy\,g(\tau,y)V(\alpha_{K^*}(-m^2\tau),\alpha_\rho(-m^2y)), \quad (7)$$

with

$$g(\tau,y)=\frac{2y}{(1+\tau)^3}\ln\frac{y+(y^2+4y)^{1/2}+\tau[(y^2+4y)^{1/2}-y]}{2(1+\tau+y)}+\frac{2+y-(y^2+4y)^{1/2}}{2(1+\tau+y)(1+\tau)^2}\mathfrak{A},$$

$$\mathfrak{A}=\frac{[\tau^3+4\tau^2+5\tau+2+y(\tau^2+3\tau+2)](1+4/y)^{1/2}-[\tau^3+4\tau^2+3\tau+y(\tau^2+3\tau-2)]}{y+(y^2+4y)^{1/2}+\tau[(y^2+4y)^{1/2}-y]},$$

and $\tau=-t/m^2$. In the same way, we get for the nondiffractive part of the vector form factor $f_+(t)$ the following expression:

$$f_+^V(t)=\frac{3}{2}\frac{\beta}{16\pi^2}\int_0^\infty dy\,g_1(\tau,y)V(\alpha_{K^*}(-m^2\tau),\alpha_\rho(-m^2y)) \quad (8)$$

with

$$g_1(\tau,y)=\frac{2\tau y-(1+\tau)^2}{(1+\tau)^3}\ln\frac{y+(y^2+4y)^{1/2}+\tau[(y^2+4y)^{1/2}-y]}{2(1+\tau+y)}+\frac{2+y-(y^2+4y)^{1/2}}{2(1+\tau+y)(1+\tau)^2}\mathfrak{A}_1,$$

$$\mathfrak{A}_1=\frac{-\tau^3+2\tau^2+5\tau+2+\tau y(3-\tau)+\tau(1+\tau)(1+\tau+y)(1+4/y)^{1/2}}{y+(y^2+4y)^{1/2}+\tau[(y^2+4y)^{1/2}-y]}.$$

By examining Eqs. (7) and (8) we discover that $f_+(t)$ and $f_0(t)$ possess double poles lying on the K^* trajectory. In addition to the function $\Gamma(1-\alpha_{K^*}(t))$ already factored out of the integrals, the remaining integrals also have poles at the same points $\alpha_{K^*}(t)=n$ which are produced by the behavior of $g(\tau, y)$ and $g_1(\tau, y)$ for large y.

III. NUMERICAL RESULTS

a. The $\xi(0)$ parameter. The derivative of Eq. (7) taken at $t=0$ gives us

$$\lambda_+ - \lambda_0 = \frac{3}{2}\frac{\beta\mu^2}{16\pi^2 m^2}\int_0^\infty dy\, g(0,y)V(\alpha_{K^*}(0), \alpha_\rho(-m^2y)) \qquad (9)$$

By taking $\beta=8\pi$, $\alpha_{K^*}(0)=0.28$, $\alpha_\rho(0)=0.48$, $\alpha'=0.89\ \mathrm{GeV}^{-2}$, we thus obtain $\lambda_+-\lambda_0=0.027$ which then yields $\xi(0)=-0.32$ through Eq. (1).

b. The λ_+ parameter.

$$f_+(t)=1+f_+^V(t)+f_+^P(t),$$

where $f_+^V(t)$ is already given by Eq. (8). Since we have no reliable way to evaluate the diffractive part $f_+^P(t)$, it is only a subject of speculation. However, as shown by GGH in their model, $f_+^P(t)$ would represent a small correction except at large values of t. Since in the K_{l3} decays, we are only interested in small t, we assume that $f_+^P(t)$ is nearly flat, so that λ_+ can be approximately given by the slope of $f_+^V(t)$. We thus have

$$\lambda_+ = -\frac{\mu^2}{m^2}\frac{3}{2}\frac{\beta}{16\pi^2}\int_0^\infty dy\, V(\alpha_{K^*}(0), \alpha_\rho(-m^2y))[G(y)], \qquad (10)$$

with

$$G(y)=g_1'(0,y) + \alpha' m^2 g_1(0,y)[\psi(0.72)-\psi(0.24+\alpha' m^2 y)], \qquad (11)$$

where $\psi=\Gamma'/\Gamma$ is the logarithmic derivative of the Γ function. With the same numerical values of β, $\alpha_{K^*}(0)$, $\alpha_\rho(0)$, and α' as above, we get $\lambda_+=0.05$. Evaluating $f_+^V(0)$ from Eq. (8), we find that it is quite different from zero $[f_+^V(0)=0.25]$, then it is necessary to assume that $f_+^V(0)+f_+^P(0)=0$ in order to preserve the Ademollo-Gatto theorem $f_+(0)\simeq 1$.

IV. CONCLUSION

A recent experiment by Chien *et al.*[12] indicates that $\lambda_+=0.05\pm0.01$. If this result is confirmed, the use of a dipole form factor for $f_+(t)$ offers an attractive explanation for the strong t dependence which is observed and suggests that vector-meson dominance of form factors might be expressed through second-order poles rather than simple poles.

A theoretical explanation for the appearance of double poles is provided by the GGH model and here this idea is extended to K_{l3} decays. By studying the difference $\lambda_0-\lambda_+$, it is hoped that some of the difficulties of the model will be diminished. Although the value of λ_+ is indeed enhanced, the value of λ_0 is also increased by the appearance of double poles and the net result $\xi(0)=-0.32$ is still small (in magnitude) in comparison with experiment. The model produces a larger and improved magnitude for $\xi(0)$ than do conventional theories which assume that $f_+(t)$ and $f_0(t)$ are dominated by simple K^* and κ poles, but does not provide a complete explanation of the data.

ACKNOWLEDGMENTS

One of us (R.O.) would like to thank Professor M. Lévy and Professor R. Vinh Mau for the hospitality received at the Laboratoire de Physique Théorique et Hautes Energies. One of us (P.X.Y.) acknowledges Professor K. Huang for a helpful correspondence that provided the motivation for this investigation.

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$$\frac{\Gamma(1-\alpha_{K^*}(u))\Gamma(1-\alpha_\rho(q^2))}{\Gamma(1-\alpha_{K^*}(u)-\alpha_\rho(q^2))}$$

is actually very small because of the denominator $\Gamma(1-\alpha_{K^*}(u)-\alpha_\rho(q^2))$. For particles on mass shell, one has $1-\alpha_{K^*}(u)-\alpha_\rho(q^2)\simeq\alpha_{K^*}(t)-0.5$. Then for t small [in the region of K_{l3} decays, $0<t<(m-\mu)^2$] $\alpha_{K^*}(t)-0.5$ is nearly zero.

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PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Can the Pion's Charge Radius be Large? *

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Analyticity has been used to derive dispersive inequalities which bound the spacelike behavior of the pion's charge form factor in terms of the timelike variation of the modulus of the form factor and the p-wave $\pi\pi$ scattering phase shift. The large charge radius, suggested by the Serpukhov-UCLA measurement of πe scattering, is only compatible with timelike data which involve a large p-wave $\pi\pi$ phase shift just above threshold.

I. INTRODUCTION

In recent years there has been a rapid accumulation of data on the behavior of the pion's charge form factor, $F(t)$.[1] Colliding beam measurements[2] of $\sigma(e^+e^-\to\pi^+\pi^-)$ at Novosibirsk, Orsay, and Frascati have furnished information on the size of $|F(t)|$ for timelike momentum transfer: $16m_\pi^2\lesssim t\lesssim 4.4$ GeV^2. On the elastic cut, $t_0\equiv 4m_\pi^2\leq t\leq 16m_\pi^2$, the phase of $F(t)$ is equal to the $J=T=1$ $\pi\pi$ phase shift, $\delta_1(t)$ (modulo π). Chew-Low extrapolation techniques[3] can be used to infer $\delta_1(t)$ from observations of $\pi N\to\pi\pi N$. On the spacelike interval, $-1.2\lesssim t\lesssim -0.2$ GeV^2, the behavior of the form factor has been extracted from pion electroproduction experiments.[4] In addition, the recent Serpukhov-UCLA measurements[5] of pion-electron scattering have provided *direct* access to $|F(t)|$ at small spacelike momentum transfer, $-0.04\lesssim t\lesssim -0.02$ GeV^2.

These experimental results in the spacelike and timelike regions should be correlated by the analyticity of the form factor. The standard method of displaying that correlation is to write an ordinary dispersive equality[6]; i.e., Cauchy's theorem is used to express $F(t)$ at spacelike momentum transfer in terms of a polynomial ("subtractions") and an integral of Im$F(t)$ over the timelike domain. Thus, the computation of $F(t)$ at spacelike t requires knowledge of the following input: the number and size of subtraction constants as well as the behavior of the modulus and phase of $F(t)$ over the entire timelike cut. In the case of the problem considered here, this approach has the disadvantage that it is usually necessary to construct a model in order to estimate the subtraction constants[7] and to estimate the phase of the form factor for $t>16m_\pi^2$.

The correlation of spacelike and timelike experiments can be expressed in a more model-independent way by using analyticity and a smaller amount of timelike information to derive *bounds* on the spacelike form factor. The resulting dispersive inequalities[8-10] typically take the following form: Knowledge of $|F(t)|$ or an upper bound on $|F(t)|$ on the entire timelike cut is used to put upper and lower limits on the value of $F(t)$ for spacelike momentum transfer. This technique has several important advantages over ordinary dispersive equalities. First of all, it is not necessary to build models for the (experimentally inaccessible) phase of $F(t)$ for $t>16m_\pi^2$. Secondly, the input information includes only an upper bound on (not the value of) $|F(t)|$ in the timelike region. This is significant since two-photon effects[11] may be sizable at high timelike t; in that case, colliding beam measurements of $\sigma(e^+e^-\to\pi^+\pi^-)$ determine *only* an upper bound[12] on $|F(t)|$ (provided that the effects of