PHYSICAL REVIEW D VOLUME 6, NUMBER 11 1 DECEMBER 1972

Asymmetry Properties of Longitudinal-Momentum Distributions for the Reaction $\pi^+p \to p8\pi$ at 11 GeV/c*

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(Received 31 May 1972)

Data are presented for the deeply inelastic high-multiplicity exclusive reaction $\pi^+p \to p4\pi^+3\pi^-\pi^0$ at 11 GeV/c. It is shown that asymmetry moments of the longitudinal-momentum distributions of the final-state particles can be interpreted qualitatively in terms of the additive quark model, but that some contribution from double scattering or more complex interactions may be indicated. It is shown also that similar results can be obtained from models not involving quarks. Specific cases of the Zieminski $F(t)$ model and the Chan-Łoskiewicz-Allison model are discussed.

I. INTRODUCTION

In the study of longitudinal-momentum distributions of pions resulting from high-energy reactions, it has been noted[1-4] that a striking asymmetry exists in the center-of-mass system in pion-proton collisions, particularly for nonbeamlike pions. It appears that these distributions approach symmetry in a reference frame in which the ratio

$$R = -p_p / p_\pi \tag{1}$$

is very nearly 1.5. Here p_p and p_π are the momenta of the incident proton and incident pion, respectively, evaluated in the chosen reference frame.

The effect has been interpreted in the light of quark-model diagrams[5] of the form of Fig. 1. In the reference frame in which $R = 1.5$, which shall be referred to as the Q system, all incident pion and proton quarks have equal momenta. It is assumed that only one quark from each incident particle participates in the reaction, the remainder continuing with essentially unchanged momentum as spectators. It is assumed moreover that the interacting quarks have zero average momentum in their respective quark-quark center-of-mass systems at the time they recombine with the spectator quarks to re-form physical particles.

Under these circumstances, a "leading pion" and a "leading nucleon" would be expected, which would consist of those particles formed by recombination of interacting quarks with the spectator quarks. The leading particles could each differ in charge from the incident particles by up to ± one unit. The "produced particles," consisting of everything else emitted, would be expected to exhibit a forward-backward symmetry in the Q system, in agreement with at least some of the data.[1-4]

In this communication we report on a similar analysis applied to a single ultrahigh-multiplicity reaction, $\pi^+p \to p8\pi$. Although the actual number of events used is relatively small, almost no work has been done heretofore on such high-multiplicity processes with a sample of events even as large as this one. The relevance of the results to the quark model will be discussed. It will be shown, first, that an analysis of the data in terms of the simple additive quark model[5] gives qualitative but not exact agreement. A new measure of asymmetry is introduced. The second discussion concerns our attempt to achieve the same results using the multiperipheral model. Simple versions of the model are shown to yield agreement at least as good as that obtained with the additive quark model. Hence, it is shown that quark-model interpretations of this type of result are not unique.

Various other aspects of these data have previously been reported.[6,7]

II. EXPERIMENTAL DETAILS

The BNL 80-in. bubble chamber, filled with H_2, was exposed to an 11-GeV/c π^+ beam for 25 000 exposures. All interaction events with eight visible tracks were measured, processed through TVGP-SQUAW-ARROW-SUMX, as modified at Florida State, and checked on the scanning table for ionization consistency. A total of 213 events were found which fit the reaction

$$\pi^+p \to p4\pi^+3\pi^-\pi^0 \tag{2}$$

with ionizations which were consistent between the fit and the scanning table. The kinematic fit to reaction (2) was required to have a confidence level of 3% or greater. Proton-π^+ ambiguities in the fits to (2) were largely removed by the ionization consistency requirement.

The center-of-mass longitudinal-momentum dis-

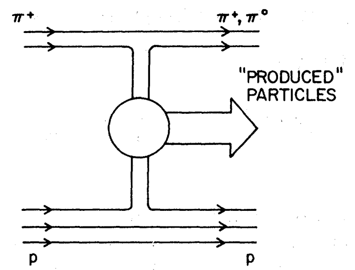

FIG. 1. Quark-model diagram, showing a model whereby the interaction between single quarks dissociated from each incident hadron is responsible for the production of particles.

tribution of the final-state proton, $p_l^*(p)$, is presented in Fig. 2. In addition to a primary peak located around -500 MeV/c, there is a secondary peak located above $+300$ MeV/c. There is no evidence from the data of other experiments known to us for such a secondary peak in $p_l^*(p)$. The protons in this peak have a laboratory momentum too high to be distinguished from π^+ or K^+ particles by ionization. In addition, the kinematic fitting procedures provide poor mass resolution on such tracks. Such events could belong to a final state which was not fitted due either to measurement errors or to the presence of multiple missing neutrals.

A calculation based on the CŁA (Chan-Łoskiewicz-Allison)[8] model, which has enjoyed success in fitting many multibody reactions, was compared with the experimental distributions. The parameters used in this calculation are the same as originally used by Chan *et al.*[8] Only diagrams involving nonexotic meson exchange were included. It

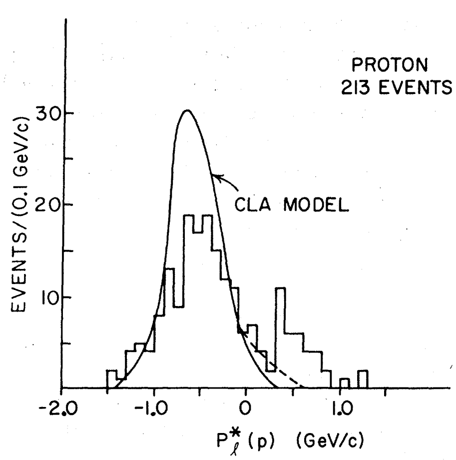

FIG. 2. Center-of-mass longitudinal-momentum distribution of the proton, showing the CŁA (Ref. 8) prediction (solid line), and an extrapolation (dashed) above 300 MeV/c based on data just below that region.

was found that if baryon-exchange diagrams were included, the calculation actually gave poorer fits to most experimental distributions[9] than if they were excluded. The requirement of nonexotic exchange introduces differences between pion charge states, since, for example, no π^- can be emitted at the top vertex.

The CŁA prediction, normalized to all 213 events, is sketched on Fig. 2 (solid curve). This calculation predicts almost no events above 300 MeV/c. While inclusion of baryon-exchange diagrams does predict events with $p_l^*(p)$ above 300 MeV/c, no combination of such diagrams was found which was able to reproduce the dip near 250 MeV/c.

Finally, at least 25% of the events with protons in the region $p_l^*(p) > 300$ MeV/c have a π^0 with a lab momentum below 250 MeV/c. In the center-of-mass angular distribution of the π^0's, these events show up as an anomalous peak in the most backward bins (i.e., antiparallel to the incident π^+). Such slow π^0's are easily "manufactured" by the fitting program from measurement uncertainties, which can be considerable at this energy.

As a result of these considerations, it was decided to remove all events with $p_l^*(p) > 300$ MeV/c from the sample, on the grounds that they are most likely spurious fits. Under the assumption that the remaining 171 events are genuine reaction (2) events, a smooth extrapolation was made into the region above 300 MeV/c based on the data between 0 and 300 MeV/c and it is shown in Fig. 2 (dashed line). It predicts that 2–3% of the genuine events were lost by the $p_l^*(p)$ cutoff. Inasmuch as some of the events on which this extrapolation is based may also be contaminated, we feel this represents an estimate of remaining contamination. When renormalized to the remaining 171 events, the CŁA estimate is that there are approximately 6% misidentified events lying in the region between zero and 300 MeV/c. It is our conclusion that uncertainties of this magnitude can cause no bias significantly affecting the results of this paper.

A number of tests for other experimental biases in the data were made. For example, the distributions of so-called "stretch" or "pull" quantities (which relate measured and fitted values) for the parameters in the fits to the events, usually an indicator of measurement or fitting bias, show no significant deviation from the normally expected behavior. The same observations were made for a (smaller) sample of events of the more highly constrained reaction $\pi^+p \to p4\pi^+3\pi^-$. In addition, studies of lower-multiplicity events of various types (with much higher statistics) in the same film indicate that the experimental parameters are well understood. We have therefore reached the conclusion that there exists no experimental bias of sufficient magnitude to affect the results presented here.

Many previous studies [1-4] in this field have made use of "inclusive"[10] experiments, in which one studies reactions of the form

$$A + B \rightarrow C + \text{anything}, \tag{3}$$

thereby summing over all final states in which C is produced. This approach is of great value in comparison with certain model calculations.

The study of "exclusive" reactions, i.e., those with specific final states, has value in that all tracks are identified, so that there is, for example, little or no proton-π^+ ambiguity problem. Moreover, the actual distribution of all protons and neutral particles is known, and can be included in the analysis. The choice of the specific reaction (2) enables investigation of the extreme "deeply inelastic" region, which might be expected to exhibit different production characteristics than less inelastic cases. As a specific example, most of the clear-cut resonant behavior observed in reactions of lower multiplicity is not observed in these data.

III. EXPERIMENTAL ASYMMETRY PROPERTIES OF THE LONGITUDINAL-MOMENTUM DISTRIBUTIONS

Only the π^- particles from reaction (2) can be unambiguously identified as "produced" particles. The center-of-mass longitudinal-momentum distribution, p_l^*, of these pions is displayed in Fig. 3. The best fit (in the least-squares sense[11]) to the symmetric function

$$\frac{dN}{dp_l^*} = A \exp(-B\,|p_l^*|), \tag{4}$$

where A is a normalization and B is an adjustable parameter, is indicated on the figure. The lack of symmetry in the data is clearly visible. The χ^2 probability of this fit is 0.0016, with $B = (4.1 \pm 0.2) \times 10^{-3} (\text{MeV}/c)^{-1}$.

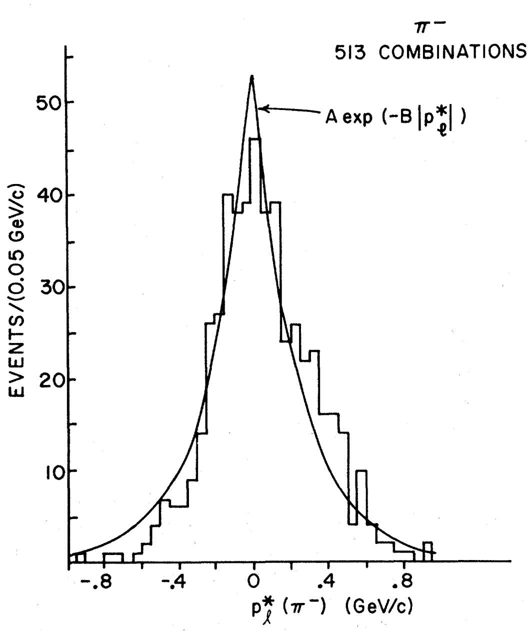

FIG. 3. Center-of-mass longitudinal-momentum distribution of the π^-, showing the best fit to the function $\exp(-B|p_l^*|)$ to illustrate the asymmetry.

As a measure of asymmetry, the first, third, and fifth moments about $p_l = 0$ are calculated for a number of different Lorentz frames, each frame corresponding to a particular value of R [defined in Eq. (1)]. The first moment is the mean of the distribution; the third moment is the mean of $p_l^{\,3}$; and the fifth moment is the mean of $p_l^{\,5}$. Use of these odd moments should reveal structural features which might remain hidden if the first moment only were used. As a test for asymmetry, a moments calculation has several attractive features not always present in other tests: (a) Each moment is a monotonic function of R, making possible an unambiguous study of symmetry. A point of maximum symmetry for the data may be defined, being in principle that value of R at which all three moments are zero, if such a point exists. (b) The values of the moments are very weak functions of the number of data points, so quantitative comparisons between, for example, π^+ (four per event) and π^0 (one per event) at a given value of R are possible. (c) The moments are direct physical observables, hence deductions based on them do not depend upon particular models or parametrizations of the data for their validity. In addition, direct model calculations can sometimes be made (often for at least the first moment) without the necessity of calculating the exact distribution. (d) Errors in the moments can be calculated conveniently by the propagation of errors technique.[12] Hence statistical uncertainties in the location of the point of maximum symmetry are easily obtainable, as will be demonstrated below.

Figures 4(a)–4(c) demonstrate the behavior of each moment as a function of R for the π^-. The shaded region indicates $\pm$ one standard deviation (statistical uncertainty only). These limits do not represent the scatter of the individual data points, since all points are evaluated using the same events. Rather, this is an estimate of the fluctuations to be expected from one independent experiment to the next.

The solid curve for each moment represents a second-degree polynomial fit which was used to interpolate to zero.[13] Table I lists, with errors,

TABLE I. Data. Zero crossing point.

Particle	Moment 1	Moment 3	Moment 5	Best estimate[a]
Proton	0.37 ± 0.02	0.38 ± 0.02	0.39 ± 0.03	0.38 ± 0.01
π^+	1.36 ± 0.07	1.49 ± 0.10	1.60 ± 0.16	1.43 ± 0.06
π^-	1.34 ± 0.08	1.30 ± 0.11	1.27 ± 0.19	1.32 ± 0.06
π^0	1.73 ± 0.20	1.69 ± 0.24	1.62 ± 0.36	1.70 ± 0.14
π^-,π^0	1.43 ± 0.08	1.44 ± 0.10	1.43 ± 0.17	1.43 ± 0.06
All π's	1.39 ± 0.05	1.46 ± 0.07	1.50 ± 0.12	1.42 ± 0.04

[a]From Eq. 5. See discussion in the text.

the value of R at which each moment passes through zero for all particles from reaction (2). A "point of maximum symmetry" can be defined for each particle as a weighted average of the symmetry points of each moment:

$$R_{\max} = \left(\sum_{i=1}^{3} x_i/\sigma_i^2\right)\left(\sum_{i=1}^{3} 1/\sigma_i^2\right)^{-1}, \qquad (5)$$

where x_i is the value of R at which the ith odd moment passes through zero, and σ_i^2 is the variance in this estimate. The error in $R_{\max}$ can be estimated from

$$\mathrm{Var}(R_{\max}) = \left(\sum_{i=1}^{3} 1/\sigma_i^2\right)^{-1}. \qquad (6)$$

Equation (5) describes a maximum likelihood estimator of the mean of three independent Gaussian distributions with the same mean but different (known) variances, from which it is hypothesized that the x_i are sampled. Equation (6) is the propagation of errors variance of $R_{\max}$ under the same assumption.

Since each data point, on Fig. 4, for example, involves exactly the same set of events as every other data point, the errors in these points are highly correlated. The values of the errors listed in Table I and subsequent tables are estimated under the assumption that these correlations are all equal to one. Hence, if one data point for a given moment is off by one standard deviation, then they all are. An estimator for the variance of x_i is therefore given by

$$\sigma_i^2 = \left(\frac{dR}{dy_i}\right)^2\bigg|_{y_i=0} \times \mathrm{Var}(y_i)|_{y_i=0}, \qquad (7)$$

where dR/dy_i is obtained from the polynomial fit, and y_i is the ith moment.

An important feature of the results of Table I is that the moments for a given type of particle all pass through zero at very nearly the same value of R. The consequence of this is that no significant asymmetry remains at the point of maximum symmetry defined by Eq. (5). Clearly, the concept of a "point of maximum symmetry" has physical validity. It follows also that Eqs. (5) and (6) provide a useful approximation to more exact expressions which would not neglect correlations between the moments.

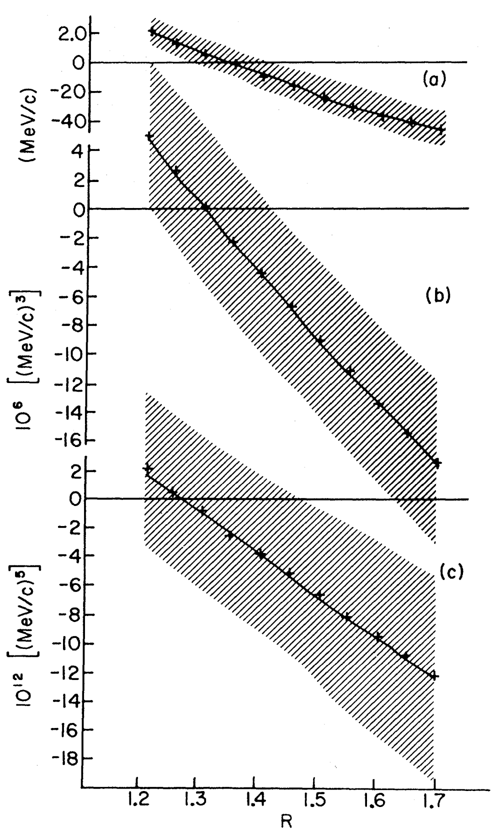

FIG. 4. (a) First, (b) third, and (c) fifth moment as a function of R for the π^- in the vicinity of the region in which these moments become zero. The data points are indicated by crosses, with the ± one standard deviation region shown shaded. The "best" second-degree polynomial fit to the data points is indicated as a solid line. Note that the data points are correlated; hence the width of the shaded area does not represent the scatter of individual data points.

The maximum error introduced by neglect of such correlations can be deduced by inspection of the tables, since a better estimator cannot be larger than the largest x_i or smaller than the smallest x_i for a given type of particle. Typically, this maximum error is less than 10% of R_{max}. It is reasonable to assume, since all pions have similar distributions, that such correlations affect each type of pion in the same way. Hence, comparisons between pion moments are still assumed to be valid, even in the presence of strong correlations. Some caution should be exercised, however, because the errors in the "best estimate" might be slightly larger than those given on the table.

IV. ADDITIVE QUARK MODEL INTERPRETATION

The data appear to indicate that the produced pions are most nearly symmetric for a value of R somewhat less than 1.5. This is consistent with other data,[1-3] for which it appears that the R of maximum symmetry decreases as the number of produced particles increases, for fixed beam energy.

In the framework of the quark model, this can be understood in terms of the inelasticity of the reaction. So much energy is expended in particle creation that it is reasonable to assume that double quark scattering or more complicated interactions account for a significant portion of the amplitude, either directly or through interference with the single scattering terms. Franco[14] has considered this problem for π^+p scattering at 12 GeV/c incident pion momentum. The result of his calculation is that double-scattering terms account for almost $\frac{2}{3}$ of the total cross section. Hence a sizable contribution from double scattering in the particular reaction (2) is a reasonable expectation.

As further experimental evidence for this, it can be pointed out that if single scattering alone dominates and if the spectator quark from the pion recombines with a quark which is on the average at rest in any reference frame with $R>0$, there will be a leading pion of average momentum greater than $\frac{1}{2}$ of the incident pion momentum in the lab (note that $R=0$ corresponds to the lab reference frame). In this reaction, this implies the existence of pions with longitudinal momentum of at least 5.5 GeV/c in the lab. The maximum value which occurs experimentally is about 3.5 GeV/c.

The same analysis applies at the proton vertex. In the simplest picture, two proton quarks remain at rest in the lab, one in the quark center-of-mass system (Q system). Therefore in the Q system, the final proton will have longitudinal momentum equal to approximately $\frac{2}{3}$ its initial value of -2.728 GeV/c. The experimental value for $\langle p^Q(\text{prot})\rangle$ is -0.790 ± 0.030 GeV/c, or less than the momentum which even even one of its constituent quarks had before the collision.[15]

Further (but related) evidence for processes other than single scattering is provided by the data of Table I. In the absence of such effects, it is relatively straightforward to calculate[16] that R_{max} for the proton should have a value of 0.2 several standard deviations less than the experimental result.

It was shown by Elbert *et al.*[1] that in π^-p at 25 GeV/c there is evidence that the role of the leading pion is shared approximately equally between a π^- and a π^0, averaged over all final states except elastic scattering. If a similar effect obtains in reaction (2), the leading pion should be a π^+ or π^0 with approximately equal probability. Then the $4\pi^+$ distribution should behave approximately the same as the summed $3\pi^-$ plus π^0 distribution, since each of these groups would contain a single leading pion about one-half of the time. Table I indicates good agreement with this hypothesis. It is useful to compare also the actual distribution of these two samples, shown in Fig. 5 in the center-of-mass reference frame. No systematic differences can be observed. Pearson's χ^2 for the hypothesis that the two distributions are sampled from the same probability density function is 21.8 for 26 degrees of freedom,[17] which indi-

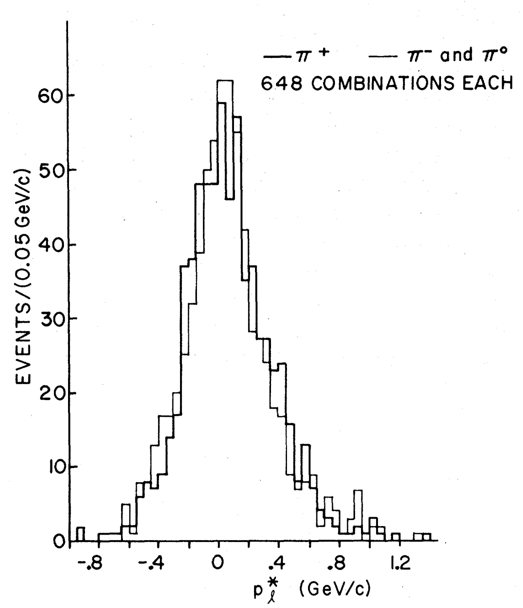

FIG. 5. Center-of-mass longitudinal-momentum distribution of the π^+ compared with that of the π^- and π^0 summed.

cates good agreement.

In the language of Fig. 1, this suggests that the replacement of an $\overline{\mathfrak{N}}$ ($\mathcal{P}$) quark in the π^+ by a $\overline{\mathcal{P}}$ ($\mathfrak{N}$) quark occurs with the same probability as replacement of a given quark by an identical one. This picture assumes that higher-order terms do not cause the replacement of more than one pion quark.

It is easy to propose additional experimental checks on this idea, to see whether it applies to all types of quarks:

(i) Reactions involving strange particles. This could involve reactions with strange beams and also associated production reactions. If strange quarks are involved anywhere in the reaction, one of these could perhaps substitute for a quark in the incident particle. In the same way, experiments with strange beams could look for non-strange leading particles.

(ii) Reactions involving neutrons can be utilized to test the application of this principle to baryons. Here again, a complete discussion would involve reactions with neutron targets and also reactions with proton targets involving charge exchange at the baryon vertex.

(iii) To complete the list, the behavior of strange quarks in baryons could be studied. This would presumably involve production of Λ and Σ hyperons, in the simplest form.

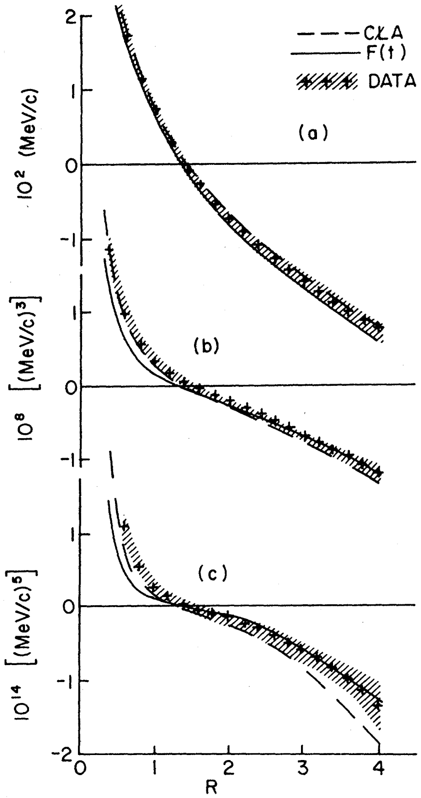

FIG. 6. (a) First, (b) third, and (c) fifth moment as a function of R for all pions summed. CLA model (Ref. 8) and $F(t)$ model (Ref. 12) predictions are shown.

V. PERIPHERAL MODEL ANALYSIS

A simple form of peripheral model known as the $F(t)$ model has been introduced by Zieminski.[18] This assumes the major features of the data of high-multiplicity reactions can be described by a matrix element which has as its only variable the momentum-transfer dependence of the proton. If the square of this matrix element is represented by $F(t)$, where t is the four-momentum transfer to the proton, then the distribution of a given quantity is given by $F(t)\times$a phase-space factor, integrated over all remaining variables. This model is assumed to be most useful in cases in which many particles are produced.

No specific features to describe the production of pions are included, except insofar as this production is affected by $F(t)$ and kinematics. A one-parameter fit to the experimental t distribution of the form

$$F(t) \propto \exp(At), \tag{8}$$

with $A = 1.24(\mathrm{GeV}/c)^{-2}$, was adequate for this data. The $F(t)$ prediction for the data can be evaluated with the aid of Fig. 6. The solid curve shown there represents a hand-smoothed approximation to a Monte Carlo distribution with an order of magnitude more "events" than the actual data. Inasmuch as the $F(t)$ model makes no effort to distinguish among pions, the prediction is compared with the data only for an average over all charge states. The quantitative agreement with the data is reasonable for the first moment, but poor for the higher

TABLE II. $F(t)$ model.[a] Zero crossing point.

Particle[b]	Moment 1	Moment 3	Moment 5	Best estimate[c]
Proton	0.39 ± 0.005	0.406 ± 0.007	0.414 ± 0.010	0.398 ± 0.004
All π's	1.34 ± 0.01	1.32 ± 0.02	1.31 ± 0.03	1.33 ± 0.01

[a] Based on a Monte Carlo study with over 1700 "events." Estimated statistical errors on this sample are included for completeness.

[b] No distinction is made between pion charge states.

[c] From Eq. (5). See discussion in the text.

TABLE III. CLA model.[a] Zero crossing point.

Particle	Moment 1	Moment 3	Moment 5	Best estimate[b]
Proton	0.336 ± 0.004	0.360 ± 0.006	0.382 ± 0.011	0.347 ± 0.003
π^+	1.68 ± 0.03	1.53 ± 0.03	1.42 ± 0.05	1.58 ± 0.02
π^-	1.22 ± 0.02	1.24 ± 0.03	1.22 ± 0.04	1.23 ± 0.02
π^0	1.17 ± 0.04	1.22 ± 0.05	1.20 ± 0.07	1.19 ± 0.03
π^-, π^0	1.21 ± 0.02	1.23 ± 0.03	1.21 ± 0.04	1.22 ± 0.02
All π's	1.42 ± 0.02	1.38 ± 0.02	1.31 ± 0.03	1.38 ± 0.01

[a] Based on a Monte Carlo study with over 1700 "events." Estimated statistical errors on this sample are included for completeness.

[b] From Eq. (5). See discussion in the text.

moments at low R. From Table II, it is clear that the model shows qualitative agreement for the point of manimum symmetry. Clearly, individual pion charge states exhibit sizable discrepancies, but the $F(t)$ model approximately reproduces the features found in the data with respect to a reference frame of maximum symmetry. In particular, excellent agreement is observed for the produced (i.e., negative) pions.

This would suggest a hypothesis that models which describe the behavior of the proton, with pion dynamics included only in an average sense or not at all, will simulate the $R_{\max}$ behavior of "produced" pions in the quark model. In the present case, any agreement between the $F(t)$ model and the *nonproduced* π^+ and π^- distributions is related to the absence of a clear-cut "leading pion."

We have tested this hypothesis with the CLA model. As discussed previously, this model correctly reproduces many of the features of proton production in this reaction.

The CLA prediction for all pions, summed, is indicated in Fig. 6. This too was obtained using the Monte Carlo technique with an order of magnitude more "events" than in the data. The agreement with the data is somewhat better than for the $F(t)$ model. Differences between pion charge states are, however, again not well reproduced. The values of $R_{\max}$ for the CLA model are listed in Table III. As in the case of the $F(t)$ model, agreement with the data is qualitatively good, though detailed correspondence is lacking.

A recent study by Caneschi[19] of the inclusive data of Elbert *et al.*[1] is in agreement with this peripheral-model analysis. Agreement with the quark model, which is achieved with a different multiperipheral model than used here, is therein interpreted as being a consequence of the expectation that few particles are emitted backward in the rest frame of either initial particle.

VI. CONCLUSIONS

It has been shown that high-multiplicity exclusive reactions can be used to study effects often studied in inclusive reactions. Longitudinal-momentum techniques were applied to the reaction $\pi^+p \to p(8\pi)^+$ to draw conclusions relevant to the quark model. The results indicate that the simplest form of this model is not adequate to explain the data quantitatively. Significant amounts of double scattering or more complex interactions appear to be required.[20] A further result is that the duties of the "leading pion" are apparently shared approximately equally between π^+ and π^0 in this reaction. This is consistent with the results of Elbert, Erwin, and Walker[1] in π^-p inclusive reactions at 25 GeV/c.

It was shown further that, whereas a quark-model interpretation may have certain attractive features, the same results can be obtained from models not involving quarks. The particularly simple case of the Zieminski $F(t)$ model[18] was shown to be in qualitative agreement with the data. This would appear to suggest that the most critical feature of data in the deeply inelastic limit for a model which desired to predict the reference frame of maximum symmetry for produced particles is the *proton-t* distribution. In other words, most models which successfully reproduce this distribution may be in qualitative agreement with the quark-model predictions for produced pions. This hypothesis was successfully tested with the CLA model,[8] which was also shown to be in good agreement with the data.

The quark and multiperipheral pictures may be compatible ways of viewing the same phenomena.

Until detailed formalisms with universal appeal have been created for each type of model, it may not be possible to make a distinction.

ACKNOWLEDGMENTS

The data for this analysis were gathered some time ago, so we wish at this point to acknowledge the contributions of many people who helped in various ways in the exposure at the Brookhaven National Laboratory and in the early stages of the measurements here at F.S.U. In particular, we want to express our appreciation to E. B. Brucker, Ben Harms, W. C. Harrison, J. S. O'Neall, and W. H. Sims for their assistance. In addition, we must compliment the scanning and measuring technicians at F.S.U. who strained to measure carefully the many tracks in each event.

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*Research supported in part by the U. S. Atomic Energy Commission under Contract No. AT(40-1)-3509. Computational costs were supported in part by National Science Foundation Grant No. GJ-367 to the Florida State University Computing Center.

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[11]The fitting was done by SUBROUTINE STEPIT, copyright 1965 by J. P. Chandler, Department of Computer Science, Oklahoma State University. Error estimates were made using SUBROUTINE FIDOUCHE, copyright 1966 by J. P. Chandler, Department of Computer Science, Oklahoma State University.

[12]A Monte Carlo study was made, in which the variance of each moment estimated from five independent samples of Monte Carlo "events" was compared with a calculated value of the standard deviation. This study indicated that the propagation of errors approximation was adequate.

[13]A simple three-parameter fit was sufficient to evaluate the point at which the data passed through zero.

[14]V. Franco, Phys. Rev. Letters 18, 1159 (1967).

[15]In fact, the final proton does not have momentum equal to a single one of the incident proton quarks for any (positive) R less than 2.8.

[16]This can be derived by using the following arguments: Define a coordinate system with the $+Z$ axis parallel to the incoming pion beam in the laboratory, assume we discuss only Lorentz transformations in the $\pm Z$ directions, and refer at all times only to the (signed) Z components of momentum. Then, if we write p_i as the incident proton momentum and π_i as the incident pion momentum, evaluated in the chosen reference frame, $R=-p_i/\pi_i$. If $\langle p\rangle$ is the average proton momentum in that reference frame after the collision, and $\langle q\rangle$ is the same for a single quark which is at rest in the Q system on the average, then we must solve for R such that $\langle p\rangle=\frac{2}{3}p_i+\langle q\rangle$ is zero. The solution is straightforward, but inelegant. It is amusing to note that R must be a positive number less than 0.3 in any inelastic reaction, under the simple additive quark model assumptions. Under these assumptions, $\langle q\rangle$ is positive for R less than 1.5. But p_i is negative for positive R only, so the requirement $\langle q\rangle=-\frac{2}{3}p_i$ can be satisfied only for $0<R<1.5$. From conservation of momentum $\langle\pi\rangle+\langle p\rangle+\langle\Sigma\rangle=\pi_i+p_i$, where $\langle\pi\rangle$ and $\langle\Sigma\rangle$ are the final average momenta of the leading pion and produced particles, respectively. The quark model requires $\langle\pi\rangle=\frac{1}{2}\pi_i+\langle q\rangle$. Therefore, setting $\langle p\rangle=0$ and doing the algebra,

$$R=-p_i/\pi_i=0.3-2\langle\Sigma\rangle/5\pi_i\,.$$

From $0<R<1.5$, both $\langle\Sigma\rangle$ and π_i are positive. For an elastic reaction $\langle\Sigma\rangle=0$, and $R=0.3$.

[17]Bins containing fewer than five events for either set of data were combined with others, for adequate statistics.

[18]A. Zieminski, Nucl. Phys. B14, 75 (1969).

[19]Luca Caneschi, Phys. Rev. D 3, 2865 (1971).

[20]Conjectures that under such circumstances abnormally large transverse momenta would be observed for the produced pions cannot be supported by the current data. In fact the negative pions have slightly less transverse momenta than the others: $\bar{p}_T(\pi^-)=(267\pm 7)$ MeV/c, as opposed to $\bar{p}_T(\pi^+)=(290\pm 7)$ MeV/c and $\bar{p}_T(\pi^0)=(296\pm 13)$ MeV/c.