Lee, Nucl. Phys. B9, 649 (1969), and by J.-L. Gervais and B. W. Lee, Nucl. Phys. B12, 627 (1969).

[15]V. B. Berestetskii, Sov. Phys.–Usp. 5, 7 (1962); T.-T. Chou and Max Dresden, Rev. Mod. Phys. 39, 143 (1967).

[16]The contribution to the W vacuum-polarization tensor from the graph under consideration here is real on the mass shell; but this is not so for the (νe)-loop contribution, for example. We discuss this case in more detail in Sec. IV E.

[17]One can verify that in the ϕ^3 theory all one-loop graphs are correctly evaluated by dispersing the whole graph in the manner we explain. We discuss the validity of the procedure here because the complications of spin are not entirely trivial.

[18]It is the coefficients of $J_\rho^{(e)} J_\sigma^{(\mu)\dagger} g^{\rho\sigma}$ and $J^{(e)} J^{(\mu)\dagger}$ which are actually dispersed.

[19]T. D. Lee, Phys. Rev. Letters 26, 801 (1971).

[20]We note that the singularity as $R \to 0$ ($m_Z \to \infty$) implies that models such as that of S. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970), which attempt to remove the neutral currents without introducing additional leptons, are nonrenormalizable. Although for μ decay the singularity is only logarithmic, one-loop contributions to non-charge-exchange processes will diverge quadratically in this limit.

[21]It is important to emphasize that we are calculating in terms of renormalized coupling constants. Thus we do not explicitly include external line corrections. This is to be contrasted with earlier calculations (cf. T. Kinoshita and A. Sirlin, Ref. 7) of the radiative corrections to the Fermi theory where the answer is expressed in terms of the unrenormalized Fermi constant G.

[22]Because of the vanishing neutrino mass the divergent part of the first two graphs of Fig. 4(d) has exactly the right structure to be removed by renormalization. This is not true for the analogous graphs for the process $e\mu \to e\mu$, for example. There one must combine graphs such as these with graphs involving scalar particles in order to cancel a divergent contribution which does not correspond to renormalization subtraction. Cf. Ref. 6.

[23]The identities

$$(\bar{e}\gamma_\gamma\gamma_\rho\gamma_\beta P_- \nu_e)(\bar{\nu}_\mu P_+\gamma^\gamma\gamma_\sigma\gamma^\beta\mu) = 4J_\eta^{(e)} J^{(\mu)\dagger\eta} g_{\rho\sigma}$$

and

$$(\bar{e}\gamma_\gamma\gamma_\rho\gamma_\beta P_- \nu_e)(\bar{\nu}_\mu P_+\gamma^\beta\gamma_\sigma\gamma^\gamma\mu) = 4J_\sigma^{(e)} J_\rho^{(\mu)\dagger}$$

are helpful in calculating these contributions. Also note that we hold s fixed in dispersing graphs 3(f) and 3(g), and u fixed in dispersing graphs 3(h) and 3(i), as noted following Eq. (23). We consistently neglect lepton masses in obtaining the expressions given; note that $m_e^2 < s$, $u < m_\mu^2$.

[24]H. H. Chen and B. W. Lee, Phys. Rev. D 5, 1874 (1972).

[25]One problem which must be considered in introducing such particles is the absence of triangle anomalies. Actually, the Weinberg model has such anomalies and is consequently not renormalizable unless the anomalies are canceled by the introduction of heavy leptons or hadrons. We have ignored the anomaly problem in this paper since it does not arise until one goes to higher orders in perturbation theory. For further discussion see C. Bouchiat, J. Iliopoulos, and P. Meyer, Phys. Letters 36B, 519 (1972); D. Gross and R. Jackiw, Phys. Rev. D 6, 477 (1972), and H. Georgi and S. L. Glashow, *ibid.* 6, 429 (1972).

[26]The T^* product is defined by subtracting a c-number part:

$$T^*(AB) = T(AB) - \langle T(AB)\rangle_\Omega\,.$$

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Reciprocal Bootstrap on the Light Cone

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We introduce dynamical considerations onto the light cone in the form of the static bootstrap. We obtain (1) the prediction that the asymmetry in the deep-inelastic electron scattering on polarized deuteron targets is small, and (2) a relation between $F_2^{\nu p} + F_2^{\nu n}$, $F_2^{\gamma p}$, and $F_2^{\gamma n}$. The "physical origin" of these results is discussed. The result (2) also follows as a "chiral-limit theorem."

I. INTRODUCTION

Just as our understanding of electromagnetic and weak interaction allows the measurement of the matrix elements of the local currents,[1] the assumption of light-cone dominance[2] enables the matrix elements of an infinite collection of local operators to be measured in deep-inelastic scattering experiments. These are the local operators contained in the so-called bilocal operators defined on the light cone. Unhappily, what one can say about these operators has so far been limited. Their Lorentz tensor property is presumed known.[3] They are also believed to transform like SU(3) singlets and octets.[4] In order to make further theoretical statements about these operators, one has to intrrduce "dynamics" at some point. In this paper our dynamical input appears in the form of the reciprocal bootstrap, as studied at length by Dashen and Frautschi.[5,6] It will become apparent that the static approximation is applicable for our purposes. Our main results are

(1) the relation

$$\tfrac{1}{3}[F_2^{\nu p}(\omega)+F_2^{\nu n}(\omega)] = (1+c)F_2^{\gamma p}(\omega)+(1-c)F_2^{\gamma n}(\omega), \quad (1)$$

where $c \sim 1.7$ is a constant to be specified later in Sec. III, and

(2) the prediction that the asymmetry in deep-inelastic electron scattering on a polarized deuteron target is much smaller than the corresponding asymmetry on a polarized nucleon target.

Several other results, less accessible to experiments, may be found in Secs. III and IV. On the basis of previous experience with the reciprocal bootstrap, we expect these predictions to hold to within 20% or 30%.

We briefly recall the reciprocal bootstrap in Sec. II. The predictions (1) and (2) are explained in Secs. III and IV, respectively. In Sec. V we discuss the chiral-limit theorems and other related approaches to our problem. Some technicalities are relegated to an appendix.

II. RECIPROCAL BOOTSTRAP

In the standard SU(3) Chew-Low model,[7] one considers $\Pi B \rightarrow \Pi B$ scattering and imposes the condition that the potential due to the exchange of octet (B) and decuplet (Δ) baryons in the crossed (u) channel produces the octet and decuplet baryons as resonances and bound states in the direct (s) channel. As is well known, the self-consistency requirement leads to the correct f/d ratio for ΠBB coupling. In the limit of small Π mass, a u-channel partial-wave amplitude with orbital angular momentum L crosses only into an s-channel amplitude with the same orbital angular momentum L. The static model is tractable because of this crucial feature.

Dashen and Frautschi[5] generalize the Chew-Low model to cover the reaction $\Pi B \rightarrow \chi B$ where χ is "anything" that has a definite mass, angular momentum, and SU(3) transformation property. The mass of B is taken to be much heavier than the mass of Π. The "mass" of χ is supposed to be small enough for the static approximation to hold, the standard self-consistency requirement may be imposed to determine the f/d ratio of the χBB coupling and the ratio of χBB coupling to $\chi B\Delta$ couplings (Fig. 1).

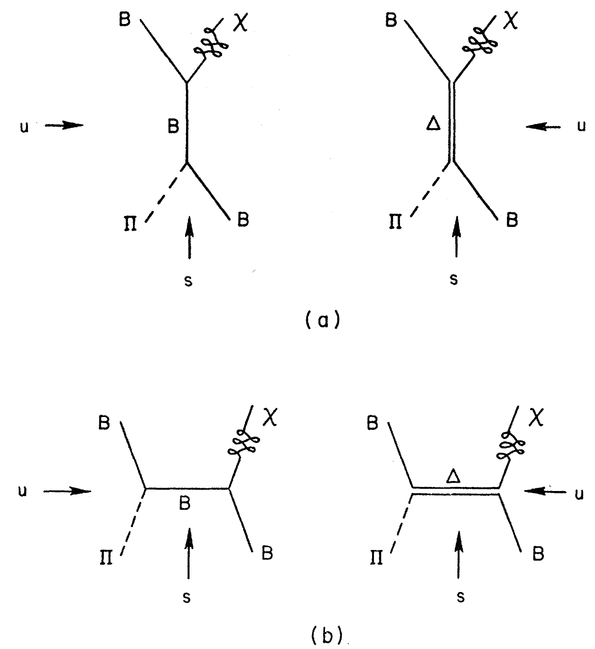

FIG. 1. The exchange force of octet B and decuplet Δ in the u channel (a) is required to produce the octet B and decuplet Δ in the s channel (b) in $\Pi B \rightarrow \chi B$ scattering.

Suppose χ has spin S. Then the χBB coupling may be defined in the rest frame of one of the baryons B as follows: Couple $\vec{S}$ to the orbital angular momentum $\vec{L}$ of the χ around B to form $\vec{K}=\vec{L}+\vec{S}$ and then couple $\vec{K}$ to the spin $\vec{\sigma}$ of B to form a total angular momentum of $\frac{1}{2}$. The allowed values of K for χBB coupling are $K=0$ and 1. (Any given Lorentz-invariant χBB coupling reduces to the form $Su_p^\dagger u_p + \vec{V} u_p^\dagger \vec{\sigma} u_p$ in the nonrelativistic limit, where u_p = Pauli two-component spinors. S corresponds to $K=0$, while V corresponds to $K=1$.) The $\chi B\Delta$ coupling proceeds similarly, with K allowed to be 1 and 2.

The crucial feature of the static approximation now states that couplings of a given K cross only to couplings with the same K. The crossing matrix depends only on K and not on S and L. Heuristically speaking, the baryon B at the χBB and $\chi B\Delta$ vertices only knows that it was hit by a bundle of angular momentum K and not the composition of the bundle. The total crossing matrix is a direct product of the K crossing matrix and the SU(3) crossing matrix.

In particular, if χ belongs to an octet, the f/d ratio of the χBB coupling depends only on whether χ has $K=0$ or $K=1$. This fact empowers Dashen and Frautschi to derive a number of interesting results, all of which are in rough agreement with available experimental information.[5,6] Since many of these results had been reviewed elsewhere,[6] we mention here only the two relations:

$$\frac{\frac{d\sigma}{dt}(\pi^- p \to \pi^0 n)}{\frac{d\sigma}{dt}(\pi^+ p \to \pi^0 \Delta^{++})} = \frac{2}{3}, \tag{2}$$

$$\frac{\frac{d\sigma}{dt}(\pi^- p \to \rho^0 n)}{\frac{d\sigma}{dt}(\pi^+ p \to \rho^0 \Delta^{++})} = \frac{2}{3}. \tag{3}$$

These follow from the static prediction for the ratio of $K=1$ χBB and $\chi B\Delta$ couplings and are supposed to work for t small enough so the static approximation holds and for s large enough so Regge exchanges are relevant. The relation (2) was rediscovered recently by saturating the Reggeon-particle amplitudes in the triple-Regge formula and was shown to agree well with experiment.[8] The relation (3) also appears to be consistent with present data.[9] Combined with the more standard successes,[5,6] the essential correctness of these relations contributes to our confidence that the static bootstrap (and our predictions) contain some truth.

III. THE LIGHT CONE

Let us now turn to the light-cone commutator of two currents. Define[10]

$$J_\mu^a(r,x) = V_\mu^a(x) + rA_\mu^a(x), \quad r=\pm 1;\ a=0,1,\ldots,8. \tag{4}$$

Then

$$[J_\mu^a(r,x), J_\nu^b(r',y)] = 0, \quad r \neq r' \tag{5}$$

$$[J_\mu^a(r,x), J_\nu^b(r,y)] = if_{abc}\{S_{\mu\nu\lambda\alpha}J_c^\lambda(r,S,X,\Delta) - ir\epsilon_{\mu\nu\lambda\alpha}J_c^\lambda(r,A,X,\Delta)\}\frac{\partial}{\partial\Delta_\alpha}\epsilon(\Delta_0)\delta(\Delta^2)$$
$$+ d_{abc}\{S_{\mu\nu\lambda\alpha}J_c^\lambda(r,A,X,\Delta) - ir\epsilon_{\mu\nu\lambda\alpha}J_c^\lambda(r,S,X,\Delta)\}\frac{\partial}{\partial\Delta_\alpha}\epsilon(\Delta_0)\delta(\Delta^2), \tag{6}$$

where $X=\frac{1}{2}(x+y)$ and $\Delta=\frac{1}{2}(x-y)$. The bilocal operators $J(S)$ are expressible in terms of local operators

$$J_\mu^a(r,S,X,\Delta) = \sum_{n\ \text{even}} \Delta^{\alpha_1}\cdots\Delta^{\alpha_n}(V^a_{\mu\alpha_1\cdots\alpha_n} + rA^a_{\mu\alpha_1\cdots\alpha_n}). \tag{7}$$

Exactly the same expansion holds for $J(A)$ except that the sum runs over odd n. The local operators $V^a_{\mu\cdots\alpha_n}$ and $A^a_{\mu\cdots\alpha_n}$ are Lorentz tensors and pseudotensors, respectively. Although we have written down the light-cone commutator in the form given by the quark-gluon model, it will become apparent to the reader that our essential results are much more general.

The experimental structure functions are given by

$$\omega^{-1}F_2^{ab}(r,\omega) = if_{abc}G_S^c(r,\omega) + d_{abc}G_A^c(r,\omega), \tag{8}$$

$$F_3^{ab}(r,\omega) = r[if_{abc}G_A^c(r,\omega) + d_{abc}G_S^c(r,\omega)], \tag{9}$$

where

$$\langle p|J_\mu^a(r,S;0,\Delta)|p\rangle = \frac{p_\mu}{m}\int_{-1}^{+1} d\omega\, e^{2i\omega p\Delta}G_S^a(r,\omega)$$

and

$$\langle p|J_\mu^a(r,A;0,\Delta)|p\rangle = \frac{p_\mu}{m}\int_{-1}^{+1} d\omega\, e^{2i\omega p\Delta}G_A^a(r,\omega). \tag{10}$$

As is well known, these equations express six structure functions in terms of five unknown bilocal functions and thus imply the relation first given by Llewellyn Smith.[11]

To proceed further we now substitute for χ (as introduced in Sec. II) the local operators $V^a_{\mu\cdots\alpha_n}$ and $A^a_{\mu\cdots\alpha_n}$ and immediately learn that these operators transform as SU(3) singlets and octets. This fact has already been presumed in writing down Eq. (6). That octet enhancement at short distances is intimately connected to the hadron spectrum and the reciprocal bootstrap is of course well known.[12] Note that the static approximation is eminently suitable here since we are interested in the value of C_J in the zero-momentum-transfer spin-averaged matrix element

$$\langle p|V^a_{\mu_1\cdots\mu_J}|p\rangle = p_{\mu_1}\cdots p_{\mu_J}C_J + \cdots \tag{11}$$

and the operators $V^a_{\mu_1\cdots\mu_J}$ "carry" zero mass. The coupling in Eq. (11) obviously corresponds to $K=0$ for all J and the discussion in Sec. II immediately informs us that $V^a_{\mu_1\cdots\mu_J}$ has the same f/d ratio independent of J. This universality of f/d ratio on the light cone means that

$$\frac{G_A^3(\omega)}{G_A^8(\omega)} = \frac{G_S^3(\omega)}{G_S^8(\omega)} = C, \quad \text{a constant independent of } \omega. \tag{12}$$

(To avoid convergence questions, we note that relations of this type should be interpreted as relations between the direct measures of the light-cone operators, namely the integral moments, rather than between the structure functions. Thus Eq. (12) would read in part

$$\int d\omega\, \omega^n G_A^3(\omega) \Big/ \int d\omega\, \omega^n G_A^8(\omega) = C$$

independent of n. For notational simplicity we continue to deal with the structure functions.) The prediction given in Eq. (1) now follows, after consulting Eq. (8). Since this relation holds just as well in the Regge limit $\omega \to 0$, the universal f/d ratio on the light cone is in fact the f/d ratio of the spin-nonflip coupling of the tensor trajectories. Indeed, in the Dashen-Frautschi schema, all $K=0$ couplings have the same f/d ratio and these include also the medium-strong, electromagnetic, and weak mass differences.[5,6] The reciprocal bootstrap determines f/d to be about -4, which should be interpreted to mean that f/d is large. For f/d varying between -2.5 and -3.3, we find the constant c ranging from 1.9 to 1.57.

Unlike the Llewellyn Smith relation,[11] which involves $F_3^{\nu p} - F_3^{\nu n}$, our relation involves $F_2^{\nu p} + F_2^{\nu n}$ and presumably may be tested directly on the deuteron with due allowance for Glauber corrections (or on heavy nuclei). There is some hint that our relation Eq. (1) may not be far wrong.[13] Clearly, universality of the f/d ratio on the light cone also leads to relations involving deep-inelastic $\Delta S=1$ neutrino processes beyond those implied by SU(3).[14] The interested reader may work these out for himself using Eqs. (8) and (9).

IV. POLARIZED TARGET

We next consider deep-inelastic scattering on polarized targets. One is then interested in the $K=1$ coupling $u_p^\dagger \vec{\sigma} u_p$ of the operators $A^a_{\mu_1 \cdots \mu_J}$ to octet baryons. Reasoning as before, we immediately deduce that the f/d ratio of this coupling is the same as the f/d ratio of the ΠBB coupling, say. But this does us little good until the somewhat futuristic experiments of deep-inelastic neutrino scattering on polarized targets are done. However, we now recall that in the reciprocal bootstrap the singlet operator does not have $K=1$ couplings. This may be understood heuristically as follows. The determination of the χBB coupling in the reciprocal bootstrap amounts to projecting out the s-channel octet baryon in the diagrams of Fig. 1(a). The group-theoretic structure may then be summarized by the self-consistent diagram in Fig. 2. Now $K=1$ couplings are largely generated by decuplet Δ exchange (rather than B exchange) in the crossed channel. (This is well known for $\Pi B \to \Pi B$.) Since $\underline{1}$ does not appear in $\underline{8}\times\underline{10}$, the singlet $K=1$ coupling to octet baryons is suppressed. This fact also manifests itself physically in the absence of low-mass SU(3)-singlet pseudoscalar mesons[12] and of an axial-vector baryonic current.

Thus we find

$$G_1^{\gamma p}(\omega) = \left(1 + \frac{3-4\alpha}{\sqrt{3}}\right) A(\omega) \simeq (1+\tfrac{1}{9}) A(\omega),$$
$$G_1^{\gamma n}(\omega) = \left(-1 + \frac{3-4\alpha}{\sqrt{3}}\right) A(\omega) \simeq (-1+\tfrac{1}{9}) A(\omega), \qquad (13a)$$

where A is some unknown function and $G_i (i=1,2)$ denotes the spin-dependent structure functions as defined in Ref. 15, for example. We have used the value $\alpha \simeq \frac{2}{3}$ as determined from ΠBB couplings. Physically, the small number $\frac{1}{9}$ reflects the smallness of the ηNN coupling to the $\pi^0 NN$ coupling. Hence

$$\frac{G_1^{\gamma p}(\omega) + G_1^{\gamma n}(\omega)}{2G_1^{\gamma p}(\omega)} \simeq \tfrac{1}{9} \simeq 0. \qquad (13b)$$

Aside from Glauber corrections, which we presume can be taken into account, deep-inelastic electron scattering from polarized deuterons will show almost no asymmetry. We remark that partonlike models predict large asymmetry for deep-inelastic electron scattering from a polarized proton target,[16] as is evident from angular momentum

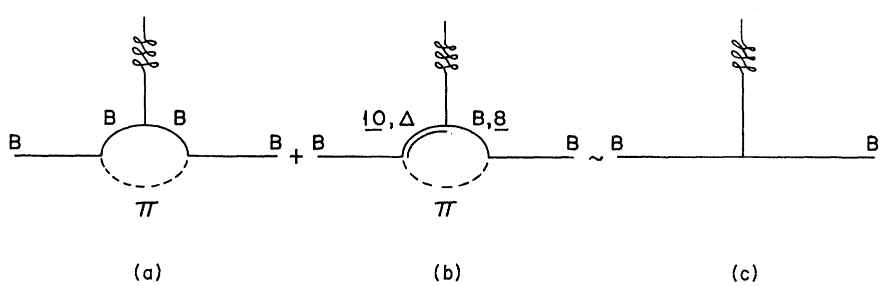

FIG. 2. Projecting out the s-channel baryon pole in Fig. 1(a) is equivalent, as far as group-theoretic structure is concerned, to coupling the s-channel ΠB state to B. The schematic relation (a) + (b) ~ (c) is only a part of a matrix equation.

conservation.

In fact, unlike the discussion in Sec. III this result already follows from an SU(2) bootstrap since $\frac{1}{2}\times\frac{3}{2}$ does not contain an SU(2) singlet. To estimate the accuracy of our prediction we note that the same physical mechanism[5] is responsible for the smallness of the isoscalar magnetic moment $\mu_p + \mu_n$ (and for the fact that the ω trajectory is largely spin-nonflip).[17] (Here μ_p and μ_n denote the total magnetic moment.) Experimentally

$$\frac{\mu_p + \mu_n}{\mu_p - \mu_n} \simeq \frac{0.88}{4.70} \sim 20\% .$$

This sort of error margin is consistent with the error found in other predictions of the reciprocal bootstrap.

The reason why the SU(3) bootstrap is mentioned in this section at all is to show that, if one should so fancy, one may derive relations involving neutrino scattering on polarized targets. The reader is again invited to work[18] these out for himself.

V. THRESHOLD DOMINANCE AND OTHER APPROACHES

Let us try to produce another argument for a universal f/d ratio on the light cone. Recently, Li and Pagels[19] have stressed that, in the chiral limit $\mu/M \to 0$, intermediate states consisting of a few Goldstone mesons may dominate certain dispersion relations. This enables them to make a number of interesting observations. In particular, by assuming that the octet-baryon matrix elements $\langle p' | \partial^\mu V_\mu^a | p \rangle$ satisfy an unsubtracted dispersion relation in $q^2 = (p - p')^2$, they obtained the f/d ratio of the baryon mass differences in terms of the f/d ratio of the ΠBB coupling.

Following these authors, we write an unsubtracted dispersion relation for $H_{cb}^a(q^2)$ where

$$\langle p_c' | V_{\mu_1 \cdots \mu_J}^a | p_b \rangle = P_{\mu_1} \cdots P_{\mu_J} H_{cb}^a(J, q^2) + \cdots .$$

Here $P = p_b + p_c'$ and $q = p_b - p_c'$. Now suppose the two-meson state dominates $\text{Im} H_{cb}^a(q^2)$ as illustrated in Fig. 3. Write

$$H_{cb}^a(J, 0) = f_L^J(-if_{cba}) + d_L^J d_{cba}$$

(the subscript L reminds us of the light cone). Let $\Gamma_{cba}(\lambda) = \lambda(-if_{cba}) + d_{cba}$ be proportional to the $\bar{B}^c B^b \Pi^a$ coupling with its f to d ratio denoted by $\lambda = f_\pi/d_\pi = (1-\alpha)/\alpha$, with $\alpha \approx \frac{2}{3}$.

According to Fig. 3,

$$H_{cb}^a(J, 0) = d_{aef} \Gamma_{cge}(\lambda) \Gamma_{gbf}(\lambda) C_J , \tag{14}$$

where C_J is a "dynamical" constant dependent on some cutoff mass which varies with J. By now it should be abundantly obvious that the universality of f/d ratio on the light cone is valid. It also

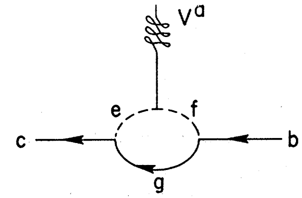

FIG. 3. The contribution of the two–Goldstone-meson state to $H_{cb}^a(J, q^2)$. The dashed lines represent Π.

should not surprise the reader that we obtain the same f/d ratio Li and Pagels obtained for the baryon mass differences, namely that

$$(f/d)_L = \frac{10}{3} \frac{(f/d)_\pi}{3(f/d)_\pi^2 - 1} . \tag{15}$$

(We shall show how this follows from Fig. 3 in the Appendix.) This expression for $(f/d)_L$ has a pole at $(f/d)_\pi = 1/\sqrt{3}$, which is close enough to the experimental value $(f/d)_\pi \simeq \frac{1}{2}$ to inhibit us from drawing any conclusion from Eq. (15) other than that $(f/d)_L$ is large.

How may the above discussion fail? The assumption of unsubtracted dispersion relations for H_{cb}^a amounts to saying that the structure functions may be calculated in terms of more fundamental parameters. The universal f/d ratio depends critically on the dominance of the two-meson states. It would appear unlikely that if four-meson states are important, the ratio of their contribution to that of the two-meson states is independent of J. Also, the structure of Fig. 3 suggests that it represent a vestigial manifestation of a Compton scattering graph in which the virtual photon scatters on a meson. Whether this is consistent with the Callan-Gross relation[3] is hard to say and depends on the specific model. (Since the Callan-Gross relation is satisfied for meson targets, one may argue that this picture is in fact not inconsistent with data.)

VI. DISCUSSION

Our program here is a simple and modest one. Recognizing that deep-inelastic scattering measures the static properties of light-cone operators and that only two couplings to baryons, $u_p^\dagger u_p$ and $u_p \vec{\sigma} u_p$, are allowed in the static limit, we proceed to draw up a list of relevant phenomenological facts. For $K=1$, say, the list reads as follows: (1) smallness of the isoscalar magnetic moment, (2) the absence of low-mass SU(3)-singlet pseudoscalar mesons, (3) smallness of the ηNN coupling, (4) the absence of an axial baryonic current, (5) the fact that the ω trajectory and the Pomeranchukon are largely spin-nonflip. One may then argue in favor of the following statement: Whatever the dynamics of physics in the static limit may be, the same dynamics that is responsible for the phenomenological pattern above would also lead to a suppression of the isoscalar spin-asymmetry in deep-inelastic scattering. Similar reasoning applies to the $K=0$ couplings. From this point of view a gross violation (larger than the expected 20–30%) of our relations by data may be interpreted as an indication that the concept of local operators on the light cone may not be a useful one.

In this paper we present a specific example of low-energy dynamics, namely the Chew-Low static bootstrap. How good is the static approximation? There has been no known study of the accuracy of the static approximation as a function of the spin S of χ. However, one may anticipate that for large S, L becomes large and the approximation becomes poorer and poorer. Thus, in terms of moment sum rules over the structure functions our results are presumably more accurate for the lower moments [cf. Eq. (10)]. In other words, Eq. (1) and Eq. (13b) are expected to break down in the threshold region ($\omega \to 1$).

We note that the group-theoretic structure in Fig. 3 bears a certain resemblance to that in Fig. 2(a). This formal connection will be spelled out in detail in the Appendix. Since the static-model bootstrap and the chiral-limit theorems are both based on the smallness of μ/M, it is not surprising that they may be related. Another approach based on $\mu \ll M$ is the sidewise dispersion relation in the threshold dominance approximation,[20] which amounts to calculating the three diagrams in Fig. 4. Two of the diagrams have appeared formerly [in Figs. 2(a) and 3]. The static-model bootstrap also shares the approximation $M_B/M_\Delta \simeq 1$ with (nonrelativistic) SU(6). Sure enough, SU(6) would also predict[21] the universal (f/d) ratio on the light cone (to be ∞ for $K=0$ and to be $\frac{2}{3}$ for $K=1$) and the ratio of χBB coupling to $\chi B\Delta$ coupling (to be roughly the same value predicted by the static bootstrap). It is also well known[22] that some of the SU(6) results are reproduced by saturating the chiral algebra by octet and decuplet baryons. [In this light then, the rediscovery[8] of Eq. (2) is not surprising at all.] It is tempting to conjecture that the light-cone operators generate an algebra. Our results may then follow by approximate saturation. We close by expressing the hope that the (somewhat mysterious) connection between all these approaches of the last two decades will become better understood.

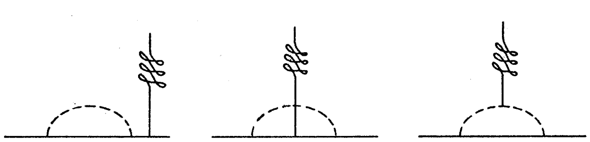

FIG. 4. Sidewise dispersion relation calculation of χBB coupling ignoring the existence of the Δ.

ACKNOWLEDGMENTS

We have enjoyed valuable conversations with Roger Dashen. We thank Dr. Carl Kaysen for his hospitality at the Institute for Advanced Study.

APPENDIX

As explained in Sec. IV the octet $K=0$ coupling H_{cba} (corresponding to $\bar{B}^c B^b \chi^a$) is the solution of the bootstrap equation, a piece of which reads

$$H_{cba} = \gamma H_{gha} \Gamma_{cge}(\lambda) \Gamma_{hbe}(\lambda) . \tag{A1}$$

Here γ is a "dynamical" constant and $\Gamma_{cba}(\lambda)$ is the ΠBB coupling introduced in Sec. V. This equation is schematically represented in Fig. 5 and appears as a piece of the mass-difference calculation of Dashen and Frautschi.[23] These authors discuss in detail all the effects ignored in (A1). Define the 8-by-8 matrices

$$(F^a)_{bc} = -if_{abc} \quad \text{and} \quad (D^a)_{bc} = d_{abc}$$

and write $H_{cba} = f(F^c)_{ba} + d(D^c)_{ba}$. With the aid of the identities

$$\operatorname{tr} D^a D^b D^c = -\tfrac{1}{2} d^{abc} , \tag{A2}$$

$$\operatorname{tr} D^a F^b F^c = \tfrac{3}{2} d^{abc} , \tag{A3}$$

$$\operatorname{tr} D^a D^b F^c = \tfrac{5}{6} i f^{abc} , \tag{A4}$$

$$\operatorname{tr} F^a F^b F^c = \tfrac{3}{2} i f^{abc} , \tag{A5}$$

we evaluate (A1) to give[24]

$$\begin{pmatrix} f \\ d \end{pmatrix} = \gamma \begin{pmatrix} \frac{1}{6}(9\lambda^2+5) & -\frac{5}{3}\lambda \\ -3\lambda & \frac{1}{2}(3\lambda^2-1) \end{pmatrix} \begin{pmatrix} f \\ d \end{pmatrix} . \tag{A6}$$

For α varying between 0.66 to 0.69, this equation gives f/d varying between -1.3 to -1.4. This crude calculation barely suggests that f/d may

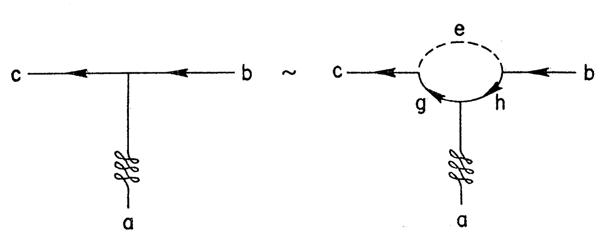

FIG. 5. Schematic representation of partial bootstrap equation for $K=0$ coupling.

be large. The much more sophisticated calculation of Ref. 23 gives f/d ranging from −1.5 to −3.

In the threshold dominance calculation, however, H_{cba} is the solution to Eq. (14). Using the symmetry property $\Gamma_{cba}(\lambda)=\Gamma_{cba}(-\lambda)$ we see that

$$\begin{pmatrix} f \\ d \end{pmatrix} = C_J \begin{pmatrix} \frac{5}{3}\lambda \\ \frac{1}{2}(3\lambda^2-1) \end{pmatrix} \tag{A7}$$

is given by the second column of the matrix in Eq. (A6) with $\lambda \to -\lambda$. This reproduces the result of Li and Pagels cited in Eq. (14). For α varying between 0.66 and 0.69 this equation gives f/d varying between −6.6 and −3.3. In the sidewise dispersion approach presumably the two (f, d) vectors in (A6) and (A7) are to be added. The precise connection between these considerations is unclear to us. Within the context of the reciprocal bootstrap, there exists an argument that the contribution of t-channel singularities is small.[5, 23] On the other hand, in the chiral-limit theorem there is no motivation for considering the baryon-antibaryon intermediate states either.

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[7]For an exposition, see, for example, S. Gasiorowicz, *Elementary Particle Physics* (Wiley, New York, 1966), Chap. 25.

[8]J. Finkelstein, Stanford report, 1972 (unpublished). See also R. Carlitz, CERN report, 1972 (unpublished).

[9]See Figs. 6 and 7 of G. Wolf, Phys. Rev. 182, 1538 (1969). We note that $d\sigma/dt\,(\pi^- p \to \rho^0 n)$ peaks at the value of ~ 2 mb/GeV^2 (at $t \sim 0$) for $P_L = 8.0$ GeV, while $d\sigma/dt\,(\pi^+ p \to \rho^0 \Delta^{++})$ peaks at ~ 6 mb/GeV^2, ~ 4 mb/GeV^2, and ~ 2 mb/GeV^2 at $P_L = 4.0$ GeV, 6.95 GeV, and 16 GeV, respectively. A_2 exchange is still insignificant at these energies, however, so the agreement may simply be accounted for by the fact that one-pion-exchange models adequately fit the data. The slopes in $|t|$ ($0 \lesssim |t| \lesssim 0.5$ GeV^2) differ by a factor of 2, showing the breakdown of the static approximation.

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[24]This matrix may be recognized as part of Table VI of Ref. 23.