Spontaneously Broken Gauge Symmetry and Elementary Particle Masses*

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There are a number of approximate relations among elementary particle masses. In conventional theories, these are explained by assuming a hierarchy of interactions with the property that the stronger interactions have the more extensive symmetry. In renormalizable theories based on spontaneously broken gauge symmetry, this view is untenable. Manifest breaking of the gauge symmetry would destroy the renormalizability. We explore the possible origin of approximate mass formulas in theories of this kind.

The elementary particle masses satisfy a number of approximate relations which seem to reflect various underlying symmetries; some examples are the Gell-Mann–Okubo mass formulas, the approximate equality of the proton and neutron masses, and the smallness of the electron mass. The recently developed class of models based on spontaneously broken gauge symmetries[1] present us with the possibility of understanding these relations and indeed of calculating departures from them. In this paper we will describe two classes of models in which departures from naive mass relations can be calculated.

The models we consider are based on a gauge algebra $\mathcal{G}$ with vector bosons W_a corresponding to the infinitesimal generators. In addition, there is a multiplet φ_i of real spinless meson fields and a multiplet ψ of spin-$\frac{1}{2}$ fermion fields. The gauge symmetry is spontaneously broken when φ_i develops a vacuum expectation value $\langle\varphi_i\rangle_0=\lambda_i$. Vector bosons corresponding to the unbroken subalgebra remain massless, while all others develop mass. No massless Goldstone bosons survive as physical states. We shall be concerned with the fermion masses and the relations which they satisfy at various levels of approximation.

ZEROTH-ORDER MASS RELATIONS

We will call a mass or mass difference calculable when it need not appear as a renormalization counterterm in the Lagrangian. The required fermion mass counterterms are of two types:

(1) Bare mass terms. Any term $\overline{\psi}m_0\psi$ which is invariant under the gauge group and all other symmetries of the Lagrangian (before spontaneous symmetry breaking) must be regarded as a counterterm.

(2) Zeroth-order tadpoles. The zeroth-order vacuum expectation of φ, which we will call λ^0, is computed by maximizing the spinless-meson part of the Lagrangian. The true vacuum expectation value λ, is obtained by minimizing the action, to which the negative of the renormalized Lagrangian is just the zero-loop approximation. Suppose that the Yukawa coupling of the mesons to the fermions is $\varphi_i\overline{\psi}\Gamma_i\psi$ (where Γ_i may involve γ_5). Then $\lambda_i^0\overline{\psi}\Gamma_i\psi$ must also be regarded as a counterterm.

The sum of these two terms gives the zeroth-order mass matrix, $m_0+\lambda_i^0\Gamma_i$. A zeroth-order mass relation is a relation between the masses in the zeroth-order mass matrix which is stable under small changes in the renormalized parameters. It is these zeroth-order mass relations which we expect to correspond to the approximate mass relations observed in elementary particle masses. Furthermore, any departures from these mass relations must be finite higher-order effects.

We can distinguish three types of zeroth-order mass relations:

(1) Mass relations determined by an unbroken subgroup of the symmetry group of the Lagrangian.

(2) Mass relations determined by the representation content of the spinless-meson multiplet.

(3) Mass relations not determined by group-theoretical properties.

Type (1) is the familiar exact mass relation associated with an exactly conserved symmetry. For example in the absence of weak and electromagnetic interactions, we might imagine a spontaneous breakdown of SU(3) which left isospin invariance unbroken. We will show in the Appendix that except in pathological cases, this type of mass relation will be maintained in higher orders. Since the mass formulas observed in nature are only approximate (with a few well-known exceptions like the vanishing of the photon mass), type (1) mass formulas cannot yield a complete description. The conventional way out of this familiar problem is to introduce explicit symmetry-breaking terms into the Lagrangian. However, in the gauge theories which we are considering, this would destroy the renormalizability of the theory.[2]

We can most easily explain mass relations of types (2) and (3) by illustrating them in simple Abelian models.

ABELIAN MODELS

For simplicity we will consider parity-conserving models with an axial-vector gauge boson, and various fermion and spinless-meson fields. Under the gauge group, the fields transform like

$$B_\mu \to B_\mu - (1/g)\partial_\mu \rho \,,$$
$$\psi_a \to \exp(i\rho\alpha_a\gamma_5)\psi_a \,,$$
$$\varphi_i \to \exp(i\rho\beta_i)\varphi_i \,.$$

The real spinless-meson fields are defined by $\varphi_i \equiv (S_i + iP_i)/\sqrt{2}$, where S_i and P_i are scalar and pseudoscalar, respectively. The Yukawa coupling term

$$\varphi_i[\bar\psi_a(1+\gamma_5)\psi_b + \bar\psi_b(1+\gamma_5)\psi_a]/\sqrt{2} + \text{H.c.} = \bar\psi_a(S_i + i\gamma_5 P_i)\psi_b + \bar\psi_b(S_i + i\gamma_5 P_i)\psi_a$$

is invariant if $\alpha_a + \alpha_b + \beta_i = 0$, and the fermion bare-mass term $\bar\psi_a\psi_a$ is invariant only if $\alpha_a = 0$.

The first model we will consider has two fermion fields and two complex spinless-meson fields with $\alpha_1 = 1$, $\alpha_2 = -2$, $\beta_1 = 1$, and $\beta_2 = -2$. If the spinless-meson Lagrangian is such that S_1, and P_1, have positive mass squared, then only φ_2 develops a vacuum expectation value in zeroth order. Furthermore, the Lagrangian is invariant under the transformation $\varphi_1 \to -\varphi_1$, $\psi_1 \to -\psi_1$; and this symmetry is not broken by the spontaneous gauge-symmetry breaking. Therefore there will be no φ_1 vacuum expectation value in any order and in addition the ψ_1 and ψ_2 fermions cannot mix.

Since φ_2 couples to the ψ_1 mass term, ψ_1 develops a mass in zeroth order. (Note that we can take $\langle\varphi_2\rangle_0$ to be real.) Because there is no meson in the model with $\beta = 4$, there can be no ψ_2 mass term in zeroth order. It is forbidden by the representation content of the scalar-meson multiplet. Thus this is an example of a type (2) zeroth-order mass relation.

Now this mass relation is changed by higher-order effects. In particular, the ψ_2 can emit an S_1 or P_1 and become a ψ_1, which reabsorbs the meson. This gives rise to a finite contribution to the ψ_2 mass which is proportional to the zeroth-order ψ_1 mass and also depends on the S_1-P_1 mass difference. We find it convenient to think about such contributions diagrammatically in stage zero, in which the gauge-symmetry breaking is introduced explicitly as tadpoles. In this language, the lowest-order contribution to the ψ_2 mass comes from the diagram in Fig. 1, where each line is labeled with the gauge quantum number it carries. In stage zero, this diagram contributes a term like

$$\bar\psi_2(1+\gamma_5)\psi_2\varphi_2^{*2} + \text{H.c.}$$

to the action, which gives a ψ_2 mass due to the spontaneous symmetry breaking.

Weinberg[3] has considered type (2) zeroth-order mass relations for an arbitrary gauge group and arbitrary meson and fermion multiplets. He has given an elegant demonstration that the one-loop corrections to type (2) mass relations are finite in the *unitary* gauge. He also gives some interesting examples of calculable weak and electromagnetic mass differences based on this type of zeroth-order mass relation.

We can illustrate zeroth-order mass relations of type (3) with a model which is superficially similar. It has two fermion fields with $\alpha_1 = 1$ and $\alpha_2 = -4$, and four complex spinless-meson fields with $\beta_1 = 1$, $\beta_2 = -2$, $\beta_3 = 3$, $\beta_4 = 8$. The scalar-meson Lagrangian can be chosen so that only φ_2 develops a zeroth-order vacuum expectation value. Furthermore, the vanishing of the φ_1, φ_3, φ_4 zeroth-order vacuum expectation values persists for any small changes of the parameters in the Lagrangian. In zeroth order, ψ_1 develops mass while ψ_2 remains massless. The masslessness of ψ_2 is an example of a type (3) zeroth-order mass relation, since we have introduced a meson S_4, which couples to $\bar\psi_2\psi_2$, but it develops no zeroth-order vacuum expectation value. This mass relation is not merely a consequence of the representation content of the meson multiplet, nor does it follow from any symmetry of the spinless-meson Lagrangian, but depends on the particular dynamics of the spinless-meson system.

In higher orders this mass relation is broken. The contributions to the ψ_2 mass are of two kinds. There is a contribution from the diagram shown in Fig. 2, similar to Fig. 1. In addition φ_4 develops a calculable vacuum expectation value in higher orders. Thus, there is a contribution to the ψ_2 mass from the diagram shown in Fig. 3. In fact,

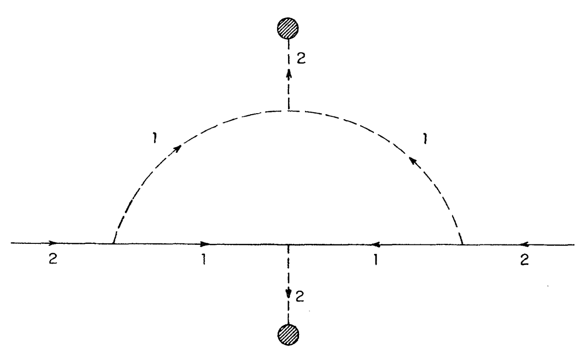

FIG. 1. Feynman diagram contributing a correction to a type (2) mass relation. In these figures, continuous lines denote fermions, broken lines denote spinless mesons, and wavy lines denote gauge mesons.

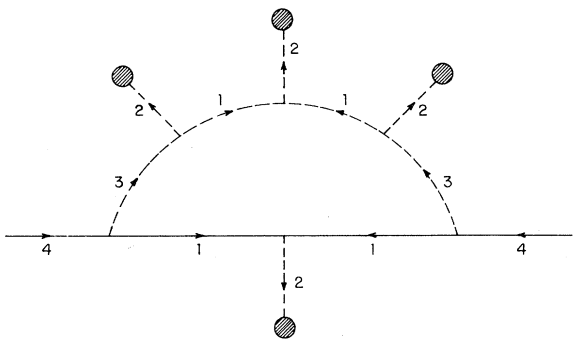

FIG. 2. Feynman diagram contributing a correction to a type (3) mass relation.

even if the original model does not contain a ψ_1 field, so that in zeroth order no massive fermion field appears in the problem, the diagram in Fig. 3 will generate a calculable ψ_2 mass in higher orders.

This type of model can also provide a natural mechanism for the generation of a hierarchy of mass scales. For example, suppose in the model that we are discussing that there are additional fermion and meson fields ψ_n for $n \geq 3$ and φ_m for $m \geq 5$ with $\alpha_n = -2^n$ and $\beta_m = 2^{m-1}$. None of these fermions have mass in zeroth order, but ψ_n gets a mass from a diagram involving at least 2^{n-2} loops.

While these Abelian models are interesting as illustrations, they are a long way from a realistic theory. In particular, in these models, the gauge bosons play no essential role in the mass-generating mechanism. Their only role is to eliminate the Goldstone boson arising from the spontaneous

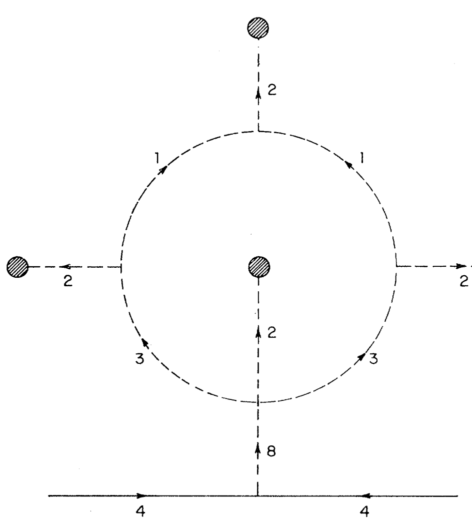

FIG. 3. Another type of Feynman diagram contributing a correction to a type (3) mass relation. This tadpole diagram can give the dominant contribution.

symmetry breaking. To go further, we must look at non-Abelian models.

NON-ABELIAN MODELS

The first non-Abelian model we will consider is based on the gauge group SU(3). There are eight vector bosons with appropriate self-interactions. Suppose that the spinless mesons are an octet. The spontaneous symmetry breaking in this system has a rather remarkable property. It automatically leaves an SU(2)×U(1) subgroup of the original gauge group invariant. To see this, let us consider the most general scalar-meson Lagrangian. We denote the octet of mesons by φ, a three-by-three, traceless, Hermitian matrix. Then

$$\mathcal{L}(\varphi) = \tfrac{1}{4}\alpha_1 \operatorname{tr}(\varphi^4) + \tfrac{1}{3}\alpha_2 \operatorname{tr}(\varphi^3) + \tfrac{1}{2}\alpha_3 \operatorname{tr}(\varphi^2)\,.$$

Note that the invariants $\det\varphi$ and $[\operatorname{tr}(\varphi^2)]^2$ are not independent but can be expressed in terms of the three traces appearing in $\mathcal{L}$. Now suppose φ develops a vacuum expectation value. By making a gauge transformation, we can choose it to be diagonal, so

$$\langle\varphi\rangle_0 = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & -(a+b) \end{bmatrix}.$$

The conditions which a and b must satisfy in order that $\varphi = \langle\varphi\rangle_0$ be an extremum are obtained by requiring

$$\frac{\partial}{\partial\varphi}\mathcal{L}(\varphi)\bigg|_{\varphi=\langle\varphi\rangle_0} = 0\,.$$

This gives two equations:

$$[\alpha_1(a^2 + 2ab + b^2) - \alpha_2 b + \alpha_3](2a + b) = 0\,,$$

$$[\alpha_1(a^2 + 2ab + b^2) - \alpha_2 a + \alpha_3](2b + a) = 0\,.$$

But these imply that either $2a + b = 0$, $2b + a = 0$, or $a = b$. Thus we can take

$$\langle\varphi\rangle_0 = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & -2a \end{bmatrix},$$

which commutes with isospin and hypercharge. By the arguments in the Appendix, this means that the isospin and hypercharge gauge symmetry remains unbroken to all orders, and the corresponding vector bosons remain massless.

It is instructive to note that this form for the zeroth-order vacuum expectation value is not the result of any group-theoretical property of the meson system, but is a consequence of the restrictions placed upon the scalar-meson Lagrangian by the requirement of renormalizability. To see this, imagine a similar model in three-dimensional space-time. Here the scalar-meson Lagrangian can involve terms like $[\mathrm{tr}(\varphi^3)]^2$, and one can construct Lagrangians which are maximized at $\varphi = \lambda$ for arbitrary values of $\mathrm{tr}(\lambda^2)$ and $\mathrm{tr}(\lambda^3)$. Of course, in the four-dimensional model, such terms are generated in the action by higher-order effects, but the argument in the Appendix shows that they do not displace $\langle\varphi\rangle_0$ away from its zeroth-order form. What can happen, however, is that an entirely new maximum can develop, far away from the zeroth-order maximum. We will not discuss this kind of higher-order spontaneous symmetry breaking here.

Now suppose there is an octet of fermions in the problem, which couples to the scalar mesons. The fermions will exhibit two types of zeroth-order mass relations. Since, as we have shown, the spontaneous symmetry breaking conserves isospin, there is a type (1) mass relation which guarantees equality of masses within isospin multiplets. If we interpret this system as a model of strong interactions, then we have, for instance, equal mass for the proton and neutron; presumably because we have not included the weak and electromagnetic interactions. On the other hand, since the meson multiplet does not contain any $\underline{27}$, $\underline{10}$, or $\overline{\underline{10}}$, there is a type (2) relation which is nothing but the Gell-Mann–Okubo formula for the nucleon masses. This mass relation is broken in higher orders, and the corrections are calculable in the sense defined above; that is they are cutoff-independent.

We now wish to examine the problem of the electron mass. In particular, we want to examine models of weak and electromagnetic interactions in which the electron mass vanishes in zeroth order while higher-order processes give calculable contributions of order αm_μ. An example of a model in which this seems possible has been given by Weinberg.[4] In this model the gauge group is chiral $SU(3)\times SU(3)$ and the leptons are a triplet, (μ^+, ν, e^-). The observed structure of the weak and electromagnetic interactions is imposed by a "superstrong" spontaneous symmetry breaking which gives a very large mass to the vector bosons coupled to unobserved currents.

In this model, the electron can emit a doubly charged vector boson W, and become a muon, so there is a natural mechanism for the generation of an electron mass of order αm_μ in diagrams like that in Fig. 4. The problem is that the mechanism cannot be this simple. It is easy to show, in the unitary gauge, that if the vector-boson masses are such that the quadratic divergence in Fig. 4 cancels, the entire contribution vanishes. However, as we shall see, it is possible to construct models of this type in which two-loop diagrams involving vector bosons can contribute to the electron mass.

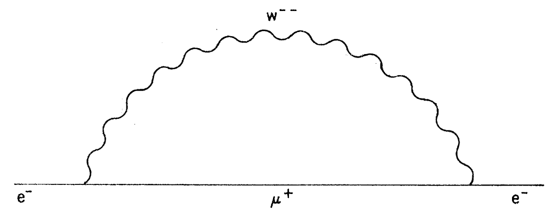

FIG. 4. This kind of Feynman diagram suggests the possibility of obtaining an electron mass of order αm_μ.

For simplicity, instead of dealing with the Weinberg model, we will try to abstract out those features which are essential to the mass-producing mechanism. We will consider a model based on the gauge group $SU(2)\times U(1)$, with gauge vector bosons $\vec{W}^\mu$ and B^μ. The left-handed fermion fields,

$$\psi_L = \tfrac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} e \\ \mu \end{pmatrix}_L ,$$

are an SU(2) doublet with U(1) quantum number $-\frac{1}{2}$, and the right-handed fermion fields,

$$\psi_R = \tfrac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} \mu \\ e \end{pmatrix}_R ,$$

are an SU(2) doublet with U(1) quantum number $\frac{1}{2}$. The meson fields are irreducible SU(2) tensors φ_i, χ_{ij}, ξ_{ij}, and M_{ijkl} with U(1) quantum numbers 1, $\frac{1}{2}$, $\frac{3}{2}$, and 2, respectively (i,j,k,l run from 1 to 3). Only the φ field couples to fermions. The Yukawa coupling is $\overline{\psi}\vec{\tau}\cdot\vec{\varphi}(1+\gamma_5)\psi + \mathrm{H.c.}$ The spinless-meson Lagrangian can be chosen so that the only zeroth-order vacuum expectation values are $\langle\varphi_i\rangle_0 = aZ_i$ and $\langle M_{ijkl}\rangle_0 = bZ_iZ_jZ_kZ_l$ where $\vec{Z}$ is the complex, null vector $(1, i, 0)$. Under a small change of the parameters in the spinless-meson Lagrangian, a and b will change, but the tensor structure of the zeroth-order vacuum expectation values, and the nullity of

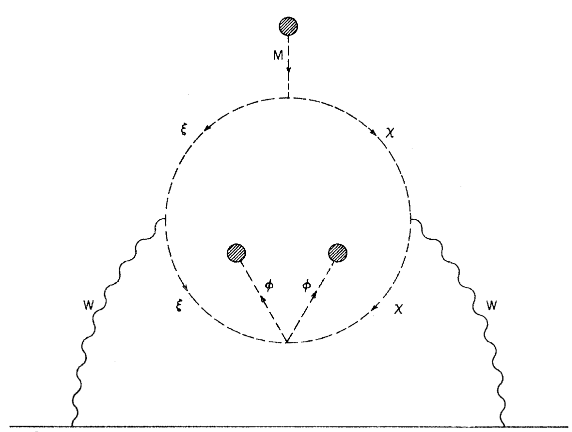

FIG. 5. A two-loop Feynman diagram realizing the suggestion of Fig. 4.

Z will persist. Then to zeroth order, only the muon develops a mass. This is a zeroth-order mass relation of type (3), since there is a scalar meson which couples to $\bar{e}e$, but it lacks a zeroth-order vacuum expectation value.

The vector mesons which contribute to the diagram in Fig. 4 are W_1^μ and W_2^μ which are vector and axial-vector, respectively. In zeroth-order they have equal mass and the diagram gives no contribution to the electron mass. However, there are one-loop diagrams which contribute to a calculable W_1-W_2 mass difference and hence there are contributions to the electron mass in two loops. Such a contribution comes from the diagram shown in Fig. 5. There are also contributions to the electron mass in one loop; but these do not involve the vector bosons, and for some choices of the parameters of the theory, their contribution can be made small compared to the vector-boson contributions.

This kind of construction can be extended to models like the SU(3)$\times$SU(3) model proposed by Weinberg[4] [although to implement the scheme there we have had to enlarge the gauge group to include extra U(1) factors]. This provides a possible, though rather complicated and inelegant, class of models in which the electron mass is, in principle, calculable. In practice, of course, the situation is not very satisfactory since the calculation is both difficult, in that it involves two loops, and useless, in that it will depend on a number of inaccessible parameters. It would be gratifying to find similar models based on zeroth-order mass relations of type (2), where the higher-order corrections are usually less complicated.

ACKNOWLEDGMENTS

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APPENDIX

We show here that zeroth-order mass relations of type (1) are necessarily exact mass relations: they persist to all orders. Suppose $\mathcal{L}(\varphi)$ is a spinless-meson Lagrangian invariant under some Lie group $\mathcal{G}$. The mesons φ transform under a real representation of $\mathcal{G}$. Let $\mathcal{A}$ be the algebra of infinitesimal generators of $\mathcal{G}$ in this representation: The elements of $\mathcal{A}$ are imaginary antisymmetric matrices A_{ij}. Suppose $\mathcal{L}(\varphi)$ has a local maximum at $\varphi=\lambda$, so that the spinless mesons develop a zeroth-order vacuum expectation value $\langle\varphi\rangle_0=\lambda$. Suppose further that this spontaneous breakdown leaves intact a subalgebra $\mathcal{B}$ of $\mathcal{A}$, giving rise to zeroth-order mass relations of type (1). The true vacuum expectation of φ is obtained by minimizing the action. For sufficiently weak coupling, the true vacuum expectation value will lie close to λ. We show that the true vacuum expectation value also leaves $\mathcal{B}$ unbroken.

The condition that λ is an extremum of $\mathcal{L}$ is just

$$\mathcal{L}_i(\lambda)=0\,, \tag{1}$$

and that it is a local maximum is that the mass matrix, $-\mathcal{L}_{ij}(\lambda)$, is positive semidefinite, where we use the notation $\mathcal{L}_{ij}...(\varphi)=[(\partial/\partial\varphi_i)(\partial/\partial\varphi_j)\cdots]\mathcal{L}(\varphi)$. The invariance of $\mathcal{L}(\varphi)$ under $\mathcal{G}$ yields

$$\mathcal{L}_i(\varphi)A_{ij}\varphi_j=0\,. \tag{2}$$

Differentiating (2), we obtain

$$\mathcal{L}_{ki}(\varphi)A_{ij}\varphi_j+\mathcal{L}_i(\varphi)A_{ik}=0\,. \tag{3}$$

Setting $\varphi=\lambda$, we have $\mathcal{L}_{ki}(\lambda)A_{ij}\lambda_j=0$, which is just the statement[5] that the Goldstone mesons are massless. At this point, we make a technical assumption: We assume that there are no massless spinless mesons but the Goldstone mesons.[6] More precisely, we assume that if $\mathcal{L}_{ij}(\lambda)\chi_j=0$ then χ is a Goldstone meson $\chi_j=A_{jk}\lambda_k$ for some A_{jk} in $\mathcal{A}$.

Now let us imagine making a small change of the Lagrangian, $\delta\mathcal{L}(\varphi)$ (such as might be induced by taking radiative corrections into account). Let $\delta\mathcal{L}$ also be invariant under $\mathcal{G}$. Denote the displaced maximum of $\mathcal{L}+\delta\mathcal{L}$ by $\lambda+\delta\lambda$. Since $\mathcal{L}+\delta\mathcal{L}$ is invariant under $\mathcal{G}$, $\delta\lambda$ may be chosen orthogonal to the tangent space of $\mathcal{G}$ at λ,

$$\delta\lambda_i A_{ij}\lambda_j=0\,. \tag{4}$$

The condition that $\lambda+\delta\lambda$ maximizes $\mathcal{L}+\delta\mathcal{L}$ may be obtained, to first order in small quantities, by differentiation of (1),

$$\mathcal{L}_{ij}(\lambda)\delta\lambda_j+\delta\mathcal{L}_i(\lambda)=0\,. \tag{5}$$

Let B_{ij} denote an arbitrary element of the subalgebra $\mathcal{B}$ which remains unbroken in zeroth order,

$$B_{ij}\lambda_j=0\,. \tag{6}$$

To prove that $\mathcal{B}$ remains unbroken in all orders, we must show that

$$B_{ij}\delta\lambda_j=0\,. \tag{7}$$

From (5), we have

$$B_{ki}\mathcal{L}_{ij}(\lambda)\delta\lambda_j+B_{ki}\delta\mathcal{L}_i(\lambda)=0\,. \tag{8}$$

But, from (6) and the invariance of $\mathcal{L}(\varphi)$ and $\delta\mathcal{L}(\varphi)$, we obtain $B_{ki}\delta\mathcal{L}_i(\lambda)=0$ and $B_{ki}\mathcal{L}_{ij}(\lambda)-\mathcal{L}_{ki}(\lambda)B_{ij}=0$, so that (8) becomes

$$\mathcal{L}_{ki}(\lambda)B_{ij}\delta\lambda_j=0\,.$$

Thus, $B_{ij}\delta\lambda_j$ is annihilated by the mass matrix, and by our assumptions, there must be an A_{ij} such that $B_{ij}\delta\lambda_j = A_{ij}\lambda_j$. Then $|B_{ij}\delta\lambda_j|^2 = \delta\lambda_j B_{ji} A_{ik}\lambda_k$ becomes, with (6), $|B_{ij}\delta\lambda_j|^2 = \delta\lambda_j [B,A]_{j,k}\lambda_k$. Since the matrices form a Lie algebra, $[B,A]$ is also in $\mathfrak{A}$, and the right-hand side vanishes by (4). This establishes (7) and completes the proof.

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[6]This condition is actually necessary. Sidney Coleman and Erick Weinberg (unpublished) have considered massless scalar electrodynamics in the one-loop approximation and found symmetry-breaking solutions which go continuously to the symmetric solution as the coupling constants go to zero.

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Constraints on Anomalies*

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The various coupling-constant-dependent numbers describing anomalous commutators are constrained by the nonrenormalization of the axial-vector–current anomaly. The axial-vector current continues to behave anomalously even if the underlying unrenormalized field theory is finite due to the vanishing of the Gell-Mann–Low eigenvalue function.

I. INTRODUCTION

It has now been established that the canonical formalism of quantum field theory frequently yields results that are not verified in perturbation theory.[1] These "anomalies" are of two distinct kinds. Firstly there are failures of the Bjorken-Johnson-Low (BJL)[2] limit: Equal-time commutators between operators, when evaluated by the BJL technique in perturbation theory, usually do not agree with the canonical determination of these commutators. A well-known consequence is the failure of the Callan-Gross sum rule for electroproduction.[3] Secondly there are violations of Ward identities associated with exact or partial symmetries; the two known examples being the triangle anomaly of the axial-vector current and the trace anomaly of the new improved energy-momentum tensor.[1] (When a Ward identity is anomalous, there is also a corresponding BJL anomaly.) The Sutherland-Veltman low-energy theorem for neutral pion decay is falsified as a consequence.[4] Both categories of anomalies arise from the divergences of unrenormalized perturbation theory, which require the introduction of regulators to define the theory. The BJL anomaly reflects the noncommutativity of the BJL high-energy limit with the infinite regulator limit which must be taken to define renormalized, physical amplitudes. Failures of Ward identities arise when no regulator