

eighth component of an octet, we have an effective correction term to the exact SU(3) matrix element which is of the form $\langle \beta | V^i S_8 | \alpha \rangle$ where S_8 is the symmetry-breaking operator. Thus we have not only matrix elements between octet states of an octet operator but also an operator whose SU(3) structure is $\underline{8} \times \underline{8} = \underline{1} + \underline{8}_A + \underline{8}_S + \underline{10} + \underline{10}^* + \underline{27}$. Requiring G parity and time-reversal invariance for the matrix elements eliminates the F_3 form factor in the absence of SU(3) breaking and means that only the $\underline{8}_A$ and $\underline{10} \pm \underline{10}^*$ terms in the above expansion can contribute in first order. The structure of the matrix element is then as given in Eq. (3) with the b_{10}

term coming solely from the decuplet operator. For details see, e.g., R. Gatto, in *Strong and Weak Interactions: Present Problems*, 1964 International School of Physics "Ettore Majorana," edited by A. Zichichi (Academic, New York, 1965).

⁵Another obvious consequence is that the slopes of the F_1 's for weak processes, normalized by their $q^2 = 0$ values, equal that of the proton charge form factor. This is rather large and should be readily seen in an experiment such as $\bar{\nu} + p \rightarrow \Sigma^0 + \mu^+$, providing a new test of octet conserved vector current.

Form Factors, Dimensionalities of Operators, Fixed Singularities, and Bjorken Limit in Exclusive Processes*

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A study of exclusive processes is presented. A simple one-current dual model is used as a theoretical laboratory for abstracting properties of the exclusive process which are also confirmed from general considerations. The form of the amplitude in the scaling limit is $\nu^\delta F(\omega, t)$ with δ being independent of ω and t . The parameter δ is also related to the rate of convergence of form factors as well as to fixed singularities in the angular momentum plane. Comparison with the light-cone-dominance approach is given and δ is shown to be simply related to a parameter d , which is given by the dimensions of the operators involved in the expansion of the product of the weak current with a strong source, and determines the singularity strength near the light cone. Possible determinations of d (or, alternatively δ) are outlined. An example of "evaluating" d from coupling constants and Regge intercept is presented, which strongly supports the notion of noncanonical dimensions. It is remarked that, in the model discussed, the annihilation channel scales in the same way as the electroproduction process.

I. INTRODUCTION

The scaling property, proposed by Bjorken,¹ finds a support from the existing available data on $e + p \rightarrow e + X$, where X is the hadronic missing mass. Various approaches² have verified the Bjorken scaling in $e + p \rightarrow e + X$. The observation that the scaling property is closely related to the singularity structure, near the light cone, of a product of local operators has³ led to many interesting theoretical works.⁴

Applying light-cone dominance to exclusive processes, it was proposed⁵ that in the scaling limit the amplitude would have the factorized form $\nu^{-d-2} f(\omega, t)$. In this approach d is a constant, independent of t and ω , which is related to the dimensions of the operators involved in the expansion and measures the singularity strength, near the light cone, of the product of the weak current with a strong source.

As discussed by Frishman (Ref. 4) the light-cone-dominance assumption for exclusive processes is not unambiguous, in contrast with the inclusive case where such a problem does not arise.

The possibility of d being a noncanonical dimensionality has been raised in Ref. 5. The nature of the parameter d could not be determined in the context of light-cone expansion unless an explicit model is assumed from which the currents are constructed.

A theory with anomalous dimensions is known⁶ to arise in the two-dimensional Thirring model. On the other hand it has been argued⁷ that the anomaly disappears in four-dimensional models. It will be interesting to learn more, in a specified model, on the possible nature of the parameter d .

Since d controls the exclusive-process amplitude in the Bjorken limit it can, in principle, be measured directly from data in that limit. In coexistence with the light-cone approach, the parameter

d may appear in other physical quantities (e.g., form factor) and show itself in other limits (e.g., Regge limit). As a result d can, in principle, be determined from other data in addition to the scaling-limit data. Such considerations will be discussed later in more detail.

In order to shed light on the structure of exclusive processes in the various limits and on the nature of the parameter d , we employ a dual model with *one* current only. The merits of *one*-current dual amplitudes, as compared with amplitudes involving *more* than one current, were discussed by Freedman⁸ and would not be repeated here. We shall only stress that the model is used as a theoretical laboratory for abstracting, hopefully, some physical results. Indeed such an approach has had a considerable number of successes; the latest striking one is perhaps the prediction of the transverse-momentum cutoff in multiparticle production processes.

It will be shown that the simple one-current dual model leads, in the Bjorken limit, to the same form as obtained from the light-cone approach. Moreover the parameter appearing in that limit (analogous to d in the light-cone expansion) affects the rate of convergence of form factors as well as the J -plane structure, thus providing other ways for studying that parameter. A closed expression for that parameter will be given in terms of coupling constants and Regge trajectory intercept. Comparing with the light-cone result we then learn that it is more likely for d to be noncanonical dimensionality.

The above results are derived from fairly general arguments, thus lending a support to the conclusions drawn from the model.

The kinematics of the exclusive process and the presentation of the model amplitude, with some of its properties, are given in Sec. II. The Regge limit and the structure of the amplitude in the J plane are discussed in Sec. III. In Sec. IV the Bjorken limit is investigated and its relation with the Regge limit is studied. Fairly general arguments are provided in Sec. V which support the re-

sults derived in Secs. III and IV. These arguments show, in a natural way, why and how the parameter appearing in the Bjorken limit is related to the asymptotic behavior of form factors as well as to possible fixed singularities in the angular momentum plane. In Sec. VI we summarize and discuss the main results, emphasizing the likeliness for anomalous dimensions in exclusive processes and pointing out possible determinations of the parameter d . A brief discussion of the annihilation process, including its relation to the electroproduction one, is also given.

II. THE MODEL AMPLITUDE

Since the exclusive "electroproduction process" will be studied, the following reaction is considered:

$$\gamma(q) + \sigma(k_2) \rightarrow \sigma(k_3) + \sigma(k_4). \quad (1)$$

The virtual "photon" γ with momentum q will be represented by a scalar current. The other identical particles are scalar ones with the corresponding momenta in the parentheses. The kinematics is given in Fig. 1.

The invariant variables to be used are

$$\begin{aligned} \nu &= \frac{k_2 \cdot q}{m}, \\ s &= (q + k_2)^2 = q^2 + 2m\nu + m^2, \\ t &= (k_2 + k_3)^2. \end{aligned} \quad (2)$$

Here m is the scalar particle mass and all particles are incoming, as shown by Fig. 1.

The one-current dual amplitude to be studied here is directly constructed from the well-known five-point function⁹ with the first and fifth particles forming the current as described in Fig. 2. The trajectories in a channel containing one of the lines 1 or 5 are taken as fixed and because of symmetry they are determined by the same parameter δ . Then the amplitude^{10,11} corresponding to the ordering of Fig. 2 is

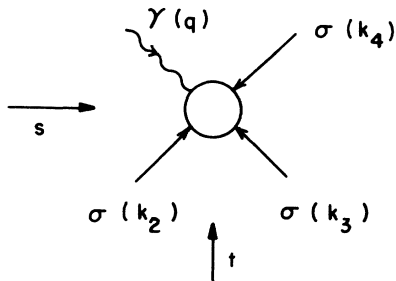


FIG. 1. The kinematics of the exclusive process.

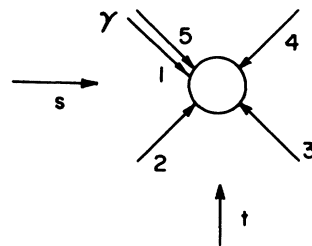


FIG. 2. A five-point amplitude from which the one-current dual model [Eq. (3)] is constructed.

$$A(s, t, q^2) = C \int_0^1 \int_0^1 dx dy x^{-\alpha(s)-1} (1-x)^{-\alpha(t)-1} \times y^{-\alpha_\gamma(q^2)-1} (1-y)^{-\delta-1} (1-xy)^{\alpha(t)}, \quad (3)$$

where C is an over-all normalization factor and we take $C=1$. The trajectories α_γ (in the current channel) and α have not necessarily the same intercept (however, a universal slope will be assumed).

The full amplitude is a combination of 6 terms corresponding to the possible ordering of the s , t , and u variables [$u = (k_2 + k_4)^2$]. However, performing the projective change of variables

$$x' = \frac{1-x}{1-xy}, \quad y' = y$$

one immediately finds that $A(s, t, q^2) = A(t, s, q^2)$, etc. The total amplitude is therefore

$$T(\nu, t, q^2) = 2[A(s, t, q^2) + A(u, t, q^2) + A(s, u, q^2)]. \quad (4)$$

We shall discuss only some general properties

III. THE J -PLANE STRUCTURE AND HIGH-ENERGY BEHAVIOR

As expected from the presence of the parameter δ in (3) there will be, in addition to moving Regge poles, also another kind of singularity. In the discussion below also the type of the singularity will be determined.

The leading singularities in the angular momentum plane are easily obtained from the following useful integral¹² representation of the amplitude in (3):

$$A(s, t, q^2) = \frac{\Gamma(-\alpha_\gamma(q^2))\Gamma(-\delta)}{\Gamma(\alpha(s) - \alpha_\gamma(q^2) - \delta)} \frac{1}{2\pi i} \int_{\eta-i\infty}^{\eta+i\infty} \frac{\Gamma(-\beta)\Gamma(\beta-\delta)\Gamma(\beta-\alpha(t))}{\Gamma(\beta-\delta-\alpha_\gamma(q^2))} \frac{\Gamma(\beta+\alpha(s)-\alpha_\gamma(q^2)-\delta)}{\Gamma(\beta-\alpha(t)-\delta)} (-1)^\beta d\beta, \quad (6)$$

where the integration contour runs parallel to the imaginary axis, lies to the left of the poles in $\Gamma(-\beta)$ ($\eta < 0$) and to the right of the other singularities stemming from the Γ functions in the numerator.

Now the high- $|s|$ limit is taken in (6), with q^2 fixed, and one obtains

$$A(s, t, q^2) \underset{|s| \rightarrow \infty}{\sim} \Gamma(-\alpha_\gamma(q^2))\Gamma(-\delta) \frac{1}{2\pi i} \int_{\eta-i\infty}^{\eta+i\infty} \frac{\Gamma(-\beta)\Gamma(\beta-\delta)\Gamma(\beta-\alpha(t))}{\Gamma(\beta-\delta-\alpha_\gamma(q^2))\Gamma(\beta-\alpha(t)-\delta)} (-\alpha's)^\beta d\beta. \quad (7)$$

In fact the coefficient of $(-\alpha's)^\beta$ is the Mellin transform of the amplitude, from which the singularity structure can be read. As was anticipated, an additional type of singularity emerged, namely a fixed singularity of the multiplicative kind located at δ (the lower lying singularities will not be considered).

Picking the rightmost singularities (the contour, of course, being closed to the left), the high-energy behavior is then

$$A(s, t, q^2) \underset{|s| \rightarrow \infty}{\sim} \frac{\Gamma(-\alpha_\gamma(q^2))\Gamma(\alpha(t)-\delta)}{\Gamma(\alpha(t)-\alpha_\gamma(q^2)-\delta)} \Gamma(-\alpha(t)) (-\alpha's)^{\alpha(t)} + \Gamma^2(-\delta) \frac{\Gamma(\delta-\alpha(t))}{\Gamma(-\alpha(t))} (-\alpha's)^\delta. \quad (8)$$

Note that the residue of the fixed pole is q^2 -independent. Moreover at the leading pole in the current channel [$\alpha_\gamma(q^2) \approx 0$] the second term in (8) does not contribute and the residue of the first term has a pure Regge behavior with no depen-

of the amplitude in (3) which are presumably features of the parent level and avoid ourselves from going into details characterizing daughter states.

Let us study the role played by the parameter δ . This parameter appears in the scattering amplitude of real "photon" where for this case $q^2=0$ in Eq. (3). Denoting the leading scalar pole, in the current channel, by σ_γ one derives the strong amplitude for $\sigma_\gamma + \sigma - \sigma + \sigma$ as a residue of that pole. This residue is simply obtained from (3) by substituting $y=0$ in the nondivergent terms and integrating on the divergent one with the proper trivial analytic continuation in mind. The parameter δ then does not appear in this residue and the original Veneziano representation is recovered. Similarly the σ particle form factor is determined from the $x \approx 0$ region of integration and is given by

$$F_{\gamma\sigma\sigma}(q^2) = \frac{\Gamma(-\alpha_\gamma(0) - \delta)}{\Gamma(-\alpha_\gamma(0))} \frac{\Gamma(-\alpha_\gamma(q^2))}{\Gamma(-\alpha_\gamma(q^2) - \delta)} \quad (5)$$

with the normalization $F_{\gamma\sigma\sigma}(0) = 1$.

Additional circumstances, in which the parameter δ shows itself, will be discussed in the following sections.

dence on δ . This result is in accord with the fact that the parameter δ is a feature of weak processes and not of strong ones as $\sigma_\gamma + \sigma - \sigma + \sigma$.

In a later section we shall provide intuitive arguments for the high- q^2 dependence in (8).

The signature in the present model is obtained from $A(u, t, q^2)$ [see Eq. (4)] simply by replacing $-\alpha's$, in Eq. (8), by $\alpha's$.

The term $A(s, u, q^2)$ has an amusing high-energy behavior. It does not vanish exponentially as in the strong amplitude case. This term exhibits a fixed pole of the additive type located at 2δ with a residue being independent of q^2 . However, since $\delta < 0$ [see, e.g., Eq. (5)] this pole is nonleading, as compared with the multiplicative singularity at δ , and therefore the $A(s, u, q^2)$ term will not be considered. It is obvious that $A(s, u, q^2)$ will not contribute at all to the residue of the leading pole in the current channel.

It should be remarked that although each term in (8) separately diverges for $\alpha(t) = \delta$ their sum does not and the net energy dependence is $(\alpha's)^\delta \times \ln|\alpha's|$. The additional logarithmic term is a manifestation of the multiplicative nature of the fixed pole.

This section furnishes another example in which the parameter δ shows itself in a definite way, namely in the J -plane spectrum of the amplitude.

IV. THE BJORKEN SCALING LIMIT

From reasons stated in the Introduction, it will be of importance to see whether the light-cone derivation in the scaling limit of exclusive processes,⁵ namely $\nu^{-d-2}f(\omega, t)$, could be obtained in a different approach. Having this in mind our model amplitude [see Eqs. (3) and (4)] will be analyzed in the Bjorken limit.

The scaling variable is defined as

$$\omega = \frac{2m\nu}{-q^2} \quad (9)$$

leading to

$$s = m^2 + 2m\nu(1 - 1/\omega) \quad (9')$$

from which the range of ω , in the electroproduction process, is obtained, namely, $1 \leq \omega$.

The integral representation, given in (6), which was used for the high-energy behavior, is also suitable for studying the scaling limit. For large s and $|q^2|$ with ω fixed ($\omega > 1$) one obtains from Eq. (6)

$$A(s, t, q^2) \xrightarrow{B} (2\alpha'm\nu)^\delta \frac{\Gamma(-\delta)}{\omega^\delta} \frac{1}{2\pi i} \int_{\eta-i\infty}^{\eta+i\infty} \frac{\Gamma(-\beta)\Gamma(\beta-\delta)\Gamma(\beta-\alpha(t))}{\Gamma(\beta-\alpha(t)-\delta)} (-\omega)^\beta d\beta, \quad (10)$$

where B stands for the Bjorken limit. Similarly for the $A(u, t, q^2)$ term the limit is

$$A(u, t, q^2) \xrightarrow{B} (2\alpha'm\nu)^\delta \frac{\Gamma(-\delta)}{\omega^\delta} \frac{1}{2\pi i} \int_{\eta-i\infty}^{\eta+i\infty} \frac{\Gamma(-\beta)\Gamma(\beta-\delta)\Gamma(\beta-\alpha(t))}{\Gamma(\beta-\alpha(t)-\delta)} (\omega-1)^\beta d\beta. \quad (10')$$

As in the high- $|s|$ limit (discussed in Sec. III) also here the term $A(s, u, q^2)$ is nonleading and will not be considered.

Then, from (10) and (10'), the total amplitude in (4), of the process $\gamma + \sigma \rightarrow \sigma + \sigma$, has the following form in the scaling limit:

$$T(\nu, t, q^2) \xrightarrow{B} \nu^\delta F(\omega, t). \quad (11)$$

This is exactly the same form as derived from light-cone expansion.⁵ By comparing the two approaches one is led to the identification

$$\delta = -d - 2. \quad (12)$$

Then the parameter d , which is related to dimensions of operators, shows itself in many circumstances; in the behavior of form factors, in a multiplicative fixed singularity in the J plane and in the scaling limit of the exclusive process $\gamma + \sigma \rightarrow \sigma + \sigma$. Therefore, in principle, one can measure d from several aspects of the data. This observation is supported by intuitive and fairly model-independent arguments as discussed in Sec. V.

We conclude this section with a remark on Eqs. (10) and (10'). As opposed to the case in (7), here ω is not necessarily asymptotic and therefore one cannot assume the dominance of the leading singularities. Many nonleading singularities could contribute significantly for nonasymptotic ω and the exact expressions for (10) and (10') involve hypergeometric functions. We shall not write down these expressions because of their sensitivity to the daughter levels. However, for $\omega \gg 1$ (large $|q^2|$ and $s/|q^2|$), which is the "high-energy limit in the scaling region," Eq. (10) reduces to

$$A(s, t, q^2) \xrightarrow{B} (2\alpha'm\nu)^\delta \{ \Gamma(\alpha(t) - \delta) \Gamma(-\alpha(t)) e^{-i\pi\alpha(t)} \omega^{\alpha(t)-\delta} + \Gamma(\delta - \alpha(t)) \Gamma(-\delta) e^{-i\pi\delta} \} \quad (13)$$

and a similar expression for (10'). Equation (13) should be compared with the large- $|q^2|$ limit of (8), which is the "scaling limit in the high-energy region," and the same expression is obtained. Thus the two limits are interchangeable.¹³

V. HEURISTIC APPROACH

In the preceding sections we have abstracted, using a dual model as a guide, several results for the exclusive process $\gamma + \sigma \rightarrow \sigma + \sigma$. The role played by the parameter δ (or, equivalently, d) has been emphasized.

In the following discussion we shall show that fairly general considerations confirm, in a simple and natural manner, the aforementioned dual model results. The analysis is based on the form of the amplitude in the scaling limit and¹³ on the high-energy structure of the exclusive process. One therefore starts with¹⁴

$$T(\nu, t, q^2) \underset{B}{\sim} \nu^{-d-2} f(\omega, t), \quad (14)$$

where the exact definition of d is given in Ref. 5 [for its relation to δ see Eq. (12)].

We shall now pass to the "high-energy limit in the scaling region," namely, to $\omega \gg 1$. In this limit the large virtual-photon mass is small compared with the incident energy. Therefore the process can be described in terms of t -channel singularities as shown by Fig. 3. From this t -channel picture it is obvious that the transition form factor of the upper vertex in Fig. 3 will appear in the limit $\omega \gg 1$. For simplicity suppose that the moving exchanged object (leaving aside, for the moment, other types of singularities) lies on the trajectory of the external scalar particles. In such a case the asymptotic elastic form factor will be present in the t -channel description. An interdependence between q^2 and t can show up which, however, should disappear near the t -channel σ pole [$\alpha(t) \simeq 0$]. Combining the above arguments with the form in (14), for $\omega \gg 1$, it is obvious that the parameter d will also determine the asymptotic behavior of the elastic form factor. The exact derivation of this fact will be given now. For $\omega \gg 1$, the expression in (14) should give the familiar $\nu^{\alpha(t)}$ behavior. This requirement restricts $f(\omega, t)$ to behave, for $\omega \gg 1$, as

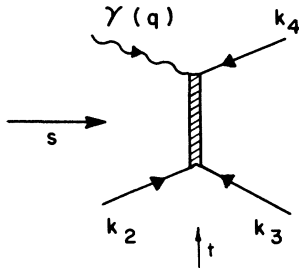


FIG. 3. The t -channel description applicable to the limit $\omega \gg 1$.

$$f(\omega, t) \underset{\omega \gg 1}{\sim} \left(\frac{\nu}{-q^2} \right)^{d+2+\alpha(t)} f_0(t) \quad (15)$$

so that

$$T(\nu, t, q^2) \underset{\omega \gg 1}{\sim} \nu^{\alpha(t)} \frac{1}{(-q^2)^{d+2}} \frac{1}{(-q^2)^{\alpha(t)}} f_0(t). \quad (16)$$

As discussed above, the interdependence between q^2 and t is indeed eliminated at $\alpha(t) \simeq 0$. The high- $|q^2|$ behavior of the elastic form factor is then

$$F_{\gamma\sigma\sigma}(q^2) \underset{|q^2| \rightarrow \infty}{\sim} \text{const} \frac{1}{(-q^2)^{d+2}}. \quad (17)$$

Recalling the identification $\delta = -d - 2$, one observes that Eq. (17) is in agreement with the dual-model result given in (5). Moreover the same asymptotic result [Eq. (17)] was predicted¹⁵ by Brandt and Preparata in the context of the light-cone approach. It should be emphasized that also in the light-cone expansion the same parameter d appears in both the asymptotic behavior of the form factor¹⁵ and in the Bjorken limit of the exclusive process.⁵

Another type of singularity, in addition to a moving Regge pole, can be introduced simply by replacing Eq. (15), with, e.g., the following expression:

$$f(\omega, t) \underset{\omega \gg 1}{\sim} \left(\frac{\nu}{-q^2} \right)^{d+2+\alpha(t)} f_0(t) + f_1(t). \quad (18)$$

The virtue of the second term is in its persistence in the limit $\omega \rightarrow \infty$ even for $\alpha(t) < -d - 2$ where the first term vanishes. With Eq. (18) one obtains, instead of (16), the expression

$$T(\nu, t, q^2) \underset{\omega \gg 1}{\sim} \nu^{\alpha(t)} \frac{1}{(-q^2)^{d+2}} \frac{1}{(-q^2)^{\alpha(t)}} f_0(t) + \nu^{-d-2} f_1(t). \quad (19)$$

The second term is due to a fixed singularity located at $\delta = -d - 2$, and Eq. (19) is in accordance with Eq. (13) [suppressing signature factors coming from $A(u, t, q^2)$]. However, here one cannot determine whether the fixed singularity is of the multiplicative or the additive type.

It is gratifying that completely different derivations have led to similar results. This fact then strongly supports the general validity of these results. We shall further discuss their significance in Sec. VI.

VI. SUMMARY AND DISCUSSION

A study of some aspects of the exclusive process $\gamma + \sigma \rightarrow \sigma + \sigma$ has been presented. A simple one-current dual model has been used as a theoretical laboratory from which several results have been

abstracted. Such a model, having only one current, has the advantage⁸ of producing the same spectrum as in the pure hadronic case in the parent level and does not suffer from the severe inconsistencies present in models involving more than one current. We have avoided ourselves from relying upon derivations which are sensitive to the structure of daughter levels and emphasized only those results which depend on the leading singularities. It is encouraging that the form of the amplitude in the scaling limit as well as the asymptotic behavior of form factors are obtained also in the theory of light-cone dominance.^{5,15} Moreover, in both approaches, the *same* parameter d [or equivalently δ , as shown by Eq. (12)] determines the large- $|q^2|$ behavior of form factors and the scaling limit of the exclusive process. The parameter d is also related to a fixed singularity in the J plane, as discussed in Sec. III. All these results were shown to emerge from simple and quite general considerations.

The question whether d is a canonical or anomalous⁶ dimension is very interesting. If it is canonical then a scale-invariant free field theory determines it and therefore no coupling constants are involved in its definition.

A possibility for "evaluating" d , *not* from high- $|q^2|$ and/or high- ν data, will be now considered. For this purpose the elastic form factor, $F_{\gamma\sigma\sigma}(q^2)$, will be studied. The *asymptotic* behavior of $F_{\gamma\sigma\sigma}(q^2)$, as given in (17), was derived from general arguments (and also from light-cone approach¹⁵). However, for the behavior in the non-asymptotic q^2 region one needs an explicit model. Suppose we assume a Veneziano-like form factor, namely,

$$F_{\gamma\sigma\sigma}(q^2) = \frac{\Gamma(-\alpha_\gamma(0) + d + 2)}{\Gamma(-\alpha_\gamma(0))} \frac{\Gamma(-\alpha_\gamma(q^2))}{\Gamma(-\alpha_\gamma(q^2) + d + 2)}, \quad (20)$$

which fulfills Eq. (17) and is normalized so that $F_{\gamma\sigma\sigma}(0) = 1$. Then by going to the first scalar pole of the current, at which the dominating diagram is depicted in Fig. 4, one obtains¹⁶

$$B(-\alpha_\gamma(0), d + 2) = \frac{1}{g_{\gamma\sigma\gamma} g_{\sigma\gamma\sigma\sigma}}, \quad (21)$$

where¹⁷ $g_{\gamma\sigma\gamma}$ is the direct weak coupling of the "photon" γ to the parent scalar pole, σ_γ , in the current channel, and $g_{\sigma\gamma\sigma\sigma}$ is self-explanatory. The importance of a relation such as (21) is two-fold; first it shows the possibility of determining d from nonasymptotic data, namely from Regge intercept and coupling constants. Secondly, as a consequence, it highly supports the presence of noncanonical dimensions. It was already men-

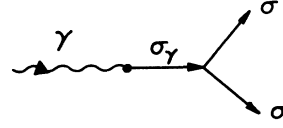


FIG. 4. The dominant contribution to the elastic form factor [given in Eq. (20)] near the first scalar pole of the current.

tioned in Ref. 5 that, in exclusive processes, an operator-product expansion involving anomalous dimensions is not excluded.

One can have information on d from direct measurements of exclusive¹⁸ processes. For example, the form of the amplitude in the scaling limit [see Eqs. (11) and (14)] offers such a way. Also if d is indeed related to a fixed singularity then it can be determined, in principle, from nonasymptotic data by using, e.g., finite-energy sum rules.

Since the *asymptotic* behavior of the form factor [Eq. (17)] was derived from general arguments (in contrast with the low- q^2 structure) it is perhaps the best property suitable for studying the parameter d . Consider for example, the pion electromagnetic form factor which appears in many reactions. As inclusive processes are relatively easier to measure, we shall consider $\gamma(q) + p \rightarrow \pi^- + X$ (the photon being virtual) in which the pion form factor can show itself. More explicitly, in the photon fragmentation region and near the kinematical boundary the dominating mechanism can be described as in Fig. 5, where the pion trajectory is assumed to contribute mostly. It is interesting to note that measurements of the same process with, however, *real* photons have indicated¹⁹ the dominance of a zero intercept exchange. The contribution of the amplitude described in Fig. 5 to the cross section of $\gamma(q) + p \rightarrow \pi^- + X$ is depicted in Fig. 6. Although the diagram in Fig. 6 vanishes with increasing $|q^2|$, nevertheless from the rate of the decrease one can extract information on the corresponding parameter d .

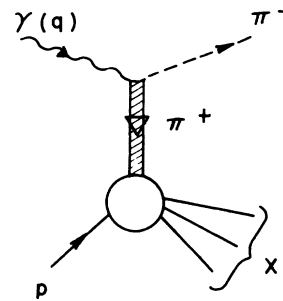


FIG. 5. Approximate mechanism in the photon fragmentation region near the kinematical boundary for the inclusive process $\gamma(q) + p \rightarrow \pi^- + X$.

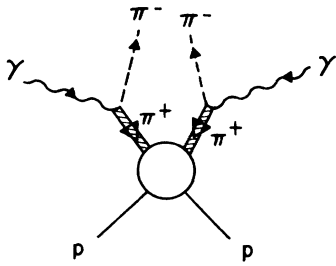


FIG. 6. The contribution of the diagram in Fig. 5 to the inclusive differential cross section of $\gamma(q) + p \rightarrow \pi^- + X$.

We shall conclude with remarks on the exclusive annihilation process $\gamma(q) \rightarrow \sigma + \sigma + \sigma$. The amplitude of this process can be derived from the amplitude of $\gamma(q) + \sigma \rightarrow \sigma + \sigma$ by analytic continuation from the region $q^2 < 0$, $t < 0$, and $\nu > 0$ ($1 \leq \omega < \infty$) to $q^2 > 0$ (timelike), $t > 0$, and $\nu < 0$ ($0 \leq \omega \leq 1$). An interesting question is whether the annihilation channel scales in the same way as the electroproduction process. The light-cone approach cannot⁴ answer this question in a simple way since a *finite* number

of terms in the operator-product expansion obviously will not generate the normal thresholds in the q^2 variable. This difficulty is not present in the discussed dual model. In fact, as one can see from the derivation in Sec. IV, the form of the amplitude in the scaling limit, given in Eq. (11), is valid irrespective of q^2 being spacelike or timelike. Thus, in this model, the annihilation process scales in the same way as the electroproduction reaction, with the variable ν having the *same* exponent in both cases.

Note added in proof. After completing this work the author learned that a similar approach has been adopted by J. H. Weis, Nucl. Phys. **B40**, 562 (1972).

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