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Consequences of Unitarity in a Veneziano Amplitude*

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A general method is presented for unitarization via a complex trajectory function, and arguments are presented for its validity in the π-π problem. The method is applied to a Veneziano amplitude with two satellites. Solutions are found to exist from threshold to slightly above the ρ mass and appear to require a markedly nonlinear Regge trajectory. Reasons for the failure of the solutions to exist above the ρ mass are discussed.

I. INTRODUCTION

In the last three years since Veneziano's[1] form of the scattering amplitude first appeared, an enormous number of papers has been published exploring most of the imaginable ramifications, extensions, and generalizations of the amplitude.[2] However, the problem of incorporating unitarity into the amplitude has remained largely unsolved, although certainly not because of any lack of effort.[2] Common to most of these approaches is the assumption that unitarity is a small correction to the Veneziano beta function plus perhaps a couple of satellites. Inelastic effects are generally ignored. At least one author,[3] however, has suggested the importance of inelastic effects in a unitary amplitude. Finally, the imposition of unitarity has until now been accomplished, if at all, only at the expense of destroying some other property, most notably crossing symmetry. This is undesirable since the explicitly crossing-symmetric nature of the Veneziano amplitude is one of its important features. As well, the violation of crossing symmetry has no obvious physical interpretation, whereas the violation of unitarity can be identified physically with a source of flux. Thus, the problem of restoring crossing symmetry is expected to be more difficult than that of imposing unitarity, simply because in the latter case, there is a physical interpretation which may be a useful guide to the correct solution.

We have devised a general method for unitarization via a complex trajectory function[4] which inputs only experimental data to the fullest possible extent. The method was applied to a Veneziano π-π amplitude in order to investigate to what extent unitarization was possible, to what extent (if any) the original form of the model was modified, and to what extent the resulting predictions agree with the experimental data. The choice of model is described in Sec. II, the method of unitarization is described in Sec. III, the results are discussed in Sec. IV, and some conclusions and speculations are given in Sec. V.

II. THE MODEL

The experimental data[5] on π-π scattering indicate that, for the energy region below about 1 GeV, the scattering is predominantly S wave for $I=0$, and predominantly P wave for $I=1$, with a relatively small contribution from the $I=2$ state. Also, the scattering in this energy region appears to be almost exclusively elastic. This suggests that in this energy region the first two partial waves of the π-π scattering amplitude ought to be a good approximation to the full amplitude. This is expected to be a good approximation at least to the extent that the higher partial waves are small relative to the S and P waves.

The most convenient theoretical description of π-π scattering is in terms of Lovelace's[6] crossing-symmetric solution which gives, for the s-channel isospin amplitudes,

$$A^{I=0}(s,t,u)=\tfrac{3}{2}[F(s,t)+F(s,u)]-\tfrac{1}{2}F(t,u), \qquad (1)$$

$$A^{I=1}(s,t,u)=F(s,t)-F(s,u), \qquad (2)$$

$$A^{I=2}(s,t,u)=F(t,u). \qquad (3)$$

The most general Veneziano form for the symmetric function $F(s,t)$ is

$$F(s,t)=\sum_{p=1}^{\infty}\sum_{n=0}^{p}C_{pn}\frac{\Gamma(p-\alpha(s))\Gamma(p-\alpha(t))}{\Gamma(p+n-\alpha(s)-\alpha(t))}, \qquad (4)$$

where $\alpha(s)$ is the exchange-degenerate ρ-f trajectory.

The amplitude is required to have a pole at $\alpha(s)=1$, but no pole at $\alpha(s)=0$, so $C_{00}=0$ and $C_{10}\neq 0$. Further, it is required that the S and P waves satisfy unitarity at the ρ mass for arbitrary ρ width so $C_{11}\neq 0$. Finally, the Adler zero[7] is required at $s=t=u=m_\pi^2$, both in order to maintain similarity with other calculations[8] and to improve the probability of obtaining reasonable scattering lengths.[9] Hence, one of $C_{2n}\neq 0$, so that it is not necessary to specify $\alpha(m_\pi^2)$ *a priori*.

The simplest form for $F(s,t)$ consistent with the above constraints is therefore

$$F(s,t)=g\left(\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}-\frac{\Gamma(2-\alpha(s))\Gamma(2-\alpha(t))}{\Gamma(n-\alpha(s)-\alpha(t))}\frac{\Gamma(1-\alpha_0)^2\Gamma(n-2\alpha_0)}{\Gamma(2-\alpha_0)^2\Gamma(1-2\alpha_0)}\right)$$
$$+gH\left(\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}-\frac{\Gamma(2-\alpha(s))\Gamma(2-\alpha(t))}{\Gamma(n-\alpha(s)-\alpha(t))}\frac{\Gamma(1-\alpha_0)^2\Gamma(n-2\alpha_0)}{\Gamma(2-\alpha_0)^2\Gamma(2-2\alpha_0)}\right), \quad (5)$$

where $\alpha_0 \equiv \alpha(m_\pi^2)$ and $n=2, 3,$ or 4. The constant g is an over-all normalization and H is essentially the ratio of the satellite term to the parent term. The second term in each bracket is constructed to ensure that $F(m_\pi^2, m_\pi^2)=0$.

III. UNITARIZATION

The principle underlying the unitarization scheme has already been suggested in Sec. II. Presumably, if a scattering process is dominated by a small number of the lowest partial waves, the sum of those partial-wave amplitudes is a good approximation to the full amplitude. Consequently, when these partial-wave amplitudes are unitary, their sum should approximate the full unitary amplitude, at least to the extent that the contributions from the nonunitary partial waves can be neglected.

Thus, in any model of the amplitude which contains a number of fixed parameters along with unknown energy-dependent variables, the experimental data can be used to establish the parameter values, and then the partial-wave unitarity equations can be used to predict the unknown variables as functions of energy. This determines everything, in principle, including phases and cross sections. For consistency, the higher partial waves must turn out to be small.

For the proposed π-π amplitude, the parameters to be fixed are g and H. The unknown energy-dependent variable is the complex trajectory function $\alpha(s)$. The real and imaginary parts of the trajectory are predicted as functions of energy by the S- and P-wave unitarity equations.

The usual objections to a complex trajectory function in a Veneziano amplitude are that ancestors are introduced and that, necessarily, all resonances occurring at a particular mass will have the same total width. The ancestors are irrelevant provided that they are small enough to be unobservable. For resonances occurring at the same mass, although the distances from the real axis to the poles in the complex plane are the same, it is only in the narrow resonance approximation that the resulting cross-section peaks measured on the real axis necessarily have the same widths. Thus, the validity of both objections for a specific model can only be decided *a posteriori* by actual calculation.

The partial-wave amplitudes are

$$f^I(s)=\frac{1}{16\pi\sqrt{s}}\int_{-1}^{1}A^I(s,z)P_l(z)dz\,, \quad (6)$$

where, as usual, s is the square of the c.m. energy and z is the cosine of the s-channel center-of-mass scattering angle. Since $z=1+t/2q^2$, where $q^2=\frac{1}{4}(s-4m_\pi^2)$, $\alpha(t)$ must be known in the region $-4q^2\leqslant t\leqslant 0$ to perform the integrations. For the energy region from threshold to the ρ mass, t will lie in the region $0\geqslant t\geqslant -0.6$ GeV^2. This information is experimentally accessible for the ρ trajectory in charge-exchange scattering.[10] The best linear fit, the best linear fit passing through one at the ρ mass, and the usual choice $\alpha(t)=0.48+0.885t$ have χ^2 values of 0.73, 1.5, and 48, respectively. As subsequent results were not very sensitive to the precise choice of $\alpha(t)$, the second fit, $\alpha(t)=0.55+0.769t$, was chosen for the detailed calculations.

The values of the parameters g and H were fixed by imposing unitarity at $s=m_\rho^2$, where the ρ mass m_ρ, and width Γ_ρ are experimentally observable.[11] $\text{Re}\alpha(m_\rho^2)$ was assumed to be unity, and initially $\text{Im}\alpha(m_\rho^2)$ was estimated to be 0.067 from the narrow-resonance approximation $\text{Im}\alpha=m_\rho\Gamma_\rho\text{Re}\alpha'$, with $m_\rho=765$ MeV and $\Gamma_\rho=118$ MeV. $\text{Re}\alpha'$ was assumed to have the same value at m_ρ^2 as input for the region $t<0$. In the event that the slope of the solution at m_ρ^2 differs significantly from the initial guess, an iteration procedure must be used to find a value of $\text{Im}\alpha$ which reproduces a width at m_ρ^2 consistent with the experimental value $\Gamma_\rho=118$ MeV.

The S and P waves were then required to satisfy the unitarity relation

$$1+2iqf_l=\eta_l e^{2i\delta_l} \quad (7)$$

at $s=m_\rho^2$. This form was used in order to include possible effects of inelasticity. For each partial wave Eq. (7) defines a family of ellipses in the g-gH plane. For particular values of the inelasticities at $s=m_\rho^2$, g and H are fixed by the intersection of the S- and P-wave ellipses. A typical pair of ellipses is shown in Fig. 1.

Certain qualitative features of the solutions are evident from the figure. For a particular partial wave, the interior and boundary of the elastic ellipse $\eta_l=1$ constitute the entire range of values for g and gH which make the partial wave unitary at that energy. For two perfectly elastic partial waves there will always be one intersection at the origin (the trivial solution). Unless the ellipses

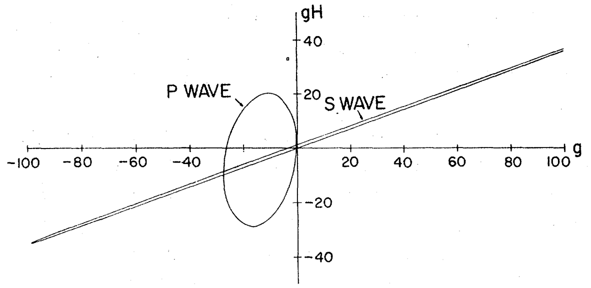

FIG. 1. S- and P-wave ellipses in the g-gH plane for $\eta_0=1$, $\eta_1=0.98$, $s=m_\rho^2$, and $\mathrm{Im}\,\alpha(m_\rho^2)=0.067$.

are tangent at the origin, there will be a second intersection near the origin (the regular solution). There will also be two intersections more distant from the origin (the exceptional solutions). These last two solutions can fail to exist if one ellipse turns out to be much smaller than the other. It is also evident that for $H=0$, the ellipses become line segments whose only common point is the origin, so at least one satellite is always necessary.

This procedure has an obvious generalization to include any finite number of partial waves, but this involves the rather undelightful prospect of finding the intersections of ellipsoids in higher dimensional spaces.

The physical content of the solutions is most evident from the simple pole approximation. Calculations to lowest order in $\mathrm{Im}\,\alpha$ predict that, in the limit of perfectly elastic resonances, the regular solution will have zero P-wave residue at the pole, and one exceptional solution will have zero S-wave residue at the pole. Only the remaining exceptional solution is predicted to have nonzero residues at the pole for both the S and P waves. Hereinafter this solution will be designated the first exceptional solution, and the other exceptional solution the second exceptional solution. It is therefore expected that the first exceptional solution offers the best chance of providing a physically reasonable unitary amplitude.

With g and H fixed by enforcing unitarity at the ρ mass, the S- and P-wave unitarity equations are now well-defined equations for the variables $\mathrm{Re}\,\alpha$ and $\mathrm{Im}\,\alpha$ at any energy. In principle, these equations completely determine $\mathrm{Re}\,\alpha$ and $\mathrm{Im}\,\alpha$ and consequently the amplitude, including phases and cross sections, as functions of energy. The only information that has been put in is $\alpha(t)$ for $t\leq 0$, the mass and width of the ρ, and the S- and P-wave inelasticities at the ρ, all of which are experimentally measurable quantities.

In practice, when the energy is varied away from the ρ mass, simultaneous solutions for $\mathrm{Re}\,\alpha$ and $\mathrm{Im}\,\alpha$ do not always exist. This can be understood as follows: For a fixed energy, unitarity defines the inelasticity parameter η_l as a function of the two real variables $\mathrm{Re}\,\alpha$ and $\mathrm{Im}\,\alpha$. A particular choice of η_l therefore implicitly defines a curve in the $\mathrm{Re}\,\alpha$-$\mathrm{Im}\,\alpha$ plane. Simultaneous solutions to unitarity correspond to the intersection of the S- and P-wave curves in this plane. In the event of multiple solutions, the solution continuous with the value $\alpha(m_\rho^2)$ is chosen. In some cases, the curves fail to intersect at a particular energy, indicating there are no unitary solutions at that energy. A representative pair of such curves is shown in Fig. 2. It may be possible to induce a solution by allowing the inelasticity parameter to vary. Then, if the curves fail to intersect for values of the inelasticity parameter less than or equal to unity, no unitary solution exists.

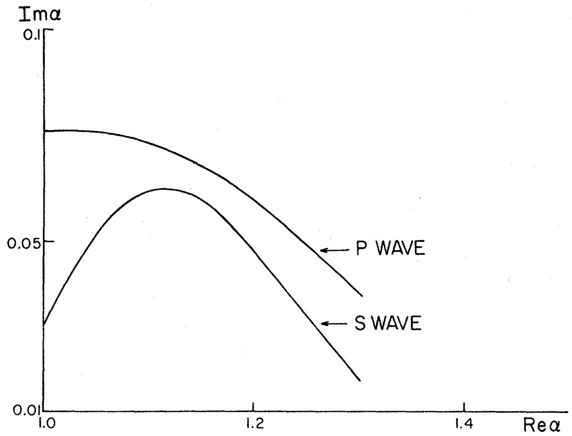

FIG. 2. S- and P-wave curves in the $\mathrm{Re}\,\alpha$-$\mathrm{Im}\,\alpha$ plane for elastic partial waves at $s=0.64\ \mathrm{GeV}^2$.

Computing $\alpha(s)$ point-by-point in this manner for $s\geq 4m_\pi^2$ has several advantages. First, no assumptions are necessary with respect to the asymptotic linearity of $\mathrm{Re}\,\alpha(s)$ or the analyticity required by a dispersion relation. Secondly, extrapolation of the predicted trajectory to negative energies provides a consistency check since the procedure should reproduce the input trajectory for $t\leq 0$ fairly closely. Finally, since the unitarity equations yield well-defined functions within a specific model, questions concerning existence, asymptotic behavior, and analytic structure could be answered, in principle, by direct calculation.

IV. RESULTS

Initially the S and P waves were assumed to be perfectly elastic below 1 GeV^2 since, experimentally, the inelastic effects do not appear to be appreciable below the $K\bar{K}$ threshold of approximately 1 GeV^2. For perfectly elastic S and P waves, the

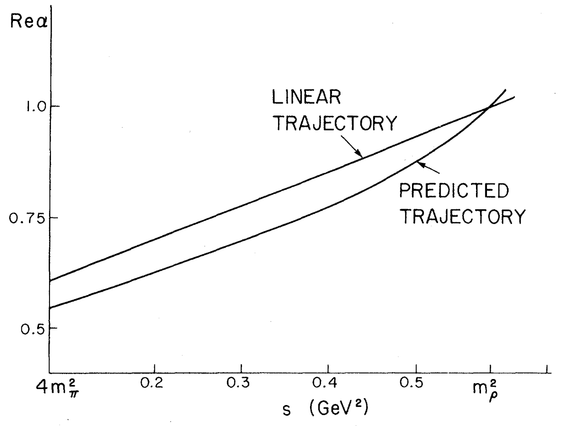

FIG. 3. $\mathrm{Re}\alpha(s)$ predicted by the first exceptional solution with $\mathrm{Im}\alpha(m_\rho^2)=0.13$.

values of H for the first and second exceptional solutions were 0.32 and 0.35, respectively. These values were relatively insensitive to variations in $\mathrm{Im}\alpha(m_\rho^2)$, changing by at most 1% for an order of magnitude variation in $\mathrm{Im}\alpha(m_\rho^2)$. The over-all normalization g, on the other hand, showed a strong direct variation with $\mathrm{Im}\alpha(m_\rho^2)$. These correlations agreed with the predictions of the simple pole approximation which stated that g was proportional to $\mathrm{Im}\alpha$, and H was independent of $\mathrm{Im}\alpha$.

Solutions for the complex trajectory function corresponding to the first exceptional solution existed for values of s from threshold up to 0.6 GeV^2, just above the ρ mass. For higher values of the energy, the solutions ceased to exist as the S- and P-wave curves no longer intersected (Fig. 2).

The outstanding characteristic of this solution was the highly nonlinear behavior of $\mathrm{Re}\alpha(s)$, especially as s increased through the ρ mass (Fig. 3).

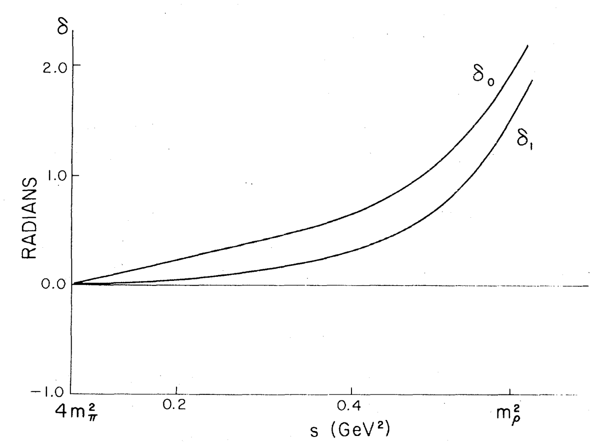

FIG. 4. Phases of the S and P waves predicted by the first exceptional solution with $\mathrm{Im}\alpha(m_\rho^2)=0.13$.

Calculation of the phases and partial-wave cross sections indeed showed resonances in the S and P waves with the P-wave cross section about three times as large as the S-wave cross section. However, the initial guess of $\mathrm{Im}\alpha(m_\rho^2)=0.067$ resulted in too narrow a ρ width, so it was necessary to raise $\mathrm{Im}\alpha(m_\rho^2)$ to 0.13 in order to obtain a ρ width of 110 MeV. Figures 4 and 5 show the phases and cross sections for $\mathrm{Im}\alpha(m_\rho^2)=0.13$, including the extrapolation of the P-wave cross section that was used to estimate the ρ width.

Assuming an elastic P wave, plots of $\eta_0(s)$ vs $\mathrm{Re}\alpha(s)$ for $\mathrm{Re}\alpha \geqslant 1$, $0.6 \leqslant s \leqslant 1$ GeV^2, revealed that the existence of the solution could be extended to between 0.7 and 0.75 GeV^2 by allowing an inelasticity in the S wave of about 15%. For $s=0.75$ GeV^2 and beyond, $\eta_0(s)$ failed to drop as low as 1.0 for $\mathrm{Re}\alpha \geqslant 1.0$.

With an elastic S wave, the same curves for $\eta_1(s)$ indicated that the existence of the solution could be extended to at least 1 GeV^2 provided that the P wave was allowed to be substantially inelastic (as much as 75%). As well, $\mathrm{Re}\alpha(s)$ would rise much more sharply with s than linearly.

The trajectory function corresponding to the second exceptional solution also existed from threshold up to $s \simeq 0.6$ GeV^2. The real part showed an even more pronounced nonlinearity than that corresponding to the first solution (Fig. 6).

The phases and partial-wave cross sections showed a resonant P wave and a nonresonant S wave about an order of magnitude smaller than the P wave. Again the initial guess of $\mathrm{Im}\alpha(m_\rho^2)$

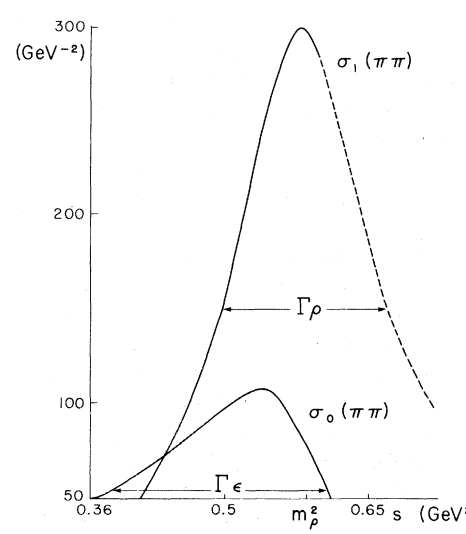

FIG. 5. S- and P-wave cross sections predicted by the first exceptional solution with $\mathrm{Im}\alpha(m_\rho^2)=0.13$. The broken line represents the extrapolation of the solution above $s=0.6$ GeV^2 used to estimate Γ_ρ.

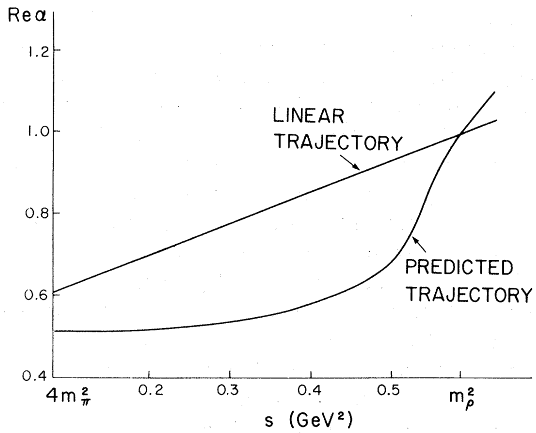

FIG. 6. Re$\alpha(s)$ predicted by the second exceptional solution with Im$\alpha(m_\rho^2)=0.33$.

$=0.067$ gave too narrow a ρ width. For this solution, however, it was necessary to raise Im$\alpha(m_\rho^2)$ to 0.33 before the ρ width became an acceptable 110 MeV.

In contrast to the first solution, this solution predicted a negative S-wave phase of about 30°. The P-wave phase was again consistent with a resonant P wave (Fig. 7). The partial-wave cross sections are shown in Fig. 8, including, again, the extrapolation used to estimate the ρ width. As with the first solution, allowing inelasticity in the S wave extended the existence of the solution up to between 0.65 and 0.7 GeV^2. Allowing inelasticity in the P wave of about 10% also extended the existence of the solution to between 0.7 and 0.75 GeV^2, and again required a very sharp rise of Reα with s.

The trajectory function corresponding to the regular solution existed from threshold up to 1 GeV^2, provided that a small P-wave inelasticity was allowed above the inelastic threshold at $s=0.32$ GeV^2. However, the phases and partial-wave cross sections revealed a nonresonant P wave. This solution was dismissed as a possible physical solution since it clearly contradicts the experimental data.

The magnitudes of the higher partial waves were calculated for the two exceptional solutions. The D wave was about 2 orders of magnitude smaller than the S or P waves, and higher partial waves were roughly an order of magnitude smaller than their immediate predecessors.

V. DISCUSSION AND CONCLUSIONS

The method by which the solutions to unitarity were obtained is felt to be exceptionally clean because only experimentally observable quantities, such as $\alpha(t)$ for $t<0$, and the mass, width, and inelasticities of the π-π resonances, were put into the model. In particular, no assumptions concerning Re$\alpha(s)$ were necessary beyond requiring Re$\alpha(m_\rho^2)=1$. Unitarity predicted Re$\alpha(s)$ and Im$\alpha(s)$, which are, essentially, only dependent on the form of the model within the context of this method. Finally, crossing symmetry and the Adler zero were maintained explicitly at all stages of the calculation.

The existence of the two exceptional solutions from threshold up to the ρ mass is, at first sight, encouraging, and even suggests that introducing unitarity into a crossing-symmetric Veneziano amplitude is a reasonable approach. However, there are serious difficulties which temper any initial enthusiasm.

The failure of the solutions to exist above the ρ mass is perhaps the most disturbing aspect of the solutions. No single reason suggests itself as the

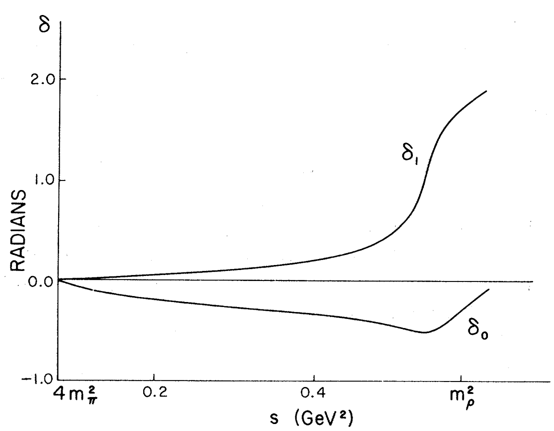

FIG. 7. Phases of the S and P waves predicted by the second exceptional solution with Im$\alpha(m_\rho^2)=0.33$.

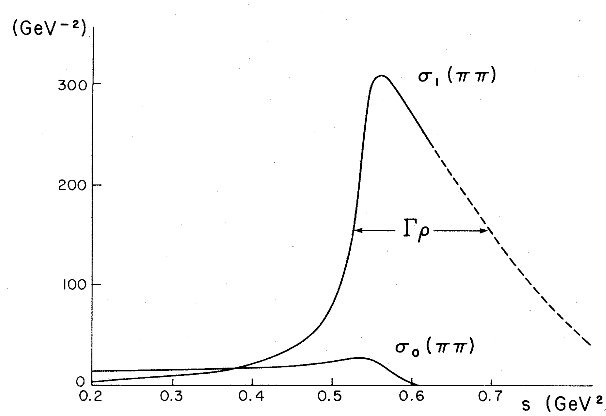

FIG. 8. S- and P-wave cross sections predicted by the second exceptional solution with Im$\alpha(m_\rho^2)=0.33$. The broken line represents the extrapolation of the solution above $s=0.62$ GeV^2 used to estimate Γ_ρ.

source of the difficulty. The calculations of η vs $\mathrm{Re}\,\alpha$ suggest that part of the difficulty may lie in a failure to properly include inelastic effects. However, this cannot be the only source of trouble since, experimentally, inelastic effects appear to be small below the $K\bar{K}$ threshold at 1 GeV^2. Also, the existence of the solutions is not expected to be particularly sensitive to small variations in the inelasticity, at least in the simple pole approximation, with which most of the results have been consistent.

The ancestors introduced into the higher partial waves by this method do turn out to be small relative to the unitary partial waves and so present no problem.

The objection that equal-mass resonances have equal widths is not as easily dismissed if the S wave is truly resonant. For the only solution which has resonances in both S and P waves the widths differ by at most 40%. Such a discrepancy could be accounted for by the extrapolation to the real axis alone, suggesting that the widths at the pole are the same. Then, if the ϵ really is a broad resonance, the model is in trouble. The case of a narrow resonance cannot be summarily dismissed since the data on this point is not clear. Since ρ widths up to 150 MeV are acceptable it should be possible by raising $\mathrm{Im}\,\alpha(m_\rho^2)$ to obtain an ϵ width of 200 MeV while still maintaining an acceptable ρ width. Then, the solution would agree with the interpretation of a narrow ϵ.

The remedy for equal widths which immediately suggests itself is to introduce a separate daughter trajectory for the ϵ, thus removing the degeneracy of the ρ and ϵ poles as they move away from the real axis. This means introducing a second trajectory about which almost nothing is known experimentally, and which, if successful, would give unequal width resonances at the cost of extra parameters plus at least a dubious theoretical interpretation. Thus, while the introduction of a daughter trajectory is a logical possibility it is not a very palatable alternative, and so was not attempted.

On the other hand, if as has been suggested[12] the S wave is nonresonant in the ρ region, but with a substantial phase of about 60°, the relevant solution is the second exceptional solution. Then the equal resonance widths present no difficulty, but the predicted S-wave phase is negative above threshold and about −30° around the ρ which does disagree with the data presented.

It may also be possible to extend the existence of the solutions and to correct the ϵ width by adding more satellites, but again, this involves introducing more parameters of dubious physical significance which is contrary to the spirit of this whole method. In fact, it seems to us that no finite number of satellites will solve these problems.

The most outstanding shortcoming of the entire scheme presented herein is the manifestly nonunitary $I=2$ amplitude. The usual procedure for obtaining $I=2$ scattering in a π-π amplitude is to add a Pomeranchukon term. This was not done for two reasons. First, there is no commonly accepted prescription for writing down a Veneziano-like Pomeranchukon amplitude. Secondly, as previously argued, to the extent that the $I=2$ scattering is small compared to the $I=0$ and $I=1$ scattering experimentally, this method should provide at least a first approximation to the unitary amplitude. Of course, a detailed quantitative comparison with the data would be expected to show some discrepancy, but no clear contradictions are expected.

These results lead to the conclusion that, first, unitarity can be introduced into the Veneziano amplitude by this method only at the cost of considerable nonlinearity in the trajectory function. Second, whether or not the $I=0$ S wave contains a broad resonance (and the experimental situation is still somewhat ambiguous about this), these results suggest that the interpretation of the Veneziano amplitude, with unitarity suitably incorporated, as a physical amplitude is probably wrong. Although the failure of this method to give a physically reasonable unitary amplitude can be attributed largely to the lack of a Pomeranchukon term, and to requiring degenerate parent and daughter poles away from the real axis, it seems much more reasonable, to us, to argue that the Veneziano amplitude must be interpreted, at best, as a type of Born approximation to the "true" amplitude. In this case it is unreasonable to expect that enforcing unitarity in the Veneziano amplitude should give physically meaningful results.

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Lepton-Hadron Symmetry Breaking and the Cabibbo Rotation

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It is shown that spontaneous breakdown of gauge-invariant $SU(2)_L \times U(1)$ lepton-hadron theories can induce hadron chiral-symmetry-breaking terms consistent with parity and hypercharge conservation and with earlier results of the $(3,3^*)+(3^*,3)$ model. The theories limit to $SU(2)\times SU(2)$ for hadrons, zero Cabibbo angle, and no isospin-breaking term (u_3) when the cross coupling between leptons and hadrons is removed or when the $SU(2)_L \times U(1)$ gauge symmetry is realized without spontaneous breakdown. The Cabibbo angle is not constrained.

Indications that spontaneously broken gauge-symmetric theories may be renormalizable [1] have stimulated a revival of Weinberg's $SU(2)_L \times U(1)$ unified theory of leptonic weak and electromagnetic interactions.[2] Theories of this type have encountered several problems, including those associated with triangle anomalies,[3] and the suppression of neutral strangeness-changing currents when hadrons are incorporated.[4] Detailed models [5] in a four-quark context have appeared which seem to avoid these difficulties. Apart from quark-antiquark bilinears carrying charm and an SU(3) $\times$SU(3) singlet, the remaining bilinears in these models belong to the $(3,3^*)+(3^*,3)$ representation suggesting again this simple multiplet for all, or at least the charm-conserving part, of hadronic chiral-symmetry-breaking terms.

Such intrinsic symmetry-breaking terms of the form $\sum_i c_i u_i$ with u_i scalar members of the $(3,3^*)$ $+(3^*,3)$ representation, whatever their origin (quark-antiquark bilinears, σ-model fields, etc.), pose, however, yet another problem [6] associated with bringing in the hadrons because they explicitly break the $SU(2)_L \times U(1)$ gauge symmetry of the weak and electromagnetic interactions. Since initial gauge symmetry of the theory appears to be an essential ingredient of renormalizability arguments,[3] these hadron chiral-symmetry-breaking terms [necessary in the canonical picture [7,8] of $SU(3)\times SU(3)$ breaking to support pseudoscalar masses and the PCAC (partially conserved axial-vector current) condition] conflict with the renormalization program. Weinberg [6] and Schechter and Ueda [9] have therefore conjectured the full $SU(2)_L \times U(1)$ invariance of the lepton-hadron theory, with hadronic chiral-symmetry-breaking terms elegantly supplied by the same mechanism which spontaneously breaks the lepton theory.

In a recent note,[10] it was shown how this mechanism succeeds in simultaneously breaking the lepton and hadron symmetry in an $SU(2)\times SU(2)$ σ-model context for nucleons and pions, joined to the leptons via an $SU(2)_L \times U(1)$-invariant mixing term. This model preserves the lepton theory, maintains the canonical view of hadron chiral $SU(2)\times SU(2)$ symmetry breaking, and realizes an intimate connection between lepton and hadron symmetries. An extension of this symmetry-breaking mechanism to strange hadrons, in a full $SU(3)\times SU(3)$ hadron context, must take into account the theoretical work [11-13] of recent years which suggests that (1) the Cabibbo rotation is related