PHYSICAL REVIEW D

PARTICLES AND FIELDS

THIRD SERIES, VOL. 6, NO. 10 — 15 November 1972

Lorentz-Invariant Formulation of Cherenkov Radiation by Tachyons

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(Received 20 March 1972; revised manuscript received 12 May 1972)

Previous treatments of Cherenkov radiation by tachyons have been in error because the prescription employed to cut off the divergent integral over frequency, namely $\omega_{max} = E/\hbar$, is not a Lorentz-invariant procedure. The resulting equation of motion for the tachyon is therefore not covariant. The proper procedure requires an extended, deformable distribution of charge and yields a particularly simple form for the tachyon's world line. A simple, covariant equation of motion is derived that describes the motion of a charged tachyon that is emitting Cherenkov radiation. It is shown that Cherenkov radiation by tachyons implies their ultimate annihilation with an antitachyon and demonstrates a disturbing property of tachyons, namely the impossibility of specifying arbitrary Cauchy data even in a purely classical theory.

I. INTRODUCTION

Since their introduction[1-3] into the literature of physics a few years ago, tachyons have stirred a lively debate among physicists. The debate has centered about the question of whether or not the existence of particles that travel with a velocity $v > 1$ (we employ units in which $c = 1$) would produce certain paradoxes concerning the concept of causality. In a field theory of tachyons[1,3,4] field commutators do not vanish for spacelike separations. This makes it impossible to specify arbitrary Cauchy data for the field and has raised questions[3,5] concerning the localizability of tachyons. Problems concerning unitarity have also been raised.[6]

Certain authors[7] have raised the question of whether the field equations that have been proposed would indeed exhibit superluminal effects and have suggested that the correct interpretation should be in terms of unstable modes. It appears[8] however that this issue is one of choosing those boundary conditions that make the solutions describe the sort of phenomena that one wishes to describe. It would appear that if a consistent, and hence paradox-free, classical theory of tachyons exists, then an equivalent quantum field theory should exist also (at least to the extent that they exist for ordinary particles). We shall therefore consider only a classical (unquantized) picture of tachyons in this paper.

The primary objection that has been raised against the existence of classical tachyons is the possibility that using these particles one could propagate information backwards in time thereby creating causal loops[9,10] as paradoxes. The sort of images that are conjured up by this possibility is illustrated by the fact that in one recent discussion[11] in the literature almost half of the references cited were to science fiction stories. All of the previous discussions of this question have employed situations in which tachyons were absorbed and reemitted (or scattered) by at least two observers in order to produce the causal loop.

We shall not discuss such causal loops in this paper, rather we shall show that if we consider the Cherenkov radiation emitted by tachyons a further example of such questions concerning causality arises in the emission of even a single tachyon.

The Cherenkov emission of electromagnetic radiation by charged tachyons has been examined by Alväger and Kreisler[12] and their results have been applied[13] in an attempt to detect such particles experimentally. The case of emission of gravitational radiation has been considered by Lapedes and Jacobs[14] and these authors have applied their results to the events recorded by Weber[15] and to the question of survival of tachyons from the "big bang" creation of the universe.

Unfortunately these authors have all made the unwarranted assumption that a tachyon cannot radiate a photon or graviton that has more energy than the tachyon itself possesses. While this requirement is quite sensible for radiation by normal particles, the existence of negative-energy states for tachyons[2,4] makes such a requirement unjustifiable in this case. In fact, this assumption leads to an equation of motion for the tachyon that is not Lorentz-invariant[16] and would hence single out a preferred reference frame if it were correct.

In Sec. II we shall derive a properly Lorentz-covariant form for the Cherenkov drag force on a charged tachyon. In Sec. III we shall show how the form of the acceleration, and hence the drag force, can be obtained, up to a multiplicative constant, from considerations of Lorentz invariance alone. In Sec. IV the effect of acceleration on the drag force will be discussed (we conclude that it has no effect), and in Sec. V we will discuss the resulting form of the world line of a "free" tachyon and estimate the effect of Cherenkov emission of gravitational waves by a neutral tachyon. We will see that this world line implies that a tachyon, if left to itself, must always annihilate with an antitachyon and as a consequence arbitrary Cauchy data may not be specified for a "free" tachyon even in the classical case.

II. LORENTZ-INVARIANT FORM OF THE DRAG FORCE

It was first pointed out by Sommerfeld[17] that a charged particle moving with a uniform velocity $v > 1$ would experience a drag force associated with the emission of electromagnetic radiation. With the advent of relativity theory Sommerfeld's result was forgotten until Frank and Tamm[18,19] showed that the phenomenon of Cherenkov radiation was essentially that investigated by Sommerfeld. Indeed, the two theories are mathematically almost identical.[20]

In the theory of the Cherenkov effect a particle whose velocity exceeds the speed of light in the medium through which it is passing, $nv > 1$, where n is the index of refraction, loses energy according to the formula[18,19]

$$\frac{dw}{dt} = -ve^2 \int \left(1 - \frac{1}{v^2 n^2(\omega)}\right) \omega d\omega, \tag{1}$$

where $n(\omega)$ is the frequency-dependent index of refraction and the integral extends over all frequencies ($\omega > 0$) for which $vn(\omega) > 1$. In the case of a charged tachyon $v > 1$ and $n = 1$ independent of frequency we therefore have

$$\frac{dw}{dt} = \frac{-e^2(v^2 - 1)}{v} \int_0^\infty \omega d\omega \tag{2}$$

which is clearly divergent. This is related to the fact well known to Sommerfeld[17] that the electromagnetic field of a point charge is singular on the shock front or "Mach cone" having the particle as its apex. This leads to a divergent expression for the Poynting vector of the radiation and hence for the retarding or drag force on the particle.

Until now the solution to this problem employed in the literature[12,14] has been to appeal to the quantum nature of the emission process and the statement that a particle cannot emit a photon with more energy than the particle possesses. With this principle we have

$$\begin{aligned}\frac{dw}{dt} &= \frac{-e^2(v^2 - 1)}{v} \int_0^{E/\hbar} \omega d\omega \\ &= \frac{-e^2(v^2 - 1)E^2}{2v\hbar^2} \\ &= -v\left(\frac{e^2\mu^2}{2\hbar^2}\right)\frac{E^2}{p^2},\end{aligned} \tag{3}$$

where μ is the "rest" mass of the tachyon and p is its momentum

$$p = \frac{\mu v}{(v^2 - 1)^{1/2}}.$$

However, this result is clearly not Lorentz-invariant[16]; a particle losing energy as in (3) would asymptotically approach zero energy. Zero energy is not an invariant notion, however; in another Lorentz frame the particle would be seen to approach some other, nonzero, energy in violation of (3). Hence Eq. (3) cannot be a law of nature describing a particle moving in free space.

On closer inspection the principle on which (3) was based can be seen to be incorrect. Since negative-energy states of tachyons can be obtained from positive-energy states by a Lorentz transformation[2,4] the requirement $\hbar\omega \leq E$ cannot be an invariant one. For if it is fulfilled in one Lorentz frame, one can always find a frame in which the recoil tachyon has negative energy and hence the condition is violated. According to the reinterpretation principle[2,4] the process would appear as a tachyon-antitachyon annihilation process in the latter frame.

One might be led to believe that the condition $\hbar\omega \leq E$ could be employed by ruling out annihilation processes from consideration, but for tachyons such a separation cannot be made in an invariant manner and any properly Lorentz-invariant treatment of one process must automatically include the other.

The essential solution to the problem was also known to Sommerfeld. He pointed out that one could obtain a finite drag force if one considered an *extended* charge distribution. He obtained a drag-force energy-loss formula

$$\frac{dw}{dt} = -\frac{9e^2(v^2-1)}{4a_0^2 v} \tag{4}$$

which is also not Lorentz-invariant.

The reason for the noninvariance of (4) is simply that Sommerfeld considered a *rigid* spherical distribution of charge with radius a_0 which is a prerelativity concept. Clearly what one must do to obtain an invariant expression for dw/dt is to consider not a rigid sphere but a *deformable* charge distribution whose shape undergoes a Lorentz extension (it is an extension for particles with $v>1$ rather than a contraction as in the usual case).[2] Such a distribution would be given by

$$\rho(v, x, y, z) = \rho(x, y, \gamma_s z), \tag{5}$$

where $\gamma_s \equiv (v^2-1)^{-1/2}$ and v is in the z direction.

In the Appendix we show that the effect of a (cylindrically symmetric) distributed charge on Eq. (2) is to replace it with

$$\frac{dw}{dt} = -v(v^2-1)\int_0^\infty \left|\tilde{\rho}\left(\frac{k_z}{\gamma_s}, k_z\right)\right|^2 k_z dk_z, \tag{6}$$

where $\tilde{\rho}(k_\perp, k_z)$ is the Fourier-Bessel transform of the charge distribution, i.e.,

$$\rho(r, z) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} dk_z \int_0^\infty k_\perp dk_\perp \times \exp(ik_z z)\, J_0(k_\perp r)\tilde{\rho}(k_\perp, k_z) \tag{7}$$

and $\omega = vk_z$. For a point charge $\tilde{\rho}(k_\perp, k_z) = e$, and we recover (2).

We shall now assume with Sommerfeld that

$$\tilde{\rho}(k_\perp, k_z) = \tilde{\rho}(k) = \frac{3e}{(ka_0)^3}[\sin ka_0 - (ka_0)\cos ka_0], \tag{8}$$

where $k = (k_\perp^2 + k_z^2)^{1/2}$ for a rigid sphere of radius a_0 and $k = (k_\perp^2 + k_z^2/\gamma_s^2)^{1/2}$ for a Lorentz-deformable sphere. For the quantity in Eq. (6) we have

$$\tilde{\rho}\left(\frac{k_z}{\gamma_s}, k_z\right) = \tilde{\rho}(k'),$$

where

$$k' = [(v^2-1)k_z^2 + k_z^2]^{1/2} = vk_z$$

for a rigid sphere and

$$k' = [(v^2-1)k_z^2 + (v^2-1)k_z^2]^{1/2} = \sqrt{2}\, k_z/\gamma_s$$

for a deformable one. Inserting expression (8) for $\tilde{\rho}$ in Eq. (6) yields upon integration

$$\frac{dw}{dt} = -\frac{9e^2}{4a_0^2}\frac{(v^2-1)}{v} \quad \text{for a rigid sphere} \tag{4}$$

and

$$\frac{dw}{dt} = -\frac{9e^2}{8a_0^2}v \quad \text{for a deformable sphere}. \tag{9}$$

We shall see that Eq. (9) is a Lorentz-invariant expression that leads to an invariant world line for a charged tachyon moving in a vacuum.

To see that (9) is invariant we note that $\gamma_s d/dt = d/ds$ where s is the proper length (not time) of the tachyon's world line and that w is the fourth component of the 4-momentum $P = (\vec{p}, w)$. We may therefore write

$$\frac{dP}{ds} = -\left(\frac{9e^2}{8a_0^2}\right)\frac{(\hat{v}, v)}{(v^2-1)^{1/2}} = \mu A, \tag{10}$$

where

$$A = -\frac{1}{\mu}\left(\frac{9e^2}{8a_0^2}\right)\frac{(\hat{v}, v)}{(v^2-1)^{1/2}}$$

is the 4-acceleration of the particle. Equation (10) is a 4-vector equation and hence covariant.

III. DERIVATION FROM LORENTZ INVARIANCE

The form of Eq. (10) may be derived directly from considerations of Lorentz invariance combined with certain assumptions of simplicity. One generally believes that in a properly Lorentz-invariant theory the laws of motion for a particle will not single out a preferred frame of reference. This does not mean that the trajectory of a given particle will not have certain frames of reference in which it will take on a particularly simple form; e.g., the rest frame of a subluminal, unaccelerated particle.

The important fact to remember about such frames is that they are singled out by the initial conditions of the trajectory, *not* by the laws of motion. We would expect a proper law of motion to enable us to compute the trajectory of a particle starting from some initial data and some fundamental constants that are characteristic of the structure of the particle. It has been pointed out[21] that the trajectory of a particle undergoing acceleration cannot be computed from a knowledge of its position and velocity only since the angle between the 3-acceleration and the 3-velocity is not Lorentz-invariant. This, however, is not unique to tachyons but is found in all cases where radiation reaction is included in the dynamics.[22] This implies that for a Cherenkov-radiating tachyon as well as for a normal particle experiencing a radiation reaction force the law of motion expressed as a differential equation must be of higher order than second in derivatives of position.

Since we believe empty space to be homogeneous we would not expect the particle's position to appear in the equation of motion. The simplest equation one could think of, therefore, is a differential equation of second order in derivatives of the velocity with constant coefficients. The coefficients must characterize the intrinsic dynamic properties of the particle and since we shall only consider scalar particles the coefficients will be constant scalars.

We are therefore led to an equation of the type

$$\frac{d^2U}{ds^2}+a\frac{dU}{ds}+bU=0\,, \tag{11}$$

where U is the 4-velocity, ds is the increment of invariant path length, and a and b are constant scalars.

Before proceeding further we must briefly discuss the notation that we shall be using in what follows. 4-vectors shall be written as ordinary capital letters and the inner product will be designated by a dot. In this dot product the vector on the right (left) will be described by its covariant (contravariant) components, i.e., $A\cdot B=A^iB_i$. This distinction is of no real importance until we consider second-rank tensors, which will always be described by their mixed components. In this case the order of the dot product is important since $T\cdot A = T_i{}^jA_j \neq A^iT_i{}^j = A\cdot T$.

If we now dot Eq. (11) from the left with U and remember that $U\cdot dU/ds=0$, we obtain

$$U\cdot\frac{d^2U}{ds^2}+bU^2=0\,,$$

or since

$$\frac{d}{ds}\left(U\cdot\frac{dU}{ds}\right)=U\cdot\frac{d^2U}{ds^2}+\left(\frac{dU}{ds}\right)^2 =0\,,$$

we have

$$\left(\frac{dU}{ds}\right)^2=bU^2 =\text{const.} \tag{12}$$

Dotting Eq. (11) with dU/ds gives

$$\frac{dU}{ds}\cdot\frac{d^2U}{ds^2}+a\left(\frac{dU}{ds}\right)^2=\frac{1}{2}\,\frac{d}{ds}\left(\frac{dU}{ds}\right)^2+abU^2 =0\,,$$

but since $(dU/ds)^2=\text{const.}$ we obtain

$$abU^2=0\,. \tag{13}$$

We must now choose whether we will set a or b equal to zero. If we choose $b=0$ it means that our 4-acceleration is a null vector from Eq. (12). With this choice Eq. (11) becomes with $A\equiv dU/ds$

$$\frac{dA}{ds}+aA=0 \tag{14}$$

whose solution is $A=e^{-as}A_0$. This solution corresponds to asymptotic states of vanishing 4-acceleration (at $s=\pm\infty$ depending on the sign of a). Such solutions are perfectly compatible with Lorentz invariance since they are solutions of a covariant equation. However, these solutions are *not* compatible with Maxwell's equations which show that a charged particle with a spacelike velocity must emit radiation and hence undergo an acceleration.

This leaves us with the equation of motion

$$\frac{d^2U}{ds^2}+bU=0\,, \tag{15}$$

where $b=(dU/ds)^2/U^2=\text{const.}$

Since (15) is a second-order differential equation for $U(s)$ the most general solution will be characterized by two arbitrary constant 4-vectors which may be taken to be the initial ($s=0$) values of the 4-velocity U_0, and the 4-acceleration A_0 subject only to the constraint $A_0\cdot U_0=0$.

If we form the initial-value tensor

$$\alpha\equiv A_0U_0-U_0A_0\,, \tag{16}$$

we have

$$A_0=\alpha\cdot U_0 \tag{17}$$

and

$$\alpha\cdot A_0=-(A_0)^2U_0\,. \tag{18}$$

Therefore in the subspace spanned by U_0 and A_0 $\alpha^2\equiv\alpha\cdot\alpha$ acts like a scalar quantity of magnitude $-(A_0)^2=-b$ and hence the general solution to Eq. (15) with the initial values U_0 and A_0 is

$$U=\exp(s\alpha)\cdot U_0\,. \tag{19}$$

The tensor α is antisymmetric in that $\alpha\cdot B = -B\cdot\alpha$ so the contravariant form of (19) is

$$U = U_0\cdot\exp(-s\alpha)\,. \tag{19'}$$

Since $A = dU/ds$ it is straightforward to verify that A is obtained from A_0 by applying the same exponential operator

$$A = \exp(s\alpha)\cdot A_0\,, \tag{20}$$

$$A = A_0\cdot\exp(-s\alpha)\,. \tag{20'}$$

It is also of interest to form the quantity

$$\begin{aligned} AU - UA &= e^{s\alpha}\cdot(A_0U_0 - U_0A_0)\cdot e^{-s\alpha} \\ &= e^{s\alpha}\cdot\alpha\cdot e^{-s\alpha} \\ &= \alpha \end{aligned} \tag{21}$$

and we note that α is an explicit constant of the motion since it may be formed from the current values of U and A as well as the initial values.

We see that we have constructed a complete dynamics, described by a covariant equation of motion the trajectories of which are completely determined by the initial values of 4-position, 4-velocity, *and* the 4-acceleration subject to the constraint $A^2 = bU^2$ where b is a constant property of the particle. If we now specialize to the case of interest, namely that of spacelike 4-velocities, $U^2 = 1$ and two possibilities remain. We consider first $b > 0$ or spacelike 4-accelerations. Since A_0 and U_0 are orthogonal spacelike vectors we may find a frame in which neither vector has a time component; we label the direction in which they point the x and y direction, respectively. In this frame we may ignore the z and t directions since the dynamics and hence the trajectory lie entirely in the (x, y) plane.

We may write α explicitly as

$$\alpha = \pm\, b^{1/2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{22}$$

We may write

$$e^{s\alpha} = I\cos(sb^{1/2}) \pm b^{-1/2}\alpha\sin(sb^{1/2})\,. \tag{23}$$

At this point it is easy to see that the trajectory is a circle in the (x, y) plane of radius $b^{-1/2}$ and this trajectory has no extent in time; it exists only at the single instant $t = t_0$. Such solutions are clearly of little interest to physics.

We turn, therefore, to the other possibility, $b < 0$, and consider the case of timelike 4-acceleration. We first note that there exists a class of Lorentz frames in which the space parts of A_0 and U_0 are parallel. This can be seen from the following considerations: Since A_0 is timelike there exists a frame in which it points in the direction of time only. U_0 being orthogonal is a pure space vector in say the x direction. All frames that may be reached via a Lorentz transformation involving only the x spatial direction will have the space parts of A_0 and U_0 lying only in the x direction and hence parallel. In the following we shall work in an arbitrary frame in this class.

In conventional notation we may write U_0 as $(\vec{v}_0, 1)/(v_0^2 - 1)^{1/2}$ and A_0 as $-\beta(\hat{v}_0, v_0)/(v_0^2 - 1)^{1/2}$ where we have chosen the over-all sign of A to correspond to a drag force and have introduced for convenience $\beta = |b|^{1/2}$. Since from (20) we have

$$\begin{aligned} A &= e^{s\alpha}\cdot\alpha\cdot U_0 \\ &= \alpha\cdot e^{s\alpha}\cdot U_0 \\ &= \alpha\cdot U\,, \end{aligned} \tag{24}$$

we may write A as

$$A = A_0(U_0\cdot U) - U_0(A_0\cdot U) \tag{25}$$

which upon carrying out the explicit calculations gives

$$A = -\beta(\hat{v}_0, \hat{v}_0\cdot\vec{v})/(v^2 - 1)^{1/2}\,. \tag{26}$$

If we now note that since the 3-acceleration is always in the direction $\hat{v}_0$ the 3-velocity must remain in this direction and so $\hat{v}_0 = \hat{v}$.

Equation (26) now becomes

$$A = -\beta(\hat{v}, v)/(v^2 - 1)^{1/2} \tag{26'}$$

and we see that this is identical to Eq. (10) with the identification

$$b = -\left(\frac{9e^2}{8ma_0^2}\right)^2. \tag{27}$$

The trajectory of the particle can be deduced quite readily by noting that since $\alpha\cdot\alpha = \beta^2$ then $\alpha^{-1} = \alpha/\beta^2$ so we have

$$\begin{aligned} X - X_0 &= \int_0^s U ds \\ &= \int_0^s e^{s\alpha}\cdot U_0 ds \\ &= (e^{s\alpha} - 1)\cdot\alpha^{-1}\cdot U_0 \\ &= \beta^{-2}(e^{s\alpha} - 1)\cdot\alpha\cdot U_0\,. \end{aligned} \tag{28}$$

If we choose the origin of our coordinates so that

$$X_0 = \alpha U_0/\beta^2\,, \tag{29}$$

we have

$$X = \beta^2\exp(s\alpha)\cdot\alpha\cdot U_0\,, \tag{30}$$

and we may obtain

$$x^2 - t^2 = X\cdot X$$
$$= \beta^{-4} U_0 \cdot \alpha \cdot \exp(-s\alpha)\cdot \exp(s\alpha)\cdot \alpha \cdot U_0$$
$$= \beta^{-4} U_0 \cdot \alpha \cdot \alpha \cdot U_0$$
$$= \beta^{-2} U_0 \cdot U_0$$
$$= \beta^{-2}. \tag{31}$$

This trajectory is hyperbola in the (x, t) plane and is invariant under the restricted class of Lorentz transformations invoking only x and t.

We see that the radiated power calculated in the previous section, Eqs. (9) and (10), can be understood as following from a properly covariant equation of motion. Those Lorentz frames in which the tachyon's trajectory are rectilinear constitute a class of preferred frames but they are singled out by the initial conditions and *not* by the law of motion hence no violation of Lorentz invariance is implied. The initial velocity *and* acceleration must be specified to determine the particle's trajectory but the acceleration is subject to the constraint that the magnitude of the 4-acceleration is fixed by the constant b in the law of motion.

It is not clear how an experimenter would determine an initial acceleration operationally. We shall assume, however, that the apparatus that produces a tachyon would do so with its initial 3-velocity and 3-acceleration parallel *in the rest frame of the apparatus*. We shall, therefore, confine our discussion in the remainder of this paper to the point of view of this preferred class of Lorentz frames.

In closing this section one final remark is in order. We are quite certain that our equation (15) is not the *only* equation of motion that would yield Eq. (10) as a particular solution. However, we believe that it is the *simplest* (i.e., linear, of lowest possible order, etc.) equation of motion that is covariant, consistent with Maxwell's equation, and that yields a solution that can be interpreted as a tachyon world line.

IV. THE EFFECT OF ACCELERATION

Our expression for the energy loss rate (9) and resulting acceleration (10) of a charged tachyon in free space have been derived in a standard manner for describing Cherenkov radiation, namely by assuming the particle to be unaccelerated. Such a derivation would appear to be inherently self-contradictory; a finite acceleration is derived by assuming no acceleration. The results of the calculation are not necessarily wrong, however, provided one can show that the acceleration of the particle does not alter the instantaneous drag force of the radiation reaction. This would not have to be true for an acceleration in general but just for the particular hyperbolic motion derived in Sec. III.

However, it is our claim that such a general solution is unnecessary. We have seen that the form of the acceleration, and thus the drag force, can be deduced, up to a multiplicative constant, from very general considerations of invariance. The only thing that must be derived from an explicit theory is this constant coefficient.

From Eq. (31) we see that the 3-acceleration becomes arbitrarily small as $v \to 1$. In this case the assumption of unaccelerated motion may be made an arbitrarily good one. The energy loss rate (9), however, remains quite finite in this case and may be calculated with arbitrary precision to yield the constant coefficient $(9e^2/8a_0^2)$. This value of the coefficient should be valid, therefore, for any value of $v > 1$ (the exact numerical factor $\frac{9}{8}$ depends upon the explicit form of the charge distribution and thus should not be taken too seriously).

Because of the above argument we therefore assert that Eq. (10) is the correct equation of motion for a charged tachyon in free space even though acceleration was neglected in its derivation.

V. DISCUSSION

We have seen in the foregoing that for a classical theory of Cherenkov radiation by a charged tachyon to be Lorentz-invariant we must consider the tachyon to be an extended, deformable particle and that annihilation with an antitachyon must be considered as an intimate part of the same process. Indeed, the only way an observer could interpret the world line of Eq. (31) under the reinterpretation principle[2,4] is as representing a particle and antiparticle approaching each other along a common line of motion, each of them losing energy via Cherenkov radiation. At the exact instant that they both become transcendent ($v = \infty$, $E = 0$) they meet and annihilate at $z = z_0$, $t = t_0 - 1/K = t_0 - (8a_0^2/9e^2)$. There is no annihilation radiation as such since at the moment of annihilation both particles have $E = 0$.

If we consider the distance to the point of inevitable annihilation as the range of the tachyon we may write an extremely simple range-energy formula. Since

$$\frac{dw}{dz} = \frac{1}{v}\frac{dw}{dt} = -\frac{9e^2}{8a_0^2}$$

the range is given by

$$R = \frac{8a_0^2}{9e^2} E. \tag{32}$$

To obtain any further results we must choose values for the size, charge, etc. of the particle. In the following we shall assume for concreteness that our tachyon has the same charge and mass as the electron and that its size is of the same order as the electron's Compton wavelength. We then have

$$\begin{aligned} R &\approx \left(\frac{\lambda_0{}^2\mu}{e^2}\right)\left(\frac{E}{\mu}\right) \\ &= \left(\frac{\lambda_0}{r_0}\right)\left(\frac{E}{\mu}\right)\lambda_0 \\ &= 137\,\lambda_0(E/\mu) \\ &\approx 5.5\times 10^{-9}(E/\mu)\ \text{cm}, \end{aligned} \tag{33}$$

where $r_0 = e^2/\mu$ the classical electron radius. For $E \approx \mu$ one obtains a range of 5.5×10^{-9} cm in contrast to the value of 5×10^{-3} cm obtained by Alväger and Kreisler.[12] Incidentally, since the energy loss per unit path length is given by the constant $9e^2/8a_0{}^2$, a tachyon in a constant electric field would not necessarily reach a steady-state energy as was claimed by these authors. If on the other hand we give our tachyon the largest energy ever seen in a cosmic-ray particle, 10^{20} eV, we obtain a range of 11 km so we may be sure that such particles do not arrive from astronomical distances.

The above theory applies not only to charged tachyons but to neutral tachyons which should emit Cherenkov gravitational radiation.[14,16,23] The wave equation of general relativity is nonlinear and such nonlinearity would be most manifest in the vicinity of the Cherenkov shock front. However, if the particle is large compared to its Schwarzschild radius Gm, nonlinear effects should be small. We may adapt our range formula (33) to the case of emission of gravitational waves merely by replacing the classical electromagnetic radius r_0 by the Schwarzschild radius r_s. We obtain

$$\begin{aligned} R &\approx \frac{\lambda_0{}^2}{r_s}\frac{E}{\mu} \\ &= -2.4\times 10^{34}(E/\mu)\ \text{cm} \\ &= 8\times 10^{15}(E/\mu)\ \text{pc}, \end{aligned} \tag{34}$$

so such particles could well be of astronomical origin.

If, on the other hand, we consider particles of protonic mass, Eqs. (33) and (34) become, respectively,

$$R \approx 3\times 10^{-12}(E/\mu)\ \text{cm}, \tag{33$'$}$$

$$R \approx 1.3\times 10^{6}(E/\mu)\ \text{pc}. \tag{34$'$}$$

We can see from the above that charged tachyons will have a range that is quite short even for energies as large as the most energetic cosmic rays. We will discuss the implications of this shortly. Neutral tachyons which are coupled only to the gravitational field have ranges that are of cosmological scale in striking contrast to their charged counterparts. Such particles, however, would be essentially undetectable since to be detectable they must have a coupling to normal matter of a reasonable strength. Such a coupling is characterized by the square of a "charge" that is shared by the particle and other matter. Such a "charge" however would mean that the tachyon would emit the appropriate intermediate field via Cherenkov radiation and therefore have a range that was inversely proportional to the square of that charge, i.e., the range against emission of mesons via the strong interactions would be ≈137 times shorter than that for photon emission. We must assume, therefore, that any tachyons that we may readily detect *or produce* in the laboratory will have ranges comparable to or shorter than the electromagnetic ones given by (33) and (33′). This leads to a curious result.

We should first of all note that the total world line of the tachyon must be of finite length. If it is not, it approaches arbitrarily near the light cone and the energy is unbounded. This means that if one can create a tachyon and send it off in a given direction, its antiparticle must be created out along that direction somewhere with just the right direction and energy to meet the original tachyon at their duly appointed place of annihilation. To see that this can cause trouble, consider the creation of a 10^{20}-eV tachyon of electronic charge and mass sent off in the direction of the moon. As we have seen, its range to annihilation is 11 km. If its antiparticle is created on the moon in order to have sufficient range to reach the annihilation point, it would need an energy at point of origin $\approx 10^{24}$ eV. This situation quickly gets out of hand if, instead of the moon, we aim our tachyon at the nearest star.

The problem is clear; we may not employ "particles coming in from infinity" when applying the reinterpretation principle here. The incoming antiparticles must have a real source at a finite distance and the closer the better. When an experimenter creates a tachyon moving in a given direction, a source of antitachyons *must* be somewhere out along that direction. The only escape from this conclusion is to assert that if there is no antitachyon source in that direction, the experimenter will be forbidden, in some as yet unknown manner, to send his tachyon in that direction.

These considerations, of course, do not establish the nonexistence of tachyons. They do indicate, however, that if they exist in a meaningful way the physics of such particles is going to appear very strange.

ACKNOWLEDGMENTS

The author would like to thank many of his colleagues in the Theoretical Studies Branch, Goddard Space Flight Center for many discussions concerning this work. Special thanks are due Dr. R. J. Drachman for discussions concerning the manuscript as well as the general problem.

APPENDIX

We begin with the wave equation for the potential

$$(\partial_x{}^2+\partial_y{}^2+\partial_z{}^2-\partial_t{}^2)\Phi=-4\pi\rho(x,y,z,t)\,, \tag{A1}$$

$$(\partial_x{}^2+\partial_y{}^2+\partial_z{}^2-\partial_t{}^2)\vec{A}=-4\pi\vec{j}(x,y,z,t)\,. \tag{A2}$$

If ρ represents a fixed charge distribution whose motion is a simple translation with velocity v in the z direction, then

$$\begin{aligned}\rho&=\rho(x,y,z-vt)\,,\\ \Phi&=\Phi(x,y,z-vt)\,,\\ \vec{j}_z&=\vec{v}\rho\,,\\ \vec{A}&=A_z\hat{e}_z=\hat{e}_z v\Phi\,.\end{aligned} \tag{A3}$$

Since from (A3) we have $\partial_t=-v\partial_z$, (A1) becomes

$$[\partial_x{}^2+\partial_y{}^2-(v^2-1)\partial_z{}^2]\Phi=-4\pi\rho(x,y,z)\,, \tag{A4}$$

where we have now transformed to a comoving coordinate system $z_{\text{new}}=z_{\text{old}}-vt$. Equation (A4) may be readily solved by making a Fourier transformation in x, y, and z to obtain

$$\tilde{\Phi}(k_x,k_y,k_z)=-\frac{4\pi\tilde{\rho}(k_x,k_y,k_z)}{k_x{}^2+k_y{}^2-k_z{}^2/\gamma_s{}^2}\,, \tag{A5}$$

where

$$\gamma_s=(v^2-1)^{-1/2}\,.$$

For an observer at a point x, y from the charge's trajectory the frequency dependence of the potential will be given by $\tilde{\Phi}(x,y,k_z)$ with $\omega=vk_z$. We have

$$\tilde{\Phi}(x,y,k_z)=\frac{-1}{\pi}\int_{-\infty}^{\infty}\int\frac{\tilde{\rho}(k_x,k_y,k_z)\exp(ik_xx+ik_yy)}{k_x{}^2+k_y{}^2-k_z{}^2/\gamma_s{}^2}dk_xdk_y\,. \tag{A6}$$

If we now assume cylindrical symmetry for the charge distribution, i.e.,

$$\tilde{\rho}(k_x,k_y,k_z)=\tilde{\rho}(k_\perp,k_z)\,,$$

where

$$k_\perp{}^2=k_x{}^2+k_y{}^2,$$

Eq. (A6) becomes

$$\begin{aligned}\tilde{\Phi}(r,k_z)&=\frac{-1}{\pi}\int_0^\infty k_\perp dk_\perp\int_0^{2\pi}\frac{d\phi\,\tilde{\rho}(k_\perp,k_z)\exp(ik_\perp r\cos\phi)}{k_\perp{}^2-k_z{}^2/\gamma_s{}^2}\\ &=-2\int_0^\infty\frac{J_0(k_\perp r)\tilde{\rho}(k_\perp,k_z)k_\perp dk_\perp}{k_\perp{}^2-k_z{}^2/\gamma_s{}^2}\,,\end{aligned} \tag{A7}$$

where $J_0(z)$ is the Bessel function of zero order.

To evaluate the integral over $k_\perp$ we first express the Bessel function in terms of Hankel functions as

$$J_0(k_\perp r)=\tfrac{1}{2}[H_0^{(1)}(k_\perp r)+H_0^{(2)}(k_\perp r)]\,. \tag{A8}$$

Hankel functions are analytic in the plane cut from $-\infty$ to 0 along the negative real axis. In this cut plane they have the following symmetry and asymptotic properties:

$$\begin{aligned}H_0^{(1)}(ze^{\pi i})&=-H_0^{(2)}(z)\,,\\ H_0^{(2)}(ze^{-\pi i})&=-H_0^{(1)}(z)\,,\end{aligned} \tag{A9}$$

$$\left.\begin{aligned}H_0^{(1)}(z)&\sim(2/\pi z)^{1/2}\exp[i(z-\tfrac{1}{4}\pi)]\\ H_0^{(2)}(z)&\sim(2/\pi z)^{1/2}\exp[-i(z-\tfrac{1}{4}\pi)]\end{aligned}\right\}z\to\infty\,. \tag{A10}$$

From (A9) we see that we may extend the integral (A7) to the negative real axis as long as we stay above (below) the branch cut for the term containing $H_0^{(1)}$ $(H_0^{(2)})$. It is readily shown that if

$$\rho(|r|>a_0)=0\,,$$

then

$$|\tilde{\rho}(k)|<K\exp(|k|a_0)\quad\text{as}\ |k|\to\infty.$$

Therefore for points outside the charge $(r>a_0)$ the term containing $H_0^{(1)}$ $(H_0^{(2)})$ vanishes exponentially on the upper (lower) infinite semicircle and the contours may be closed accordingly. The zeros of the denominator are moved off the real axis by adding $i\epsilon$ to the denominator, a prescription that guarantees outgoing rather than incoming waves. If we further assume that $\tilde{\rho}(k_\perp,k_z)$ has no poles in the finite $k_\perp$ plane the integral (A7) may be evaluated by residues in a straightforward manner to obtain

$$\tilde{\Phi}(r,k_z)=-\pi iH_0^{(1)}(k_zr/\gamma_s)\tilde{\rho}(k_z/\gamma_s,k_z)\,. \tag{A11}$$

The field strengths are given by

$$\begin{aligned}E_z&=-\partial_z\Phi-\partial_tA_z\\ &=(v^2-1)\partial_z\Phi\,,\\ H_\phi&=-\partial_rA_z\,,\end{aligned} \tag{A12}$$

which are Fourier-transformed to become

$$\tilde{E}_z = (v^2 - 1)(ik_z)\tilde{\Phi}\,,\qquad \tilde{H}_\phi = -v\partial_r\tilde{\Phi}\,. \tag{A13}$$

The radial component of Poynting's vector is given by

$$S_r(r, z) = \frac{-1}{4\pi} E_z(r, z)H_\phi(r, z) \tag{A14}$$

and the energy radiated per unit time is given by

$$\frac{dw}{dt} = 2\pi r \int_{-\infty}^{\infty} S_r(r, z)dz = -\frac{r}{2}\int_{-\infty}^{\infty} E_z H_\phi dz$$
$$= -\frac{r}{2}\int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dk_z dk_z'}{4\pi^2}\tilde{E}_z(r, k_z)\tilde{H}_\phi(r, k_z') \times \exp i(k_z + k_z')z$$
$$= -\frac{r}{4\pi}\int_{-\infty}^{\infty} \tilde{E}_z(r, k_z)\tilde{H}_\phi(r, -k_z)dk_z\,. \tag{A15}$$

Inserting (A13) in (A15), we obtain

$$\frac{dw}{dt} = -\frac{rv(v^2 - 1)}{4\pi}\int_{-\infty}^{\infty} (ik_z)\tilde{\Phi}(k_z)\partial_r\tilde{\Phi}(-k_z)dk_z$$
$$= \frac{\pi r v(v^2 - 1)}{4\gamma_s}\int_{-\infty}^{\infty} (-ik_z^2)|\bar{\rho}(k_z/\gamma_s, k_z)|^2 \times H_0^{(1)}\left(\frac{k_z r}{\gamma_s}\right)H_1^{(1)}\left(-\frac{k_z r}{\gamma_s}\right)dk_z\,. \tag{A16}$$

If we now add the positive and negative values of k_z together, we obtain

$$\frac{dw}{dt} = \frac{\pi r v(v^2 - 1)}{4\gamma_s}\int_0^{\infty} (-ik_z^2)|\bar{\rho}(k_z/\gamma_s, k_z)|^2 W\left(\frac{k_z r}{\gamma_s}\right)dk_z\,, \tag{A17}$$

where

$$W(z) = H_0^{(1)}(z)H_1^{(1)}(-z) + H_0^{(1)}(-z)H_1^{(1)}(z)$$
$$= H_0^{(1)}(z)H_1^{(2)}(z) - H_0^{(2)}(z)H_1^{(1)}(z)$$
$$= 4i/\pi z\,. \tag{A18}$$

Combining (A18) with (A17) gives

$$\frac{dw}{dt} = v(v^2 - 1)\int_0^{\infty} |\bar{\rho}(k_z/\gamma_s, k_z)|^2 k_z dk_z\,. \tag{A19}$$

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