Energies, 1971, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972), p. 239.

[4] M. Nauenberg, Phys. Rev. Letters 22, 556 (1969); S. J. Brodsky and J. Pumplin, Phys. Rev. 182, 1794 (1969); B. Margolis and C. L. Tang, Nucl. Phys. B10, 329 (1969).

[5] G. Mennessier and O. Nachtmann, Phys. Letters 34B, 309 (1971).

[6] G. V. Bochmann, B. Margolis, and C. L. Tang, Phys. Rev. Letters 24, 483 (1970).

[7] G. V. Bochmann, Phys. Rev. D 5, 266 (1972).

[8] What is important is the fraction of the hadron electroproduction cross section which goes into the $\rho^0 p$ channel, as described by D. E. Andrews *et al.*, Phys. Rev. Letters 26, 864 (1971).

[9] See D. E. Andrews *et al.*, Ref. 8; C. Driver *et al.*, Nucl. Phys. B38, 1 (1972); E. D. Bloom *et al.*, Phys. Rev. Letters 28, 516 (1972).

[10] H. Harari, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, 1971,* edited by N. B. Mistry (Cornell Univ. Press, Ithaca, N. Y., 1972), p. 303.

[11] The values of the two-body amplitudes and the nuclear parameters are the same as listed in Ref. 6, except that we have here ignored the contribution of the vector mesons ϕ and ω to $f_{\gamma\gamma}$.

[12] G. V. Bochmann, Phys. Rev. D 6, 1938 (1972).

[13] J. Swartz and R. Talman, Phys. Rev. Letters 23, 1078 (1969).

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Compulsory Resonance Formation*

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It is noted that if two mesons are allowed by the quark model to resonate, they do so for $p^* \leq p_0^{MM} \equiv 350$ MeV/c, where p^* is the c.m. momentum. The corresponding value for meson-baryon systems is $p_0^{MB} \equiv 250$ MeV/c, suggesting (in an optical picture) that the baryon is indeed bigger than the meson. Crucial tests of the rule are provided by exotic baryon-antibaryon systems, for which one expects $p_0^{BB} \cong 200$ MeV/c, and by other specific two-body modes which are predicted to resonate not far above threshold.

A dynamical theory of elementary particle resonances does not yet exist. Various models (bootstraps, linear Regge trajectories, harmonic oscillator quark model) have given some partial insights into the spectrum, but attempts to force them to be quantitative have so far met with limited success. Rather, these models are most useful as guides to a correct theory and to further relevant experiments.

In this spirit we should like to point out an approximate regularity in the way two strongly interacting particles form resonances. Tests of this regularity are easily made.

Introduce the following rules[1]:

(a) The observed mesons are made of a quark and an antiquark, and the observed baryons of three quarks.[2]

(b) Two particles may resonate when any antiquark in one can annihilate a quark in the other.

The remarkable fact is that when two particles *may* resonate according to rules (a) and (b), they do so *at least once between threshold and a low momentum* p_0 *in the center-of-mass system.* For meson-meson systems p_0 is around 350 MeV/c while for meson-baryon systems it is around 250 MeV/c. The case of baryon-antibaryon systems will be discussed presently.

Using the resonance tables of Ref. 3 we have compiled Fig. 1, which shows the center-of-mass momenta p^* for which various meson-meson and meson-baryon pairs form their first resonance above threshold. Each isospin is counted as a separate channel. Both distributions show a remarkable peaking and a rather sharp cutoff above this peak.

As shown by the partial-wave label S, P, D, . . ., in the upper right corner of each box, the first resonance above threshold is generally formed in a rather low relative orbital angular momentum state. The number of S waves and P waves is roughly equal.

The peaking in Figs. 1(a) and 1(b) undoubtedly arises in part from the regular spacing of hadron levels as predicted by various models. On the other hand, it has a simple optical interpretation as well: *Two particles A and B begin forming resonances with one another at a certain well-defined relative distance.* Set

$$p_0^{AB}(R_A + R_B) = \bar{l}\,, \tag{1}$$

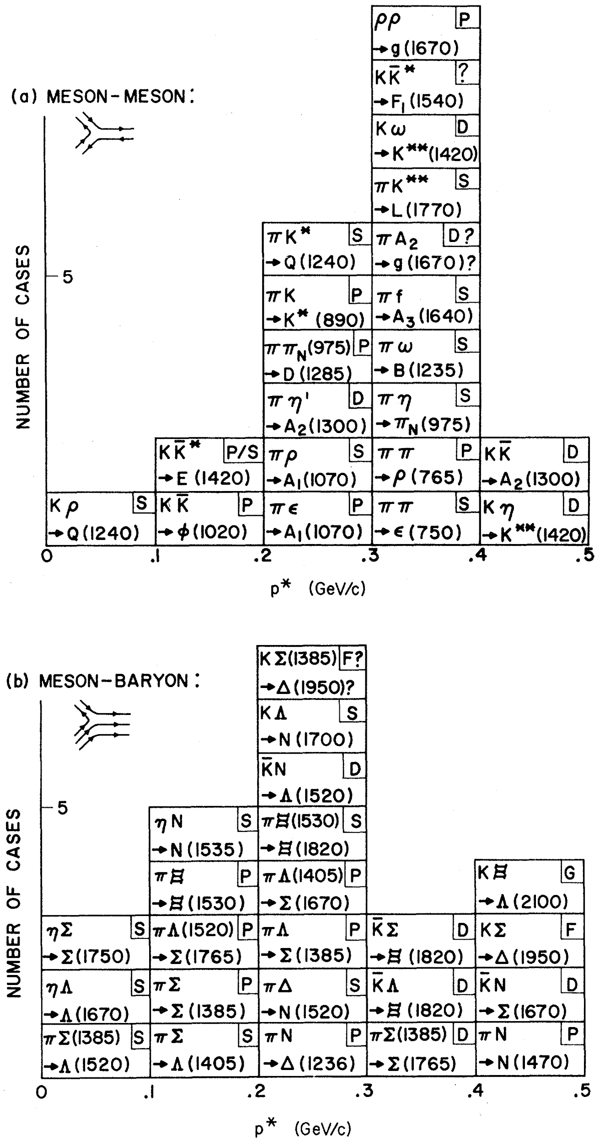

FIG. 1. Center-of-mass momenta p^* for which two particles form their first resonance above threshold if allowed to do so by the graph depicted in the inset. (a) Meson-meson systems. (b) Meson-baryon systems. In both cases, the lowest partial wave in which the given resonance can be formed is shown in the small box at the upper right corner of each rectangle. Each isospin is counted as a separate channel. The question marks are explained in the text.

where p_0^{AB} is the value of p^* at which the distributions in Fig. 1 peak, R_i is the "radius" of particle i, and $\bar{l}$ is some average orbital angular momentum (around 1, here) for which resonance formation begins. Then, in meson-baryon systems, $p_0^{MB} \cong 250$ MeV corresponds to $R_A + R_B \cong 0.8$ F, a value in rough agreement with that obtained by optical analyses of two-body elastic and quasielastic scattering.[4]

Comparing $p_0^{MM} \cong 350$ MeV and $p_0^{MB} \cong 250$ MeV, and assuming $\bar{l}$ is the same for both cases, one obtains

$$R_M/R_B \simeq \tfrac{5}{9} \tag{2}$$

or

$$\sigma_T(BB)/\sigma_T(MB) \simeq 1.6\,,$$

also reasonable values. We thus interpret the shift in peaks in Fig. 1 as saying that the baryon is larger than the meson.

It has been conjectured that various two-body amplitudes possess an imaginary part (in addition to any "black sphere" diffractive scattering) which is related to the presence of low-energy resonances in the direct channel.[5] The peripheral nature of these imaginary parts has been noted.[4] As Fig. 1 shows, the formation of "first resonances" is indeed a peripheral process as well (for meson-meson and meson-baryon systems), as it occurs for large and roughly constant values of impact parameter.

The rule we are discussing – "compulsory resonance formation" – has some particular consequences which are hard to state more economically in other ways. In particular, it predicts the formation of *exotic baryon-antibaryon resonances*[6] *not far above threshold.* Taking the optical picture seriously, one would expect $p_0^{\bar{B}B} \simeq 200$ MeV/c.[7] Systems such as $(\bar{\Delta}N)_{I=2}$ or $(\bar{Y}_1^*N)_{I=3/2}$ would then be expected to resonate somewhere in the ranges

$$2175 \leq M[(\bar{\Delta}N)_{I=2}] \leq 2215 \text{ MeV}\,, \tag{3}$$

$$2225 \leq M[(\bar{Y}_1^*N)_{I=3/2}] \leq 2265 \text{ MeV}. \tag{4}$$

Such states could conceivably be quite narrow, lying so close to threshold, requiring good resolution to observe. If formed in S or P waves, their spins would be no more than three.

The estimates (3) and (4) are considerably more stringent than ones given previously.[8] Failure to confirm them would invalidate the present simple optical picture of compulsory resonance formation.

The ideal reactions in which to check Eqs. (3) and (4) would be backward meson production[1,9]:

$$\pi^- + n \to p(\text{fwd.}) + (M)^{--}\,, \tag{5}$$

$$\pi^+ + p \to \Delta(\text{fwd.}) + (M)^{++}\,. \tag{6}$$

We would expect the selection of actual baryon-antibaryon pairs in $(M)^{--}$ in these two reactions to enhance the effects of the exotic resonances, as it has been suggested that decays of exotic mesons into any system of ordinary mesons may be forbidden.[1,8]

TABLE I. Some low-mass meson-meson and meson-baryon states predicted by compulsory resonance formation.

$I, \|Y\|$	Channel(s)	Mass (MeV)	Possible J^{PC} [a]	Remarks
0,0	$\pi\rho$	900–1250	$\underline{1^{+-}}; 0^{--}, 1^{--}, 2^{--}$	Possible SU(3) companion of B (1235)
1,0	πB πD	1400–1700	$1^{-+}; 0^{++}, 1^{++}, 2^{++}$	Hard to fit into usual quark model spectrum
0,0	$\eta\omega$ $\eta\phi$	1300–1500 1550–1750	$\underline{1^{+-}}; 0^{--}, \underline{1^{--}}, \underline{2^{--}}$	Possible SU(3) companion of B or $L=2$ quark model state
0,0	πA_2 $\rho\rho$	1450–1700 1500–1700	$\underline{2^{-+}}; 1^{++}, 2^{++}, 3^{++}$ $(0–2)^{++}; (0–3)^{-+}$	Possible SU(3) companion of $A_3(1640)$
$\frac{1}{2}$, 1	$K\eta'$	1450–1650	$\underline{0^{++}}; 1^{-+}$ [b]	Hard to fit into usual quark model spectrum
0, −2	$\bar{K}\Xi$	1800–2000	$\frac{1}{2}^-; \frac{1}{2}^+, \underline{\frac{3}{2}^+}, \frac{5}{2}^+$	Possibly related to Ω^- as $N(1470)$ ($\frac{1}{2}^+$) related to $N(938)$

[a] Based on S- or P-wave formation. Guesses based on SU(3) or quark-model systematics are underlined.

[b] If η' is a unitary singlet, SU(3) would forbid 1^{--}.

It is, of course, very important to compare reactions (5) and (6) with companion reactions in which M does not have exotic I_3 and Y, in order to demonstrate that such reactions are indeed capable of producing *any* baryon-antibaryon resonances.

If we insist that "first resonances" be formed in states of $l=0$ or 1, we are led to suspect some of the J^P assignments quoted in Ref. 3, as indicated by the question marks in Fig. 1. Most of the high-partial-wave "first resonances" occur for high p^*, however. We would then predict these systems to have lower-l, lower-p^* states too.

There are some meson-meson and meson-baryon channels in which compulsory resonance formation predicts resonances that have not been seen. Notably, if $p^* \lesssim 350$ MeV/c, one expects various low-mass states listed in Table I. Many of these will be particularly accessible in forthcoming multiparticle spectrometers at CERN and Brookhaven. The predictions are intended as a complement to the quark model. The fact that they are based on specific decay modes may make them more easily tested than similar quark-model predictions.

The predictions of Table I are all for states which have not yet been seen. There are other channels in which resonances are predicted which can be identified with observed states. In this case, compulsory resonance formation predicts the existence of various new decay modes. Some of these are related to observed decays by SU(3), and will not be discussed further. Others are new modes and are listed in Table II.

TABLE II. Some predicted new modes of observed resonances.

Resonance	I	J^{PC}	Mode		Final state
B^+	1	1^{+-}	$\pi^{+,0}\pi_N^{0,+}(975)$	→	$\pi^+\pi^0\eta$
			$\pi^{+,0}A_1^{0,+}$	→	$\pi^+\rho^+\pi^-$
				→	$\pi^+\rho^-\pi^+$
				→	$\pi^0\rho^0\pi^+$
				→	$\pi^0\rho^+\pi^0$
f	0	2^{++}	$\pi^\pm A_1^\mp$	→	$\pi^+\pi^-\rho^0$
			$\pi^0 A_1^0$	→	$\pi^0\pi^\pm\rho^\mp$
$\omega(1680)$	0	3^{--}	$\pi^\pm B^\mp$	→	$\pi^+\pi^-\omega$
Q^+	$\frac{1}{2}$	$1^{+\pm}$	$K^+\epsilon$	→	$K^+\pi^+\pi^-$
				→	$K^+\pi^0\pi^0$
$\Delta^{++}(1670)$	$\frac{3}{2}$	$J^P=\frac{3}{2}^-$	$\Delta^{++}\pi^0$	→	$p\pi^+\pi^0$
			$\Delta^+\pi^+$	→	$p\pi^0\pi^+$
				→	$\eta\pi^+\pi^+$
$L^+(1770)$	$\frac{1}{2}$	$2^{-\pm}$	$K^+\phi$	→	$K^+K^-K^+$
				→	$K^+K_SK_L$

The fact that resonance formation is possible at all for S waves runs somewhat counter to a naive optical picture, as the centrifugal barrier that usually "holds a resonance together" appears to be lacking. Instead, we envision the appropriate barrier terms to be consequences of some as yet unspecified relative orbital angular momentum of constituents. For example, if the $q\bar{q}$ annihilating pair is to have the quantum numbers of the vacuum, $J^{PC}=0^{++}$, it must be in a $^3P_0$ state.[10] This could then lead to an effective centrifugal barrier even for S-wave resonance formation.

If $N\bar{N}\to\pi\pi$ must proceed via a $^3P_0$ $q\bar{q}$ annihilation, the claim[11] for a large $l\geq 1$ contribution to this reaction at rest could be understood: The centrifugal barrier just mentioned would suppress S-wave annihilation relative to one's naive expectations.

It is amusing that the $^3P_0$ picture is actually in reasonable accord with data on partial widths and angular distributions.[12] The empirical regularity evident from Fig. 1, however, is meant to be independent of whether the annihilating $q\bar{q}$ pairs in the figure can be taken seriously except as a guide to SU(3) properties.

The present work represents an extension of the idea of duality graphs,[13] which by themselves do not tell *when* two particles must begin to resonate. Predictions of this sort do follow from arguments advanced by Schmid[14] relying on details of degenerate Regge-pole exchange. What we are suggesting here, however, is that gross features of elementary particle resonance formation may be viewed more directly in terms of quark graphs and simple optics.

Note added in proof. Implicit in the scheme mentioned here is the prediction of states increasing in exoticity as mass increases. This rate of increase should not be particularly rapid. One expects (neglecting the small Q values)

$$m(n)\cong n-2\text{ GeV} \tag{7}$$

for any hadron (meson or baryon) with a total number n of quarks or antiquarks. This rule is self-consistent since

$$m(n_{12})\cong m(n_1)+m(n_2)$$
$$=n_{12}-2\,,$$

when

$$n_{12}=n_1+n_2-2\,. \tag{8}$$

The last relation follows from our rules since in forming a hadron with n_{12} quarks out of ones with n_1 and n_2 quarks one $q\bar{q}$ pair must vanish. If one allows more than one such pair to vanish, of course, Eq. (7) should be viewed as a lower bound on $m(n)$.

The prediction (7) need not cause any particular concern since it would be testable (and worth testing) only if the lowest predicted exotic states $qq\bar{q}\bar{q}$ (the M_4) were observed.

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[1] P. G. O. Freund, R. Waltz, and J. Rosner, Nucl. Phys. B13, 237 (1969).

[2] M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, CERN Reports No. TH-401, 1964 (unpublished) and No. TH-412, 1964 (unpublished).

[3] Particle Data Group, Phys. Letters 39B, 1 (1972).

[4] H. Harari, Ann. Phys. (N.Y.) 63, 432 (1971); M. Davier and H. Harari, Phys. Letters 35B, 239 (1971).

[5] P. G. O. Freund, Phys. Rev. Letters 20, 235 (1968); H. Harari, *ibid.* 20, 1395 (1968).

[6] J. Rosner, Phys. Rev. Letters 21, 950 (1968); 21, 1422(E) (1968). It has been suggested in Ref. 1 that these four-quark ($qq\bar{q}\bar{q}$) objects be denoted M_4.

[7] The $\bar{p}p$ system seems to resonate for $p^*\cong 200$ MeV/c. See D. Cline, in Argonne National Laboratory Report No. ANL/HEP-7208 (unpublished), V. II, p. 620.

[8] J. Rosner, Phys. Letters 33B, 493 (1970).

[9] M. Jacob and J. Weyers, Nuovo Cimento 69A, 521 (1970); 70A, 285(E) (1970).

[10] J. Rosner, Phys. Rev. Letters 22, 689 (1969); L. Micu, Nucl. Phys. B10, 521 (1969).

[11] S. Devons *et al.*, Phys. Rev. Letters 27, 1614 (1971).

[12] L. Micu, Ref. 10; E. W. Colglazier and J. L. Rosner, Nucl. Phys. B27, 349 (1971); W. P. Petersen and J. L. Rosner, Phys. Rev. D 6, 820 (1972).

[13] M. Imachi, T. Matsuoka, K. Ninomiya, and S. Sawada, Prog. Theoret. Phys. (Kyoto) 40, 353 (1968); H. Harari, Phys. Rev. Letters 22, 562 (1969); J. Rosner, Ref. 10.

[14] C. Schmid, Phys. Rev. Letters 20, 689 (1968).